On the calculation of stability charts on the base of the damping and the stiffness of the cutting process

ON THE CALCULATION OF STABILITY CHARTS ON THE BASE OF THE DAMPING AND THE STIFFNESS OF THE CUTTING PROCESS.

by

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SUMMARY

This report deals with a new method for calculating stability charts. Simple experiments, only measuring frequencies, yield the values necessary to establish the threshold of stability. Now, the dynamic cutting coefficient can be determined. A very good resemblance between the calculated values and the experimental ones is shown for cutting speeds, higher than 60 m/min.

ZUSAMMENFASSUNG

Dieser Bericht beschäftigt sich mit einer neuentwickelten Methode zur rechnerischen Ermittlung von Stabilitätsdiagrammen. Die zur Bestimmung der Stabilitätsgrenze erforderlichen Werte gehen dabei aus einfachen Frequenzmessungen hervor. Sodann lassen sich die dynamischen Schnittkraftkoeffiziente bestimmen. Beispielsweise wird schliesslich für Schnittgeschwindigkeiten über 60 m/min gezeigt, dass die Übereinstimmung zwischen rechnerisch und experimentell ermittelten Werte sehr gut ist.

RÉSUMÉ

Ce rapport traite d'une nouvelle méthode de calculer des graphiques de stabilité. Des expériences simples, seulement le mesurage des fréquences, donnent les valeurs nécessaires pour fixer la limite de stabilité. Ensuite le coefficient dynamique de cisaillement peut être déterminé. Une bonne concordance entre les valeurs calculées et les valeurs obtenues par les expériences est arrivée en cas que les vitesses de cisaillement seront plus hautes que 60 m/min.

1. INTRODUCTION

Actually, there are two main directions in the field of dynamic stability-tests of machine tools {1}.
First of all there is a method characterised by measuring the transfer function of the tool.
Then, the critical depth of cut is determined with the equation

$$b_{cr} = \frac{1}{2(-R_n)k_d}$$

The quantity k_d seems to depend upon the geometry of the cutting process as well as upon the working material.
R_n is the maximum negative real part of the polar curve.
The second method simply consists of carrying out experiments in order to establish the critical depth of cut for standardized conditions.
The progress of the investigations concerning cutting stability is mainly obstructed by an insufficient knowledge of the k_d-value.
The latter value depends on many quantities such as feed, cutting speed, tool wear, geometry of the tool, and workpiece material.
Especially, the influence of the working material on cutting stability makes it difficult to compare results from cutting tests according to both methods.

2. THE INCREMENTAL CUTTING STIFFNESS

Peters and Vanherck {2} assume that it is allowed to take the incremental cutting stiffness k_i for the already mentioned k_d-value. Thus, they calculate the critical depth of cut with the relation

$$b_{cr} = \frac{1}{2(-R_n)k_i}$$

The numerical values of k_i are obtained by static cutting tests. Fig. 1 shows, for orthogonal cutting, the change ΔF of the resultant cutting force due to an increase Δh of the feed. The incremental cutting stiffness may be defined as

$$k_i = k_{st} \cos \beta$$

where

$$k_{st} = \frac{\Delta F}{\Delta h} \cdot$$

β represents the angle between the vector ΔF and the direction of motion of the tool.

Peters and Vanherck evaluated the calculated b_{cr}-values with experimental data from a special tool holder. A fairly good recemblance between both results was found. However, experiments carried out in the Laboratory of Production Engineering at the Eindhoven University of Technology, using the same tool holder, did not confirm the usefulness of the method in the same extent.

We measured the cutting forces with a three-component dynamometer which has its first natural frequency at approximately 1.5 kHz. Our results are shown in Fig. 2. In general, the calculated b_{cr}-values, are considerable smaller than the experimental ones.

Among other things, Fig. 3 shows our curves for k_i according to the method of Peters and Vanherck, belonging to the calculated stability chart of Fig. 2.

It should be noticed that all our tests are carried out for the following conditions:

- orthogonal cutting
- workpiece material: C45 N
- tool: standard throw-away type carbide
  tool tip P30
  geometry: $\alpha = 5^o$, $\gamma = 6^o$, $\kappa_o = 90^o$, $\kappa' = 30^o$,
  $r_\varepsilon = 0.4$ mm, $\lambda = 0^o$
- nominal feed: 0.072 mm/rev.

3. THE DAMPING OF THE CUTTING PROCESS

3.1. General

Many investigators in this field have already perceived the existence of damping in the cutting process. In this context the most well-known relation $\Delta F = k_1 \Delta h + k_2 \Delta\dot{x} + k_3 \Delta\Omega$ is given by Tobias {3}.

However, experiments in order to get numerical values for this damping phenomenon are supposed to be difficult. Consequently, reliable values

for the process damping caused by the workpiece material are not available at this moment.
The test rig which is used in cooperation work of the C.I.R.P. Ma-Group for investigations of susceptibility to chatter of materials gave us the possibility to carry out experiments in order to get numerical values of the damping ratio during turning operations.

3.2. Process damping and incremental stiffness as basic quantities for stability charts

Tlusty, Polaceck a.o. {4} derived

$$T_c = \frac{1}{2(-R)}$$

where T_c represents the transfer function of the cutting process while R is the real part of the transferfunction T_m of the machine tool.
When k_i is supposed to be independent of the depth of cut b the dynamic cutting force can be written as

$$\Delta F = T_c \Delta h = b k_i \Delta h$$

Hence, it follows on the threshold of stability

$$b_{cr}\, k_i = \frac{1}{2(-R_n)}$$

For a single-degree-of-freedom system we can derive

$$R_n = -\frac{1}{k}\,\frac{1}{4\ \zeta(1+\zeta)} \simeq -\frac{1}{4\zeta k} = -\frac{1}{2\rho_t \omega_o}$$

where k is the stiffness of the machine tool, ρ_t the damping constant, ζ the damping ratio, and ω_o the angular velocity at natural frequency.
Now, we can write

$$b_{cr}\, k_i = \rho_t\ \omega_o$$

If during turning operations damping is added to the system (ρ_c) it will be necessary to increase b in order to become instability, or

$$b_{cr}\, k_i = (\rho_t + \rho_c)\omega_o = \rho_s \omega_o$$

If we excite the tool by a pulse during cutting, it is possible to measure the displacement response before regeneration occurs. Now, we can calculate the total damping ratio of the system with the aid of the consecutive amplitude ratio

$$\frac{A_n}{A_o} = e^{-\frac{\pi \rho_s n}{\omega_d m}}$$

where n is the number of periods, and m is the mass. The value ω_d is characterised by

$$\omega_d = \omega_o \sqrt{1 - \zeta_s^2}$$

Thus, the amplitude ratio yields

$$\frac{A_n}{A_o} = e^{-\frac{\pi n \rho_s}{\omega_o m} \frac{1}{\sqrt{1-\zeta_s^2}}} \approx e^{-2\pi n \zeta_s}$$

