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by

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Contents

Summary

This report describes an investigation of the influence of the shape of the teeth on permeance and on the tangential force-displacement curves of slotted structures.

Chapter 1 presents the calculation procedures. First the influence of varying slotdepth is calculated for rectangular teeth, followed by the calculation of the influence of rounded slots. The influence of the toothwidth/toothpitch ratio is then investigated, the slotdepth being kept constant. These calculations are carried out for three assumed material conditions, viz.:

a Ideal iron, i.e. iron with infinite permeability, $\mu_r \to \infty$

b Normal iron, i.e. iron with a finite, non-constant permeability, excluding the influence of hysteresis and saturation of the teeth.

c Normal iron, now considering also the influence of eventual saturation of the teeth.

In chapter 2 a description is given of a measuring device by which the calculated force-displacement curves are verified. These measurements are carried out for five different airgap values, seven different toothwidths and eight values of the excitation.

Finally, in the third chapter, a comparison between the calculations and measurements is made and conclusions regarding the most suitable toothwidth for slotted structures are drawn.

INTRODUCTION

The problem that is investigated in this report is that of the magnetic behaviour of two identical slotted iron surfaces opposite each other (see figure 1). The problem is considered to be two dimensional; calculations will apply to units of 1 m perpendicular to the drawing. Further the calculations apply to an infinite repetition of this pattern, end effects not being considered. The relevant magnetic field may be generated by a constant current I in a coil of N windings giving an mmf $\Theta = NI$.

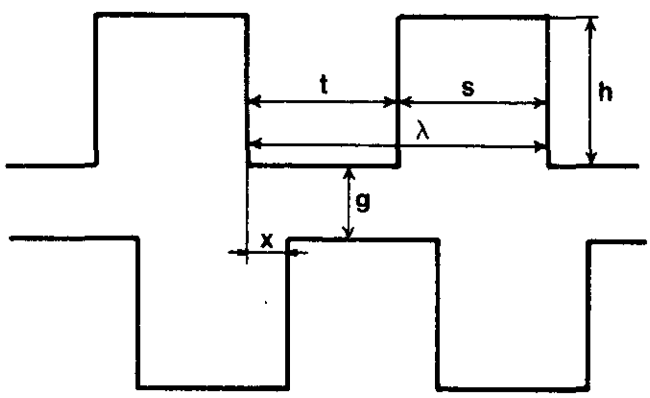


Fig. 1. Doubly slotted structure.

We are interested in the magnetic permeance between both surfaces and in the horizontal force when one of the surfaces is shifted with respect to the other, keeping the airgap g at a certain constant value. Furthermore, keeping the toothpitch λ at a constant value, the slotwidth s and the toothwidth t can be varied, resulting in various force-displacement curves. The above mentioned features can be of great importance when designing electrical machines with slots in stator and rotor, e.g. stepping motors.

The toothwidth can be made such that, for a certain value of g, one finds:

a An as large as possible average force when varying the displacement from a position with maximum permeance to a position with minimum permeance.

b An as large as possible maximum force. The displacement value at which this maximum occurs can be of importance.

c A particular shape of the force-displacement curve. This can be important if one looks for a particular dynamic behaviour of a machine.

In this report the calculation of permeances and forces is presented. The influence of varying slotdepth is calculated for rectangular teeth, followed by the influence of rounded slot bottoms. Next the influence of the toothwidth/toothpitch ratio is investigated, the slotdepth being kept constant. These last calculations are carried out for three assumed material conditions, viz.:

a Ideal iron, i.e. iron with permeability $\mu_r \to \infty$

b Normal iron, i.e. iron with a finite, non-constant permeability, excluding the influence of hysteresis and saturation of the teeth.

In a, by nature the iron/air interfaces are equipotential surfaces. In b, a similar property is assumed, which may be considered to be a fair approximation.

c Normal iron, now including the influence of saturation of the teeth.

Saturation of the teeth gives a change in the magnetic potential along the iron surface. The absolute value of this potential is higher in the slots than on the tips of the teeth, an effect that may be indicated by a virtual rounding off of the teeth corners and an increase of the airgap width. This reduces the permeance and decreases the force with respect to the case without saturation of the teeth.
To verify the calculations a measuring device was built.
The measurements were carried out for five different airgap

values, seven different toothwidths and eight different values of the potential across the airgap.
Finally the results of the calculations will be compared with the measured results and some conclusions will be drawn.

1. CALCULATIONS

1.1. Calculation of the Force

1.1.1. Introduction.

For the calculation of the force it is essential to know as precisely as possible the magnetic field in the airgap.

In this chapter we will first develop a method to calculate the force-displacement curves, requiring numerical computations and after that find expressions for the average force under static conditions, i.e. the average force developed when one iron surface is made to move with respect to the other from a position with maximum permeance to a position of minimum permeance in a quasi-static way, keeping the potential across the airgap constant.

To be able to carry out the calculations, a few simplifying assumptions will be made.

a The field problem may be treated as being two-dimensional.

b As the field pattern repeats itself every toothpitch λ, it suffices to calculate the field intensity and the force for one toothpitch. The total force can then be found by multiplying the calculated force by the number of teeth.

c First, calculations will be carried out for slotting with ideal iron ($\mu_r \to \infty$).
In this case only the magnetic field in the airgap has to be dealt with.
After that slotting with normal iron and finally slotting with normal iron and saturation of the teeth will be investigated.

1.1.2. Determining the formula for calculating the force.

To calculate the force, two, closely related methods can be used.

_a_ Make use of the total magnetic co-energy W'_m

The force is then found from [1]

$$F_x = \frac{\partial W'_m}{\partial x} \qquad (1)$$

in which x is the horizontal displacement

_b_ By integrating Maxwell's stresses over a flat plane in the middle of the airgap.

The force then follows from [8]

$$F = \frac{1}{\mu_0} \iint \{(\underline{B}.\underline{n})\underline{B} - \tfrac{1}{2} B^2 \underline{n}\} dA \qquad (2)$$

The integral should be extended over a surface that envelopes one of the members; for our purpose it is acceptable to assume contributions for areas other than that in the airgap to be negligible.

Both methods require a detailed information regarding the magnetic field intensity.

In our calculations we will make use of method _b_ as this requires evaluation of a surface integral, while in the first case a volume integral of the energy intensity has to be calculated.

Furthermore the following of Maxwell's equations will be used:

$$\text{curl } \underline{H} = \underline{J} + \varepsilon_0 \frac{\partial \underline{E}}{\partial t} \qquad (3)$$

$$\text{div } \underline{B} = 0 \qquad (4)$$

and

$$\underline{B} = \mu_0 \mu_r \underline{H} \qquad (5)$$

As there are no changing electrical charges in the airgap and because the excitation coils are not situated in the airgap region, (3) becomes:

$$\text{curl } \underline{H} = \underline{0} \qquad (6)$$

The magnetic field intensity is obtained from a scalar potential such that

$$\underline{H} = -\text{grad } U \tag{7}$$

therefore (4) and (7) lead to Laplace's equation

$$\text{div. grad } U = 0 \tag{8}$$

In a Cartesian coordinate system this can be written as

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0 \tag{9}$$

The magnetic field in the airgap can be split into two parts with respect to the chosen plane in the airgap. (see fig. 2).

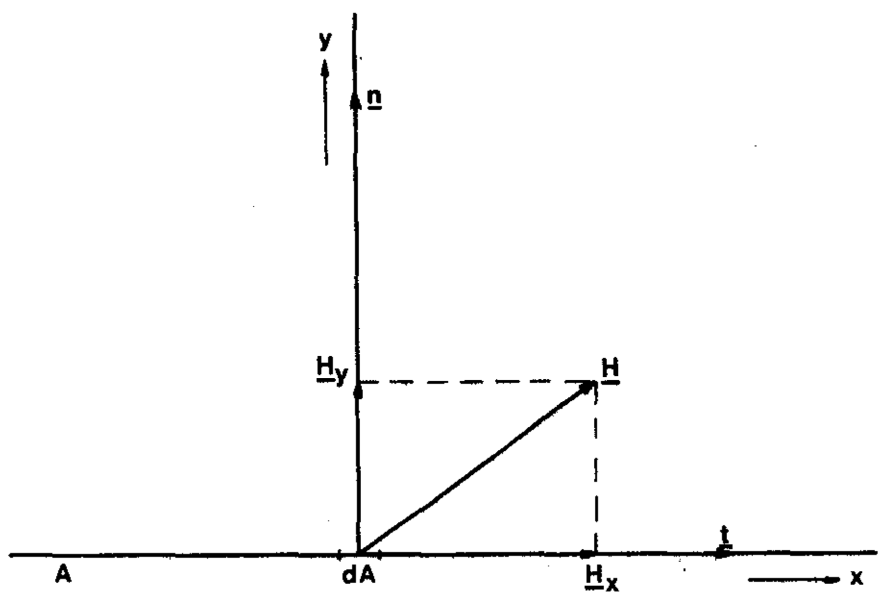


Fig. 2 Magnetic field.

a Vertical component $\underline{H}_y$

$$\underline{H}_y = (\underline{H}.\underline{n})\underline{n} \tag{10}$$

b Horizontal component $\underline{H}_x$

$$\underline{H}_x = (\underline{n} \times \underline{H}) \times \underline{n} \tag{11}$$

From (2), (5), (10) and (11) it follows that

$$F_{total} = \mu_0 \iint \{H_y \underline{H} - \tfrac{1}{2}(H_y^2 + H_x^2)\underline{n}\} dA$$

$$= \mu_0 \iint \{H_y H_x \underline{t} + \tfrac{1}{2}(H_y^2 - H_x^2)\underline{n}\} dA \qquad (12)$$

Each part of the area seems to be subjected to a vertical tension

$$\sigma = \tfrac{1}{2}\mu_0 \, (H_y^2 - H_x^2) \qquad (13)$$

and a horizontal tension

$$\tau = \mu_0 \, H_x H_y \qquad (14)$$

As we are only interested in the horizontal force F, (12) simplifies to

$$F = \mu_0 \iint H_x H_y dA \qquad (15)$$

For the two-dimensional case this results in

$$F = \mu_0 \, l_t \int H_x H_y dx \qquad (16)$$

in which l_t = toothlength perpendicular to the plane of drawing.

For our numerical computations this gives

$$F = \mu_0 l_t z.\frac{\lambda}{n} \sum_1^n H_x H_y + \text{error} \qquad (17)$$

when the airgap is divided horizontally into n equal parts per toothpitch and z is the number of teeth. The error can be made sufficiently small by a suitable large value of n.

For these field calculations one can resort to the method of conformal mapping, and to numerical methods.

The first method is not suitable for our purpose owing to the rather complex shape of the area. For most computations the finite difference method [9] was applied, but for the calculations of section 1.2.2. the finite element method [10] was chosen.

1.2. Computed results

1.2.1. The influence of the slotdepth, with ideal iron.

The influence of the slotdepth is calculated for rectangular teeth with toothpitch $\lambda = 2m$, toothwidth t = slotwidth, $s = \lambda/2$, (see fig. 1).

Furthermore $x = 0.5$ m

Θ/g = constant = 1 A/m

$l_t = 1$ m

The forces are computed for

λ/g = 40, 20, 10, 8.05 and 5.

The slotdepth for these 5 values are

0.6s, s and 2s respectively.

The results of the calculations are given in table 1.

$\frac{\lambda}{g}$	F_v	$h = 0.6s = 0.3\lambda$		$h=s=\frac{1}{2}\lambda$		$h=2s=\lambda$	
		F	dev. %	F	dev. %	F	dev. %
40	0.0208	0.0206	0.96	0.0208	0.00	0.0208	0.00
20	0.0347	0.0337	2.9	0.0345	0.58	0.0346	0.29
10	0.0469	0.0440	6.2	0.0463	1.28	0.0466	0.64
8.05	0.0479	0.0434	9.4	0.0467	2.5	0.0472	1.45
5	0.0418	0.0350	16.2	0.0398	4.8	0.0405	3.10

Table 1.

In this table the deviations with respect to F_v are given. F_v is the force calculated by Veltkamp and Brands [2]; their calculations were carried out for infinitely deep slots by means of conformal mapping, therefore considered to be exact. F and F_v are specific dimensionless force figures, derived from real forces by dividing these by $\mu_0 . l_t . \Theta^2 \lambda$.

From table 1 we see that for increasing airgapwidth g the influence of the slotdepth increases.

For further calculations a value of 0.5λ was chosen for the slotdepth, because this is a value that will often be met in practice, also because the influence of the slotbottom is slight in this case.

1.2.2. The influence of the slot shape.

Here the influence of rounded slot bottoms is calculated. These slots consist of a rectangular part with depth h' and on top of that a semicircle as drawn in figure 3.

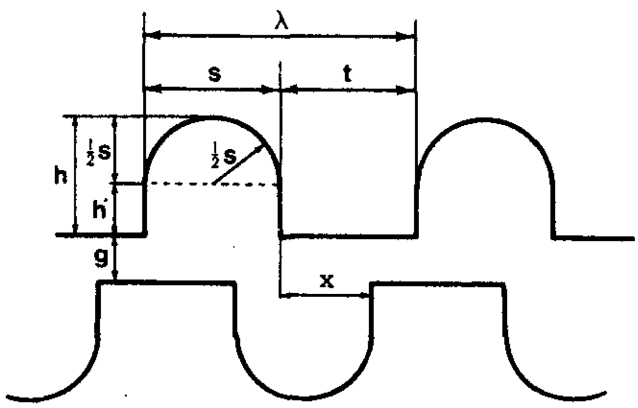


Fig. 3 Rounded slot bottoms.

The influence on the force is calculated for $\lambda/g = 20$ with $h = 0.5s$, $0.75s$ and s respectively. Furthermore

$$s = t = 1 \text{ m}$$
$$x = 0.5 \text{ m}$$
$$\Theta/g = 1 \text{ A/m}$$
$$l_t = 1 \text{ m}$$

In table 2 the calculated values are given

h	F
½s	0.0324
¾s	0.0344
s	0.0346

table 2.

When comparing this with the values of table 1 we come to the conclusion that for a slotdepth $h = s = 0.5\lambda$ rounding of the slot bottoms has hardly any effect on force F.

1.2.3. Calculation of the force-displacement curves for slotting with ideal iron.

For the slotting as drawn in fig. 1 the force is calculated for a toothlength of 1m, a toothpitch λ of 1m and a nominal magnetic field intensity H of 1 A/m, the latter appearing in the homogeneous field if the slots were non-existing. This requires fixed values of magnetic potential.
This is done for 5 values of λ/g viz. 40, 20, 10, 8.05 and 5.
Calculations for $\lambda/g = 8.05$ are carried out because, with this particular value, according to [3] and [4], for $t/g =$ $= 3.05$, $s/g = 5.00$ and Θ/g having a fixed value, the average force should be maximum.
For each value of λ/g, calculations are carried out for toothwidths of $\frac{1}{8}\lambda$, $\frac{1}{4}\lambda$ .... $\frac{7}{8}\lambda$. The results of these calculations are given in figures 4 - 8, the appropriate values being shown in tables 3 - 7.
The permeance values are also calculated for $l_t = 1m$, $\lambda = 1m$ and $\Theta = 1A$ and are shown in figures 9 - 13 and tables 8 - 12.
In the figures and tables D stands for displacement and is equal to x.

MAGNETIC FORCE (F) ON DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M, PITCH = 1M, NOMINAL MAGNETIC FIELD INTENSITY = 1A/M.

***** PITCH/G = 40 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0464	0.0307	0.0218	0.0168	0.0136	0.0114	0.0098
0.20	0.0295	0.0348	0.0254	0.0198	0.0161	0.0135	0.0116
0.30	0.0004	0.0335	0.0262	0.0206	0.0168	0.0142	0.0078
0.40	0.0000	0.0261	0.0260	0.0210	0.0172	0.0145	0.0034
0.50	0.0000	0.0036	0.0246	0.0208	0.0173	0.0140	0.0018
0.60	0.0000	0.0000	0.0211	0.0205	0.0173	0.0059	0.0011
0.70	0.0000	0.0000	0.0104	0.0196	0.0170	0.0028	0.0007
0.80	0.0000	0.0000	0.0001	0.0178	0.0105	0.0014	0.0004
0.90	0.0000	0.0000	0.0000	0.0130	0.0033	0.0006	0.0002
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 3.

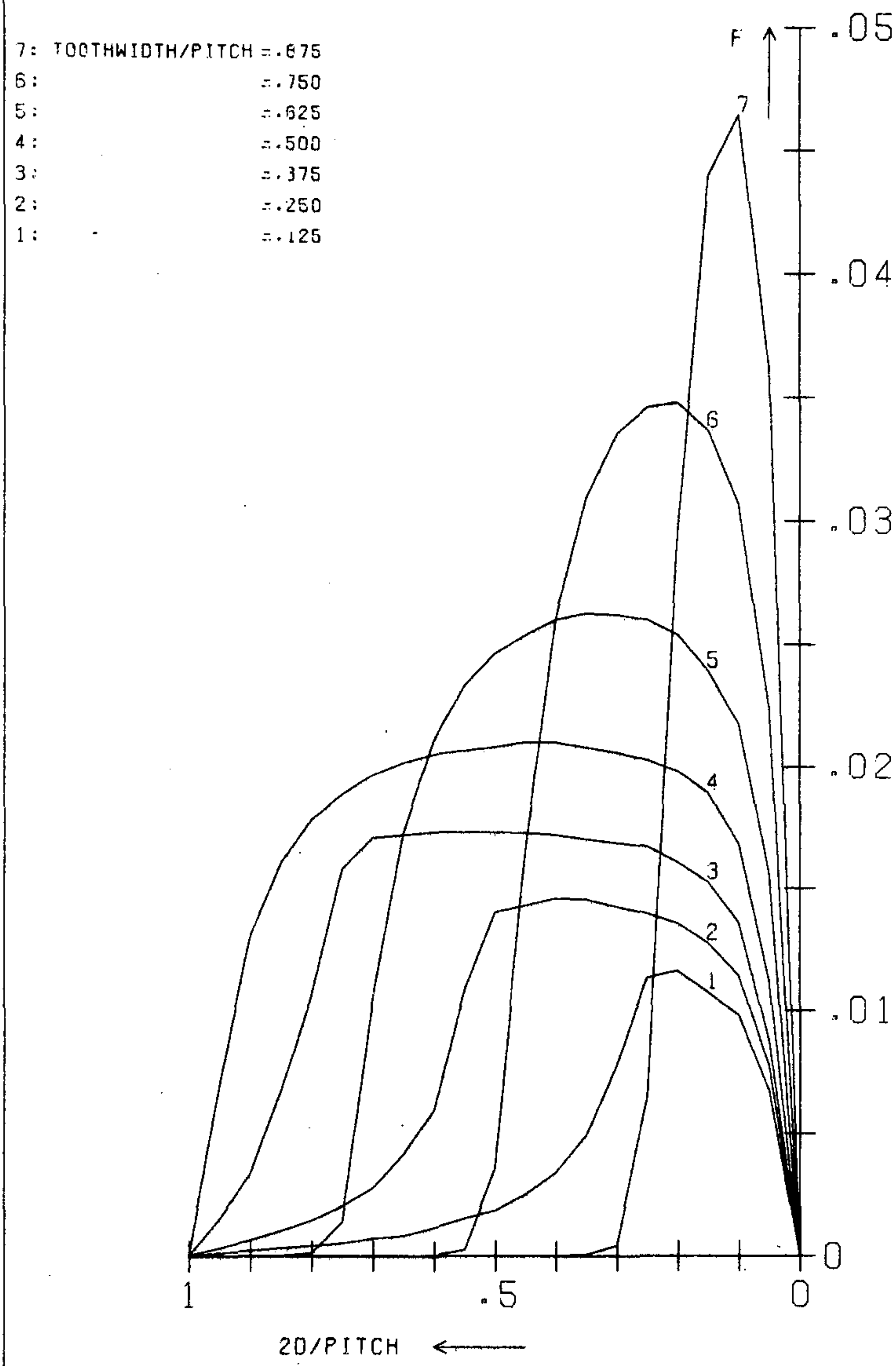


Fig. 4 PITCH/G=40

MAGNETIC FORCE (F) ON DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M, PITCH = 1M, NOMINAL MAGNETIC FIELD INTENSITY = 1A/M.

***** PITCH/G = 20 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0393	0.0356	0.0279	0.0224	0.0183	0.0156	0.0133
0.20	0.0265	0.0464	0.0378	0.0307	0.0253	0.0216	0.0182
0.30	0.0032	0.0442	0.0405	0.0336	0.0280	0.0240	0.0154
0.40	0.0002	0.0306	0.0401	0.0348	0.0293	0.0250	0.0093
0.50	0.0000	0.0078	0.0365	0.0345	0.0297	0.0233	0.0057
0.60	0.0000	0.0004	0.0284	0.0335	0.0297	0.0149	0.0037
0.70	0.0000	0.0000	0.0125	0.0309	0.0284	0.0083	0.0024
0.80	0.0000	0.0000	0.0012	0.0261	0.0206	0.0045	0.0014
0.90	0.0000	0.0000	0.0001	0.0162	0.0087	0.0020	0.0007
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4.

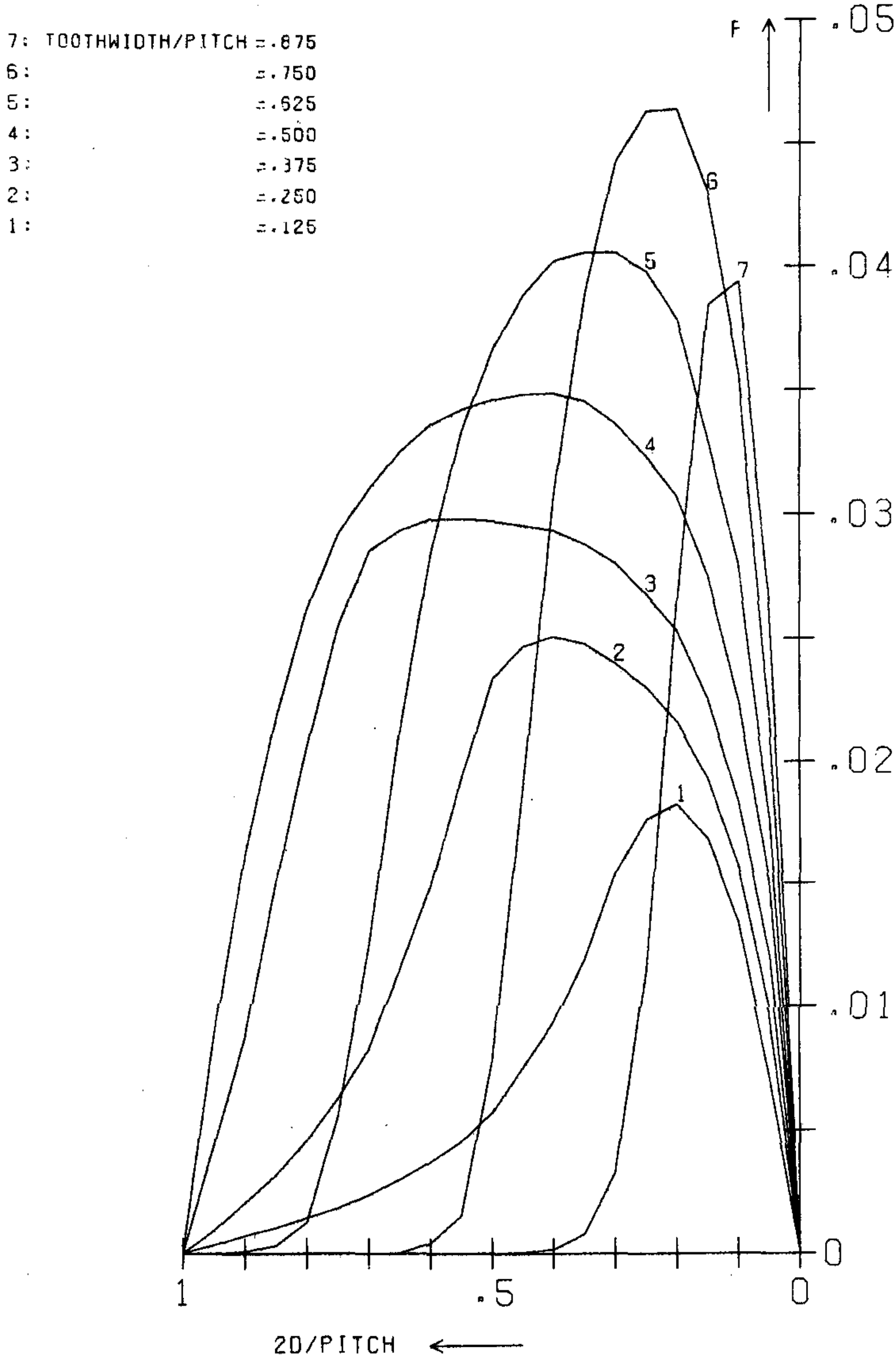


Fig. 5 PITCH/G=20

MAGNETIC FORCE (F) ON DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M, PITCH = 1M, NOMINAL MAGNETIC FIELD INTENSITY = 1A/M.

***** PITCH/G = 10 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0166	0.0267	0.0251	0.0218	0.0188	0.0163	0.0137
0.20	0.0153	0.0398	0.0404	0.0359	0.0312	0.0272	0.0218
0.30	0.0062	0.0388	0.0468	0.0430	0.0380	0.0330	0.0223
0.40	0.0016	0.0268	0.0468	0.0464	0.0417	0.0355	0.0183
0.50	0.0003	0.0116	0.0411	0.0463	0.0429	0.0334	0.0134
0.60	0.0000	0.0033	0.0302	0.0441	0.0426	0.0266	0.0095
0.70	0.0000	0.0008	0.0158	0.0387	0.0387	0.0183	0.0065
0.80	0.0000	0.0002	0.0053	0.0301	0.0293	0.0112	0.0040
0.90	0.0000	0.0000	0.0012	0.0167	0.0150	0.0053	0.0020
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 5.

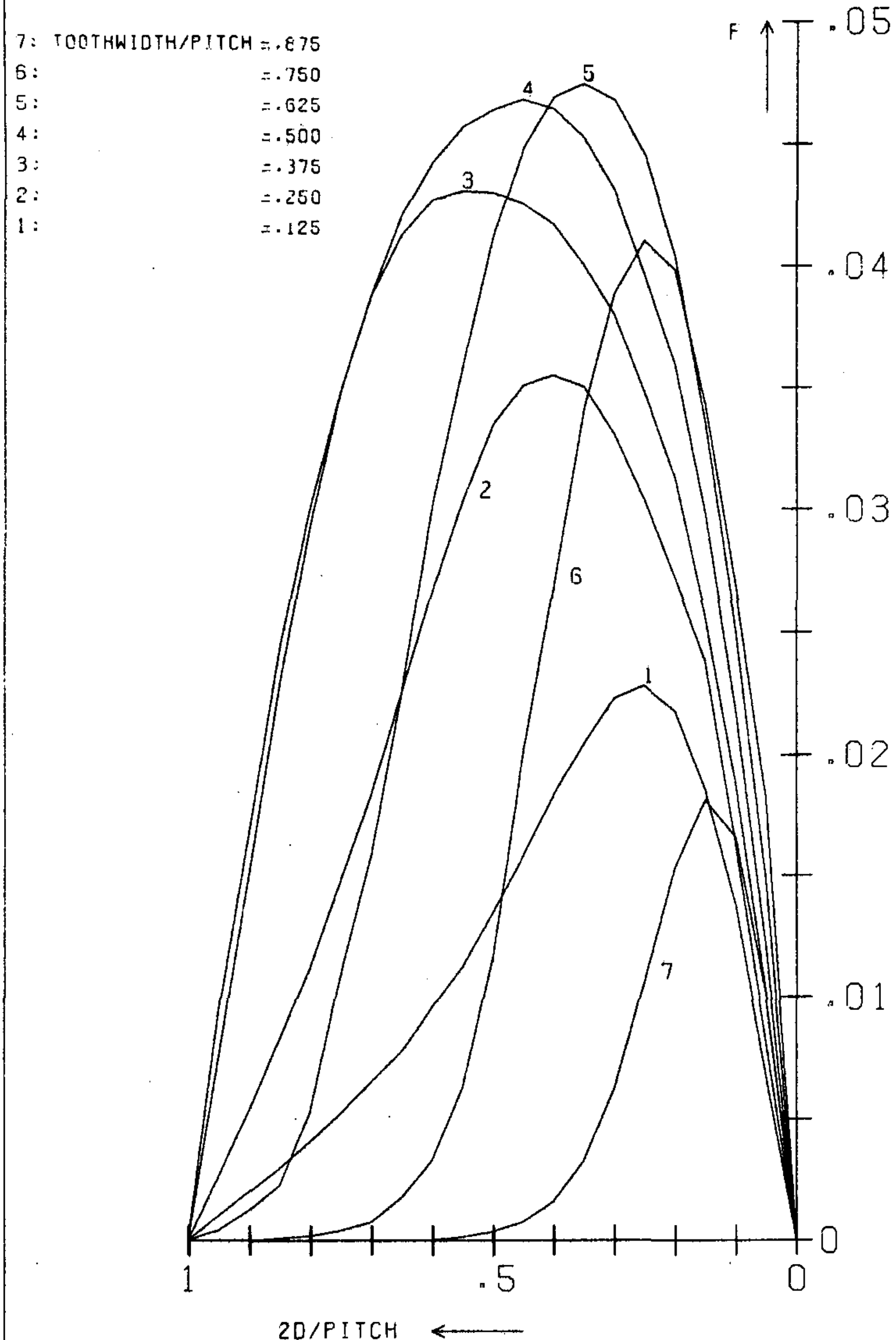


Fig. 6 PITCH/G=10

MAGNETIC FORCE (F) ON DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M, PITCH = 1M, NOMINAL MAGNETIC FIELD INTENSITY = 1A/M.

***** PITCH/G = 8.05 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0105	0.0210	0.0219	0.0199	0.0176	0.0151	0.0125
0.20	0.0112	0.0326	0.0368	0.0342	0.0305	0.0269	0.0209
0.30	0.0059	0.0326	0.0437	0.0425	0.0385	0.0336	0.0225
0.40	0.0021	0.0237	0.0442	0.0463	0.0430	0.0365	0.0200
0.50	0.0006	0.0122	0.0389	0.0467	0.0446	0.0348	0.0156
0.60	0.0001	0.0046	0.0288	0.0441	0.0438	0.0291	0.0116
0.70	0.0000	0.0015	0.0165	0.0383	0.0393	0.0210	0.0084
0.80	0.0000	0.0004	0.0069	0.0290	0.0298	0.0134	0.0051
0.90	0.0000	0.0001	0.0021	0.0158	0.0157	0.0064	0.0024
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 6.

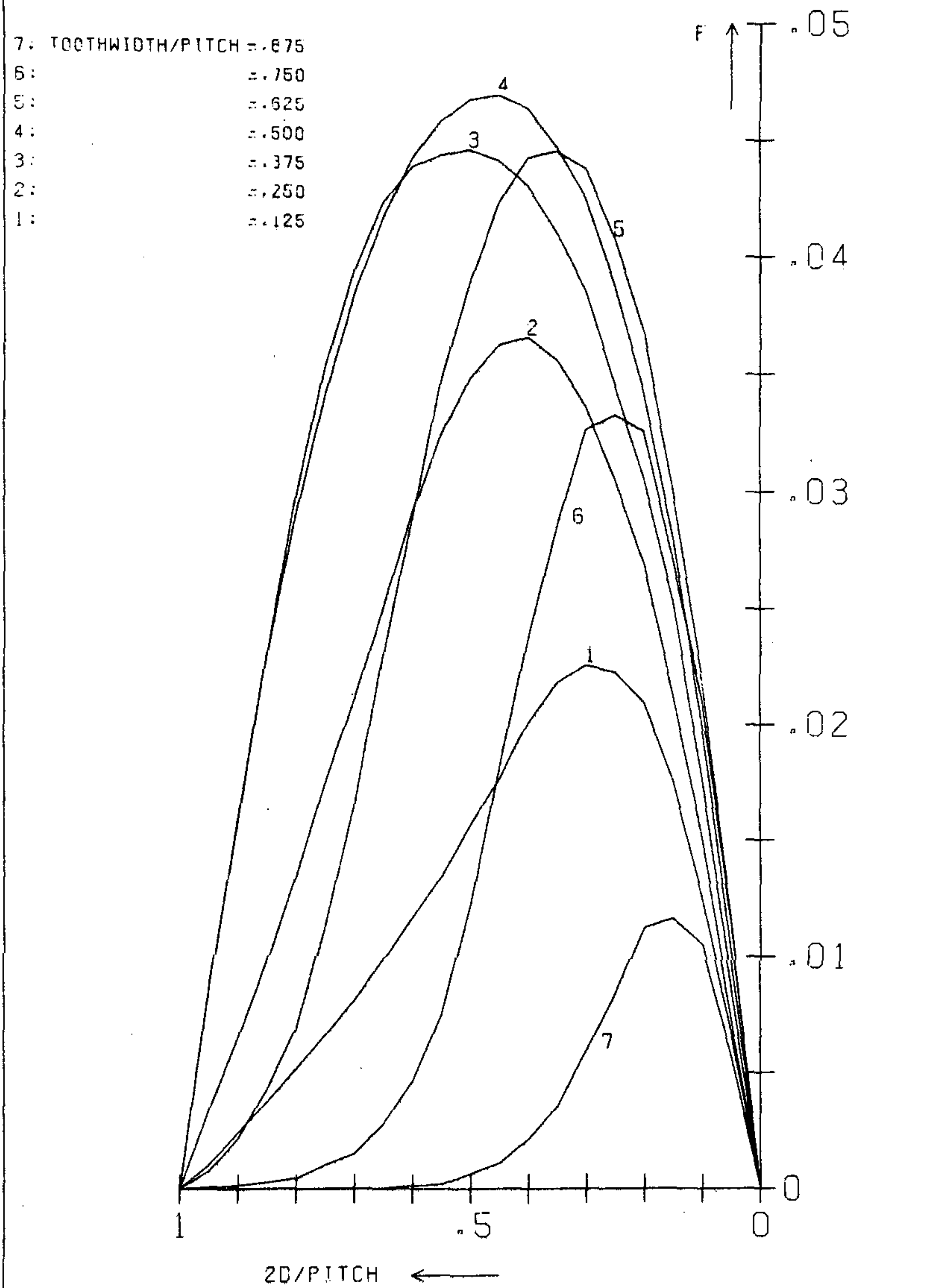


Fig. 7 PITCH/G=6.05

MAGNETIC FORCE (F) ON DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M, PITCH = 1M, NOMINAL MAGNETIC FIELD INTENSITY = 1A/M.

***** PITCH/G = 5 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE	FORCE
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0024	0.0100	0.0137	0.0143	0.0135	0.0120	0.0091
0.20	0.0033	0.0165	0.0242	0.0259	0.0251	0.0228	0.0167
0.30	0.0027	0.0181	0.0305	0.0343	0.0337	0.0296	0.0200
0.40	0.0016	0.0153	0.0318	0.0388	0.0390	0.0337	0.0204
0.50	0.0008	0.0106	0.0290	0.0398	0.0410	0.0329	0.0180
0.60	0.0005	0.0062	0.0229	0.0374	0.0398	0.0297	0.0148
0.70	0.0002	0.0032	0.0157	0.0320	0.0348	0.0232	0.0111
0.80	0.0001	0.0015	0.0089	0.0235	0.0261	0.0160	0.0073
0.90	0.0000	0.0006	0.0039	0.0125	0.0139	0.0081	0.0037
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 7.

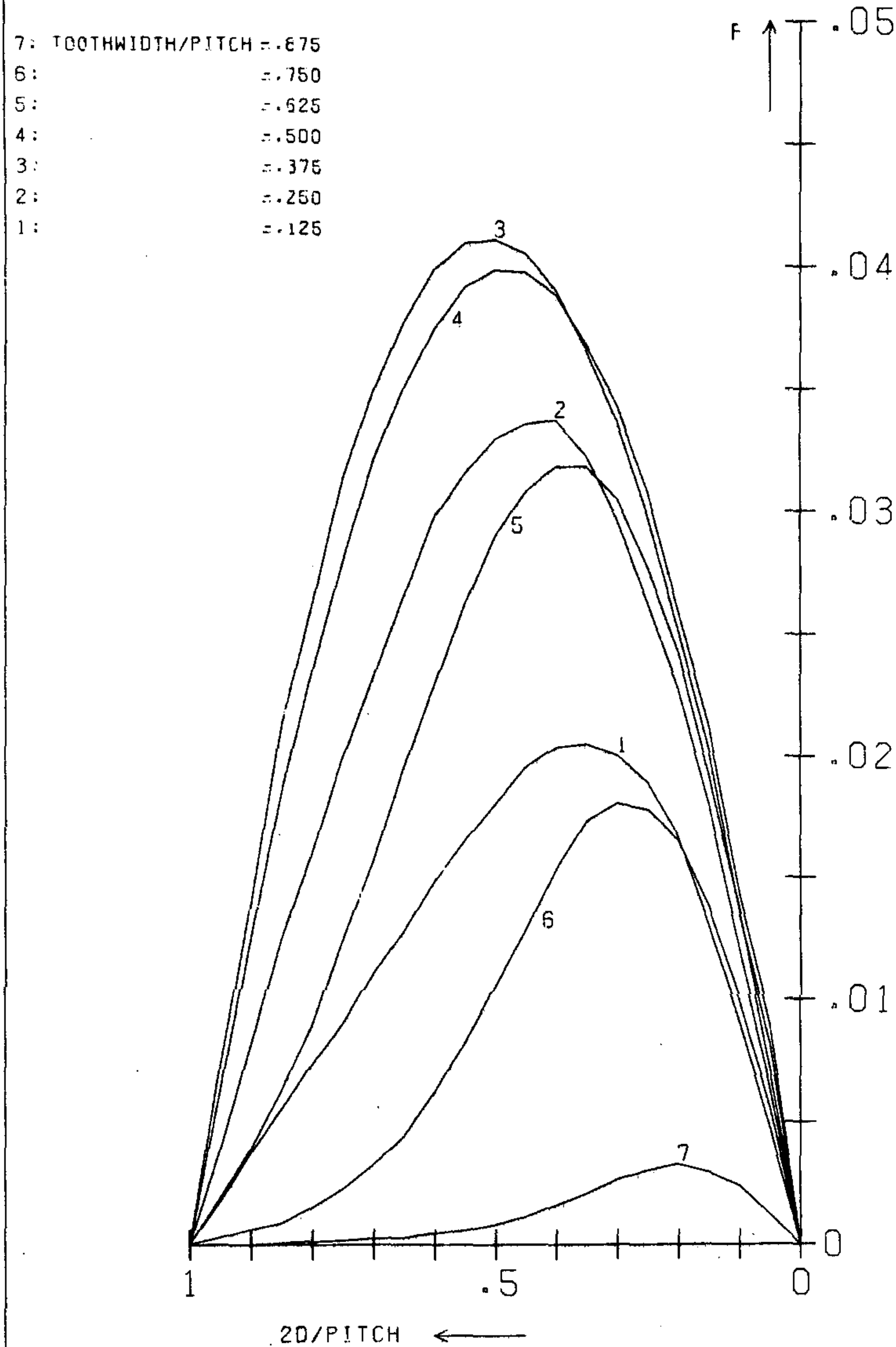


Fig. 8 PITCH/G=5

PERMEANCE OF DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M, PITCH = 1M, MAGNETIC POTENTIAL = 1A.

***** PITCH/G = 40 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.
0.00	36.77	32.22	27.64	22.84	17.99	13.14	8.27
0.10	36.11	31.39	26.74	21.90	17.06	12.18	7.32
0.20	35.26	30.05	25.28	20.38	15.53	10.63	5.76
0.30	35.06	28.67	23.72	18.77	13.87	8.97	4.26
0.40	35.06	27.45	22.13	17.08	12.15	7.22	3.52
0.50	35.06	26.84	20.61	15.43	10.44	5.49	3.20
0.60	35.06	26.82	19.20	13.75	8.69	4.28	2.97
0.70	35.06	26.82	18.18	12.16	6.98	3.81	2.87
0.80	35.06	26.82	18.01	10.63	5.42	3.55	2.76
0.90	35.06	26.82	18.01	9.38	4.82	3.45	2.75
1.00	35.06	26.82	18.01	8.81	4.65	3.39	2.74

Table 8.

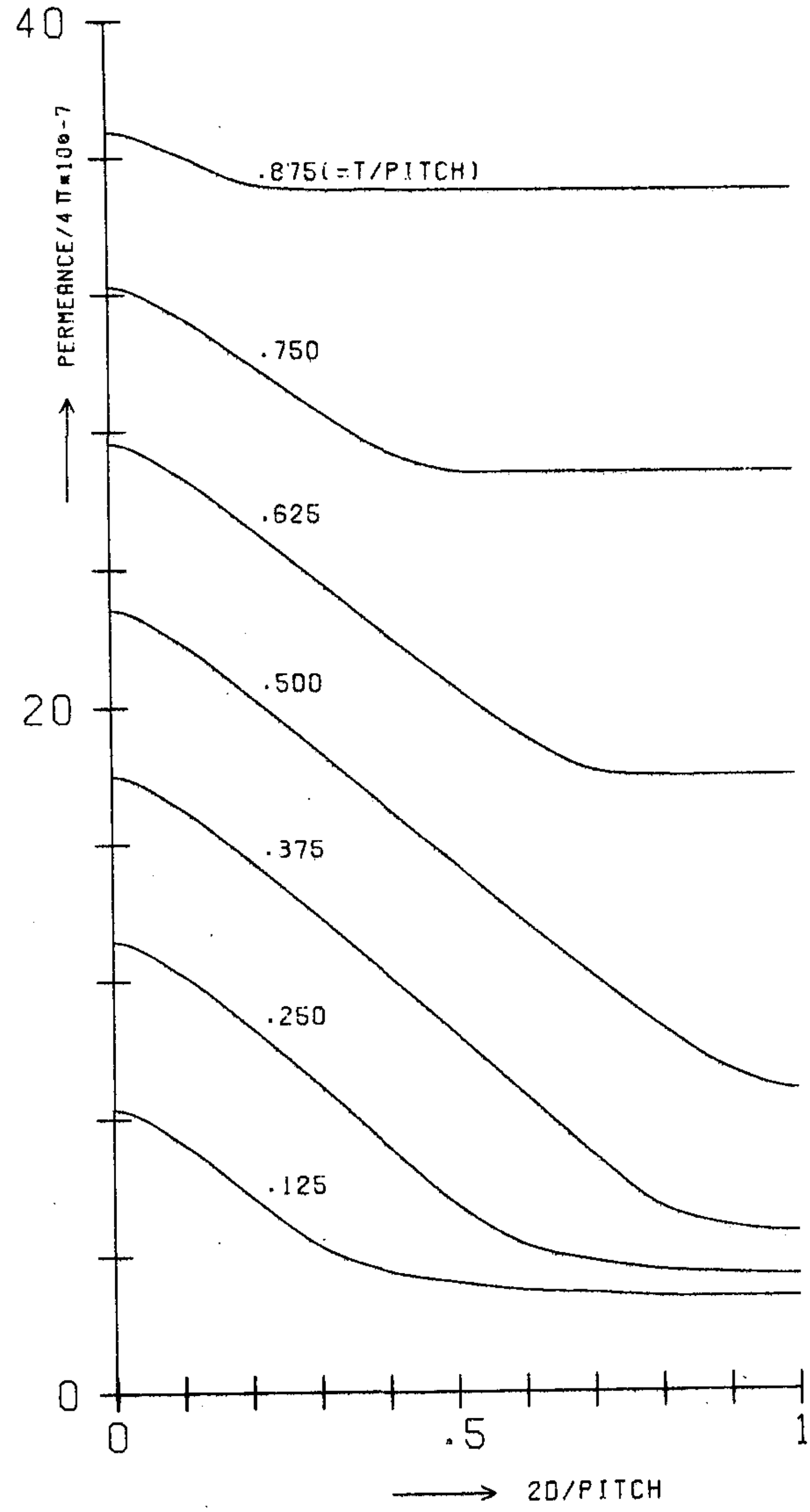


Fig. 9 PITCH/G=40

PERMEANCE OF DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M, PITCH = 1M, MAGNETIC POTENTIAL = 1A.

***** PITCH/G = 20 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.
0.00	18.83	16.72	14.53	12.22	9.87	7.52	5.15
0.10	18.70	16.51	14.28	11.95	9.61	7.24	4.87
0.20	18.51	16.07	13.76	11.39	9.03	6.66	4.28
0.30	18.46	15.62	13.17	10.75	8.37	5.97	3.68
0.40	18.46	15.23	12.55	10.05	7.63	5.22	3.23
0.50	18.46	15.07	11.98	9.37	6.91	4.49	3.00
0.60	18.45	15.04	11.47	8.67	6.14	3.88	2.81
0.70	18.45	15.04	11.16	8.05	5.43	3.56	2.74
0.80	18.45	15.04	11.06	7.45	4.77	3.35	2.64
0.90	18.45	15.04	11.07	7.04	4.43	3.27	2.63
1.00	18.45	15.04	11.06	6.87	4.31	3.22	2.60

Table 9.

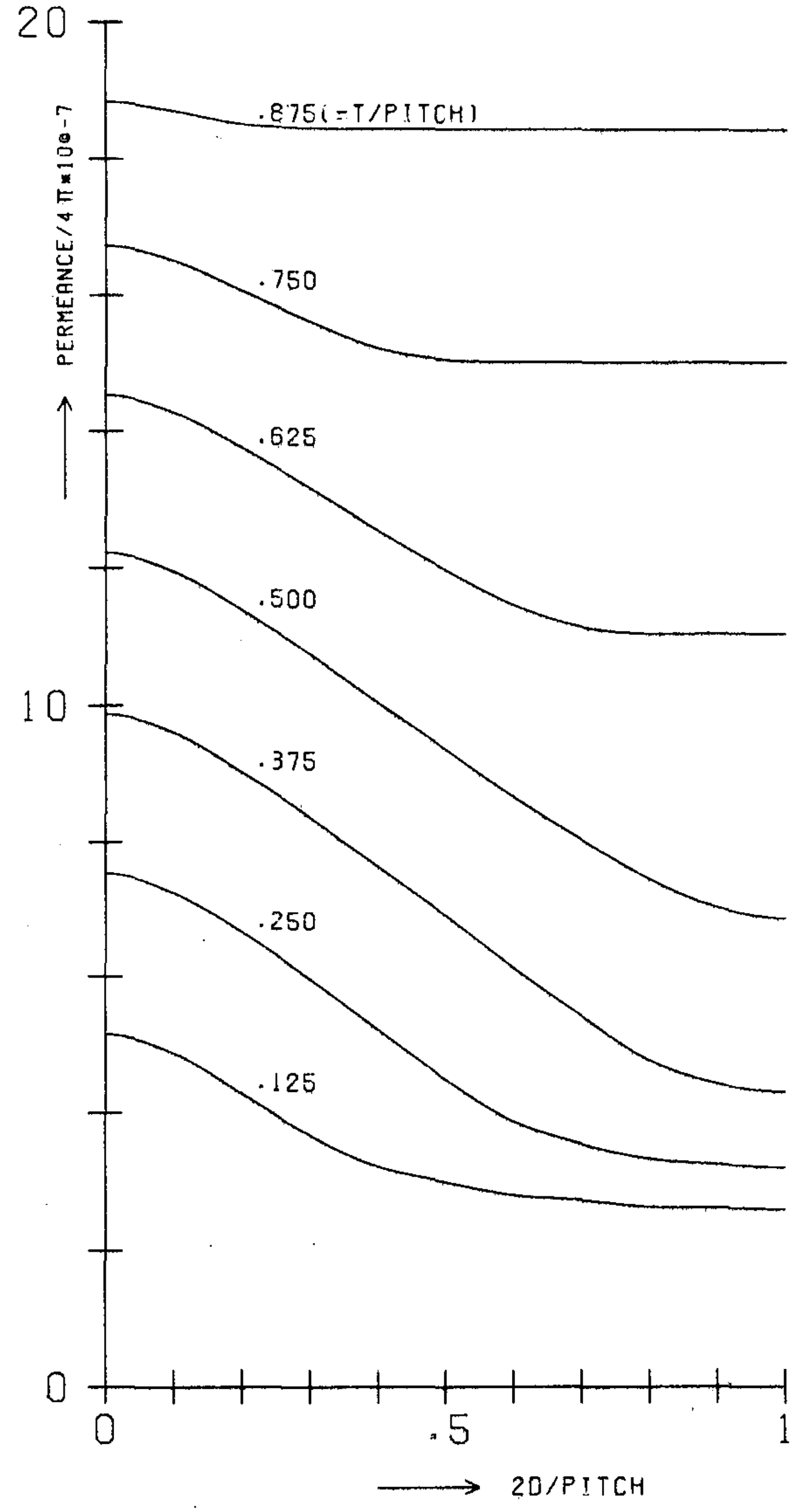


Fig. 10 PITCH/G=20

PERMEANCE OF DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M, PITCH = 1M, MAGNETIC POTENTIAL = 1A.

***** PITCH/G = 10 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.
0.00	9.60	8.77	7.77	6.71	5.61	4.51	3.39
0.10	9.59	8.73	7.72	6.65	5.55	4.44	3.32
0.20	9.57	8.65	7.59	6.50	5.39	4.27	3.16
0.30	9.55	8.54	7.43	6.30	5.17	4.05	2.96
0.40	9.55	8.46	7.25	6.07	4.92	3.78	2.77
0.50	9.55	8.41	7.09	5.85	4.66	3.52	2.64
0.60	9.55	8.40	6.95	5.61	4.38	3.28	2.53
0.70	9.55	8.39	6.87	5.41	4.14	3.12	2.47
0.80	9.55	8.39	6.83	5.23	3.91	3.00	2.41
0.90	9.55	8.39	6.82	5.12	3.78	2.95	2.39
1.00	9.55	8.39	6.82	5.08	3.72	2.92	2.38

Table 10.

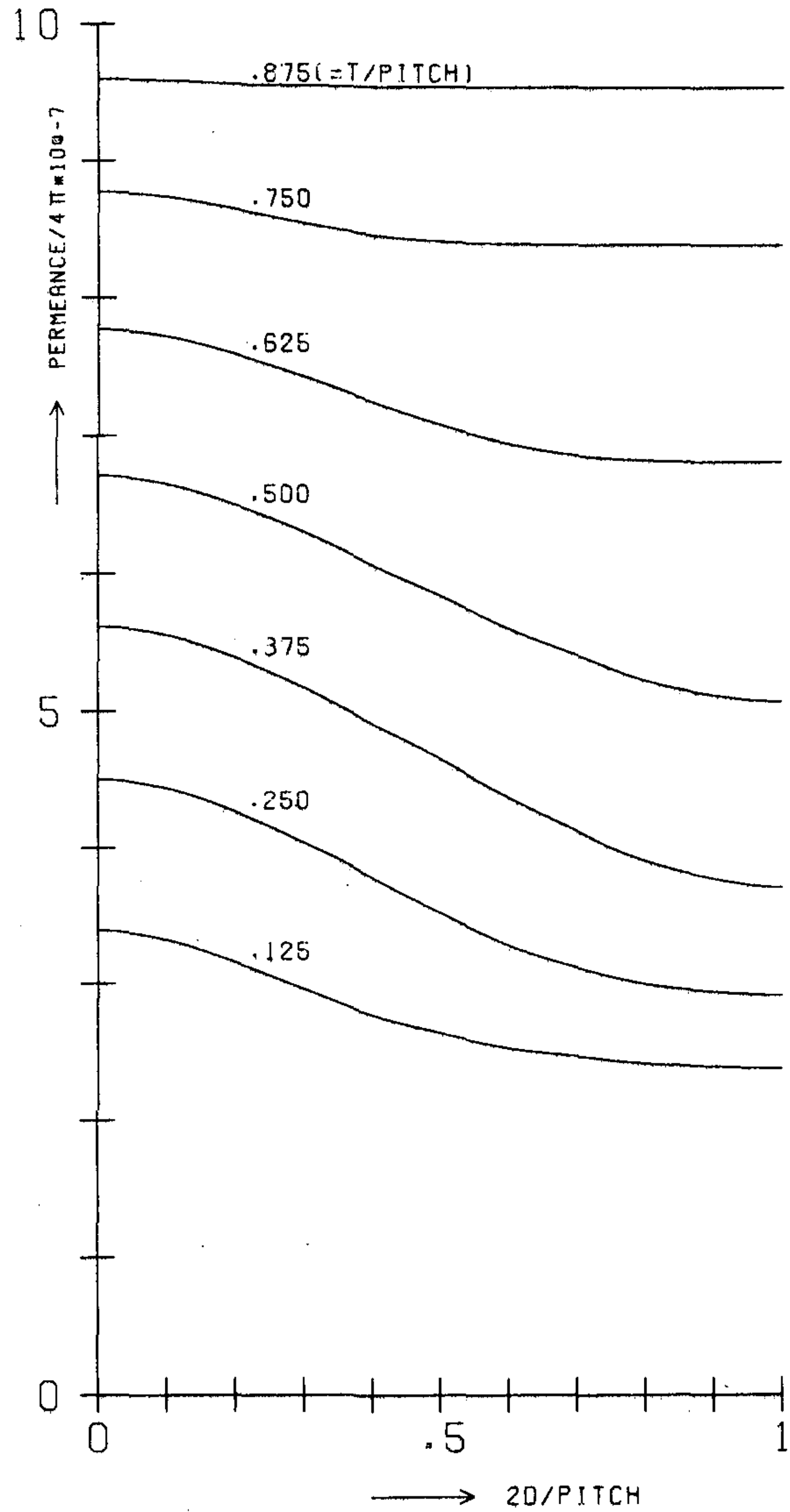


Fig. 11 PITCH/G=10

PERMEANCE OF DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M, PITCH = 1M, MAGNETIC POTENTIAL = 1A.

***** PITCH/G = 8.05 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.
0.00	7.73	7.13	6.37	5.56	4.71	3.85	2.99
0.10	7.73	7.11	6.34	5.52	4.67	3.82	2.95
0.20	7.71	7.06	6.27	5.43	4.57	3.71	2.85
0.30	7.71	7.01	6.17	5.31	4.43	3.56	2.72
0.40	7.71	6.96	6.06	5.16	4.26	3.38	2.59
0.50	7.70	6.93	5.96	5.01	4.09	3.21	2.50
0.60	7.70	6.92	5.88	4.86	3.91	3.05	2.41
0.70	7.70	6.92	5.82	4.73	3.74	2.93	2.37
0.80	7.70	6.92	5.79	4.62	3.59	2.84	2.32
0.90	7.70	6.92	5.78	4.55	3.51	2.80	2.29
1.00	7.70	6.92	5.78	4.54	3.46	2.78	2.28

Table 11.

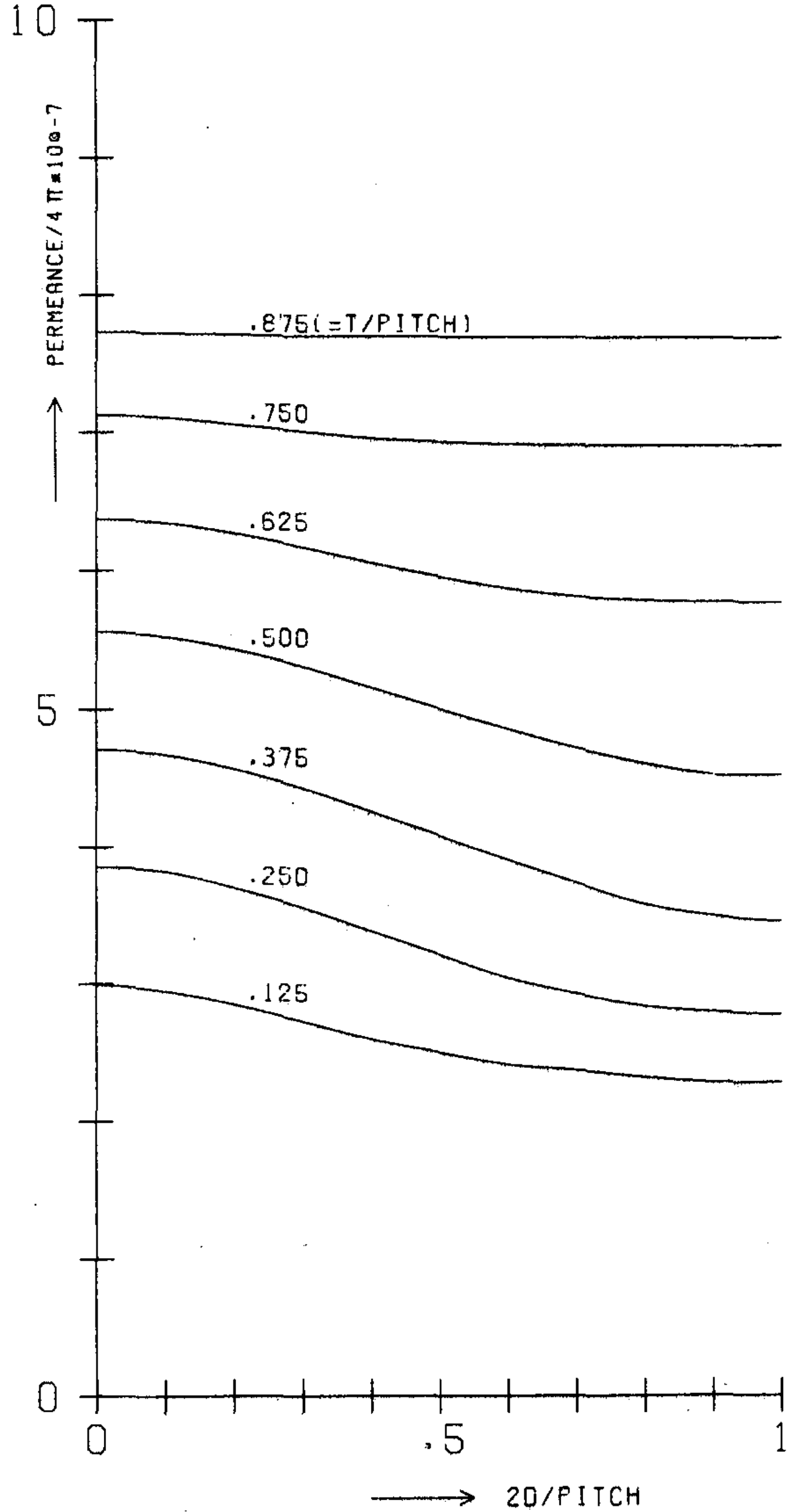


Fig. 12 PITCH/G=6.05

PERMEANCE OF DOUBLY SLOTTED STRUCTURES.