or

$$\zeta_s = \frac{\ln \frac{A_o}{A_n}}{2\pi n}$$

It has to be noticed that the overall damping ratio of the system ζ_s can be written as

$$\zeta_s = \frac{\rho_s}{2\sqrt{m\ (k+bk_i)}}$$

Consequently

$$\rho_s = 2\zeta_s \sqrt{m\ (k+bk_i)}$$

Now, on the threshold of stability the following relations will exist

$$b_{cr}\, k_i = \rho_s\, \omega_o = 2\, \zeta_s \sqrt{k^2 + b\, k\, k_i}$$

$$b_{cr}\, k_i = 2\, \zeta_s^{\,2}\, k\, \{\, 1 + \sqrt{1 + \frac{1}{\zeta_s^{\,2}}}\} =$$

$$b_{cr}\, k_i = 2\, k\, \{\, \zeta_s + \zeta_s^{\,2} + \ldots\ldots\}$$

Finally, with $k = m\, \omega_o^{\,2}$ we find

$$b_{cr}\, k_i \approx 2\, m\, \omega_o^{\,2}\, \zeta_s$$

Once more, we will assume that k_i will not be influenced by b. If the stability chart under certain conditions is known, it is possible to calculate the values of k_i with the aid of the ζ_s-values obtained by means of the logarithmic decrement. This k_i-value will be unique and not be influenced by the dynamic behaviour of the tool. The obtained k_i- and ζ_s-values can be used for predicting stability charts for every machine tool of which the transfer function is known. It should be noticed that where the direction of ΔF is unknown, it is difficult to extrapolate k_i to an other direction than the main one of the rig.
The assumption that the cutting process will add damping to the vibratory system is confirmed by the results in Fig. 4 and Fig. 5. These results are obtained from experiments measuring the pulse response of the test rig during cutting. Fig. 6 shows a typical example of such a response.

3.3. Experimental approach of the problem

Considering the system on the threshold of stability, it follows

$$b_{cr}\, k_i = \rho_s\, \omega_o\, .$$

In general, the following relation exists

$$\Delta k = m\, (\omega_n^{\,2} - \omega_o^{\,2})$$

where Δk is the amount of stiffness to be added in order to get the natural frequency of the system at ω_n.
After pulse-excitation during cutting, the angular velocity of the

motion will be

$$\omega_c = \omega_n \sqrt{1 - \zeta_s^2}$$

If b = 0 and the carriage of the lathe moves, the next relation will also be valid

$$\omega_{mt} = \omega_{om} \sqrt{1 - \zeta_{mt}^2} .$$

It has to be noticed that in this case ω_{om} represents the angular velocity at natural frequency of the tool while the carriage moves. The corresponding damping ratio is ζ_{mt}.
Consequently

$$k_m = m\, \omega_{om}^2 \quad .$$

It is established that the transfer function of the machine tool depends upon the velocity of the carriage. The magnitude of the change in compliance will be influenced by ζ_t and in some extent also by the carriage speed (5).
Assuming $\zeta_s^2 \ll 1$ and $\zeta_{mt}^2 \ll 1$, it is possible to obtain approximately the process stiffness with the equation

$$b\, k_i = m\, (\omega_c^2 - \omega_{mt}^2)$$

Thus, writing for the overall damping in practice

$$\rho_s = \rho_{mt} + \rho_c \quad ,$$

on the threshold of stability the next relation will be valid

$$m\, \{\, \omega_c^2\, (b,V) - \omega_{mt}\, (b,v) \} \;=\; \rho_s\, \omega_{om}$$

We consider the feed as a parameter for this relation.

Consequently

$$m\, (\omega_c^2 - \omega_{mt}^2) = 2\zeta_s \sqrt{m\, (k_m + b\, k_i)}\; \omega_{om}$$

$$\omega_c^2 - \omega_{mt}^2 = 2\,\zeta_s \frac{\omega_c}{\sqrt{1-\zeta_s^2}} \frac{\omega_{mt}}{\sqrt{1-\zeta_{mt}^2}} = 2A\,\omega_c\,\omega_{mt}$$

This yields

$$\frac{\omega_c}{\omega_{mt}} = A + \sqrt{1 + A^2}$$

Assuming $A^2 << 1$, it follows

$$\frac{\omega_c}{\omega_{mt}} = A + 1 + \tfrac{1}{2} A^2 + \ldots\ldots \approx 1 + A$$

$$\frac{\omega_c}{\omega_{mt}} \approx 1 + \frac{\zeta_s}{\sqrt{1-\zeta_s^2}\,\sqrt{1-\zeta_{mt}^2}} ,$$

or in practice

$$\zeta_s \approx \frac{\omega_c}{\omega_{mt}} - 1$$

If the experimental results satisfy this equation at the threshold of stability, the validity of the theory in the preceding pages has been proved.

Fig. 7 shows the stability chart for $\zeta_{mt} = 0.080$. The curve for ζ_s, made with the aid of the logarithmic decrement, for the corresponding cutting-data of the stability chart of Fig. 7, is shown in Fig. 8.

A second curve in the diagram of Fig. 8 shows the calculated values. For practical reasons, during experiments, the values for the parameter b_{cr} are replaced by values which are a shade smaller. The third curve shows that ζ_{mt} slightly depends upon the cutting speed. Fig. 9 gives the curve showing the fequency versus cutting speed at b_{cr}, while another curve shows the frequency for b = 0. A very good resemblance between the calculated values and the experimental ones can be established for cutting speeds higher than 60 m/min.

So, it points out that the right slope of the stability chart is not only defined by the compliance of the tool and an incremental cutting stiffness k_i, but also by a process damping.

Although, at low cutting speeds, the calculated values of ζ_s do not agree well with the experimental ones, it is doubtless that a process damping will also have great influence in this range of cutting speeds. Up to now, a further explanation cannot be given.

3.4. The calculation of k_i, when process damping is taken into account

From the foregoing, it is clear that only for cutting speeds larger than 60 m/min, the new method gives a good resemblance with the experimental results according to

$$\zeta_s = \frac{\omega_c}{\omega_{mt}} - 1$$

Consequently, the new method will give the best k_i-values for cutting speeds higher than the mentioned ones.
The preceeding theory shows

$$b_{cr}\, k_i = \rho_s\, \omega_{om} = \frac{\rho_s}{\sqrt{m\, k_m}}\, m\, \omega_{om}^2$$

or

$$b_{cr}\, k_i = 2\, \zeta_s \frac{\sqrt{m(k_m + b\, k_i)}}{\sqrt{m\, k_m}}\, m\, \omega_{om}^2$$

$$b_{cr}\, k_i = 2\, \zeta_s\, m\, \omega_c\, \omega_{om} \approx 2\, \left(\frac{\omega_c}{\omega_{mt}} - 1\right) m\, \omega_c\, \omega_{om}$$

with the approximation $\omega_{mt} \approx \omega_{om}$, the incremental cutting stiffness will be

$$k_i = \frac{2\, m\, \omega_c\, (\omega_c - \omega_{mt})}{b_{cr}}$$

Fig. 3 shows the calculated k_i-values.
A considerable difference is to be seen between the latter values and those calculated according to Peters-method.
The difference between both k_i-values is small at 80 m/min. As Fig. 5 shows a very low value for ζ_c at the mentioned cutting speed, this difference can be expected.

3.5. The influence of the wear of the tool on the process damping and on the incremental cutting stiffness

Fig. 10 shows the curves of the overall damping ratio and of the frequency taken from the pulse-responce, versus tool wear when the cutting speed V = 76 m/min and b = 1.25 mm. As appears from the results, the damping will not increase up to 0.2 mm. flank wear (VB) of the tool. However, for values higher than 0.2 mm, the damping will suddenly increase untill

two or three times its initial value.
This influence is taken into account during all experiments mentioned in this paper by keeping the wear of the tool within the range of 0.1 ÷ 0.2 mm.
Where the change in frequency is only small, and this change can be explained by the increasing damping in a great manner, we can conclude that the incremental cutting stiffness calculated according to the new method will not be influenced by the wear of the tool.

Conclusions

For cutting speeds higher than 60 m/min the method proposed in this paper gives, up to now, the best analytical approximation for the experimental results. Only frequencies have to be measured in order to get all the information necessary for calculation of the incremental cutting stiffness and the process damping. A further advantage of this method is, that these experiments, i.e. measuring frequencies, can be easily done in each laboratory.
Furthermore, this method gives the possibility to get real values of the damping and the stiffness of materials. These values will be unique and they will not be influenced by the dynamic data of the tool. Now, we can derive the dynamic cutting-coefficient, as we can compare materials on susceptibility to chatter.
It is possible to use the results for predicting the dynamic behaviour of machine tools during cutting if the directions of motion of the tool and of the test rig are the same, or if the direction of the dynamic force is known.
At low cutting speeds, we cannot calculate the stability chart well only with the values for the process damping and those for the cutting stiffness.
The k_i-values will not be influenced by the wear of the tool, while the damping may increase to high values.

NOTE:

In the beginning, the experiments mentioned in this paper did not

yield well reproductive values. It turned out that this was on account of the temperature of the workpiece, which increases during cutting.
The influence has been eliminated by standardizing the experimental conditions.

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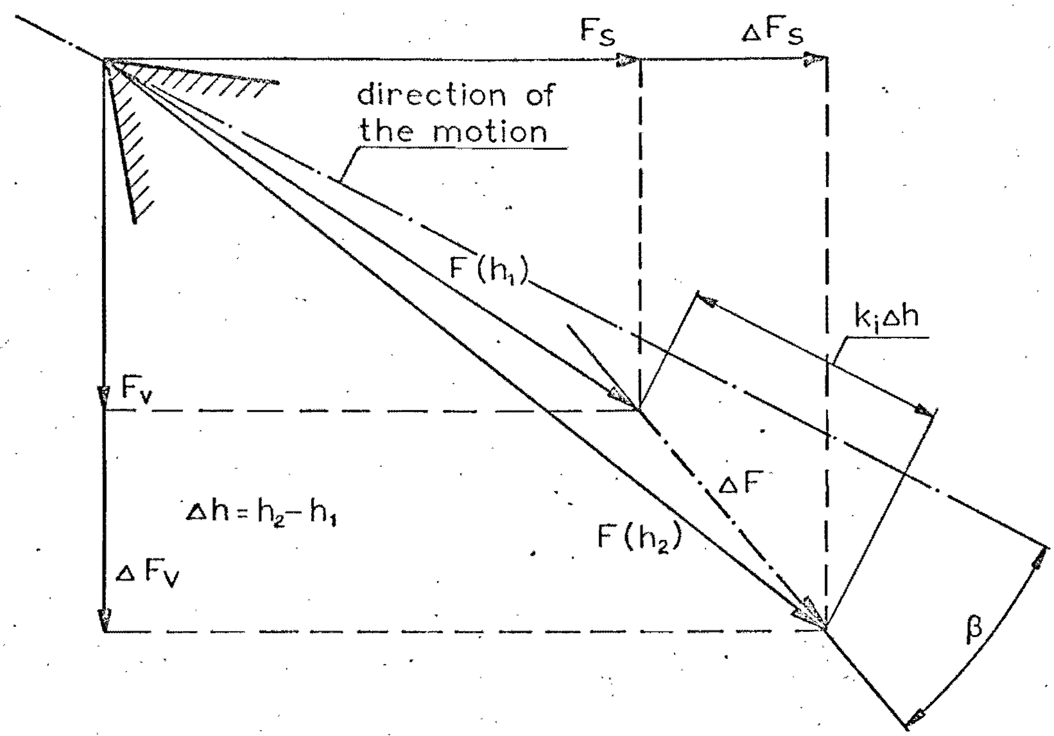

Fig. 1. Determination of the incremental cutting stiffness according to the method of Peters and Vanherck.