LENGTH = 1M. PITCH = 1M. MAGNETIC POTENTIAL = 1A.

***** PITCH/G = 5 *****

T/PITCH	0.875	0.750	0.625	0.500	0.375	0.250	0.125
2D/PITCH	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.	PERM.
0.00	4.90	4.60	4.21	3.77	3.29	2.80	2.32
0.10	4.90	4.60	4.20	3.74	3.28	2.79	2.30
0.20	4.90	4.59	4.18	3.73	3.25	2.76	2.28
0.30	4.89	4.58	4.16	3.69	3.20	2.71	2.23
0.40	4.89	4.57	4.13	3.64	3.14	2.64	2.18
0.50	4.89	4.56	4.10	3.60	3.08	2.59	2.14
0.60	4.89	4.55	4.08	3.55	3.02	2.51	2.10
0.70	4.89	4.55	4.06	3.50	2.96	2.47	2.07
0.80	4.89	4.55	4.05	3.47	2.91	2.42	2.05
0.90	4.89	4.55	4.04	3.45	2.88	2.41	2.04
1.00	4.89	4.55	4.04	3.44	2.87	2.39	2.03

Table 12.

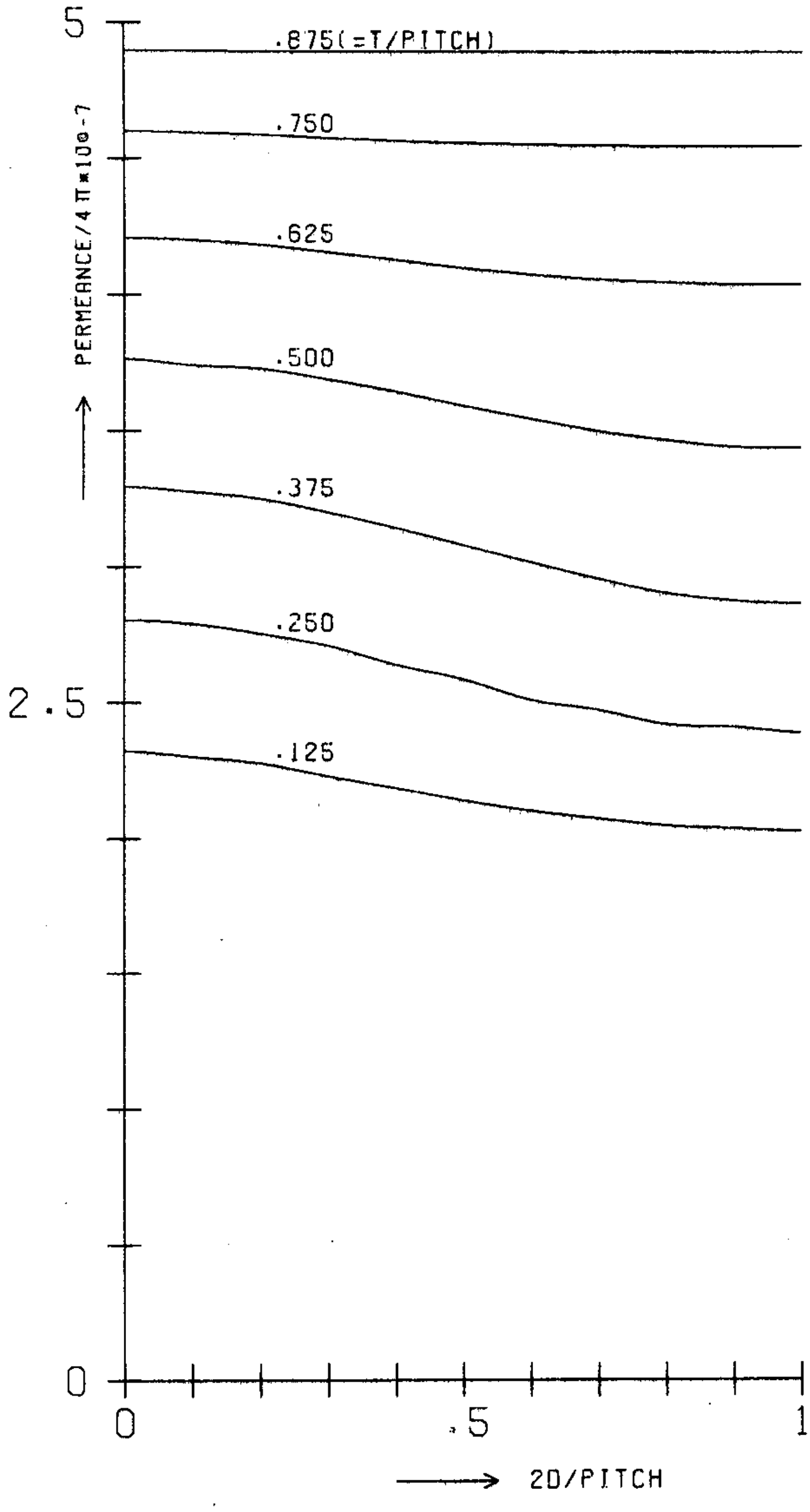


Fig. 13 PITCH/G=5

If one is only interested in the average force the following simple approach may be used, thus omitting expensive numerical computations

From the general expression for the force, see ref. [1]

$$F = \frac{\partial W'_m}{\partial x} \tag{18}$$

we may express the average force as

$$\overline{F} = \frac{W'_m(o) - W'_m(-X)}{X} \tag{19}$$

where X is the displacement between the position of minimum permeance to the next position of maximum permeance and $W'_m(\ )$ is the magnetic co-energy in the relevant extreme positions, keeping excitation at a constant value.

Now P(0) = permeance per toothpitch per meter toothlength for position 0, tooth facing tooth in Wb/A.

P(-X) = permeance for position -X, tooth facing slot.

Mukherji and Neville [3], present expressions and values for the permeances P_1 and P_2. It must be noted that these authors for simplicity omit the factor μ_0, which should be replaced for actual calculations.

To find the relevant values of P(0) and P(-X) that apply to our models it will be evident that for one set of teeth with length l_t

$$P(0) = \mu_0 . l_t . P_1$$
$$P(-X) = \mu_0 . l_t . P_2$$

Applying quantities as defined above, in reference [5] an expression is derived for the average torque based on (18), for a stepping motor. In our case this becomes

$$\overline{F} = \mu_0 H^2{}_{max} . \frac{P_1 - P_2}{\lambda^2/g^2} . l_t . \lambda . z \tag{20}$$

Here H_{max} is the fieldstrength that would appear in the homo-

geneous part of the airgap field between teeth.
Inspection of expression (20) suggests the introduction of a factor of merit of the tooth configuration

$$f_1 = \frac{P_1 - P_2}{\lambda^2/g^2} \qquad (21)$$

This figure should be as large as possible if the maximum average force is to be derived from a given slotted structure. The calculated values of f_1 are shown in table 13 and in figure 14. The maximum value is reached for λ/g = 8.05 and is equal to 0.0193.

λ/g	$10^{-3} \times (P_1 - P_2)/(\lambda/g)^2$ for t/λ =						
	1/8	1/4	3/8	1/2	5/8	3/4	7/8
5	11.55	16.41	16.98	12.89	6.69	2.09	0.10
8.05	10.90	16.57	19.24	15.67	9.11	3.26	0.37
10	10.08	15.87	18.97	16.30	9.53	3.76	0.50
20	6.38	10.74	13.91	13.37	8.68	4.19	0.95
40	3.48	6.09	8.34	8.77	6.02	3.37	1.06

Table 13.

An illustration of the optimisation is given in figure 15, showing the hatched area that represents the difference of co-energy between the two extreme positions. The hatched area should be as large as possible, with a given excitation.

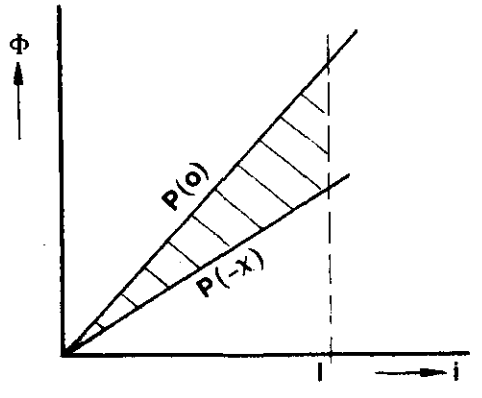


Fig. 15 Permeance diagram for airgap.

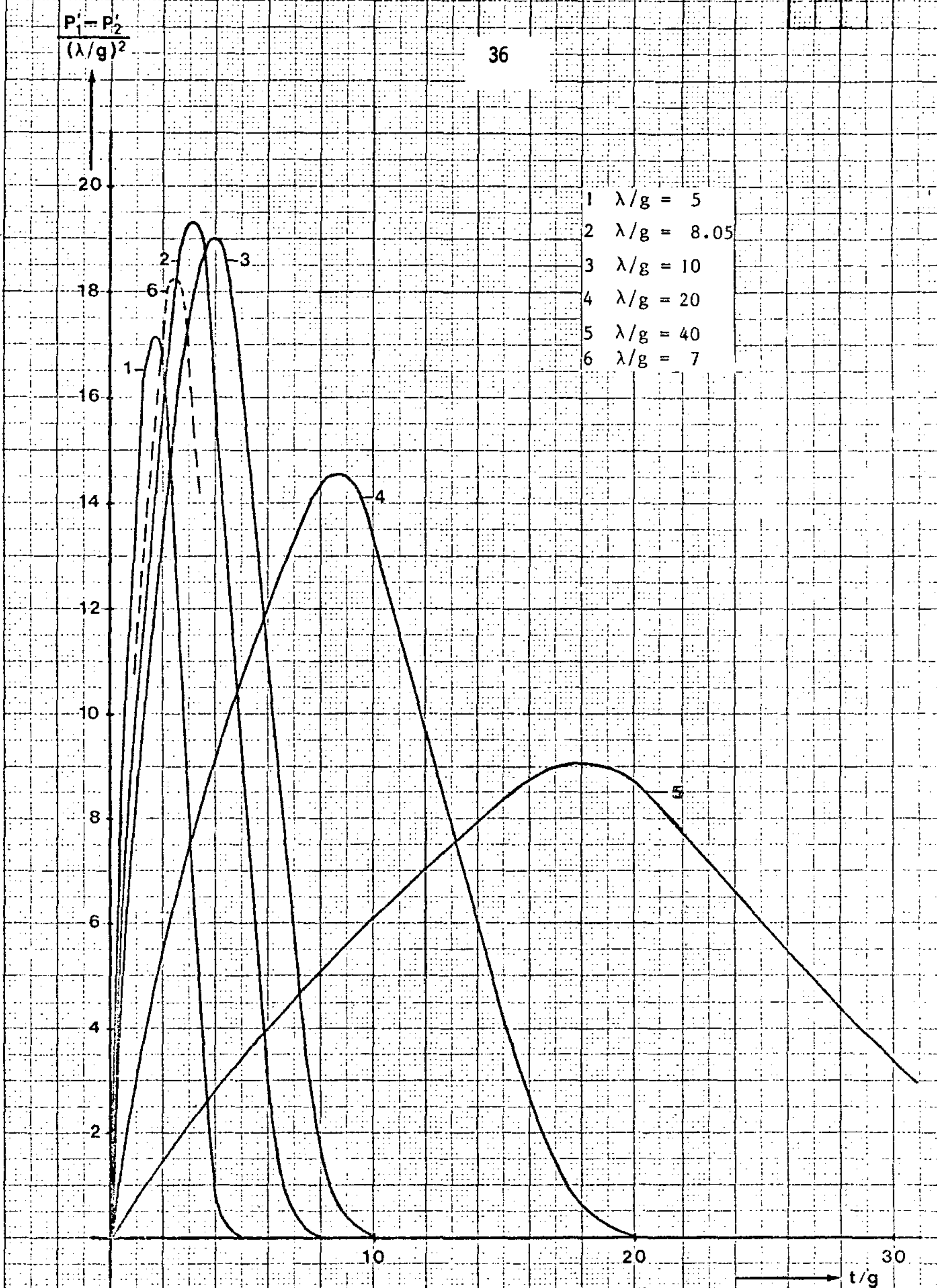


Fig.14 Factor of Merit

The elevation of the permeance lines is determined by P(0) and P(-X) respectively. Values from (21) that make f_1 maximum will optimise said area for a fixed excitation. Idealisation is only valid if the iron in the teeth and the rest of the structure is exploited far below saturation. This means that the weight and volume of the iron is not used optimally. A limitation bears on the allowed value of H_{max}, because the latter is a measure for airgap induction, which in turn determines the iron induction. In search of better force production the previous idealisation will be dropped.

1.2.4. Influence of properties of iron, excluding saturation of the teeth.

In this case W'_m contains contributions from the airgap volume as well as from the iron parts of the magnetic circuit. The total m.m.f. can be thought to be divided into two parts that follow from the line-integral of the field intensity.

$$NI = \oint \underline{H}.\underline{dl} = \int \underline{H}_{gap}.\underline{dl} + \int \underline{H}_{iron}.\underline{dl} = \Theta_{air} + \Theta_{iron} \quad (22)$$

This division is shown in figure 16.

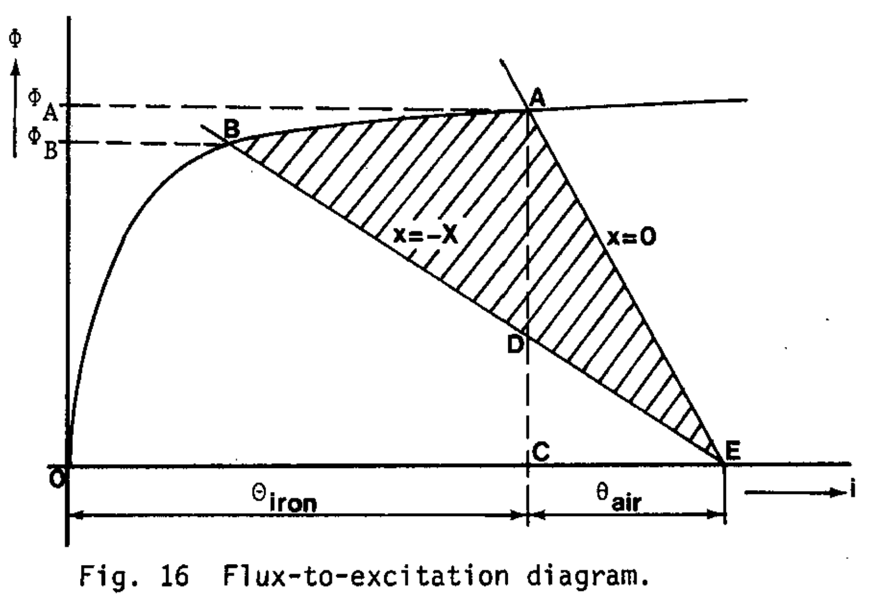


Fig. 16 Flux-to-excitation diagram.

It shows the curve for the Θ_{iron} depending on the total magnetic flux, and the lines pertaining to $\Theta_{air} = NI - \Theta_{iron}$ are also drawn. The latter emerge from point E, which indicates the value of the total available excitation NI.
Line EA illustrates the required excitation for the airgaps in the case of teeth in alignment; line EB applies to the other extreme position. The intersections of the airgap lines with the curve for the iron part, points A and B respectively, show the fluxes ϕ_A or ϕ_B that will appear in either of the mentioned positions. The perpendiculars through A or B show the relevant division of the total excitation into the main portions, as required by eqn. (22).
The area included between the horizontal axis, the iron curve and the airgap line appears to be the magnetic co-energy for the complete slotted structure assembly. For instance, in position x = 0, area EAC indicates the contribution of the airgap space to W'_m and area OBAC that of the iron parts. Triangle EAB indicates the difference of the co-energies of the two extreme positions. Its area is therefore a measure for the work performed during the motion from position x = -X to x = 0. It is also a measure for the average force.
To ascertain the exact shape of the force curve applicable to iron that is considerably saturated an expensive numerical computation would be needed; instead the following approximation is suggested.
The force curves for a slotting with idealised iron have been calculated in paragraph 1.2.3. Once the appropriate curve of the idealised iron is available, it is most interesting to learn how its shape is altered by observing real iron properties.
This may be accomplished by the use of a corollary of the basic expression for the force which can be applied in conjunction with figure 17.

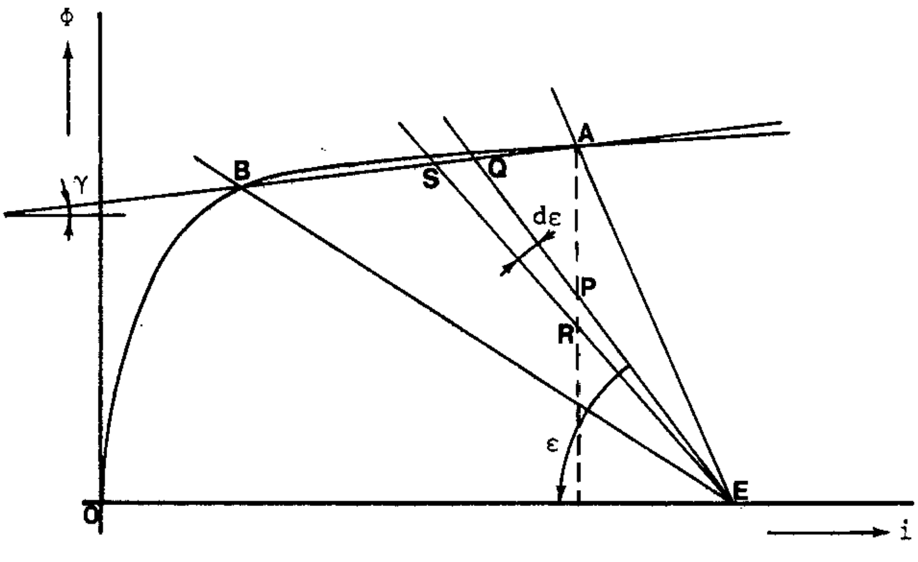


Fig. 17 Flux-to-excitation diagram.
Determination of actual force curve.

$$F(x) = \frac{dW'_m}{d\varepsilon} \cdot \frac{d\varepsilon}{dx} \quad \text{(constant excitation)} \tag{23}$$

This expression can be applied to both the ideal (triangle EPR) and the actual iron (triangle EQS). The ratio of the values of the forces then appears to be:

$$\frac{F_{actual}(x)}{F_{ideal}(x)} = \left(\frac{QE(\varepsilon)}{PE(\varepsilon)}\right)^2 \tag{24}$$

if we accept that the curve between A en B is replaced by its chord.
If the relation $\varepsilon = \varepsilon(x)$ is available, the actual force can be derived from the ideal force curve by a simple geometrical operation. This procedure has been introduced and applied in ref. [6].
This procedure was also applied (using the calculated values of the ideal forces) to a measuring device (see chapter 2). Instead of the force the torque was calculated but the force can be found by dividing the torque by the average radius of the rotor and the stator of the device. The calculated values of the torques can be found in tables 14 - 18.

CALCULATED TNORM VALUES IN NM FOR LAMBDA/G = 40.00

TOOTHWIDTH T =0.125 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	4.9727	16.2074	21.3699	23.7221	25.3303	26.5662	27.5761	28.4304
0.20	5.9927	23.9757	37.3456	43.1491	46.7934	49.4788	51.6095	53.3983
0.30	4.0885	16.3575	35.5356	47.1571	53.4769	57.7082	60.8933	63.4658
0.40	1.7825	7.1316	16.0472	25.7309	31.2331	34.6302	37.0512	38.9300
0.50	0.9706	3.8833	8.7381	15.0182	19.2650	21.8457	23.6229	24.9716
0.60	0.5970	2.3836	5.3671	9.5432	12.8945	14.9969	16.4039	17.4497
0.70	0.3719	1.4879	3.3480	5.9516	8.3052	9.7913	10.7770	11.5315
0.80	0.2167	0.8681	1.9530	3.4714	5.0121	6.0188	6.6823	7.1632
0.90	0.1026	0.4111	0.9247	1.6435	2.3825	2.8682	3.1880	3.4197
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.250 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	4.7066	7.4659	8.4503	9.0804	9.5504	9.9292	10.2464	10.5201
0.20	5.6716	11.2263	12.9121	13.9506	14.7119	15.3180	15.8271	16.2665
0.30	6.0779	15.6409	18.5565	20.2385	21.4454	22.3907	23.1707	23.8418
0.40	6.3262	21.7290	27.8101	30.9176	33.0363	34.6590	35.9789	37.1051
0.50	6.2116	24.8360	41.3213	48.3592	52.6796	55.8269	58.3143	60.3817
0.60	2.6723	10.6912	23.1223	30.5385	34.5754	37.2852	39.3341	40.9641
0.70	1.2466	4.9967	11.2380	16.5709	19.4229	21.2423	22.5730	23.6266
0.80	0.6421	2.5691	5.7808	9.1895	11.1042	12.2914	13.1393	13.7999
0.90	0.2779	1.1138	2.5050	4.0868	5.0135	5.5822	5.9844	6.2955
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.375 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	3.2386	4.1811	4.6292	4.9338	5.1654	5.3549	5.5151	5.6577
0.20	4.4034	5.8905	6.5577	7.0019	7.3399	7.6131	7.8461	8.0510
0.30	5.3431	7.6020	8.5296	9.1317	9.5872	9.9589	10.2680	10.5415
0.40	5.9218	9.8508	11.1849	12.0319	12.6595	13.1662	13.5903	13.9586
0.50	6.0576	12.9065	14.9498	16.1853	17.0911	17.8087	18.4060	18.9219
0.60	6.1597	17.3348	20.8793	22.8676	24.2709	25.3720	26.2768	27.0502
0.70	6.2055	22.7177	30.1101	33.7353	36.1643	38.0014	39.4973	40.7560
0.80	3.9002	15.6039	27.4067	32.4729	35.5293	37.7269	39.4544	40.8851
0.90	1.2402	4.9515	9.9562	12.4150	13.8066	14.7736	15.5171	16.1255
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.500 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	2.1168	2.5794	2.8271	2.9990	3.1334	3.2439	3.3378	3.4194
0.20	2.8207	3.4819	3.8248	4.0617	4.2456	4.3978	4.5249	4.6386
0.30	3.3782	4.2465	4.6812	4.9782	5.2093	5.3955	5.5560	5.6969
0.40	3.9920	5.1523	5.7051	6.0782	6.3638	6.5976	6.7951	6.9683
0.50	4.6148	6.1939	6.8991	7.3661	7.7222	8.0136	8.2572	8.4714
0.60	5.2609	7.5358	8.4568	9.0627	9.5166	9.8821	10.1924	10.4612
0.70	5.4008	8.9842	10.2010	10.9734	11.5458	12.0080	12.3948	12.7307
0.80	4.9641	10.3053	11.1904	12.8768	13.5888	14.1551	14.6274	15.0336
0.90	3.6960	9.2814	10.9694	11.9490	12.6533	13.2058	13.6678	14.0613
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 14a.

TOOTHWIDTH T =0.625 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	1.4215	1.7046	1.8577	1.9660	2.0498	2.1208	2.1809	2.2336
0.20	1.8392	2.2186	2.4208	2.5637	2.6752	2.7677	2.8468	2.9152
0.30	2.1263	2.5858	2.8273	2.9961	3.1272	3.2359	3.3300	3.4107
0.40	2.3837	2.9276	3.2061	3.4034	3.5537	3.6785	3.7861	3.8796
0.50	2.5569	3.1791	3.4902	3.7069	3.8746	4.0111	4.1283	4.2313
0.60	2.4674	3.1127	3.4266	3.6442	3.8119	3.9478	4.0643	4.1661
0.70	1.3288	1.6993	1.8756	1.9963	2.0888	2.1639	2.2291	2.2848
0.80	0.0131	0.0168	0.0186	0.0198	0.0207	0.0214	0.0221	0.0226
0.90	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.750 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.9926	1.1734	1.2740	1.3468	1.4037	1.4501	1.4908	1.5266
0.20	1.2208	1.4487	1.5746	1.6643	1.7354	1.7928	1.8435	1.8884
0.30	1.2832	1.5279	1.6630	1.7584	1.8338	1.8955	1.9492	1.9958
0.40	1.0819	1.2936	1.4090	1.4903	1.5545	1.6077	1.6524	1.6923
0.50	0.1537	0.1842	0.2007	0.2124	0.2215	0.2291	0.2356	0.2413
0.60	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.875 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.5819	0.6749	0.7316	0.7722	0.8042	0.8306	0.8539	0.8810
0.20	0.3854	0.4493	0.4866	0.5138	0.5351	0.5532	0.5683	0.5799
0.30	0.0054	0.0063	0.0069	0.0073	0.0076	0.0078	0.0080	0.0082
0.40	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.60	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 14b.

CALCULATED TNORM VALUES IN NM FOR LAMBDA/G = 20.00

TOOTHWIDTH T =0.125 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	1.7254	6.8972	13.3009	16.2696	17.9854	19.1913	20.1286	20.8946
0.20	2.3678	9.4732	20.4880	27.0592	30.6362	33.0374	34.8527	36.3148
0.30	2.0151	8.0486	18.1230	27.7771	33.0120	36.3038	38.6841	40.5560
0.40	1.2284	4.9148	11.0590	18.8967	24.0897	27.2444	29.4258	31.0841
0.50	0.7514	3.0026	6.7600	12.0116	16.0539	18.5955	20.3032	21.5764
0.60	0.4852	1.9413	4.3682	7.7659	11.0372	13.1299	14.5123	15.5216
0.70	0.3119	1.2499	2.8117	4.9970	7.2654	8.7615	9.7474	10.4605
0.80	0.1864	0.7449	1.6770	2.9799	4.4392	5.4564	6.1265	6.6064
0.90	0.0914	0.3651	0.8220	1.4604	2.1826	2.6899	3.0247	3.2638
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.250 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	1.6811	5.8017	7.4301	8.1896	8.7485	9.1790	9.5275	9.8246
0.20	2.3461	8.7880	11.8432	13.2029	14.1688	14.9015	15.4913	15.9929
0.30	2.6253	10.4945	15.6509	17.7989	19.2466	20.3179	21.1760	21.8942
0.40	2.7651	11.0535	19.8155	23.4124	25.6496	27.2552	28.5116	29.5528
0.50	2.5976	10.3838	21.7914	27.7434	31.1014	33.3929	35.1452	36.5681
0.60	1.6638	6.6565	14.9761	21.6428	25.2072	27.4996	29.1833	30.5207
0.70	0.9282	3.7135	8.3559	13.2454	15.9841	17.6817	18.8958	19.8430
0.80	0.5113	2.0429	4.6001	7.6743	9.5695	10.7239	11.5330	12.1533
0.90	0.2297	0.9189	2.0676	3.5046	4.4338	4.9985	5.3898	5.6893
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.375 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	1.6033	3.9193	4.5700	4.9708	5.2596	5.4885	5.6774	5.8397
0.20	2.2343	5.9825	7.0753	7.7261	8.1909	8.5545	8.8562	9.1155
0.30	2.4863	7.4252	8.9791	9.8635	10.4840	10.9668	11.3662	11.7045
0.40	2.6233	8.8008	11.0371	12.2332	13.0508	13.6815	14.1988	14.6373
0.50	2.6797	9.9778	13.2499	14.8693	15.9506	16.7662	17.4305	17.9886
0.60	2.7035	10.8163	15.9944	18.3243	19.8111	20.9157	21.7973	22.5378
0.70	2.6106	10.4445	18.3304	21.7139	23.7555	25.2257	26.3813	27.3385
0.80	1.9028	7.6128	15.4473	19.3773	21.5911	23.1215	24.2960	25.2560
0.90	0.8081	3.2278	6.9677	9.1647	10.3633	11.1695	11.7800	12.2715
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.500 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	1.5281	2.6212	2.9816	3.2094	3.3779	3.5139	3.6272	3.7259
0.20	2.1070	3.9170	4.4810	4.8325	5.0915	5.2984	5.4730	5.6223
0.30	2.3255	4.7338	5.4587	5.9020	6.2271	6.4846	6.7017	6.8882
0.40	2.4300	5.5023	6.4147	6.9591	7.3540	7.6678	7.9286	8.1519
0.50	2.4302	6.1077	7.2198	7.8642	8.3275	8.6929	8.9948	9.2537
0.60	2.3708	6.7023	8.0806	8.8519	9.3955	9.8225	10.1738	10.4745
0.70	2.2011	6.8690	8.4855	9.3549	9.9594	10.4271	10.8132	11.1395
0.80	1.8747	6.4174	8.2001	9.1136	9.7362	10.2140	10.6022	10.9331
0.90	1.1735	4.2643	5.6252	6.2977	6.7471	7.0897	7.3670	7.6022
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 15a.

TOOTHWIDTH T =0.625 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNCRM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	1.2747	1.7816	1.9946	2.1341	2.2397	2.3251	2.3977	2.4609
0.20	1.8033	2.5806	2.8969	3.1029	3.2585	3.3837	3.4901	3.5822
0.30	2.0519	2.9966	3.3765	3.6219	3.8056	3.9543	4.0796	4.1877
0.40	2.0497	3.2367	3.6632	3.9357	4.1386	4.3031	4.4409	4.5598
0.50	1.8738	3.2027	3.6428	3.9204	4.1266	4.2918	4.4305	4.5513
0.60	1.4601	2.6821	3.0662	3.3054	3.4828	3.6243	3.7425	3.8453
0.70	0.6446	1.2371	1.4189	1.5314	1.6145	1.6806	1.7362	1.7840
0.80	0.0639	0.1244	0.1429	0.1543	0.1627	0.1694	0.1749	0.1798
0.90	0.0034	0.0067	0.0077	0.0083	0.0088	0.0091	0.0094	0.0097
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.750 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNCRM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.8857	1.1668	1.2938	1.3795	1.4451	1.4987	1.5439	1.5831
0.20	1.1910	1.5834	1.7593	1.8766	1.9665	2.0392	2.1014	2.1558
0.30	1.1837	1.5928	1.7720	1.8920	1.9830	2.0572	2.1198	2.1746
0.40	0.8491	1.1550	1.2875	1.3751	1.4418	1.4961	1.5421	1.5822
0.50	0.2188	0.2993	0.3337	0.3566	0.3740	0.3880	0.4000	0.4104
0.60	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.875 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNCRM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.3985	0.5058	0.5576	0.5928	0.6203	0.6426	0.6617	0.6784
0.20	0.2725	0.3568	0.3826	0.4069	0.4257	0.4411	0.4542	0.4656
0.30	0.0333	0.0424	0.0467	0.0497	0.0520	0.0539	0.0555	0.0569
0.40	0.0016	0.0021	0.0023	0.0025	0.0026	0.0027	0.0027	0.0028
0.50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.60	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 15b.

CALCULATED TNORM VALUES IN NM FOR LAMBDA/G = 10.00

TOOTHWIDTH T =0.125 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.4456	1.7790	4.0057	6.7506	8.4215	9.4562	10.1773	10.7309
0.20	0.7062	2.8253	6.3572	11.3137	14.2590	16.2338	17.5864	18.6100
0.30	0.7253	2.8959	6.5208	11.5968	15.7020	18.2753	19.9997	21.2790
0.40	0.5951	2.3856	5.3643	9.5336	13.7204	16.4450	18.2398	19.5455
0.50	0.4388	1.7520	3.9450	7.0089	10.4388	12.8271	14.4008	15.5262
0.60	0.3117	1.2471	2.8062	4.9890	7.5980	9.5665	10.8801	11.8064
0.70	0.2125	0.8502	1.9131	3.4012	5.3144	6.6956	7.6815	8.3742
0.80	0.1316	0.5274	1.1860	2.1079	3.2929	4.2398	4.9064	5.3730
0.90	0.0646	0.2590	0.5825	1.0353	1.6173	2.0979	2.4355	2.6716
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.250 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.4461	1.7811	3.7720	4.8365	5.4337	5.8396	6.1492	6.3998
0.20	0.7456	2.9830	6.4913	8.5581	9.6969	10.4606	11.0362	11.5003
0.30	0.9099	3.6330	8.1805	11.2129	12.8876	13.9847	14.8007	15.4529
0.40	0.9814	3.9185	8.8113	13.1189	15.4244	16.8915	17.9611	18.8055
0.50	0.9240	3.6969	8.3187	13.3113	16.1386	17.8852	19.1327	20.1010
0.60	0.7396	2.9591	6.6492	11.2517	14.1959	15.9840	17.2282	18.1764
0.70	0.5084	2.0382	4.5831	8.1452	10.4176	11.9057	12.9198	13.6846
0.80	0.3120	1.2457	2.8049	4.9833	6.6557	7.7066	8.4134	8.9406
0.90	0.1478	0.5912	1.3285	2.3627	3.2170	3.7522	4.1102	4.3753
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.375 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.4237	1.6916	2.9042	3.3913	3.7003	3.9246	4.1017	4.2482
0.20	0.7053	2.8218	5.0332	5.9366	6.5002	6.9045	7.2224	7.4856
0.30	0.8614	3.4390	6.4465	7.7246	8.5001	9.0511	9.4808	9.8349
0.40	0.9452	3.7895	7.5024	9.2039	10.2024	10.9007	11.4405	11.8823
0.50	0.9780	3.9130	8.1540	10.3132	11.5397	12.3803	13.0228	13.5446
0.60	0.9743	3.8979	8.4892	11.2046	12.7012	13.7018	14.4579	15.0667
0.70	0.8893	3.5506	7.9839	11.0343	12.7013	13.7908	14.6007	15.2471
0.80	0.6733	2.6940	6.0618	8.9799	10.5359	11.5287	12.2539	12.8268
0.90	0.3468	1.3846	3.1135	4.7859	5.6918	6.2605	6.6717	6.9946
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.500 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNCRM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.3891	1.4935	2.0391	2.3017	2.4744	2.6049	2.7100	2.7984
0.20	0.6413	2.5073	3.4867	3.9539	4.2575	4.4850	4.6685	4.8226
0.30	0.7716	3.0808	4.3874	5.0013	5.3969	5.6911	5.9280	6.1262
0.40	0.8313	3.3260	5.0121	5.7578	6.2321	6.5824	6.8616	7.0960
0.50	0.8332	0.3335	5.2912	6.1336	6.6595	7.0452	7.3522	7.6051
0.60	0.7979	3.1857	5.3559	6.2787	6.8443	7.2558	7.5803	7.8495
0.70	0.7000	2.8007	4.9346	5.8511	6.4034	6.8004	7.1130	7.3713
0.80	0.5480	2.1800	4.0084	4.8138	5.2901	5.6297	5.8943	6.1136
0.90	0.3036	1.2148	2.2862	2.7691	3.0511	3.2508	3.4066	3.5347
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 16a.

TOOTHWIDTH T =0.625 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.3317	1.0897	1.3689	1.5153	1.6162	1.6936	1.7574	1.8122
0.20	0.5343	1.7928	2.2695	2.5170	2.6860	2.8167	2.9226	3.0135
0.30	0.6208	2.1313	2.7268	3.0316	3.2395	3.3980	3.5281	3.6380
0.40	0.6215	2.1960	2.8468	3.1742	3.3960	3.5646	3.7023	3.8188
0.50	0.5484	1.9788	2.5997	2.9069	3.1140	3.2709	3.3986	3.5066
0.60	0.4021	1.4807	1.9695	2.2087	2.3681	2.4894	2.5873	2.6701
0.70	0.2107	0.7849	1.0524	1.1822	1.2686	1.3340	1.3867	1.4313
0.80	0.0701	0.2625	0.3533	0.3973	0.4264	0.4485	0.4662	0.4813
0.90	0.0165	0.0617	0.0831	0.0935	0.1004	0.1056	0.1098	0.1133
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.750 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.2327	0.6505	0.7826	0.8568	0.9092	0.9503	0.9843	1.0132
0.20	0.3480	0.9834	1.1863	1.2999	1.3801	1.4427	1.4944	1.5365
0.30	0.3391	0.9751	1.1806	1.2949	1.3754	1.4379	1.4898	1.5338
0.40	0.2342	0.6832	0.8297	0.9106	0.9675	1.0119	1.0485	1.0796
0.50	0.1017	0.2991	0.3636	0.3994	0.4245	0.4441	0.4601	0.4738
0.60	0.0286	0.0842	0.1025	0.1126	0.1197	0.1252	0.1296	0.1336
0.70	0.0065	0.0193	0.0235	0.0258	0.0274	0.0287	0.0298	0.0307
0.80	0.0015	0.0043	0.0052	0.0057	0.0061	0.0064	0.0066	0.0068
0.90	0.0003	0.3338	0.0010	0.0011	0.0011	0.0012	0.0012	0.0013
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.875 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0715	0.1740	0.2046	0.2227	0.2356	0.2456	0.2543	0.2616
0.20	0.0661	0.1615	0.1899	0.2067	0.2188	0.2283	0.2361	0.2429
0.30	0.0268	0.0655	0.0771	0.0839	0.0888	0.0927	0.0959	0.0986
0.40	0.0068	0.0167	0.0197	0.0214	0.0227	0.0237	0.0245	0.0252
0.50	0.0015	0.0037	0.0044	0.0048	0.0051	0.0053	0.0055	0.0056
0.60	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 16b.

CALCULATED TNORM VALUES IN NM FOR LAMBDA/G = 8.05

TOOTHWIDTH T =0.125 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.2613	1.0454	2.3490	4.1776	5.7157	6.6286	7.2603	7.7281
0.20	0.4378	1.7516	3.9413	7.0070	9.9024	11.6569	12.8528	13.7291
0.30	0.4719	1.8917	4.2537	7.5599	11.0713	13.3246	14.8383	15.9315
0.40	0.4213	1.6856	3.7878	6.7364	10.1723	12.5912	14.1230	15.3681
0.50	0.3286	1.3148	2.9564	5.2596	8.2183	10.2171	11.6680	12.6599
0.60	0.2445	0.9801	2.2039	3.9168	6.1189	7.8806	9.1214	9.9692
0.70	0.1702	0.6825	1.5347	2.7275	4.2643	5.5805	6.5080	7.1555
0.80	0.1079	0.4308	0.9688	1.7235	2.6920	3.5840	4.2174	4.6590
0.90	0.0527	0.2104	0.4737	0.8415	1.3154	1.7690	2.0949	2.3226
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.250 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.2670	1.0704	2.4066	3.5414	4.1418	4.5359	4.8196	5.0439
0.20	0.4760	1.9085	4.2973	6.5350	7.7432	8.5056	9.0581	9.4923
0.30	0.5975	2.3905	5.3790	8.5211	10.2785	11.3696	12.1497	12.7580
0.40	0.6527	2.6059	5.8677	9.7076	12.0293	13.4478	14.4436	15.2109
0.50	0.6220	2.4886	5.5998	9.5994	12.2803	13.9082	15.0309	15.8846
0.60	0.5212	2.0811	4.6795	8.3242	10.9388	12.6001	13.7225	14.5637
0.70	0.3770	1.5084	3.3896	6.0282	8.2497	9.6417	10.5706	11.2579
0.80	0.2401	0.9606	2.1614	3.8427	5.4203	6.4221	7.0858	7.5715
0.90	0.1157	0.4631	1.0420	1.8538	2.6456	3.1553	3.4917	3.7370
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.375 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.2575	1.0302	2.1303	2.7036	3.0229	3.2425	3.4105	3.5471
0.20	0.4472	1.7929	3.7712	4.8508	5.4464	5.8528	6.1618	6.4128
0.30	0.5662	2.2604	4.8734	6.4032	7.2383	7.7996	8.2247	8.5664
0.40	0.6328	2.5317	5.5901	7.5646	8.6337	9.3413	9.8710	10.2973
0.50	0.6580	2.6274	5.9144	8.2759	9.5556	10.3888	11.0056	11.4984
0.60	0.6466	2.5869	5.8208	8.6046	10.0888	11.0364	11.7283	12.2764
0.70	0.5832	2.3285	5.2410	8.1251	9.7001	10.6853	11.3956	11.9529
0.80	0.4415	1.7665	3.9749	6.4100	7.8040	8.6634	9.2743	9.7489
0.90	0.2323	0.9293	2.0931	3.4459	4.2505	4.7430	5.0899	5.3581
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.500 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.2310	0.9224	1.5849	1.8673	2.0390	2.1630	2.2610	2.3420
0.20	0.3971	1.5886	2.7851	3.2985	3.6083	3.8313	4.0069	4.1519
0.30	0.4936	1.9791	3.5710	4.2643	4.6777	4.9730	5.2047	5.3963
0.40	0.5409	2.1597	4.0360	4.8730	5.3638	5.7129	5.9841	6.2086
0.50	0.5450	2.1804	4.2109	5.1497	5.6923	6.0741	6.3701	6.6127
0.60	0.5173	2.0653	4.1147	5.1076	5.6714	6.0644	6.3676	6.6155
0.70	0.4492	1.7972	3.6753	4.6314	5.1672	5.5368	5.8207	6.0517
0.80	0.3400	1.3602	2.8430	3.6339	4.0723	4.3722	4.6014	4.7874
0.90	0.1856	0.7442	1.5738	2.0325	2.2854	2.4570	2.5876	2.6936
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 17a.

TOOTHWIDTH T =0.625 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.7550	1.0648	1.2121	0.1891	1.3075	1.3786	1.4356	1.4837
0.20	1.2688	1.8224	2.0800	0.3178	2.2448	2.3680	2.4667	2.5495
0.30	1.5128	2.2233	2.5450	0.3781	2.7509	2.9036	3.0255	3.1278
0.40	1.5321	2.3094	2.6538	0.8210	2.8726	3.0343	3.1633	3.2712
0.50	1.3485	2.0853	2.4057	0.3376	2.6077	2.7567	2.8751	2.9741
0.60	1.0003	1.5765	1.8251	0.2500	1.9810	2.0951	2.1862	2.2620
0.70	0.5717	0.9134	1.0600	0.1429	1.1513	1.2182	1.2715	1.3159
0.80	0.2394	0.3853	0.4477	0.0598	0.4865	0.5149	0.5375	0.5562
0.90	0.0742	0.1197	0.1391	0.0185	0.1512	0.1600	0.1670	0.1728
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.750 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.1199	0.4316	0.5660	0.6326	0.6774	0.7116	0.7394	0.7628
0.20	0.1861	0.6744	0.8878	0.9934	1.0641	1.1179	1.1616	1.1986
0.30	0.1863	0.6802	0.8999	1.0078	1.0801	1.1350	1.1796	1.2173
0.40	0.1358	0.4989	0.6630	0.7432	0.7968	0.8375	0.8705	0.8983
0.50	0.0696	0.2570	0.3423	0.3840	0.4118	0.4329	0.4499	0.4644
0.60	0.0263	0.0971	0.1295	0.1453	0.1558	0.1638	0.1702	0.1757
0.70	0.0084	0.0311	0.0415	0.0466	0.0500	0.0525	0.0546	0.0564
0.80	0.0025	0.0092	0.0123	0.0138	0.0148	0.0160	0.0162	0.0167
0.90	0.0006	0.0024	0.0032	0.0036	0.0038	0.0040	0.0042	0.0043
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.875 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0298	0.0978	0.1229	0.1360	0.1451	0.1521	0.1578	0.1626
0.20	0.0317	0.1044	0.1312	0.1452	0.1549	0.1623	0.1684	0.1736
0.30	0.0169	0.0556	0.0699	0.0773	0.0825	0.0865	0.0897	0.0925
0.40	0.0060	0.0196	0.0247	0.0273	0.0292	0.0306	0.0317	0.0327
0.50	0.0017	0.0057	0.0072	0.0080	0.0085	0.0089	0.0093	0.0096
0.60	0.0006	0.0018	0.0023	0.0025	0.0027	0.0028	0.0029	0.0030
0.70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 17b.

CALCULATED TNORM VALUES IN NM FOR LAMBDA/G = 5.00

TOOTHWIDTH T =0.125 LAMBDA

I (AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0727	0.2902	0.6535	1.1610	1.8144	2.4402	2.8707	3.1780
0.20	0.1324	0.5286	1.1903	2.1147	3.3056	4.4753	5.2885	5.8687
0.30	0.1592	0.6370	1.4315	2.5458	3.9778	5.4608	6.5229	7.2609
0.40	0.1619	0.6476	1.4571	2.5879	4.0445	5.6177	6.7812	7.6142
0.50	0.1431	0.5726	1.2865	2.2908	3.5791	5.0187	6.1221	6.9179
0.60	0.1177	0.4708	1.0593	1.8832	2.8244	4.1605	5.1250	5.8273
0.70	0.0878	0.3513	0.7916	1.4068	2.1975	3.1643	3.8702	4.4211
0.80	0.0581	0.2331	0.5242	0.9316	1.4554	2.0955	2.5908	2.9731
0.90	0.0297	0.1189	0.2674	0.4752	0.7423	1.0688	1.3262	1.5245
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.250 LAMBDA

I (AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0810	0.3241	0.7298	1.2977	1.8565	2.2175	2.4558	2.6292
0.20	0.1537	0.6161	1.3852	2.4621	3.5593	4.2777	4.7509	5.0942
0.30	0.2010	0.8027	1.8050	3.2111	4.6986	5.6957	6.3518	6.8254
0.40	0.2284	0.9121	2.0535	3.6488	5.4353	6.6797	7.5002	8.0872
0.50	0.2234	0.8930	2.0066	3.5686	5.3763	6.6831	7.5484	8.1633
0.60	0.2014	0.8059	1.8131	3.2239	5.0362	6.2329	7.1070	7.7233
0.70	0.1573	0.6294	1.4162	2.5178	3.9342	4.9466	5.6703	6.1792
0.80	0.1083	0.4335	0.9763	1.7358	2.7117	3.4779	4.0183	4.3967
0.90	0.0550	0.2205	0.4957	0.8811	1.3765	1.7726	2.0518	2.2470
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.375 LAMBDA

I (AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0757	0.3027	0.6802	1.1767	1.5077	1.7113	1.8513	1.9575
0.20	0.1406	0.5626	1.2659	2.1970	2.8311	3.2210	3.4882	3.6904
0.30	0.1886	0.7546	1.6980	2.9705	3.8691	4.4226	4.7998	5.0846
0.40	0.2188	0.8753	1.9722	3.5051	4.5845	5.2722	5.7376	6.0865
0.50	0.2301	0.9223	2.0739	3.6858	4.9308	5.7105	6.2350	6.6256
0.60	0.2240	0.8945	2.0141	3.5783	4.8879	5.7060	6.2525	6.6575
0.70	0.1961	0.7828	1.7627	3.1348	4.3626	5.1344	5.6471	6.0244
0.80	0.1470	0.5881	1.3234	2.3503	3.3266	3.9466	4.3566	4.6569
0.90	0.0764	0.3136	0.7057	1.2546	1.7890	2.1319	2.3583	2.5234
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.500 LAMBDA

I (AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0637	0.2544	0.5730	0.8882	1.0605	1.1683	1.2459	1.3066
0.20	0.1157	0.4618	1.0398	1.6176	1.9340	2.1319	2.2743	2.3860
0.30	0.1533	0.6120	1.3781	2.1683	2.6055	2.8778	3.0729	3.2256
0.40	0.1734	0.6939	1.5593	2.4843	3.0032	3.3246	3.5545	3.7334
0.50	0.1779	0.7116	1.6012	2.5807	3.1415	3.4869	3.7326	3.9233
0.60	0.1670	0.6682	1.5035	2.4524	3.0054	3.3450	3.5854	3.7714
0.70	0.1429	0.5718	1.2884	2.1220	2.6193	2.9234	3.1375	3.3030
0.80	0.1048	0.4202	0.9449	1.5685	1.9466	2.1773	2.3392	2.4640
0.90	0.0557	0.2232	0.5019	0.8376	1.0437	1.1693	1.2573	1.3248
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 18a.

TOOTHWIDTH T =0.625 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0454	0.1821	0.4094	0.5545	0.6356	0.6889	0.7286	0.7604
0.20	0.0803	0.3221	0.7243	0.9864	1.1317	1.2273	1.2983	1.3553
0.30	0.1016	0.4057	0.9123	1.2533	1.4408	1.5635	1.6547	1.7277
0.40	0.1061	0.4236	0.9526	1.3209	1.5216	1.6527	1.7499	1.8275
0.50	0.0966	0.3859	0.8688	1.2143	1.4017	1.5237	1.6142	1.6862
0.60	0.0764	0.3049	0.6866	0.9666	1.1177	1.2159	1.2886	1.3464
0.70	0.0522	0.2084	0.4692	0.6644	0.7695	0.8376	0.8879	0.9279
0.80	0.0298	0.1191	0.2681	0.3808	0.4413	0.4804	0.5094	0.5324
0.90	0.0130	0.0520	0.1169	0.1664	0.1929	0.2101	0.2227	0.2328
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.750 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0221	0.0883	0.1856	0.2380	0.2670	0.2868	0.3019	0.3142
0.20	0.0365	0.1463	0.3071	0.3944	0.4426	0.4755	0.5006	0.5210
0.30	0.0398	0.1596	0.3358	0.4318	0.4849	0.5210	0.5485	0.5709
0.40	0.0338	0.1357	0.2860	0.3684	0.4138	0.4448	0.4683	0.4874
0.50	0.0233	0.0935	0.1973	0.2545	0.2860	0.3074	0.3237	0.3369
0.60	0.0137	0.0547	0.1156	0.1492	0.1677	0.1803	0.1899	0.1976
0.70	0.0071	0.0285	0.0603	0.0778	0.0874	0.0940	0.0990	0.1030
0.80	0.0033	0.0133	0.0281	0.0363	0.0408	0.0439	0.0462	0.0481
0.90	0.0013	0.0053	0.0112	0.0145	0.0163	0.0176	0.0185	0.0192
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TOOTHWIDTH T =0.875 LAMBDA

I(AMP)	1	2	3	4	5	6	7	8
POS.	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM	TNORM
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0027	0.0106	0.0210	0.0259	0.0288	0.0307	0.0323	0.0335
0.20	0.0036	0.0144	0.0285	0.0352	0.0391	0.0417	0.0438	0.0455
0.30	0.0029	0.0117	0.0232	0.0287	0.0319	0.0341	0.0358	0.0371
0.40	0.0018	0.0072	0.0142	0.0176	0.0195	0.0209	0.0219	0.0227
0.50	0.0009	0.0034	0.0067	0.0083	0.0093	0.0099	0.0104	0.0106
0.60	0.0005	0.0019	0.0037	0.0046	0.0051	0.0055	0.0058	0.0060
0.70	0.0003	0.0011	0.0022	0.0029	0.0031	0.0033	0.0035	0.0036
0.80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 18b.

Typical is the deformation of the original (ideal iron) curve. This deformation is brought about by the fact that eqn.(24) increases the more the unstable position is approached. The force curve of figure 18 serves as an example, it shows the curves for s = t = 20g.

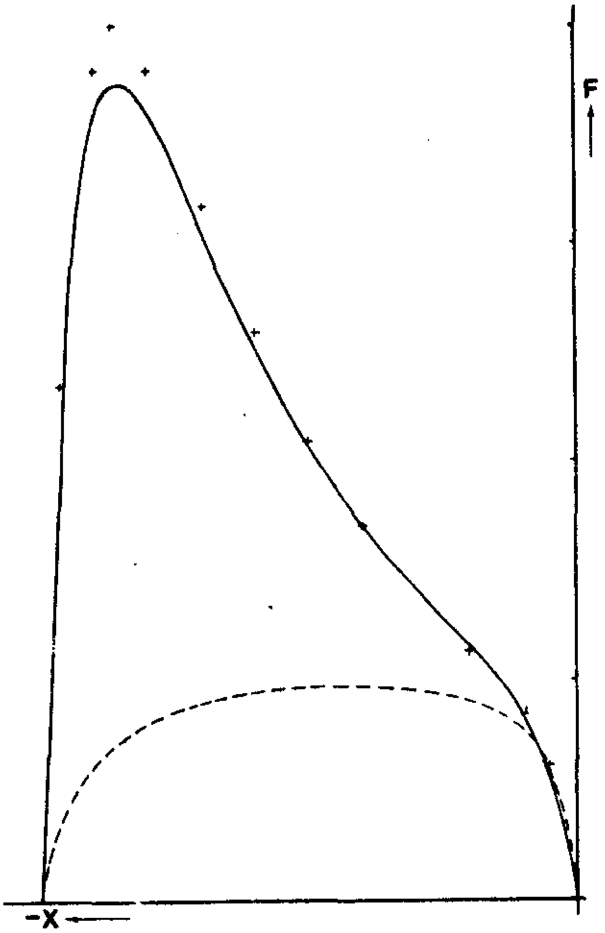


Fig. 18 Diagram of force vs. displacement.

———————— experimental force curve.
++++++++++ calculated force curve.
---------- force curve for ideal iron.

These curves apply for excitations that produce equal magnetic fluxes in position x = 0.

This alteration of the shape of the force curve can be of importance when considering the dynamic properties of a machine. It is also of interest that with a large iron reluctance the force is not proportional to the square of the excitation. In addition, for relatively higher values of the current it is even less than directly proportional. It is reasonable to assume that a certain maximum value for the induction is to be observed when considering various slottings for best performance. We shall now discuss which combination of t/g and s/g offers a maximum average force for a given size of slotting. In this case the discussion is simplified by assuming another extreme materiaal condition, which we will call ideal saturation (see figure 19).

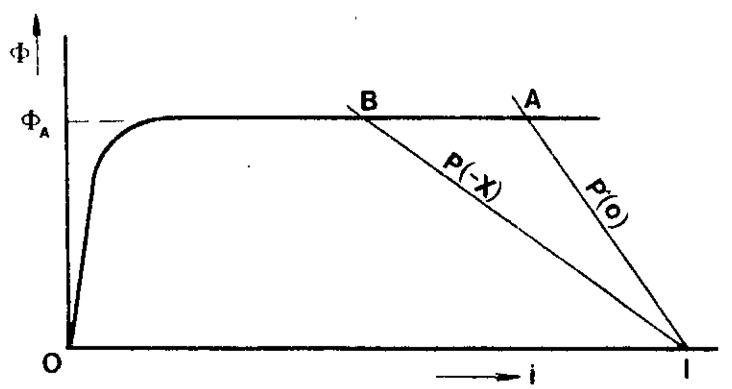


Fig. 19 Flux-to-excitation diagram, idealised saturation.

Here the essential part of the iron curve appears as the line of constant flux $\phi = \phi_A$.
Since tan AEO = P(0) and tan BEO = P (-X). the average force appears to become, using (19).

$$\overline{F} = \frac{1}{2} \frac{\phi^2 A}{X} \left[\frac{1}{P(-X)} - \frac{1}{P(0)} \right] \tag{25}$$

ϕ_A can be expressed in terms of maximum allowable induction B_{max} in the bottom of the teeth by

$$\phi_A = z.l_t.t.B_{max} \tag{26}$$

With eqn. (25) the average force, considering eqns. (26) and $P(0) = \mu_0 l_t.z.P_1$ and $P(-X) = \mu_0 l_t.z.P_2$, becomes

$$\overline{F}_{max} = \frac{B^2_{max}}{\mu_0}\left[\frac{1/P_2 - 1/P_1}{\lambda^2/t^2}\right].z.l_t.\lambda \tag{27}$$

The value to be optimised is now

$$f_2 = \frac{1/P_2 - 1/P_1}{\lambda^2/t^2} \tag{28}$$

It is not hard to see that eqn. (28) is favoured by a small P_2 and a large P_1. The latter can be increased arbitrarily by reducing the airgap as far as mechanical restriction will allow, whereas P_2 can be reduced by selecting a narrow toothwidth. However, the denominator in eqn. (28) opposes the effect of such a tendency. Calculation of the value of eqn. (28), using an extremely small gap, leads to the upper curve of figure 20.

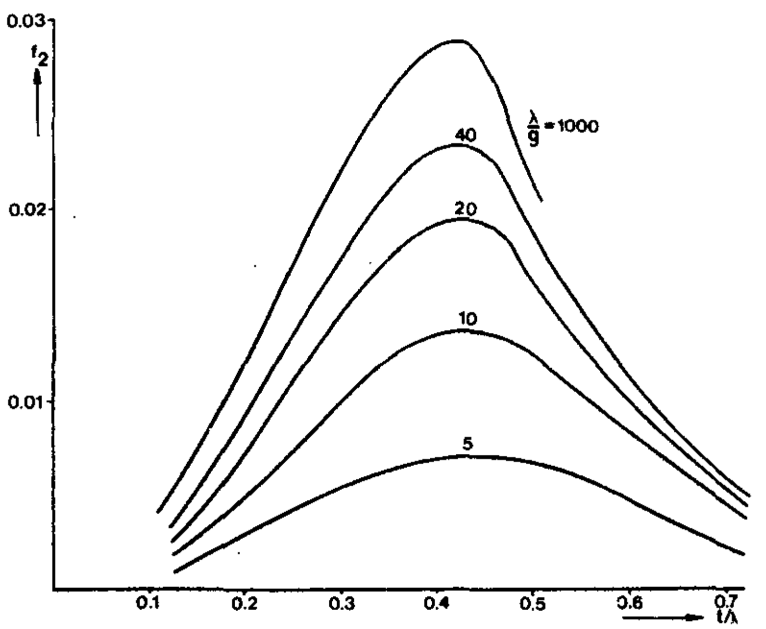


Fig. 20 Force parameter vs. t/λ.