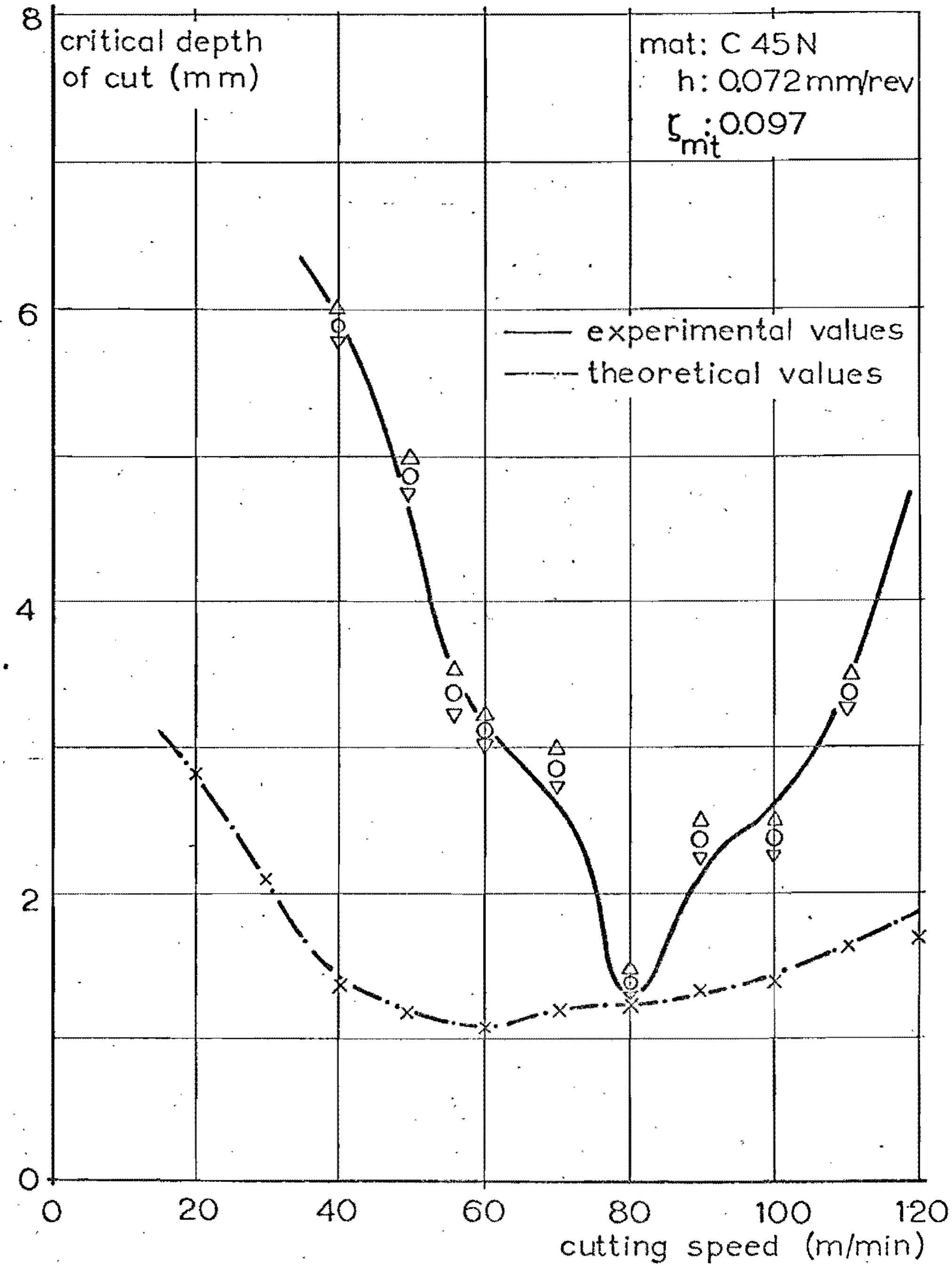

Fig. 2. The experimental and the calculated stability chart (Peters method). The quantity ζ_{mt} represents the average value of the damping ratio of the tool when the carriage is moving.

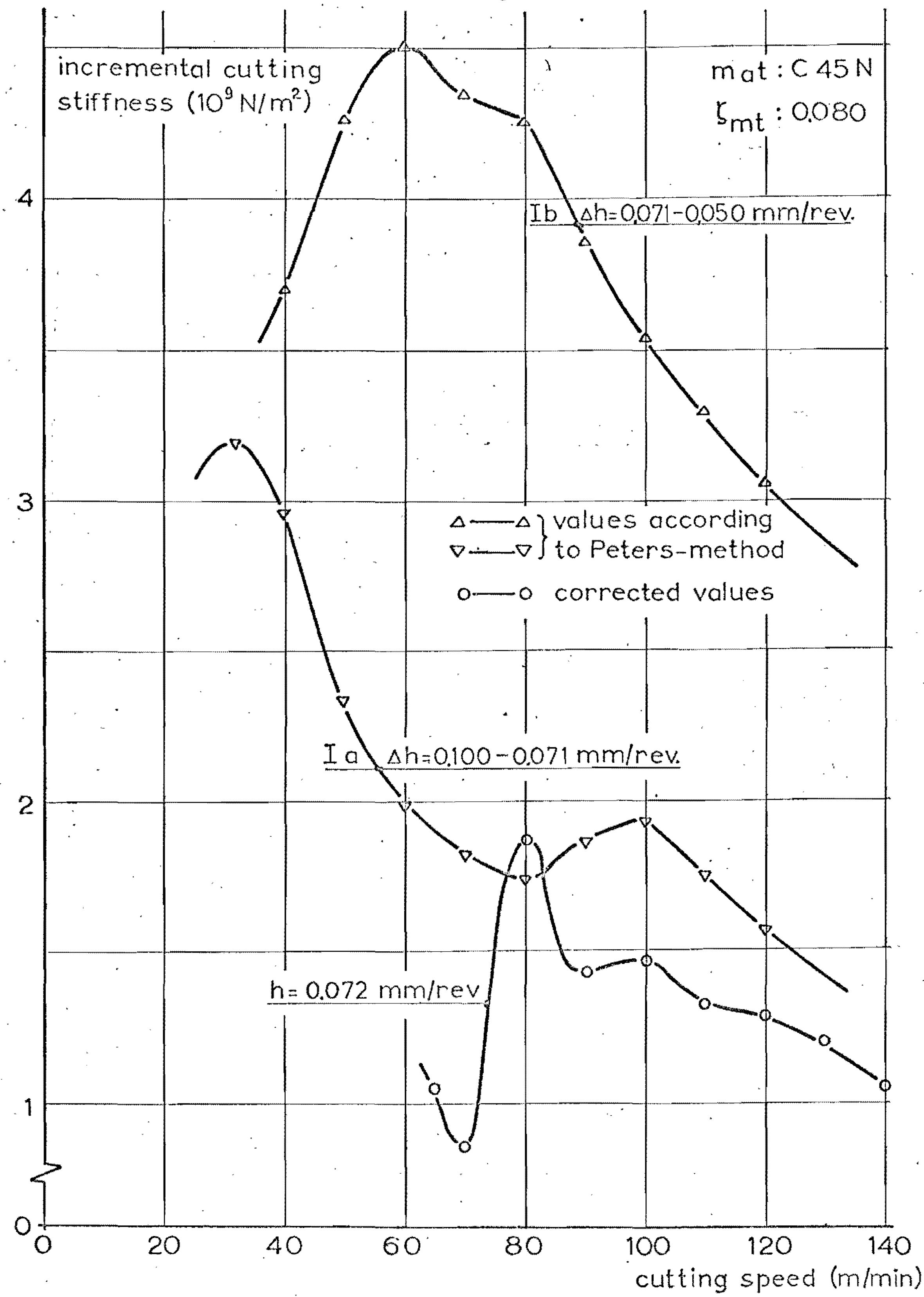

Fig. 3. The incremental cutting stiffness versus cutting speed according to the method from Peters and Vanherck (I) and according to the new method (II).