Its maximum value is approximately 0.0288, the toothwidth then being $t = 0.414\lambda$. Since, owing to mechanical restrictions, some minimum airgap is necessary and because the pitch λ is determined by external design conditions, the ratio λ/g can be considered to be a fairly independent quantity. It is therefore reasonable to derive curves for fixed λ/g ratios as shown in figure 20. Apparently, by increasing this parameter the maximum value of f_2 increases rapidly. The best ratio t/λ for any value of the parameter appears to be around 0.42.

From the previous discussion it follows that it is advantageous to design the iron circuit with a reluctance of large magnitude. This is elucidated by a study of figure 16. The hatched area consists of the fairly fixed contribution EAD from the airgap; it is augmented by the iron co-energy ADB. The greater the saturation, the larger the latter becomes. It should be pointed out that the highly saturated part should not contain the teeth themselves.

It has been stated already that the behaviour as indicated in figure 19 represents a fictitious idealisation. Figure 17 illustrates a more realistic situation.

The previous considerations may then be extended by the following correction. Let it be assumed that the essential part of the iron curve may be replaced by the straight line AB, elevation γ. The expression for the average force becomes

$$\overline{F} = \frac{B^2{}_{max}}{\mu_0} \cdot \frac{1/P_2 - 1/P_1}{\lambda^2/t^2} \cdot \frac{1 + \tan\gamma/P(0)}{1 + \tan\gamma/P(-X)} \cdot z \cdot l_t \cdot \lambda \quad (29)$$

1.2.5. Influence of saturation in teeth.

The previous assumption of the teeth not being saturated implied that the iron-air interfaces were approximately equipotential surfaces. If, owing to large inductions, low values of μ in the teeth material appear, this assumption has to be dropped and we will have to refer to more elaborate calculations in which also the varying iron permeance is accounted for.

Computer programs have been developed to calculate field quantities for configurations we are concerned with. The calculations give the permeances in the extreme positions for the airgap and its environment between equipotential lines AA en BB (see figure 21).

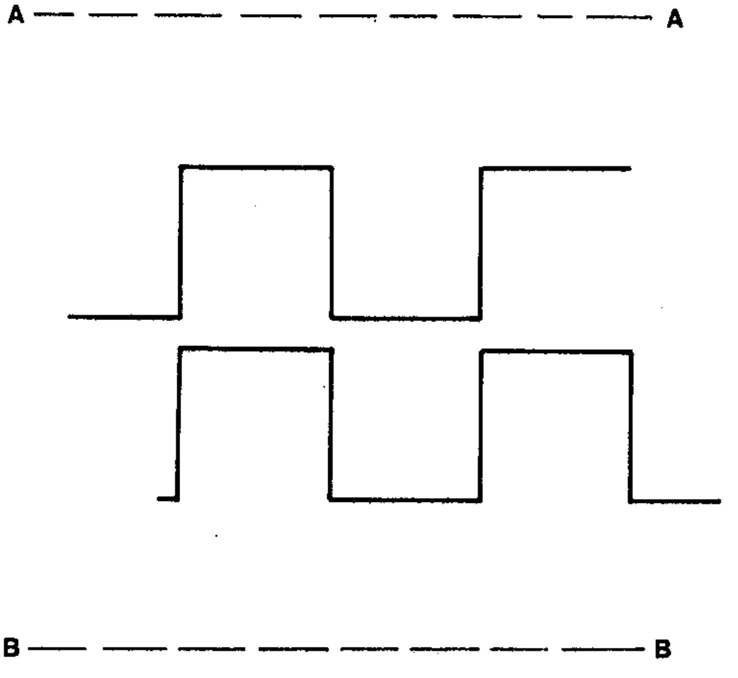


Fig. 21 Airgap and environment.

These lines are situated sufficiently far inside the iron where the field can be considered to be homogeneous and the term equipotential surface is warranted for the planes AA and BB. The calculations were carried out for iron with an induction curve as given in figure 22. It is not hard to see that at low values of the excitation, iron influence is negligible and the the theory of section 1.2.3. applies.

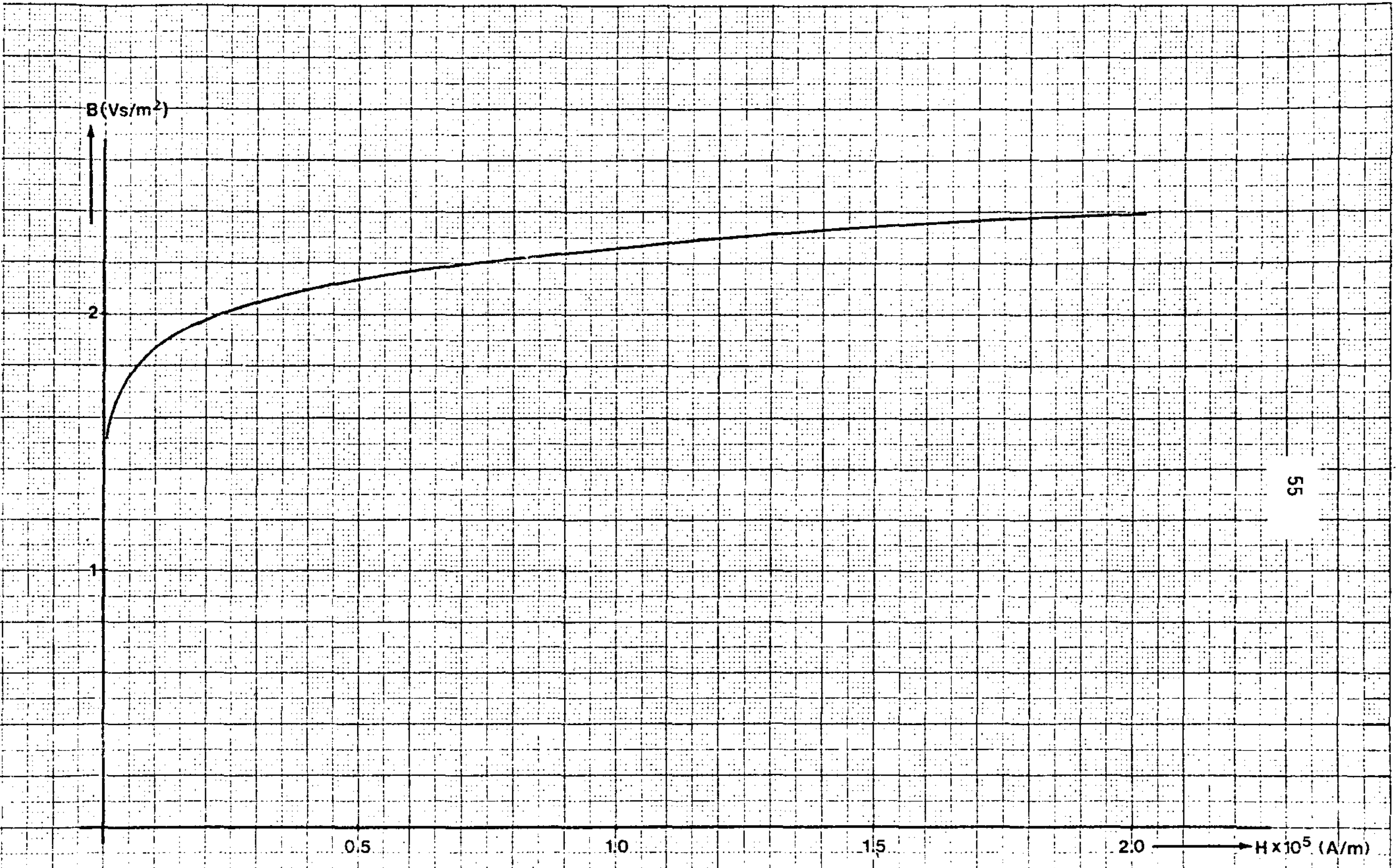


Fig. 22 B = B(H)

With increasing induction in the teeth the lowering permeability, particularly near sharp edges, is going to have effect, decreasing the overall permeance.
A typical set of permeance lines is shown in figure 23.

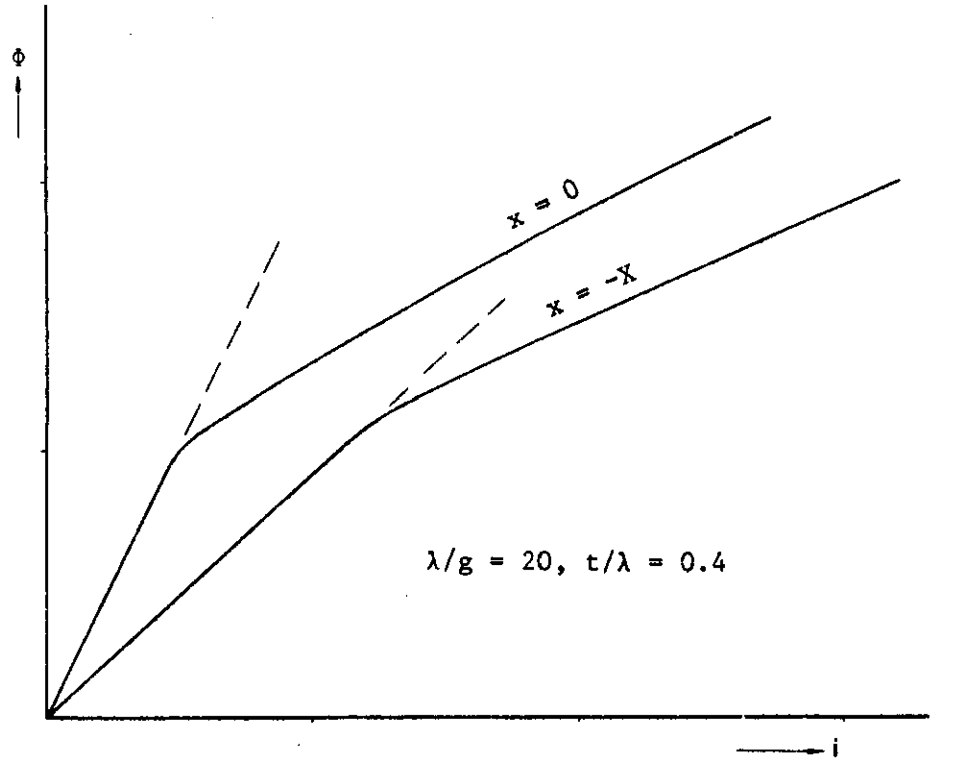


Fig. 23 Permeance diagram, computed for real iron.

The general shape of the lines is easily understood; it is however, remarkable that the two lines tend to become parallel, actually a slight deviation can be observed.
To facilitate analysis of force production and optimisation the following idealisation of these curves is introduced. Without introducing serious errors the P-curves of figure 23 may be approximated as sets of straight lines as shown in figure 24. Here again the lines are drawn in mirror fashion from the point of total excitation $\Theta = NI$.

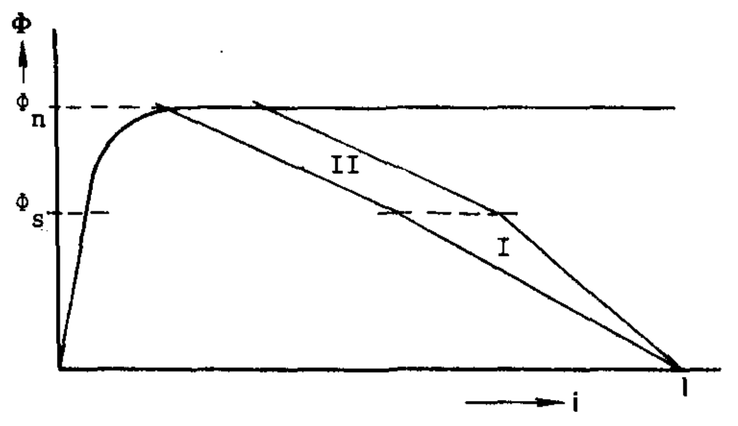


Fig. 24 Permeance diagram, saturated teeth.

The bend in both P-lines appears to lie on the same flux level (at least for the conditions we are interested in). This level appears to be determined by the cross section of the tooth base multiplied by a certain value of magnetic induction, which discriminates between "saturated" behaviour and "ideal" behaviour.

These marginal values will be indicated with s; the typical induction as B_s and the relevant flux as ϕ_s, related by:

$$\phi_s = z.t.l_t.B_s \tag{30}$$

It must be stated specifically that these simplifications are developed on grounds of observed curve shapes and do not have any underlying theoretical base.

The average force now can also be calculated along the lines of section 1.2.4.

The limiting flux through the system is taken as ϕ_n, with some geometry it follows from figure 24:

$$\text{Area II} : \text{Area I} = (\phi_n - \phi_s) : \phi_s$$

or $$\text{Area (I + II)} = \frac{2\phi_n - \phi_s}{\phi_s} \cdot \text{Area I}$$

Area I can be related to the similar triangle of figure 15. Considering that the force is proportional to the area of co-energy swept out, it is clear, with (28) and replacing B_{max} by B_s, that:

$$\overline{F} = \frac{B_s^2}{\mu_0} \; f_2 \; \frac{2\phi_n - \phi_s}{\phi_s} \cdot \lambda . z . l_t \qquad (31)$$

Calculation of the force-displacement curves for teeth-saturation was not carried out because the expenses involved with the elaborate numerical computation were not considered justifiable.

2. MEASUREMENTS

In this chapter measurements, that were carried out in order to verify the calculations, are described.
The calculations for ideal iron can only be compared with measurements carried out with very low excitations, to keep μ_r large, in the order of one hundred or preferably more throughout the entire investigated region.

2.1. Description of the measuring device

A cylindrical stator and rotor were made, as drawn in figure 25.

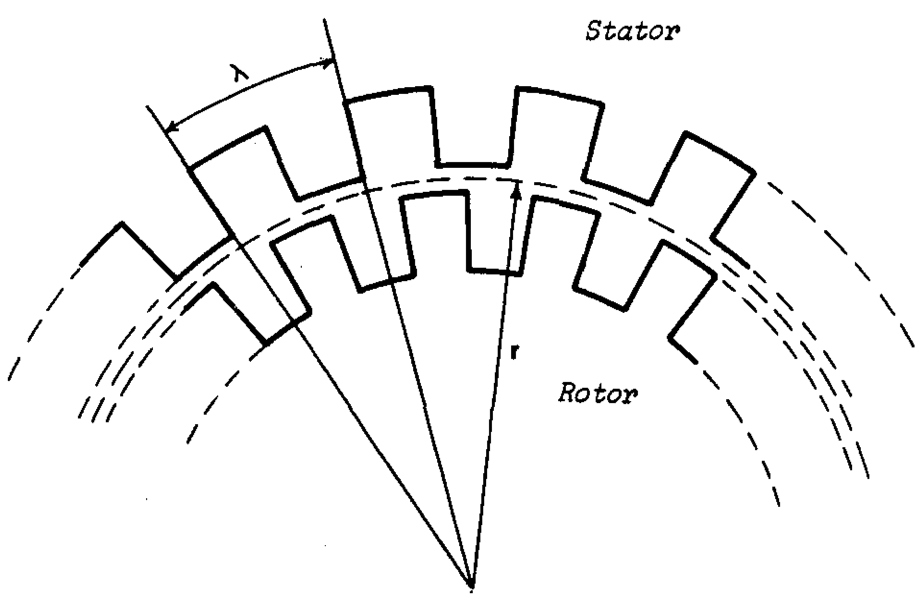
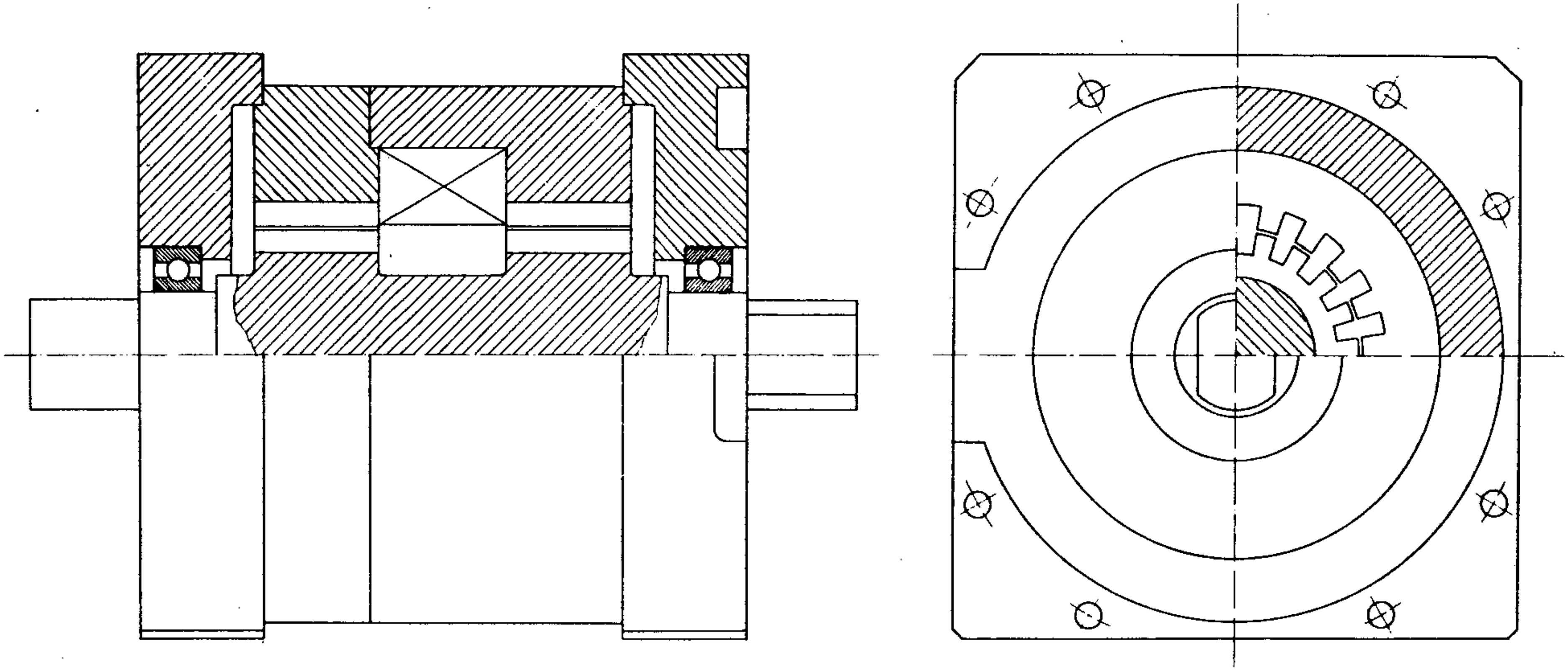


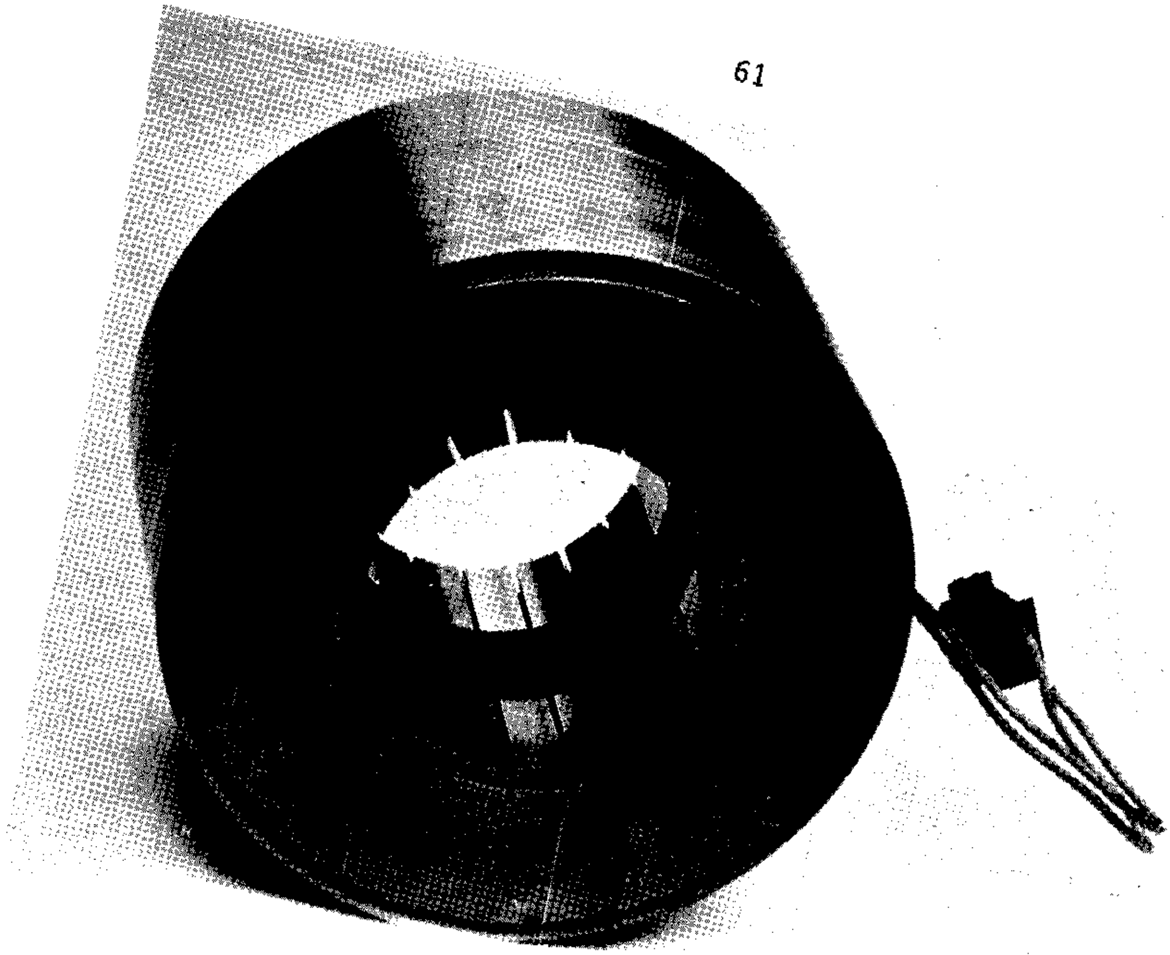
Fig. 25 Teeth configuration of measuring device.

To avoid the influence of the normal forces and end effects a cylindrical device was chosen.
For the purpose of simple calculations the curved airgap region was developed as drawn in figure 1.

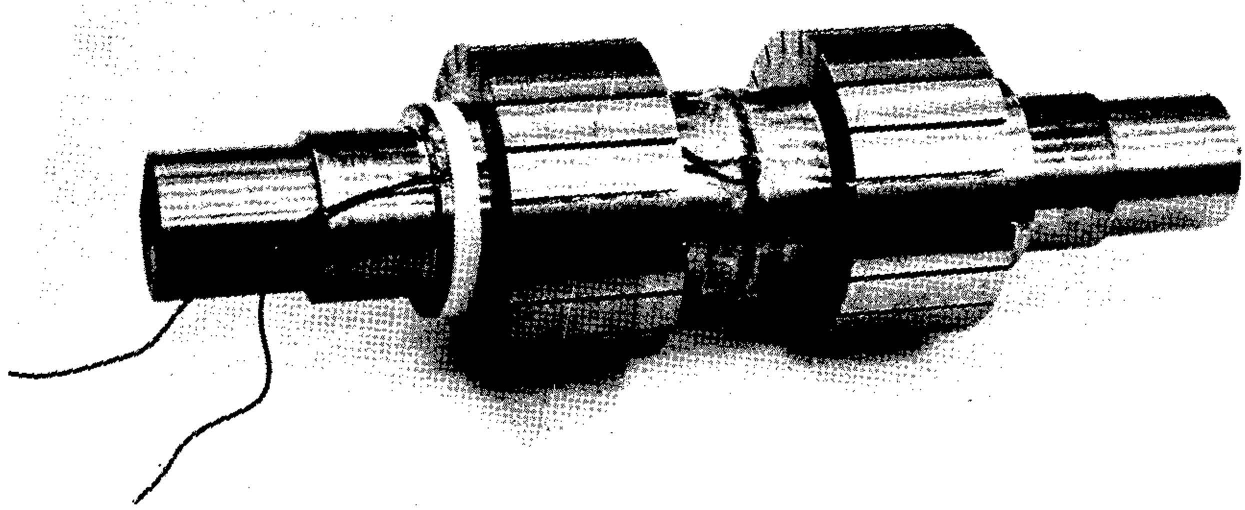


Scale 1:2.

Fig. 26 Stator-rotor combination.



Photograph 1. Stator ($t = 7\lambda/8$)



Photograph 2. Rotor ($t = 7\lambda/8$)

This development is allowed as the machine has a sufficient number of teeth ($z = 20$) and as the airgap width g is small compared with the toothpitch λ.

2.1.1. The stator.

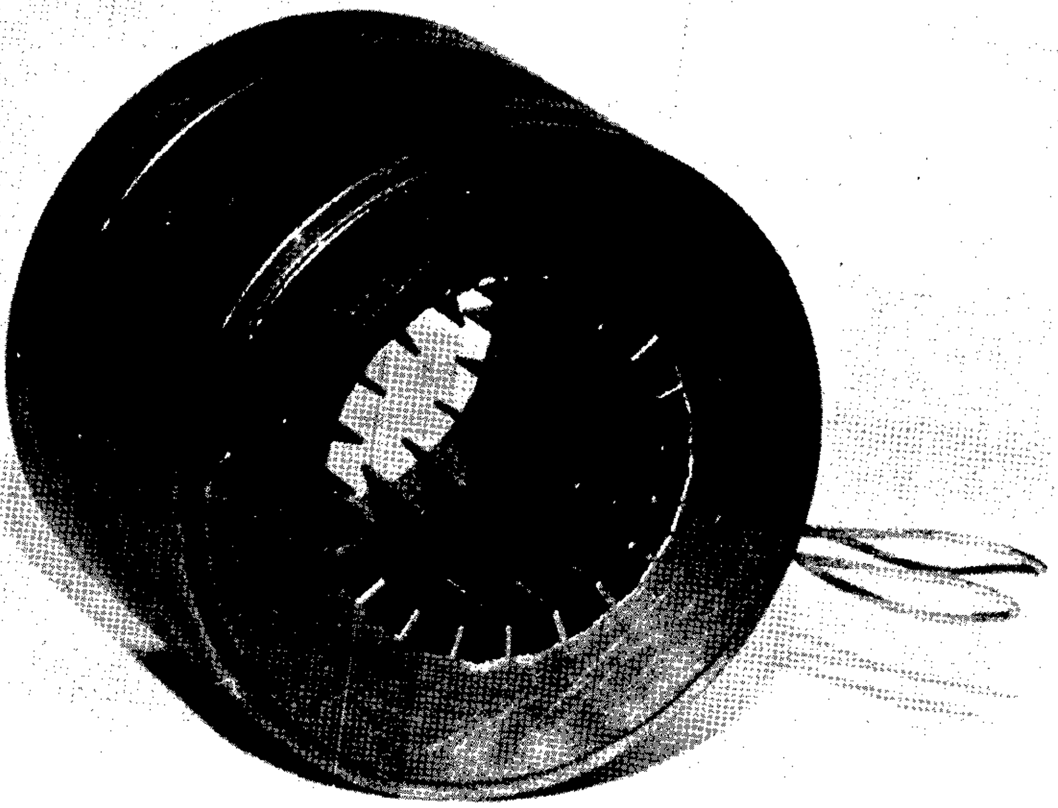
The stator is made of iron that, according to the manufacturer, contains very little impurities. With this iron very high inductions are possible before saturation becomes prohibitive (see figure 22).
The stator has a bore of 80.00 mm and 20 rectangular slots. In the first instance these were very narrow slots (0.125 λ) and for every new set of measurements they were widened. The maximum slotwidth was 0.875 λ.
Between both parts of the stator is an annular coil with 498 windings of 0.95 mm^2 copper wire.

2.1.2. The rotor.

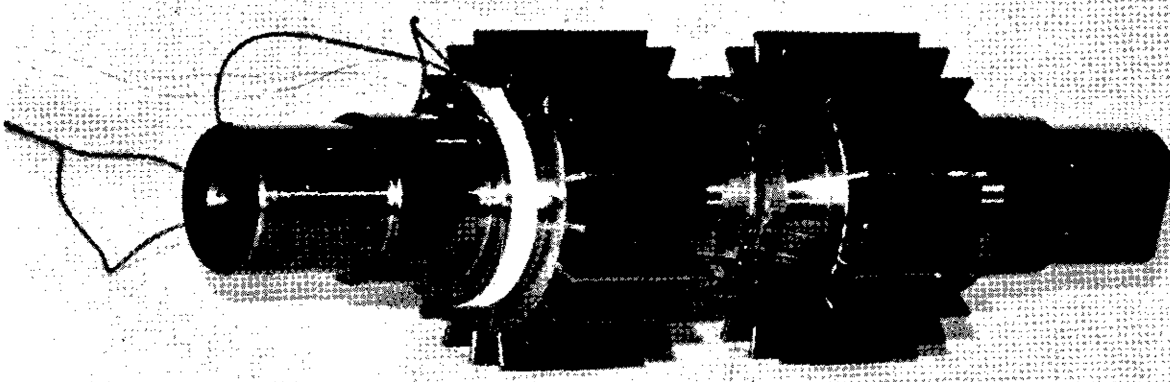
There were 5 rotors made, also of the same iron mentioned above. They have diameters as given in table 19.
Other important dimensions are: active toothlength 40 mm rotor neck diameter 50 mm.

rotor nr	λ/g	diameter (mm)	
1	5	74.99	(74.97)
2	8.05	76.89	(76.88)
3	10	77.51	(77.49)
4	20	78.75	(78.74)
5	40	79.38	(79.37)

Table 19.



Photograph 3. Stator (t = $\lambda/8$)



Photograph 4. Rotor (t = $\lambda/8$)

The values between brackets are the ones that were aimed at, the others are measured values. Stator and rotor are shown in photographs 1 - 4.
Photographs 1 and 2 show the smallest and 3 and 4 the largest slots.
The stator is fitted between two aluminium shields in which the bearings of the rotor are placed so that it can rotate freely.

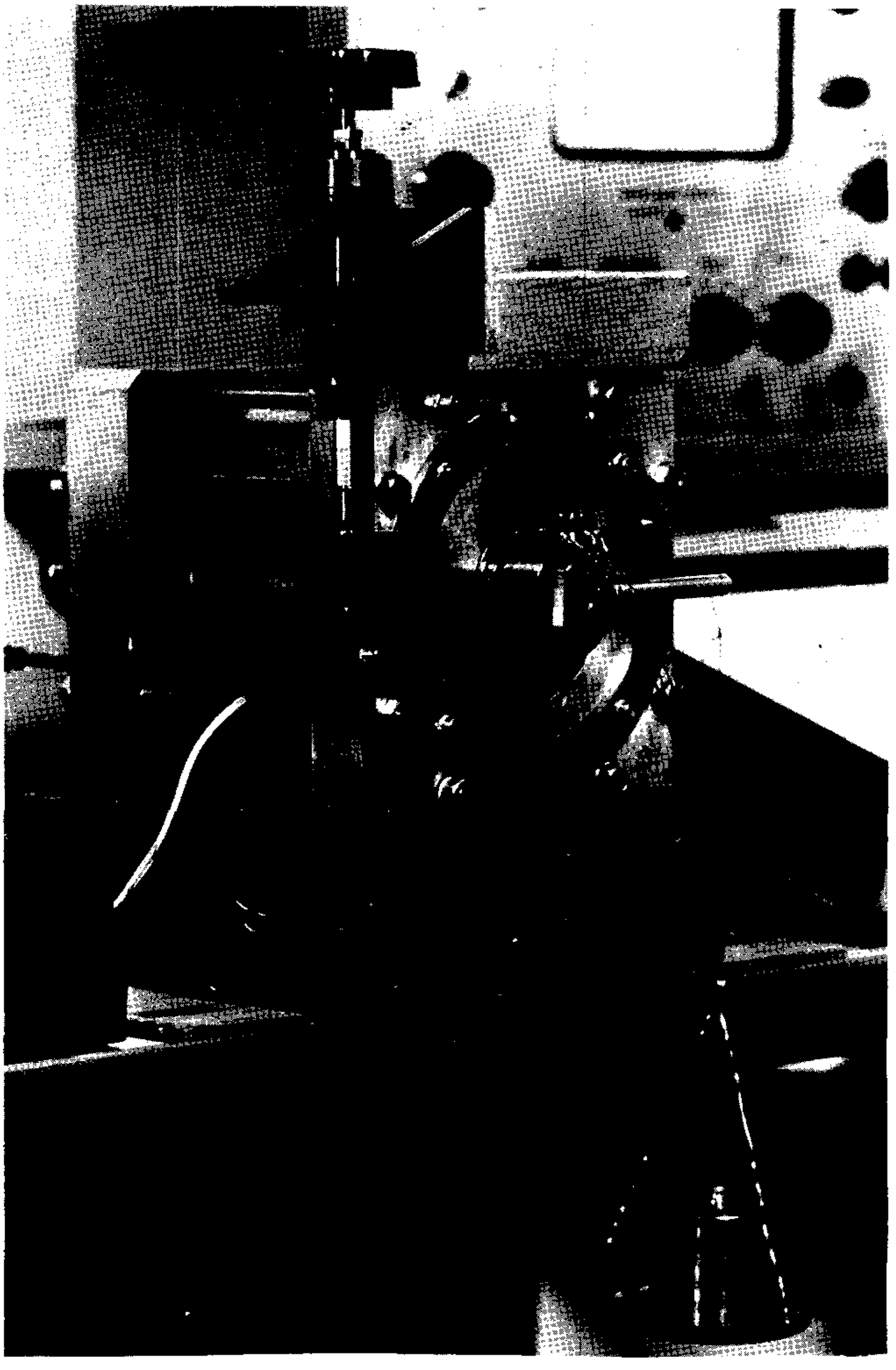
2.2. Description of the measuring equipment

2.2.1. Measurement of the force.

The force is measured by means of two straingauges. These are glued, one on each side of a metal arm that is connected to one of the rotor ends (see photograph 5 and figure 27).
The extreme end of the arm, that reaches radially from the rotor shaft, is caught by a revolvable ring, the latter being clamped to the stator. When exciting the stator coil the rotor wants to go to its equilibrium state. As the extreme end of the arm is fixed relative to the stator, a bending torque will be exerted upon the arm which results in the resistances of the straingauges becoming unequal. As these straingauges are connected in a bridge circuit this bending will result in an output voltage directly proportional to the force between stator and rotor. By moving the ring by means of an adjusting apparatus the rotorposition can be varied.

2.2.2. Measurement of the displacement.

In order to measure the displacement a potentiometer is attached to the other end of the rotor. By connecting a DC voltage across the potentiometer we can measure a voltage directly proportional to the displacement.



Photograph 5. Complete measuring device.

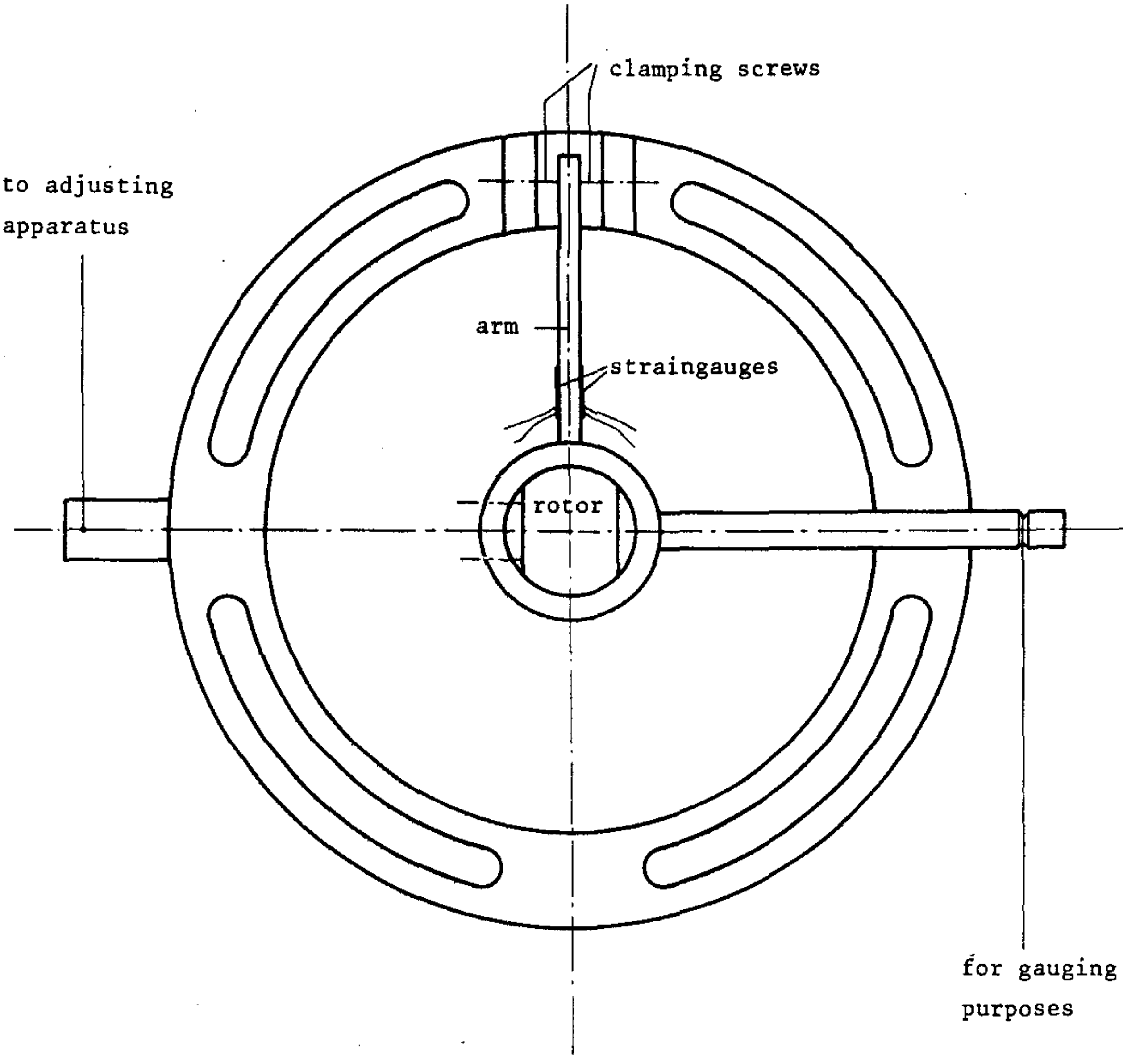
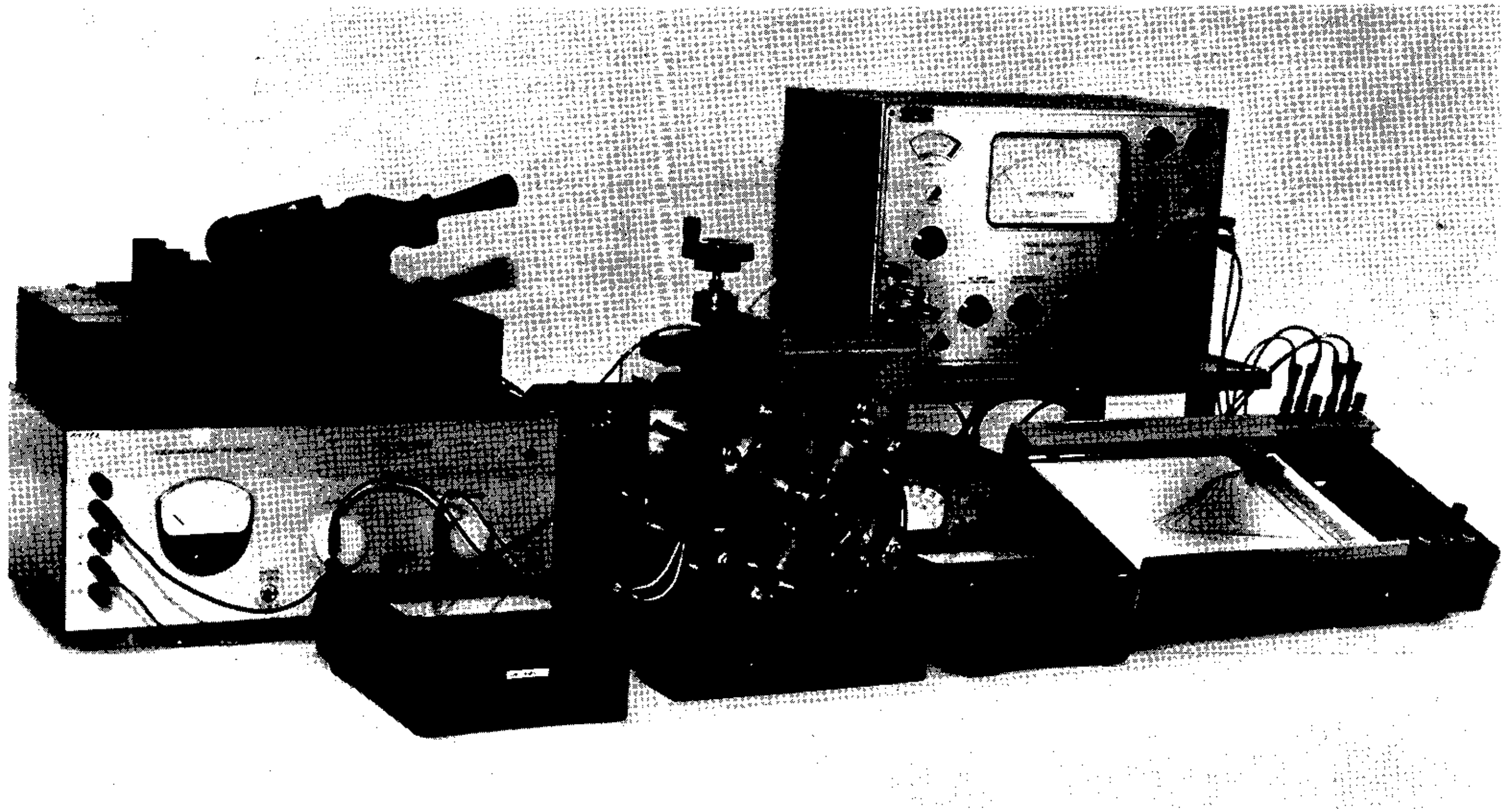


Fig. 27 Force measuring apparatus.



Photograph 6. Complete measuring apparatus.

2.2.3. Measurement of the magnetic flux in the neck of the rotor.

The magnetic flux in the neck of the rotor is measured by means of a coil around the neck consisting of two windings and a fluxmeter. By commutating the stator current, we can measure the flux in the neck of the rotor. As this is the narrowest part (excluding the teeth) of the iron circuit this flux gives an indication of saturation of the circuit. Actually the mmf for the iron can be ascribed to this part of the circuit, the remainder being of much larger area anywhere.

2.3. Measured results.

In the following figures the measured curves, together with calculated values are given.
The calculations are for normal iron, not including saturation of the teeth. The calculated values can be found in tables 14 - 18. A total of 35 sets of curves were measured. Instead of the force the torque of the machine was measured. If one is interested in the force this can be found by dividing the torque by the average radius of stator and rotor. For measurements of flux in the rotorneck and in the teeth, see ref [7].

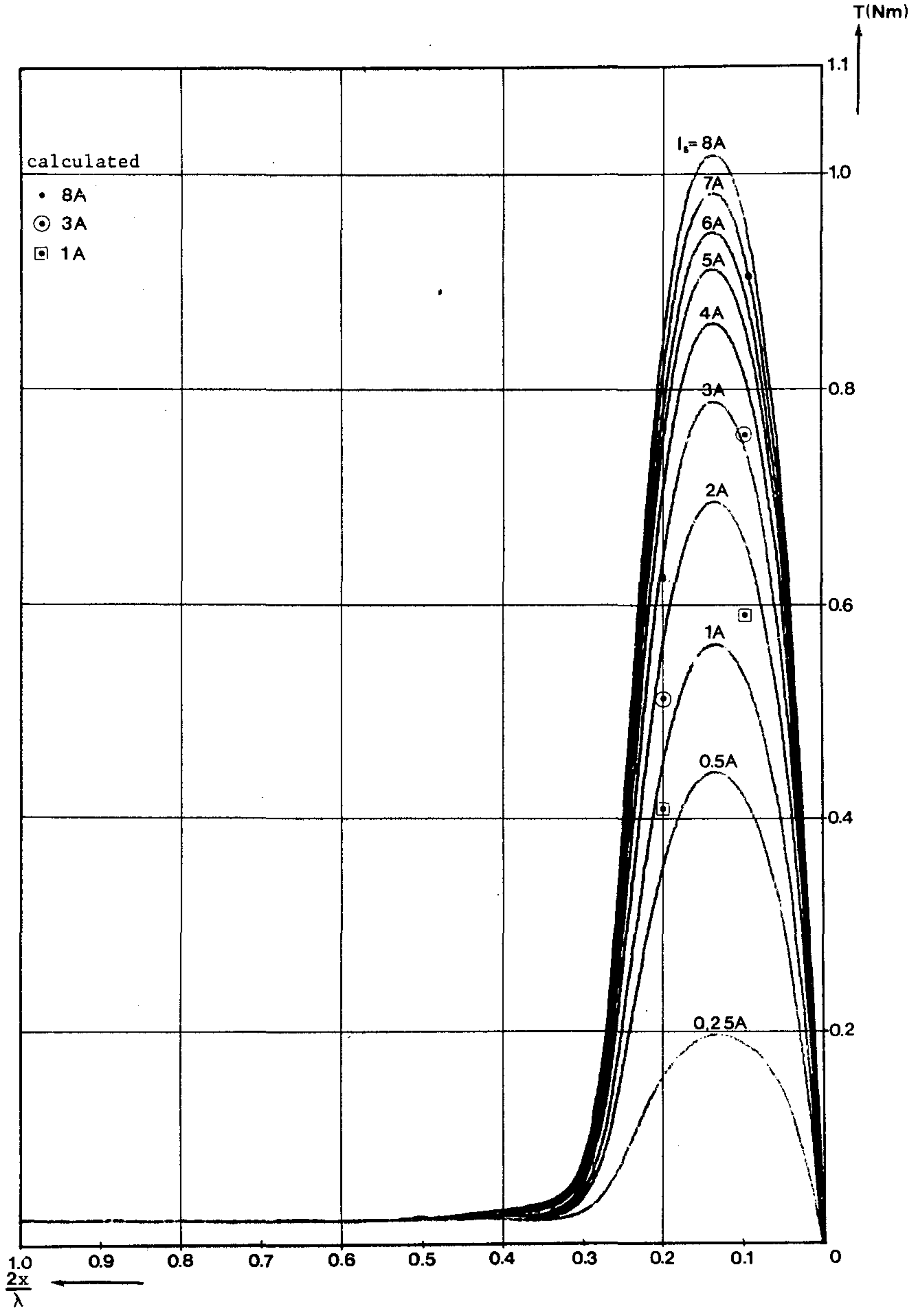


Fig. 28 $\lambda/g = 40$, $t = 7/8\ \lambda$.