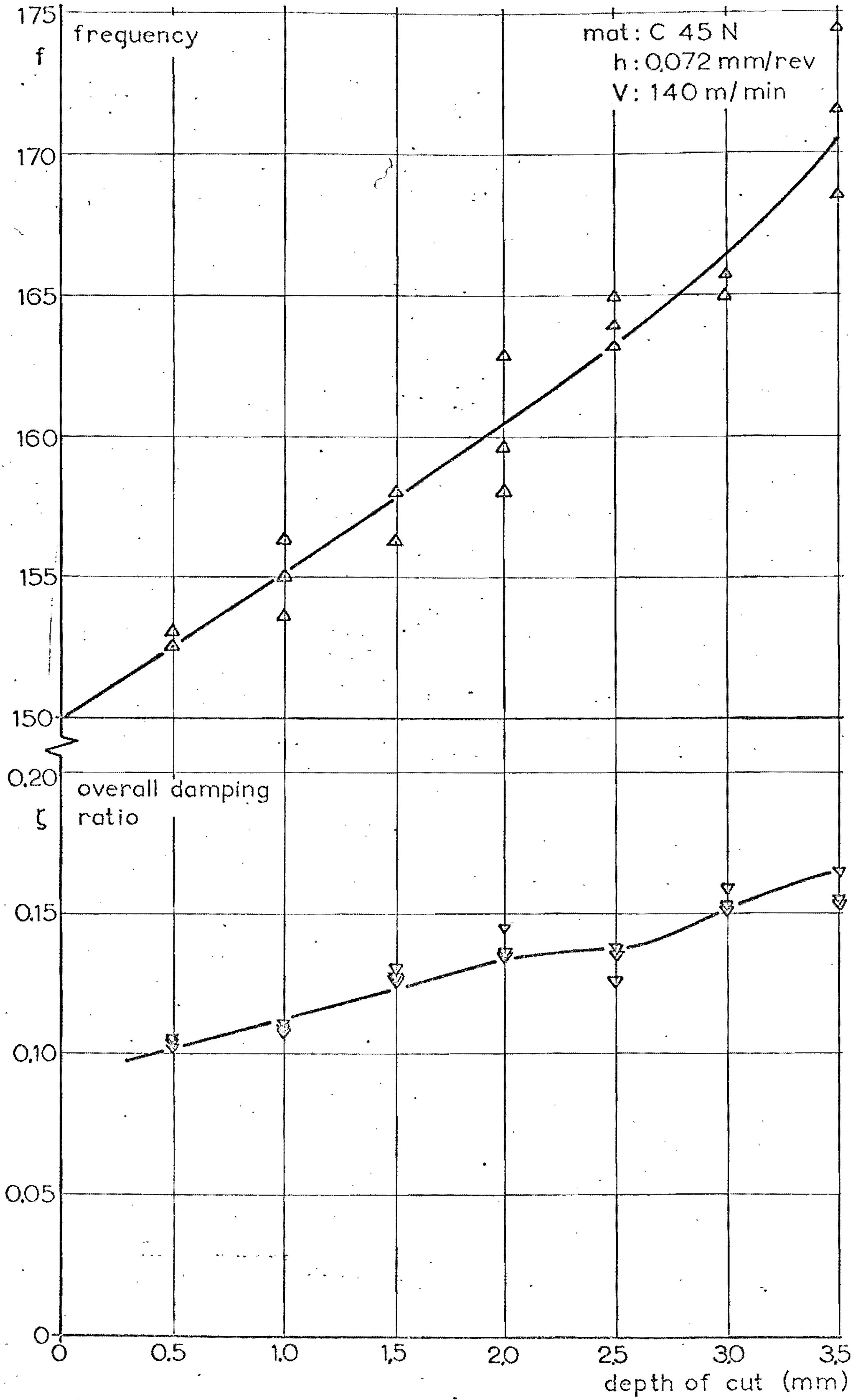

Fig. 4. The overall damping ratio and the frequency of the pulse-response versus depth of cut.

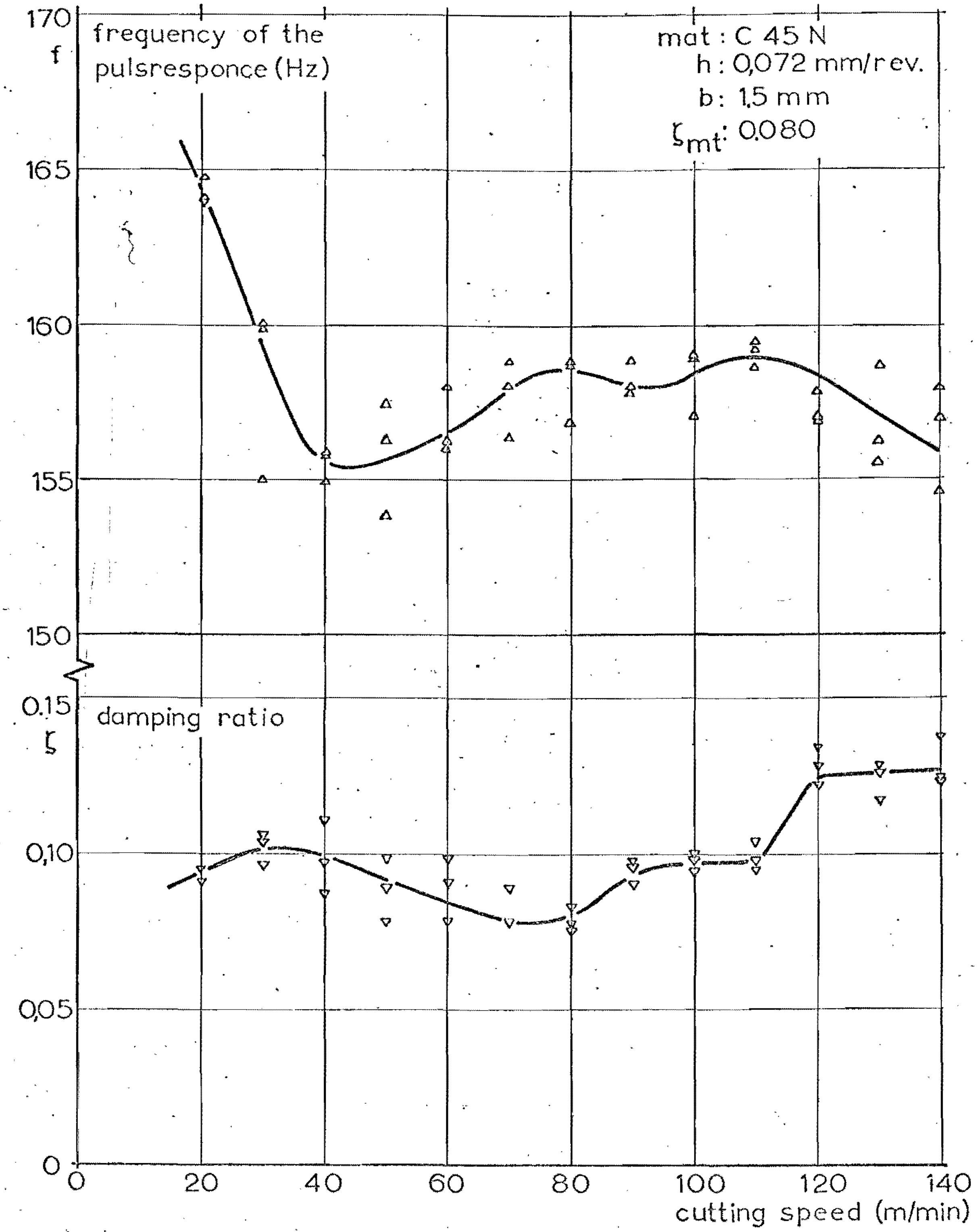

Fig. 5. The damping ratio and the frequency of the overall system versus cutting speed.