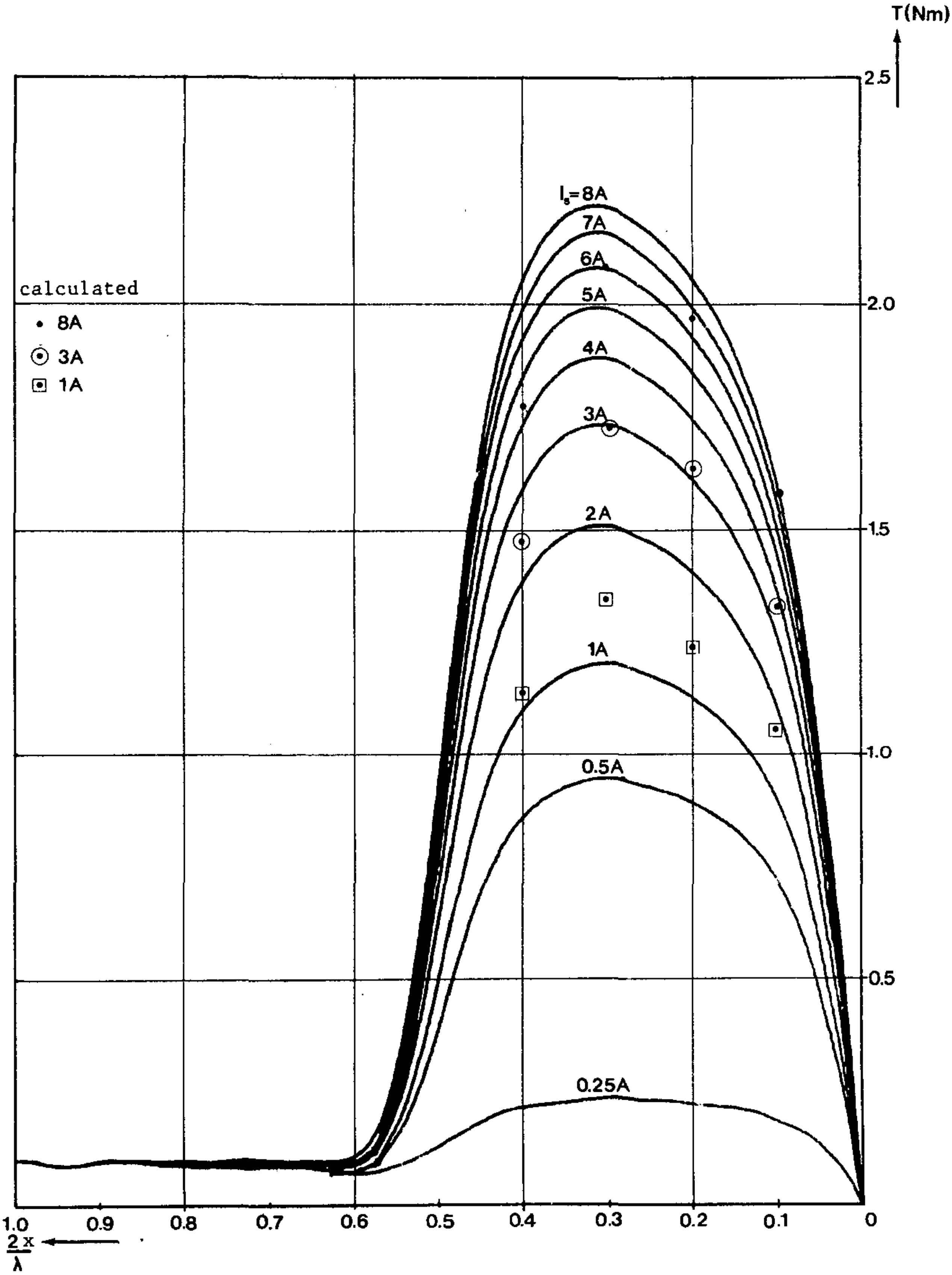


Fig.29 $\lambda/g = 40$, $t = 3/4\ \lambda$

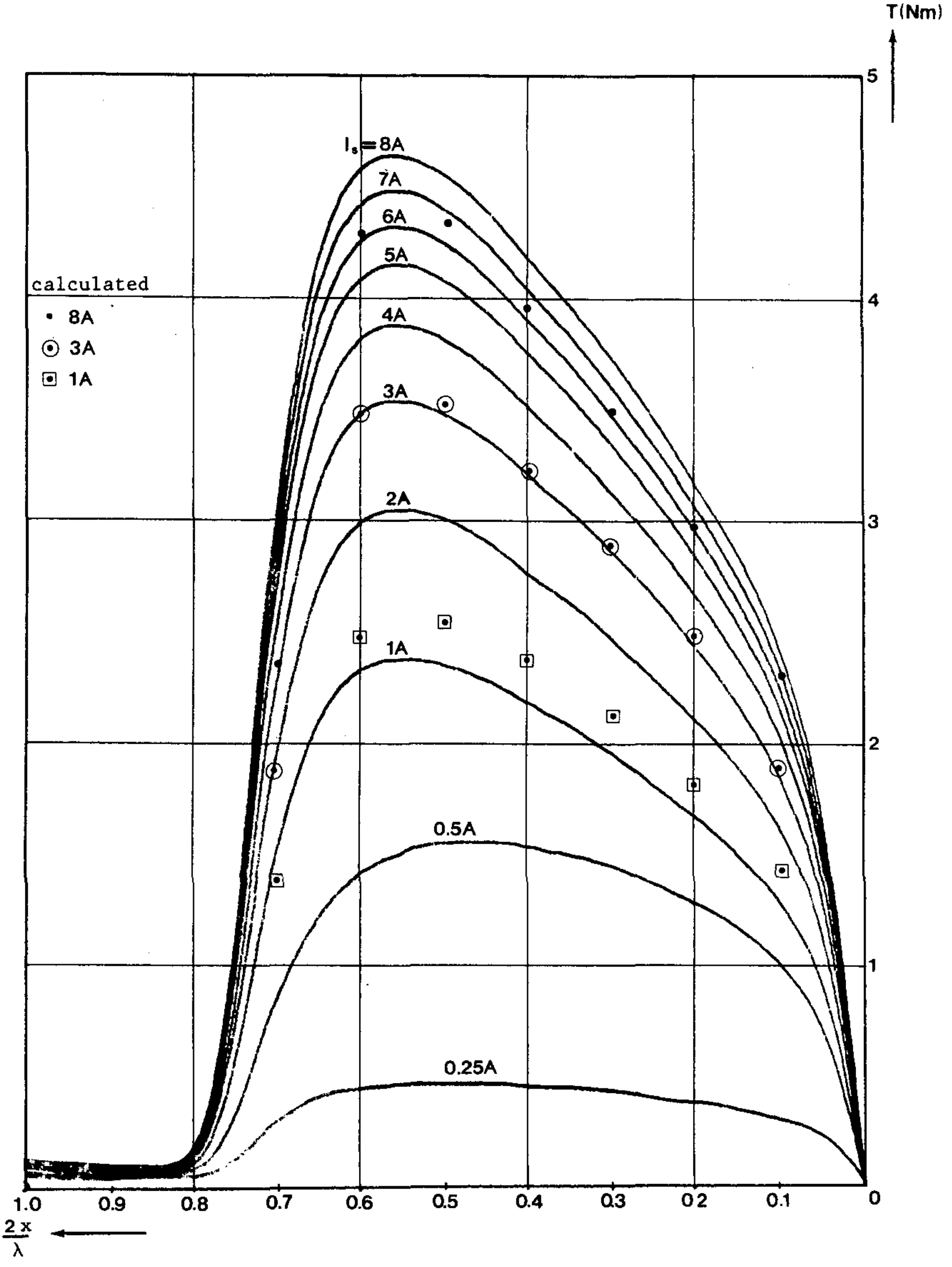


Fig. 30 $\lambda/g = 40$, $t=5/8\ \lambda$

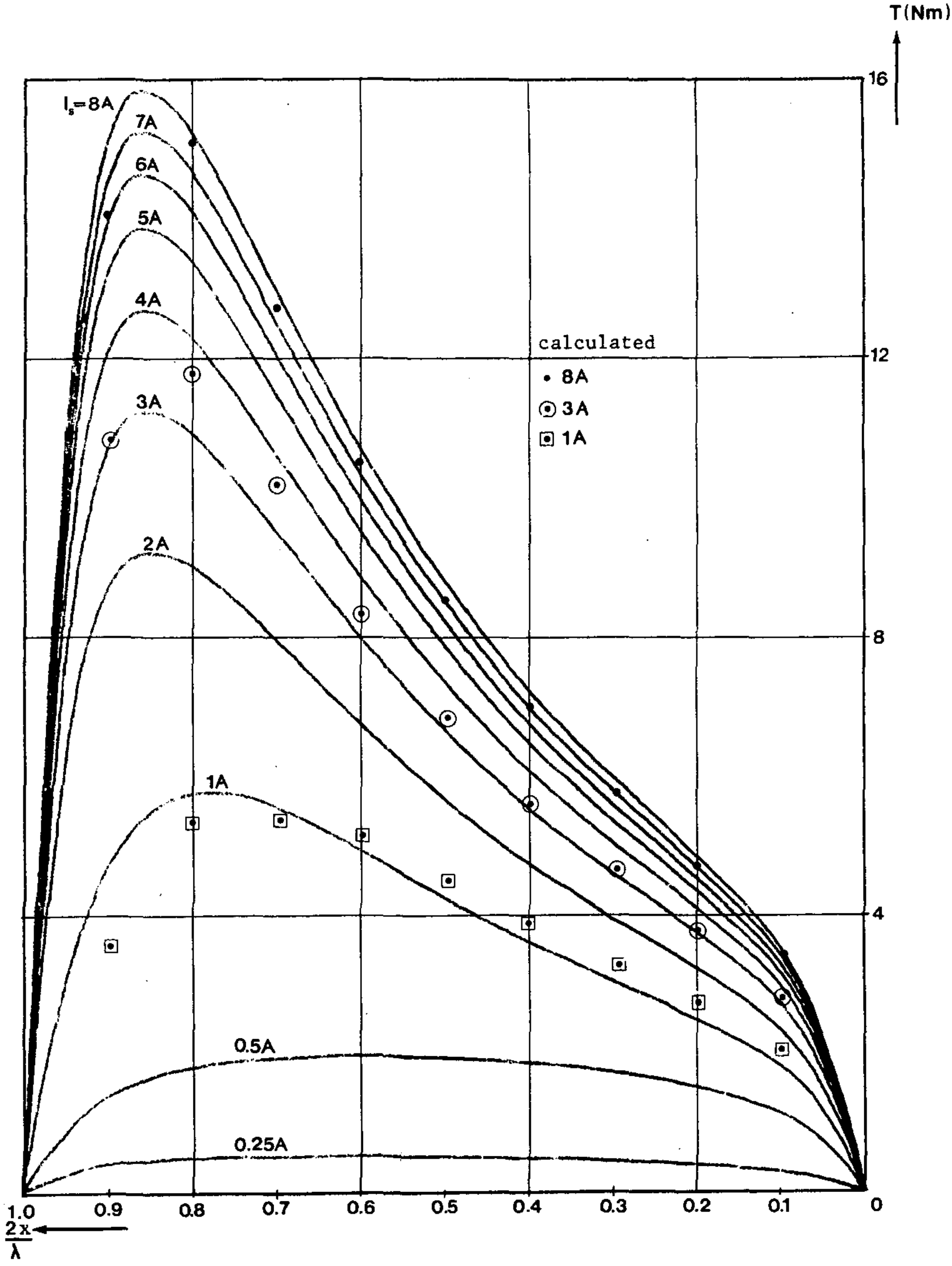


Fig. 31 $\lambda/g = 40$, $t = 1/2\ \lambda$

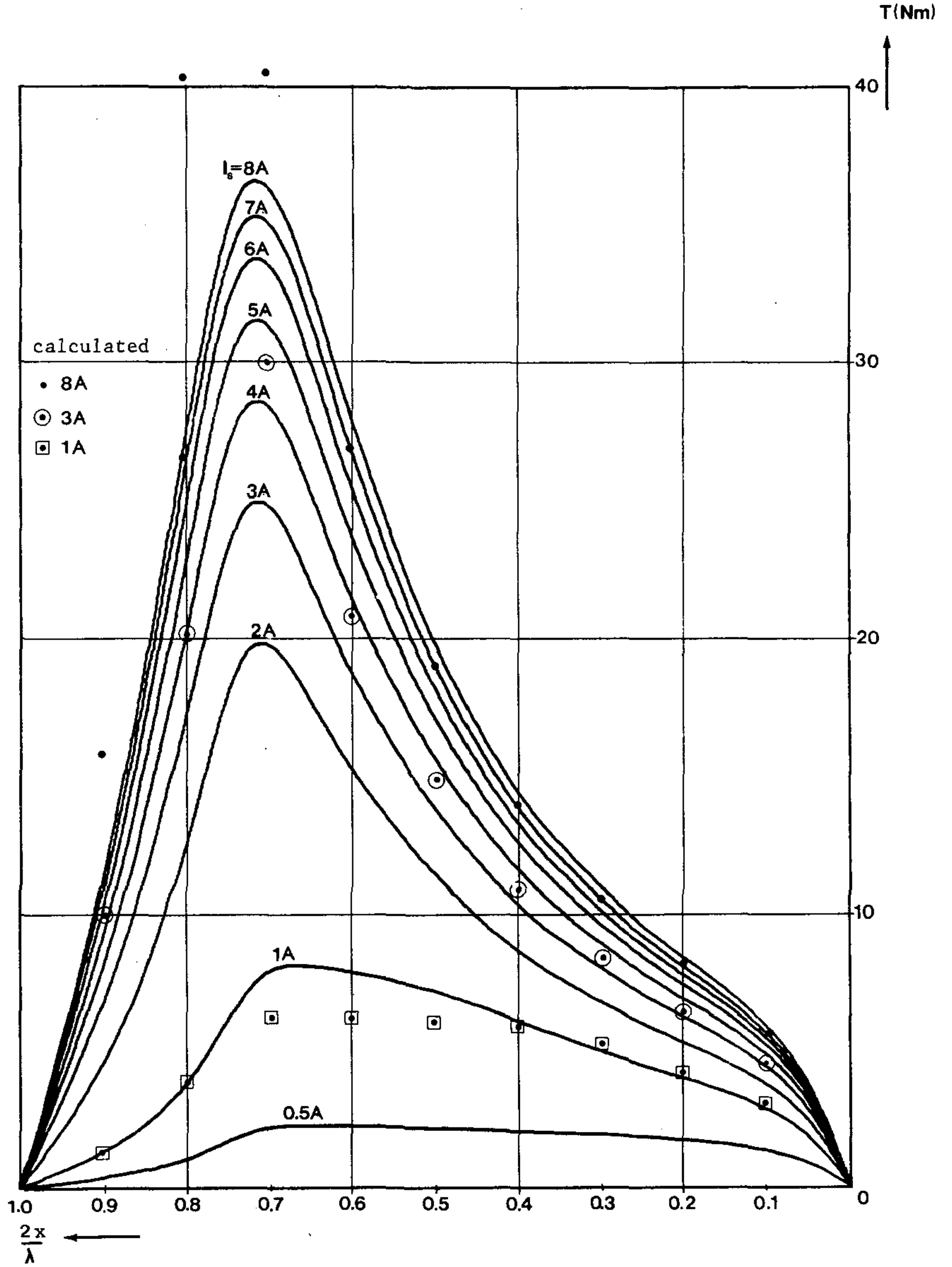


Fig. 32 $\lambda/g = 40$, $t = 3/8\,\lambda$

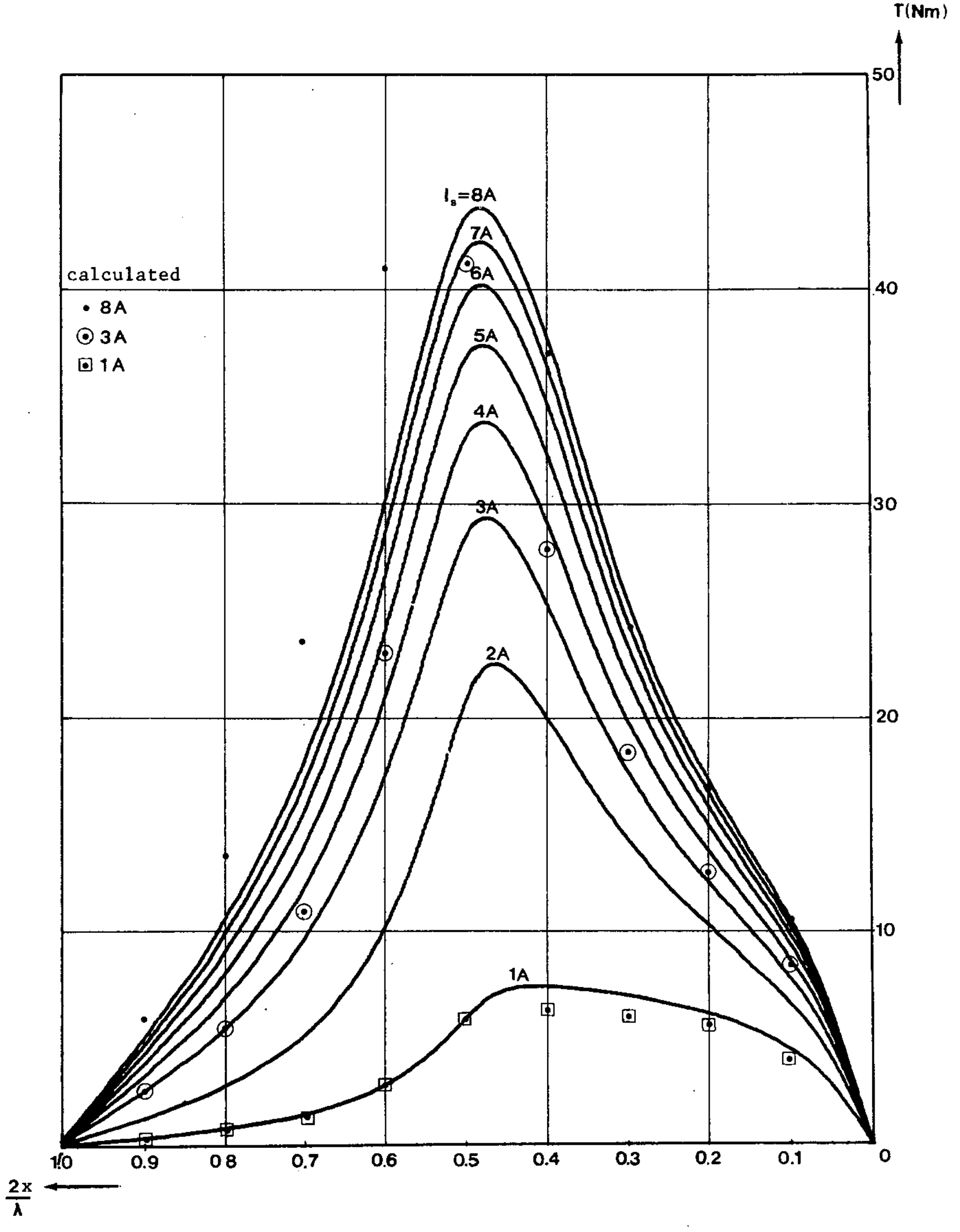


Fig. 33 $\lambda/g = 40$, $t = 1/4\ \lambda$

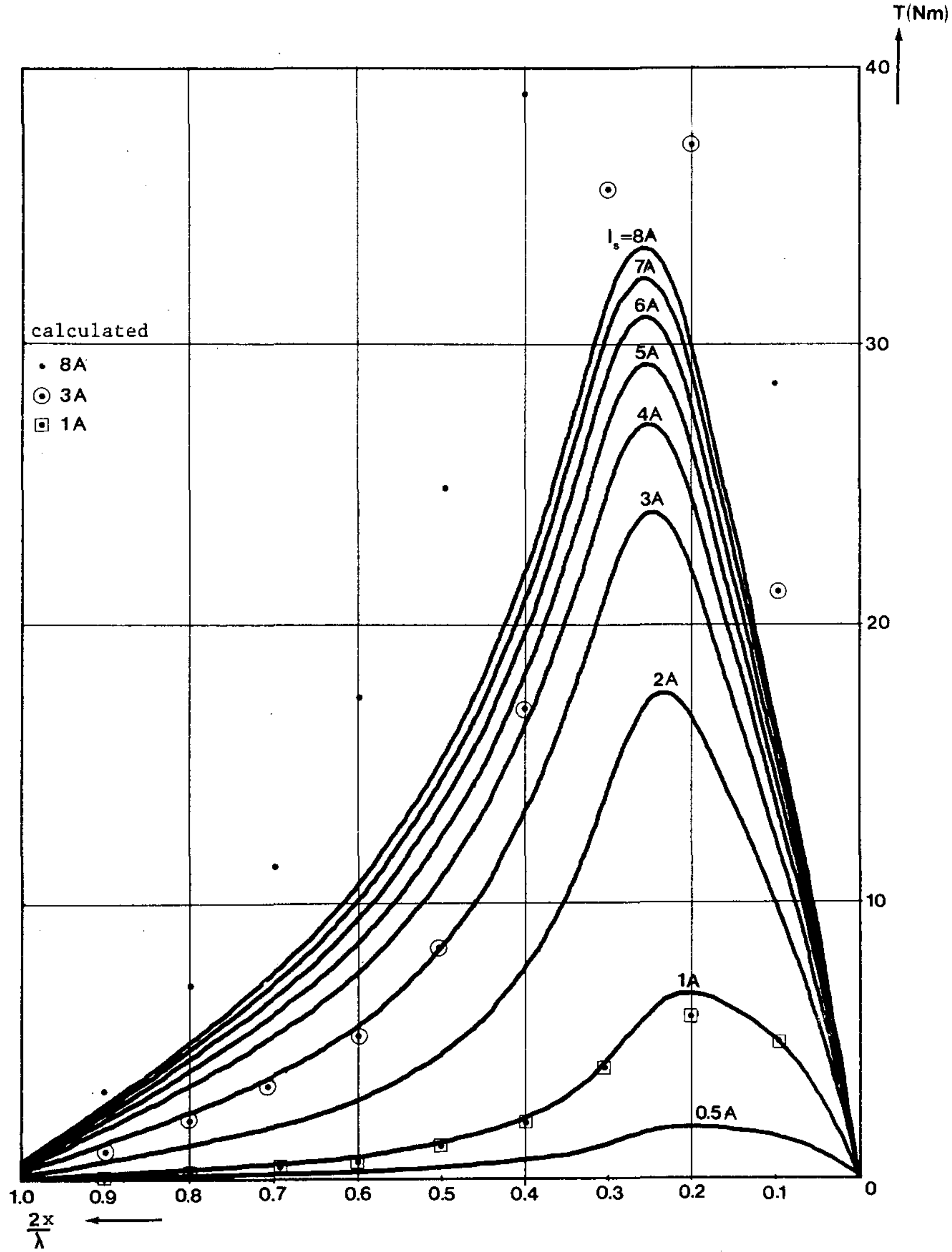


Fig. 34 λ/g = 40, t = 1/8 λ

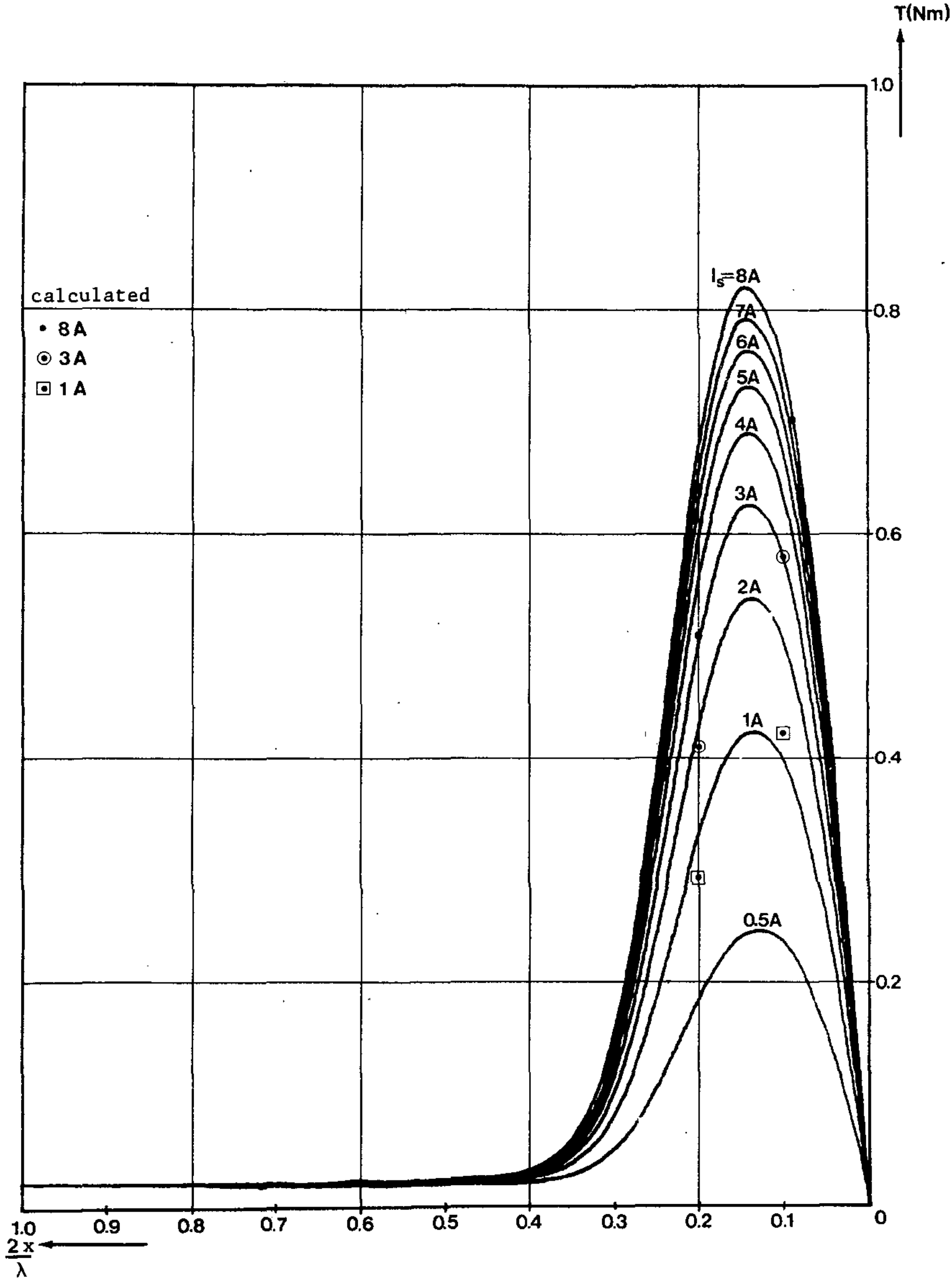


Fig. 35 λ/g = 20, t = 7/8 λ

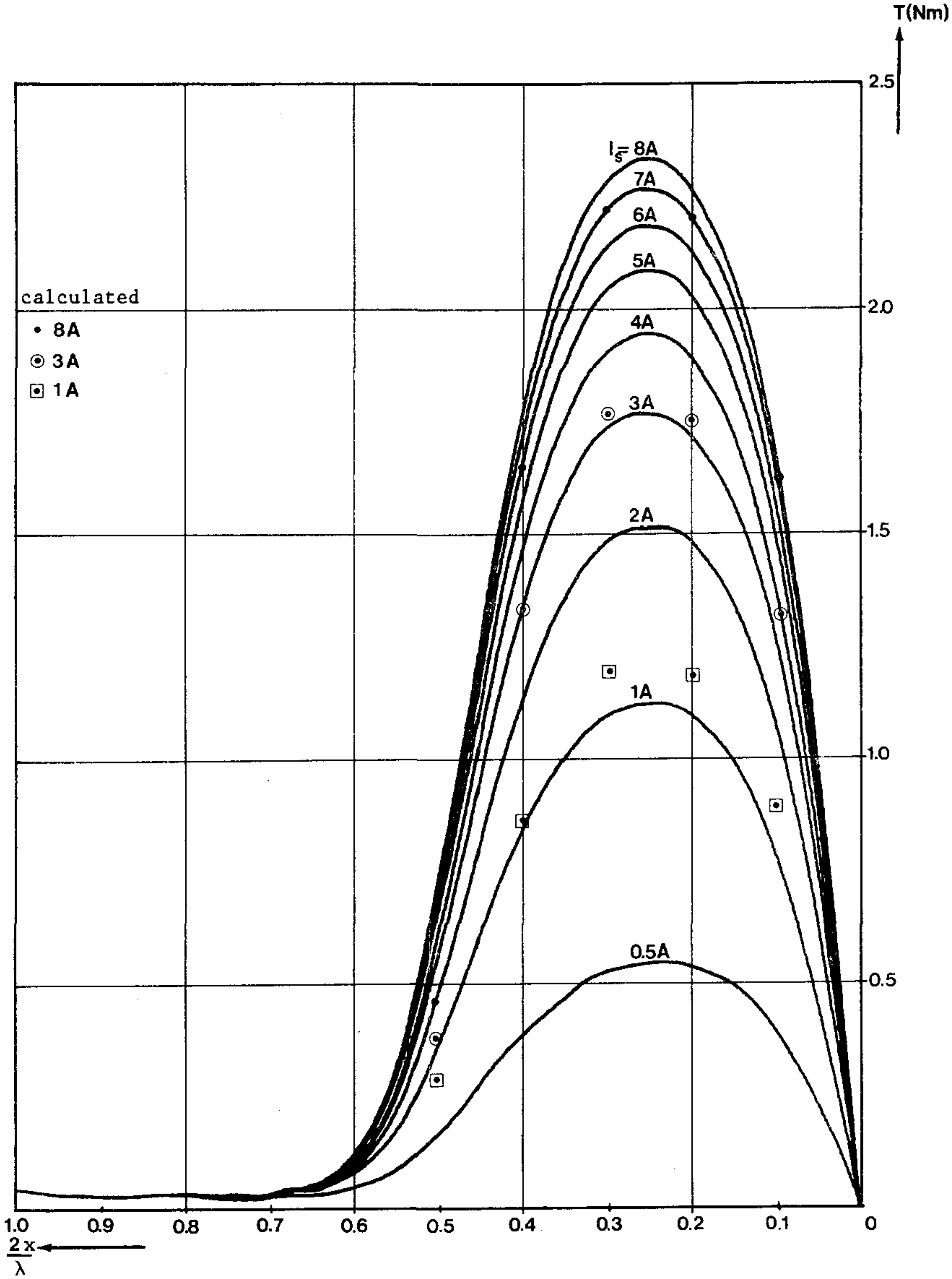


Fig. 36 λ/g=20, t=3/4 λ

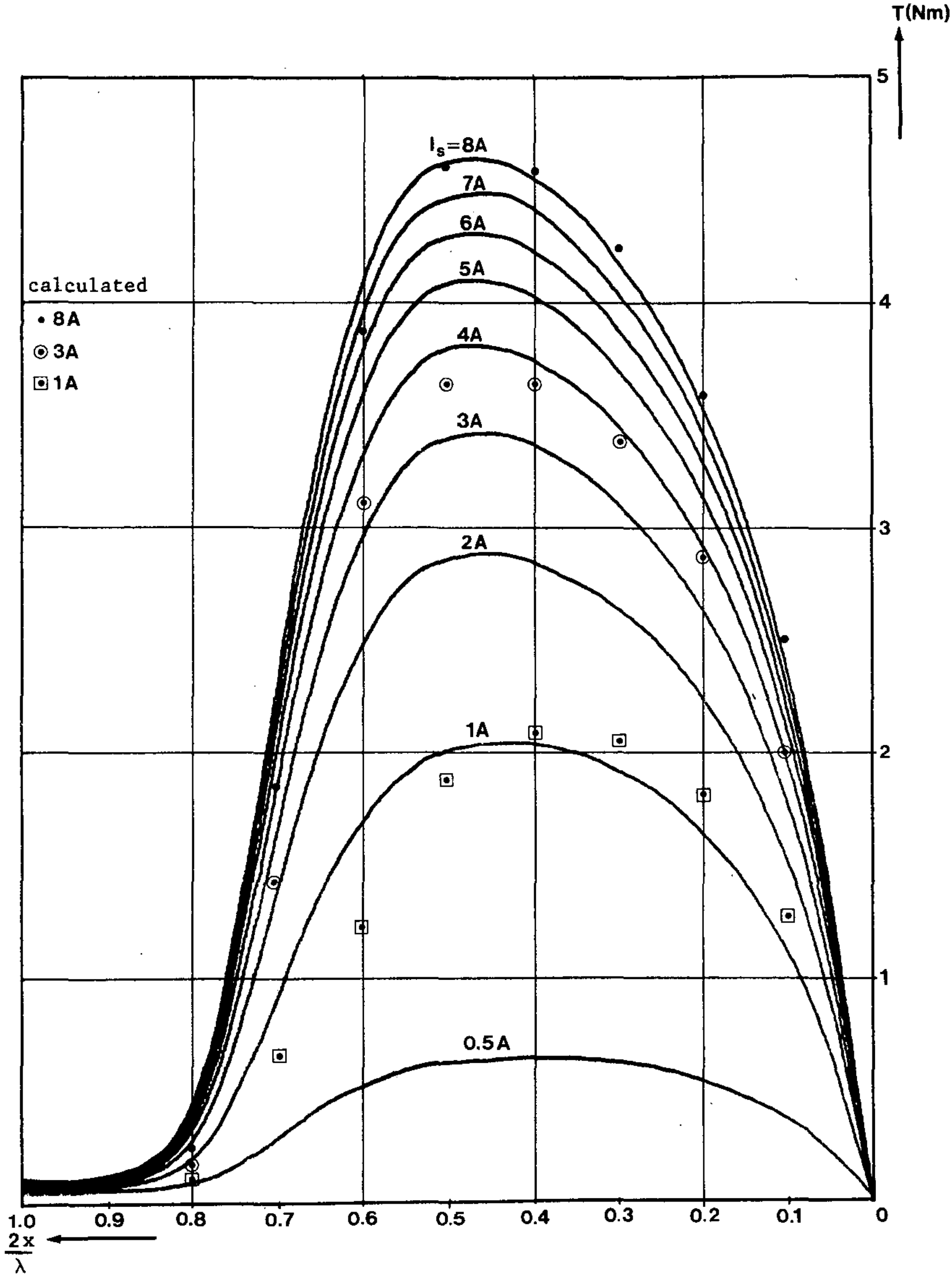


Fig. 37 $\lambda/g = 20$, $t = 5/8\ \lambda$

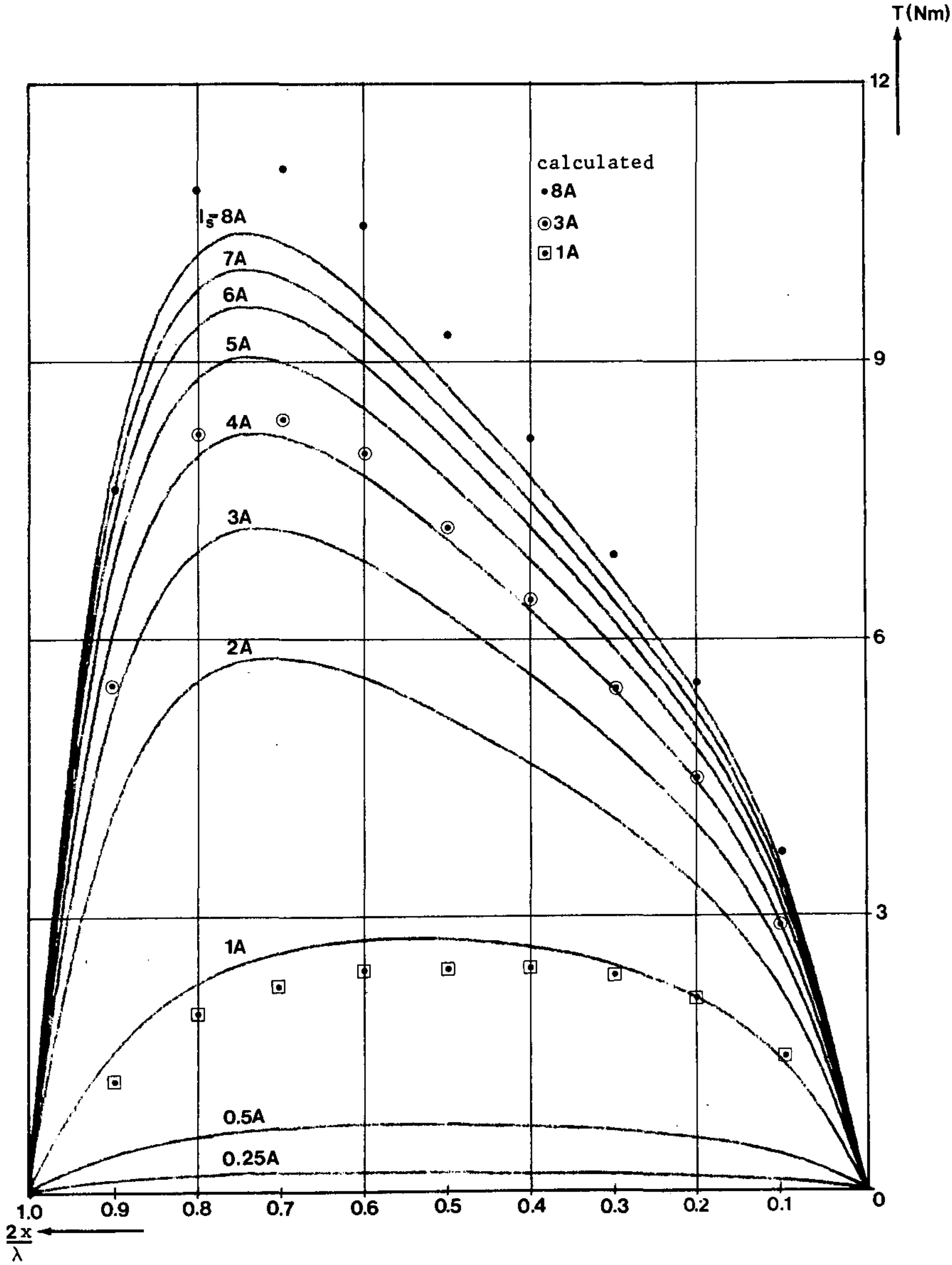


Fig. 38 $\lambda/g=20$, $t=1/2\ \lambda$.