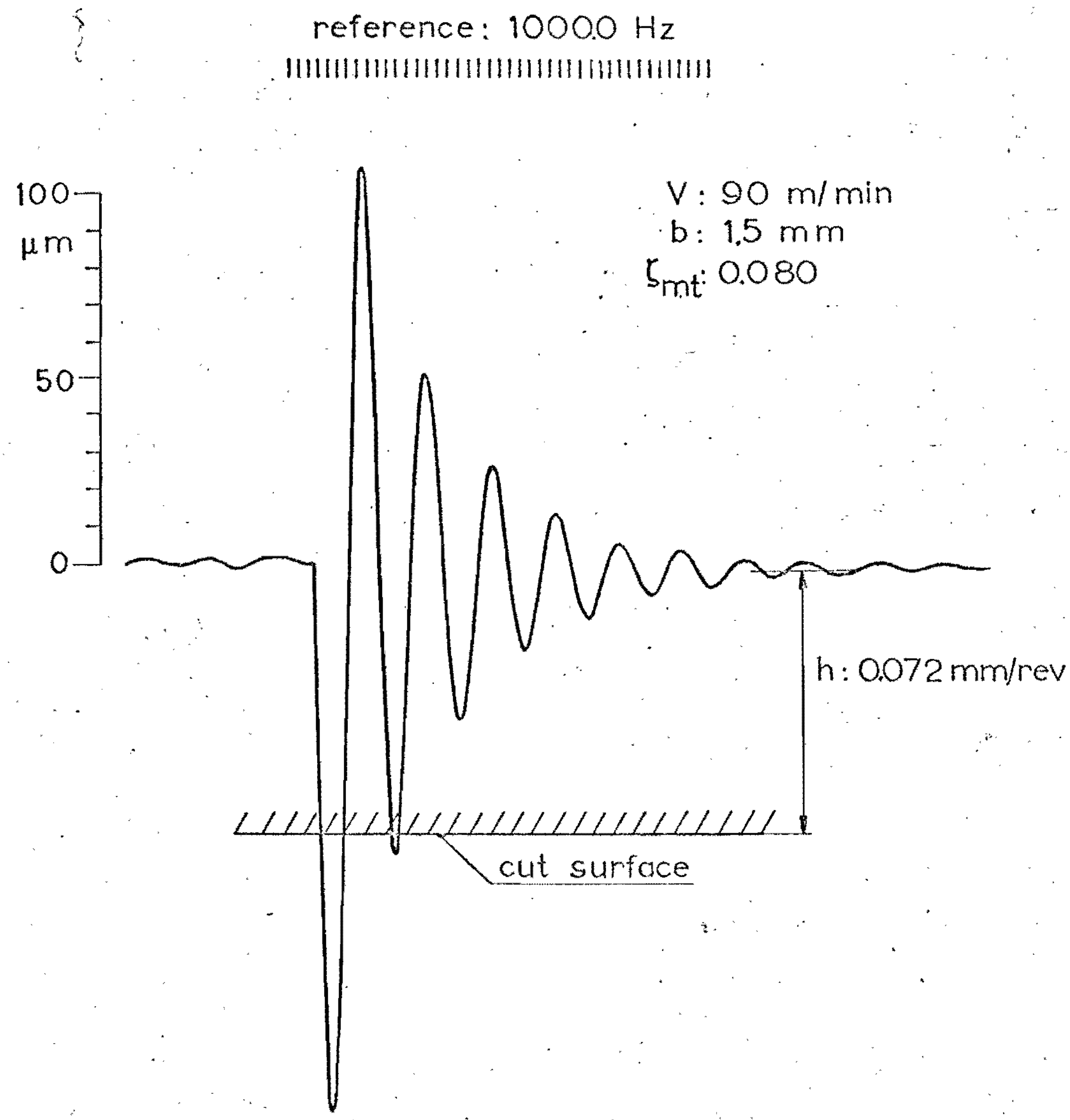

Fig. 6. Example of a pulse-response.
The mass of the tool holder = 14,35 kg.

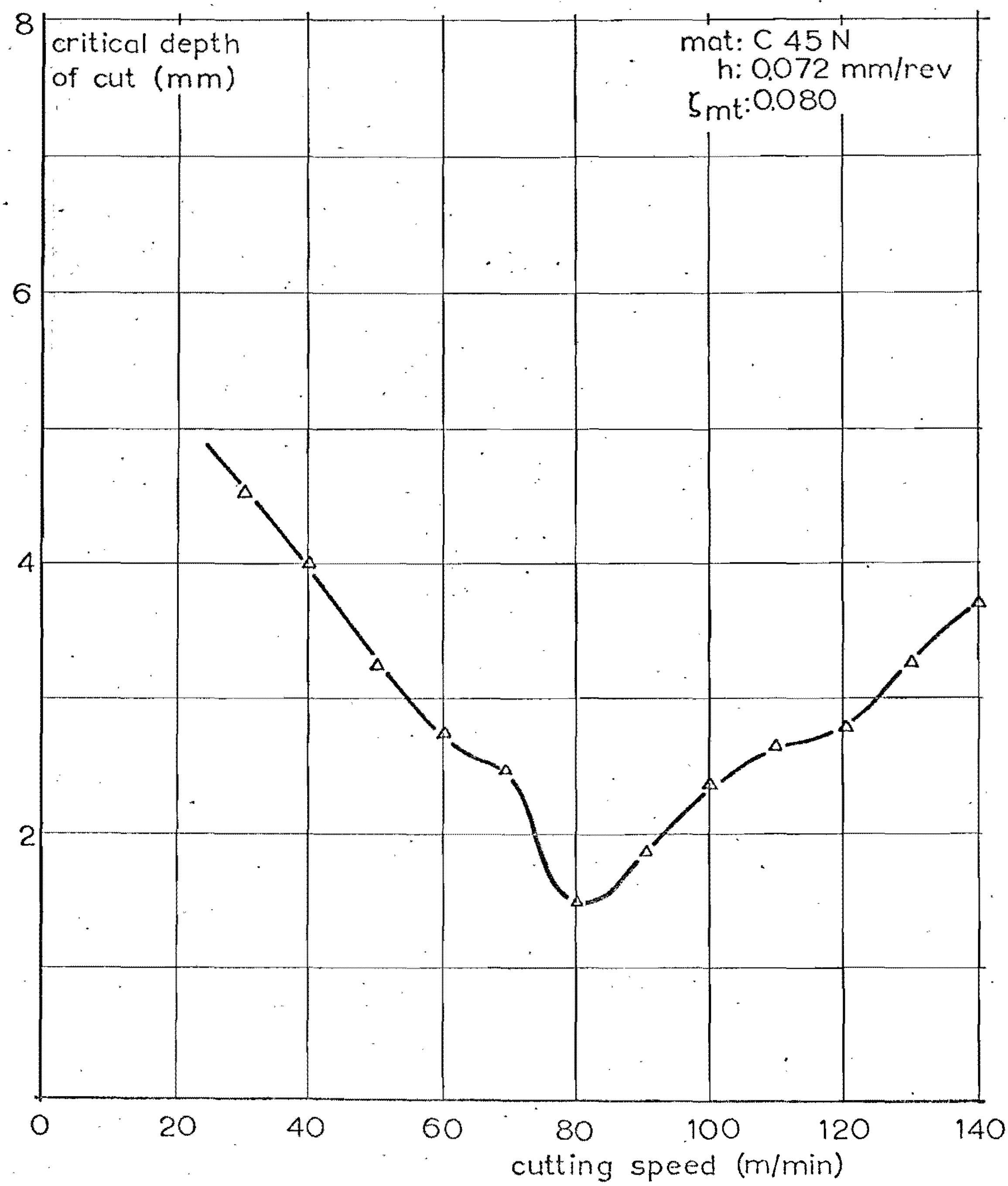

Fig. 7. The experimental stability chart for $\zeta_{mt} = 0.080$.

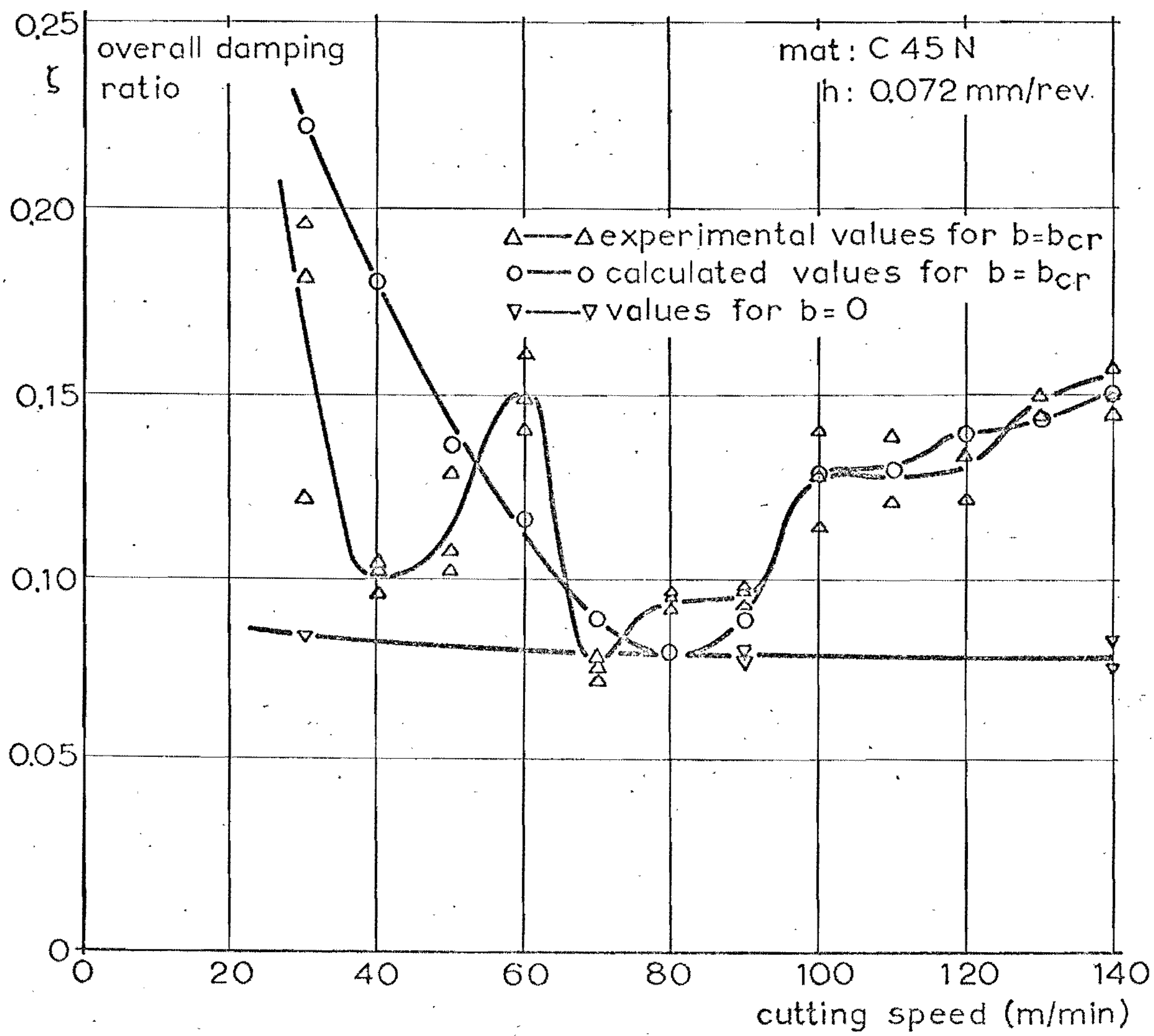

Fig. 8. The overall damping ratio at the threshold of stability, and the damping ratio of the moving tool (b=0) versus cutting speed.

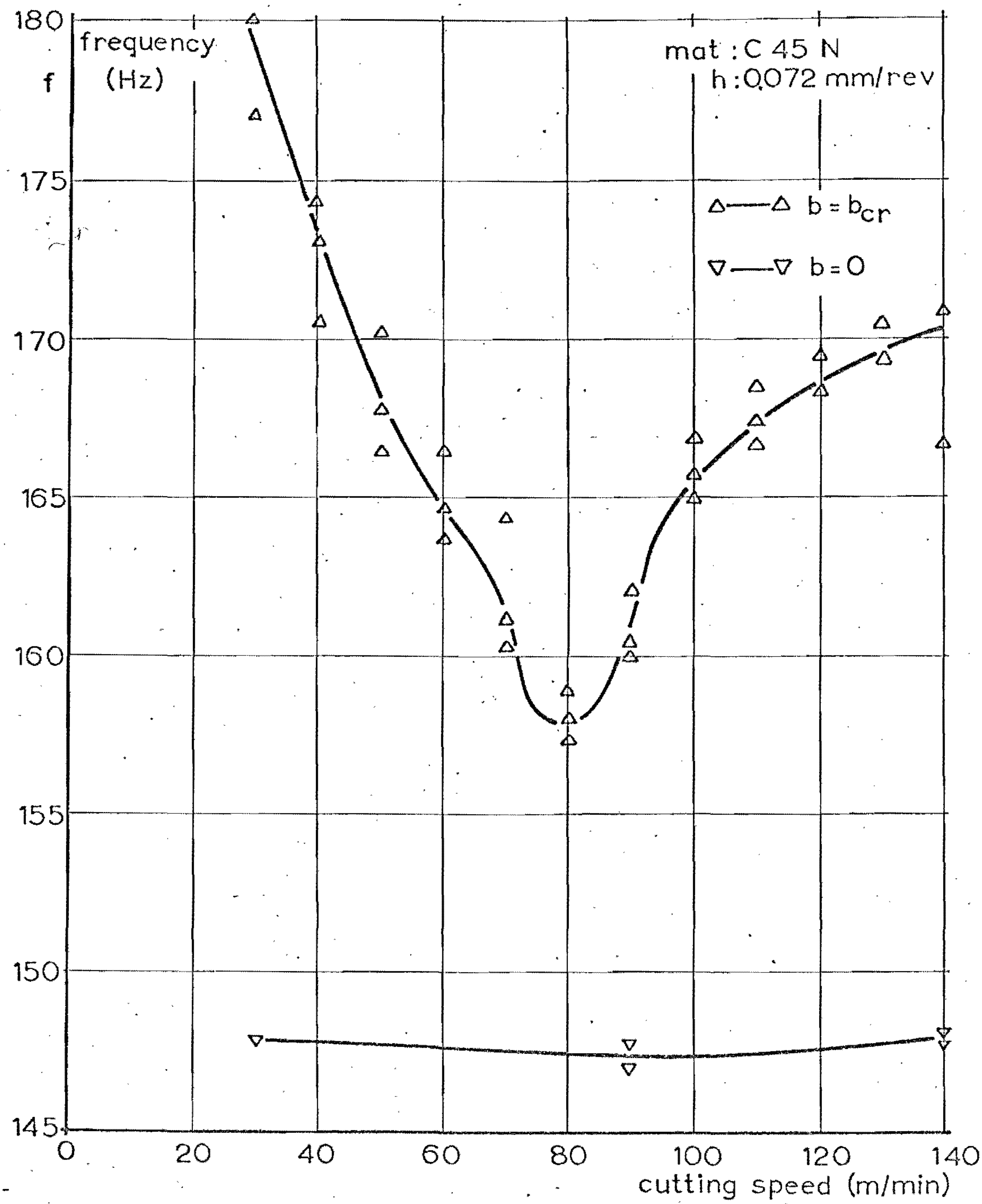

Fig. 9. The frequency at the threshold of stability, and the frequency for b=0 versus cutting speed.

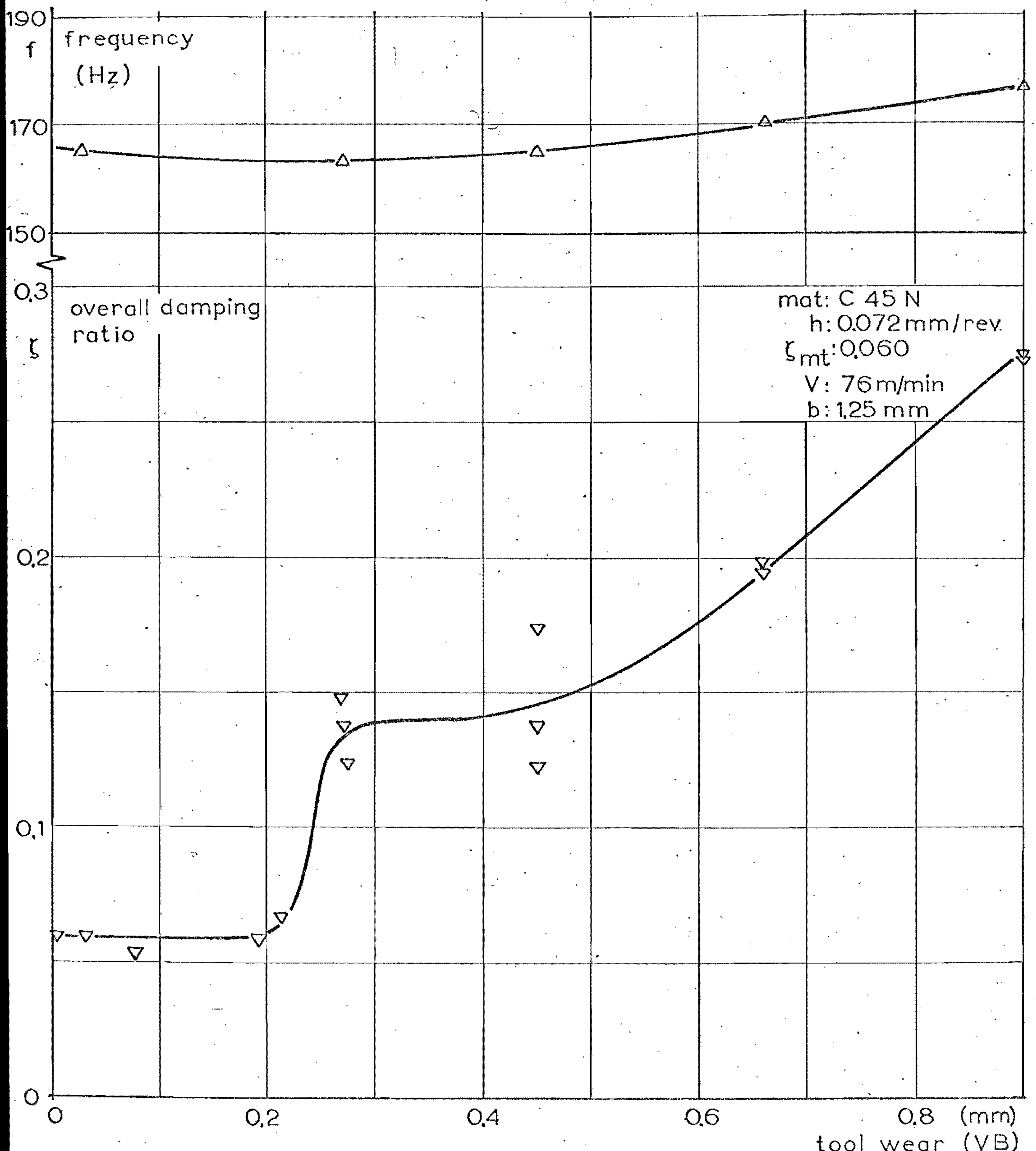

Fig. 10. The overall damping-ratio and the frequency of the system versus tool wear.