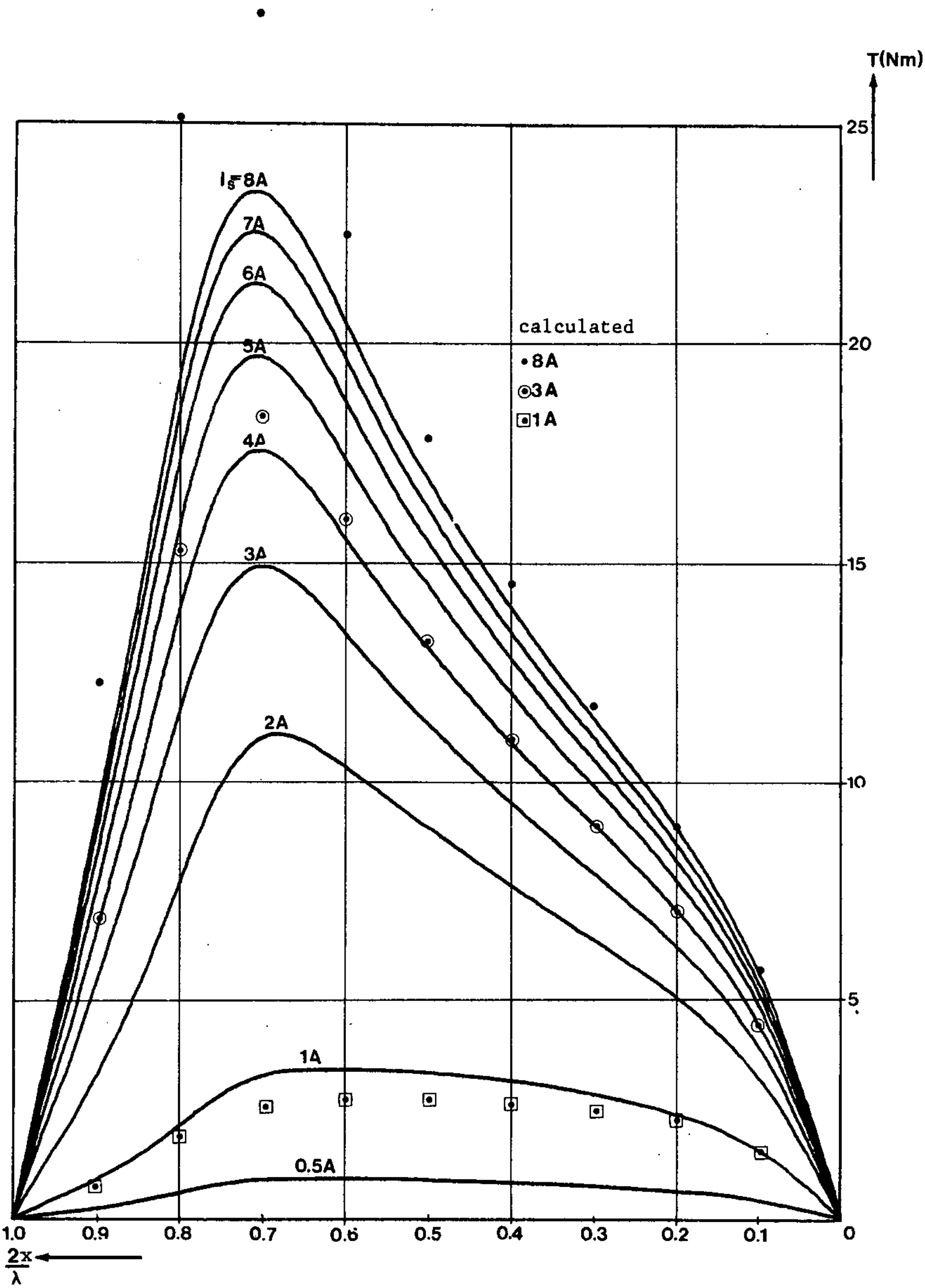


Fig. 39 λ/g=20, t=3/8 λ

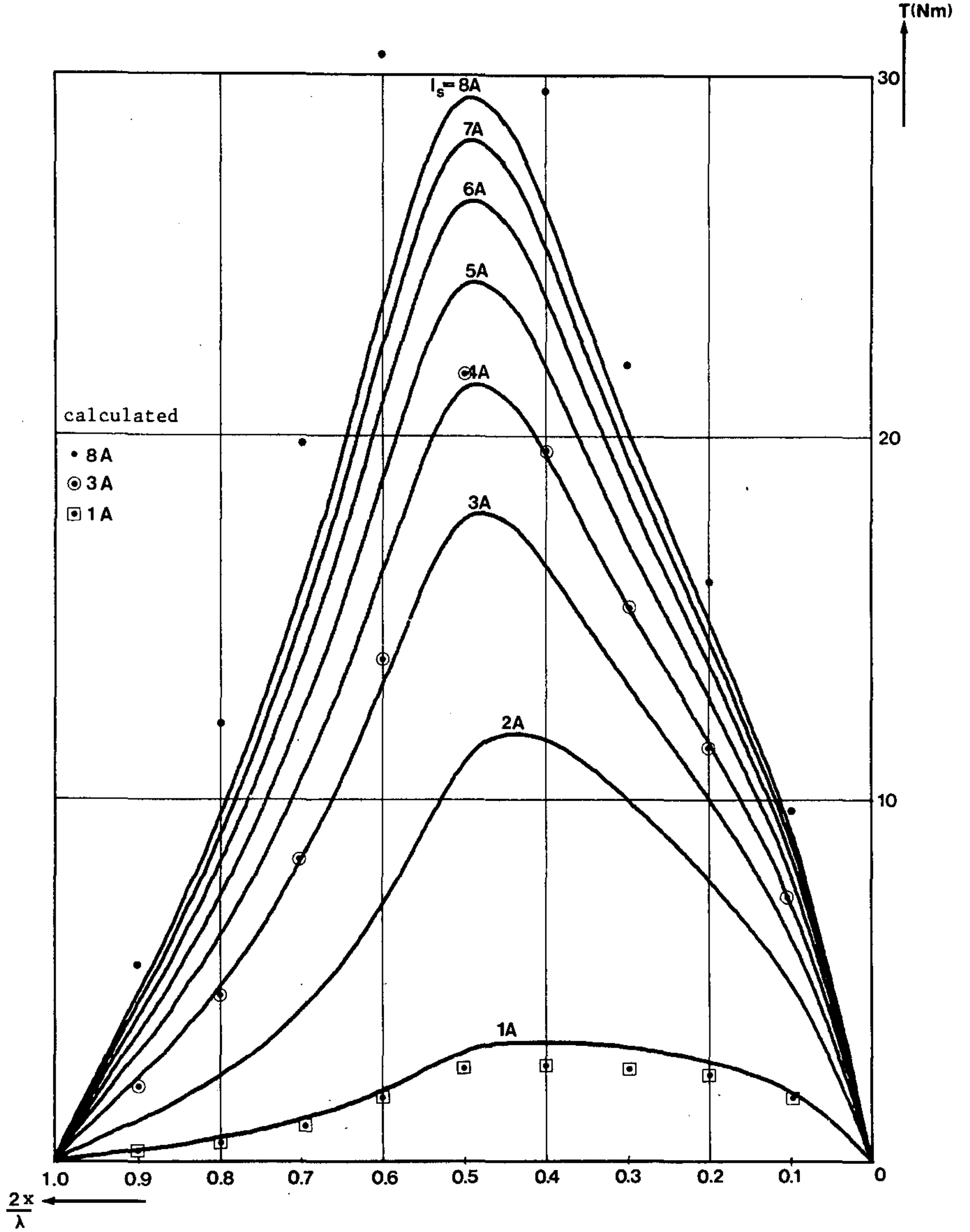


Fig. 40 λ/g = 20, t = 1/4 λ

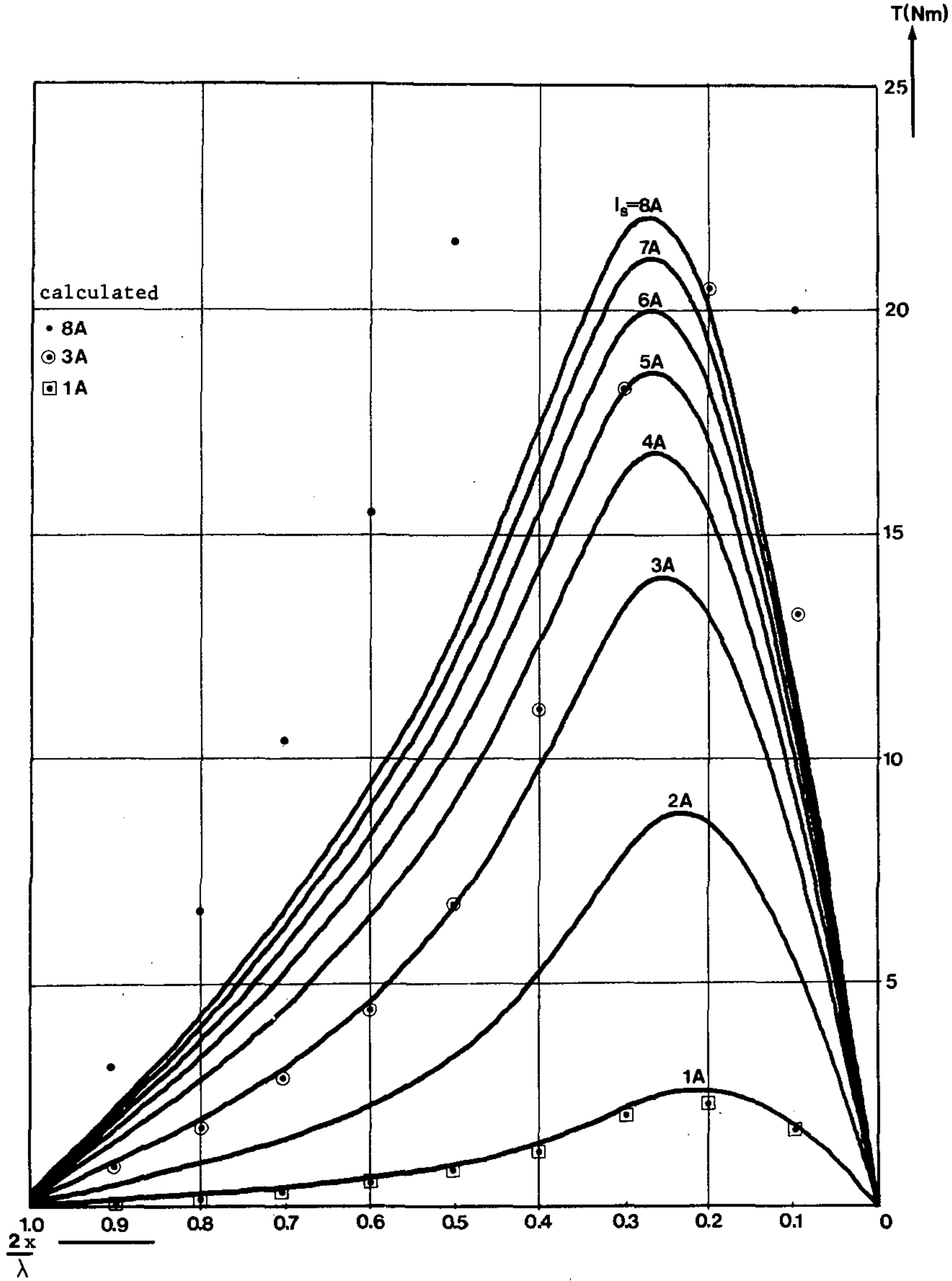


Fig. 41 $\lambda/g = 20$, $t = 1/8\ \lambda$

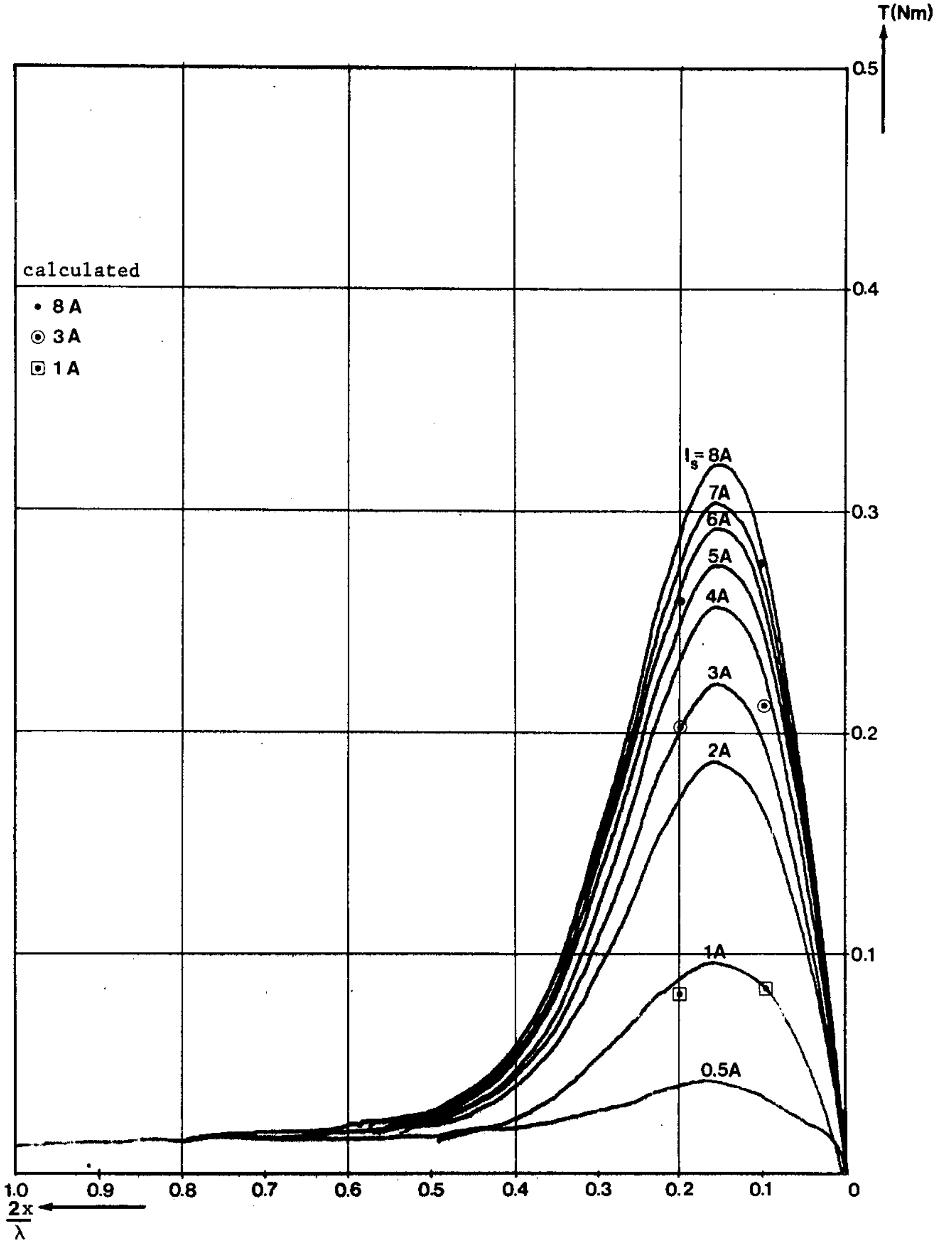


Fig. 42 λ/g = 10, t = 7/8 λ

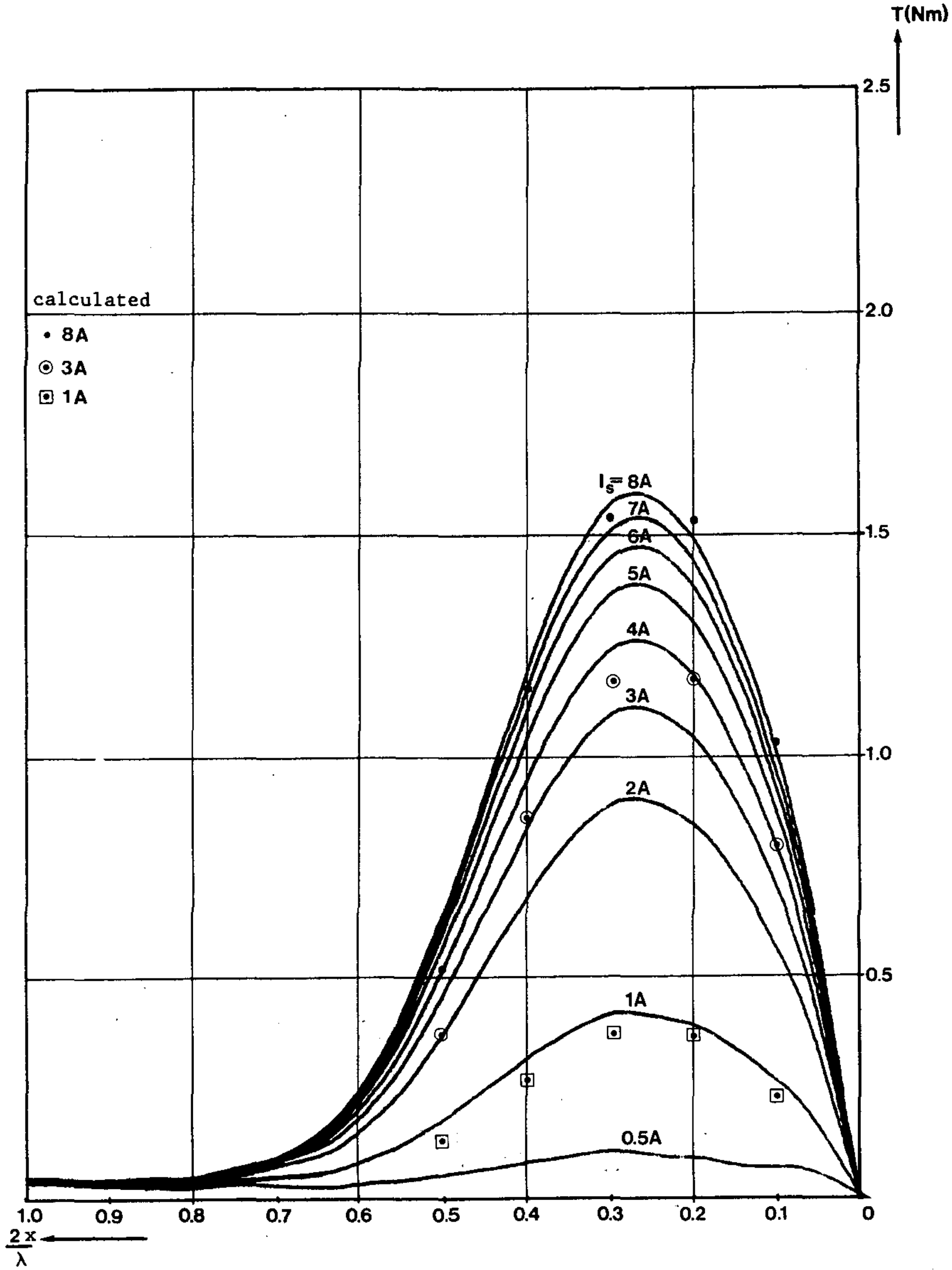


Fig. 43 $\lambda/g = 10$, $t = 3/4\ \lambda$

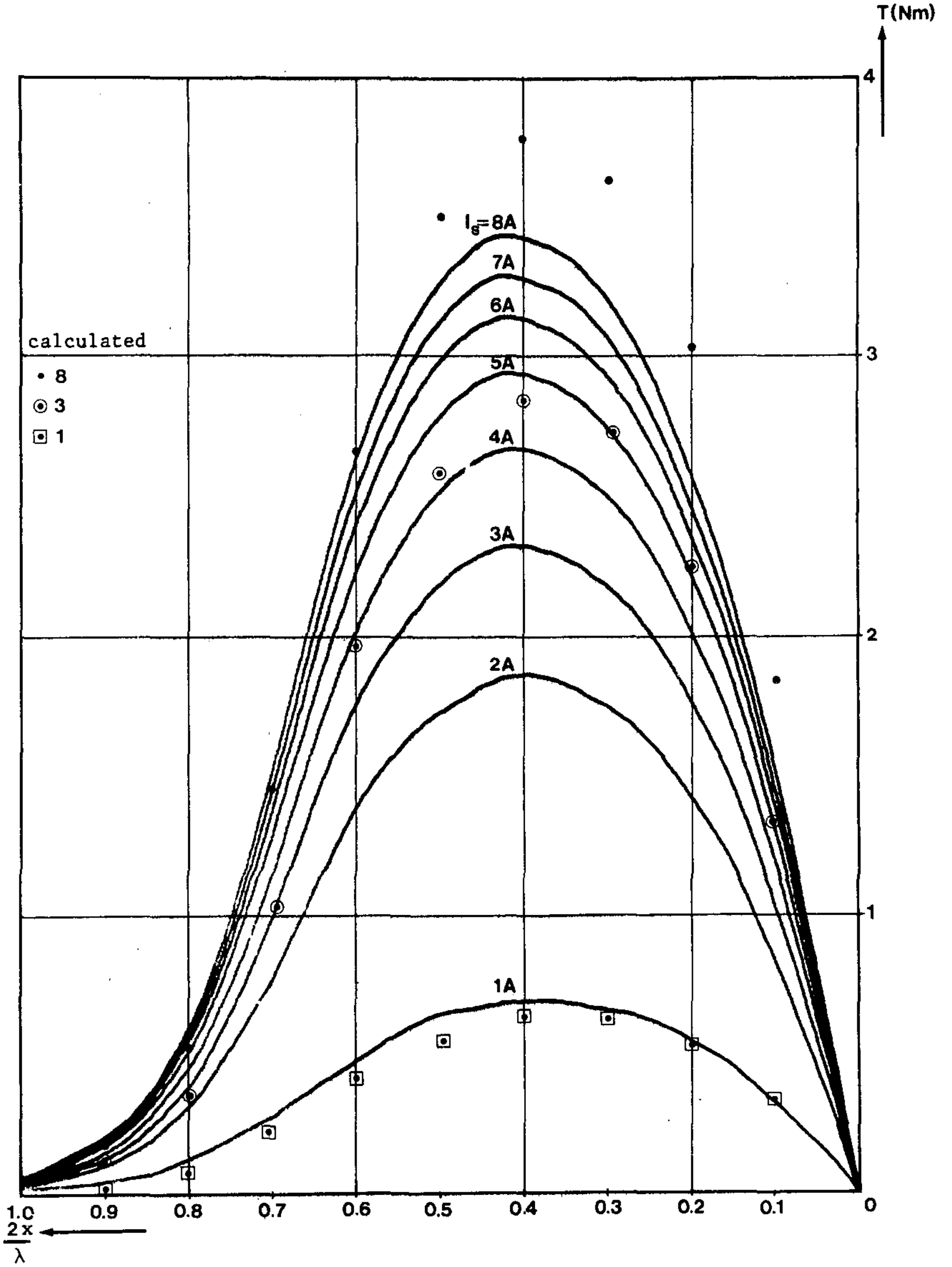


Fig. 44 λ/g=10, t=5/8 λ.

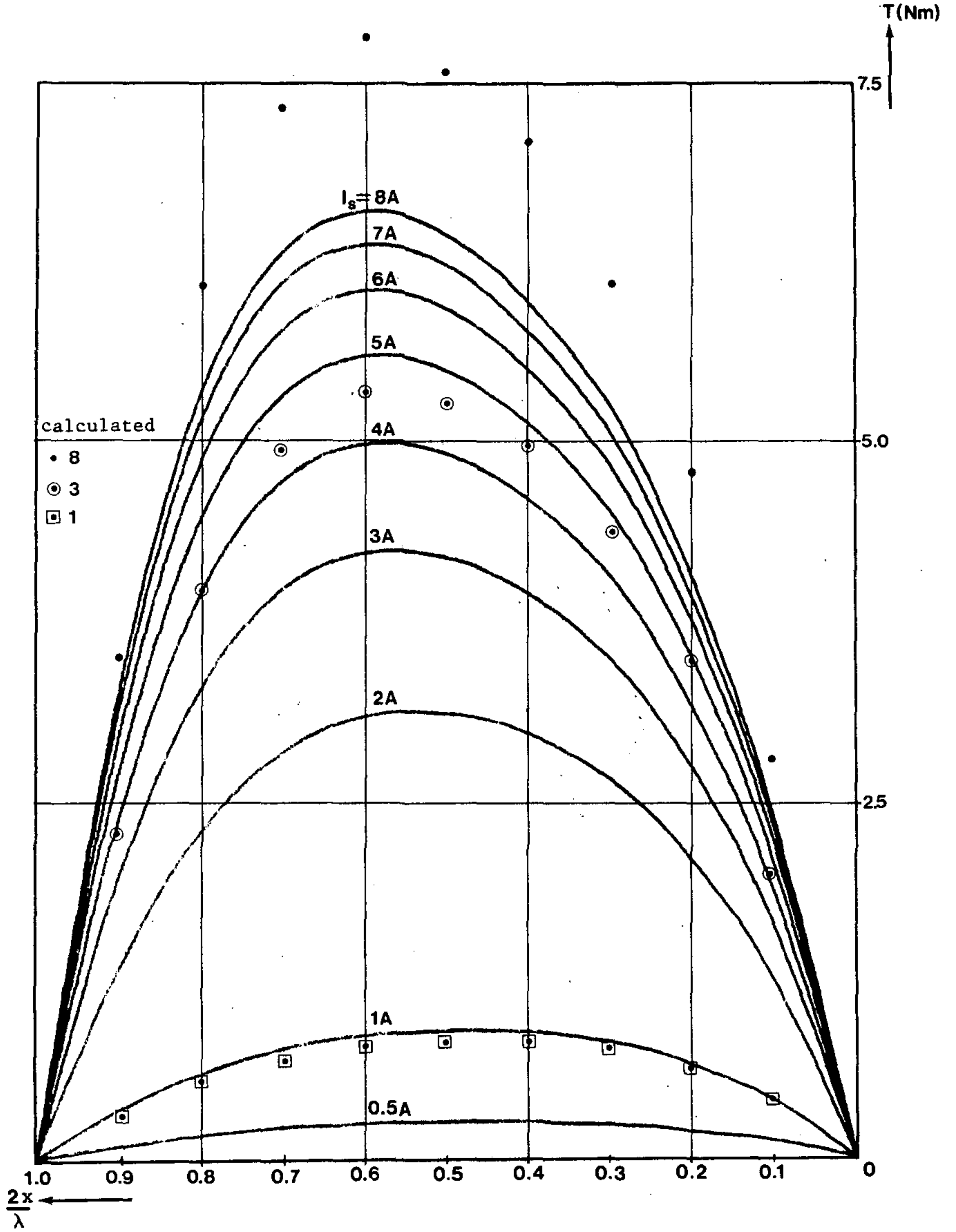


Fig. 45 λ/g = 10, t = 1/2 λ.

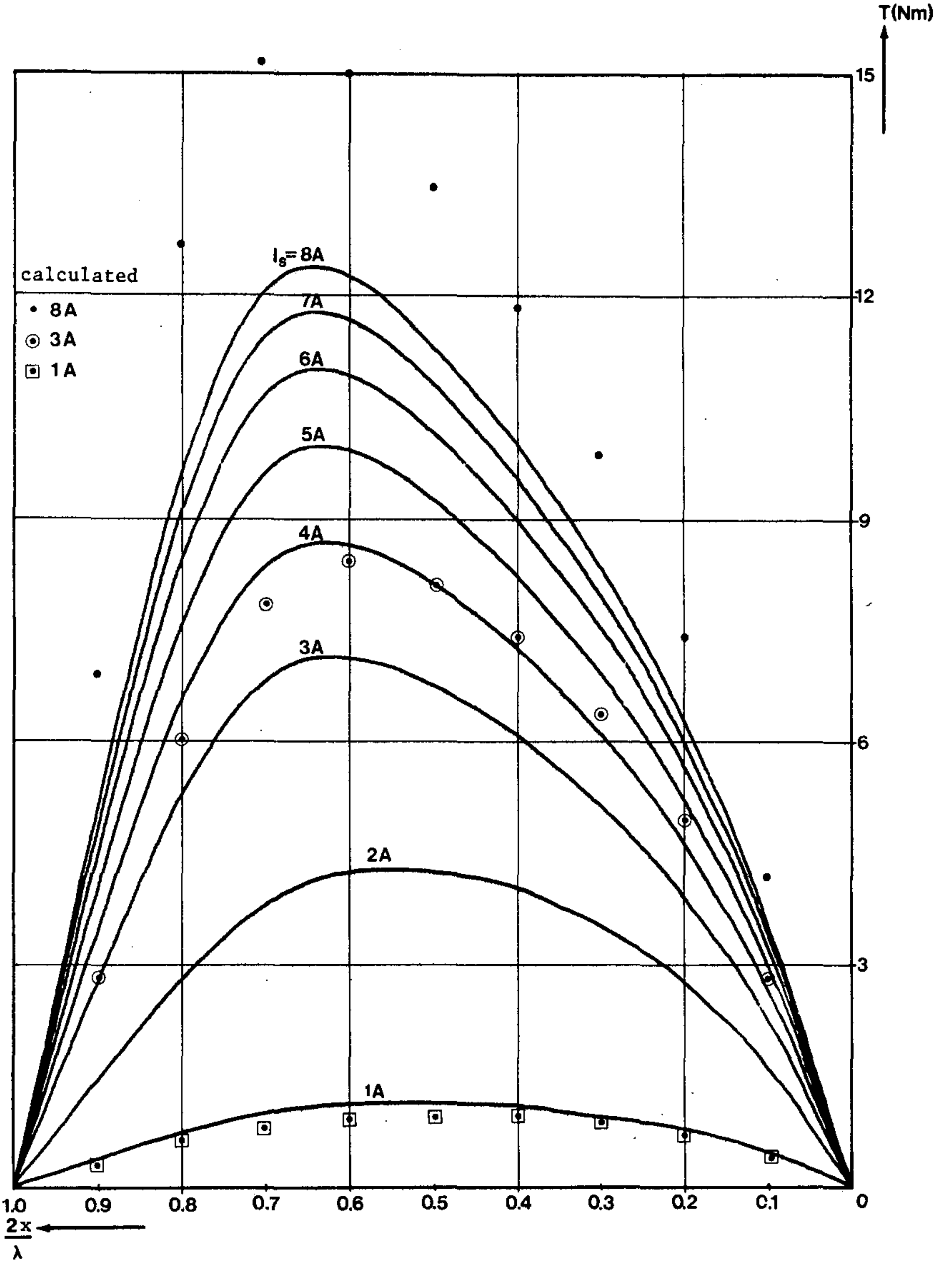


Fig. 46 λ/g = 10, t = 3/8 λ

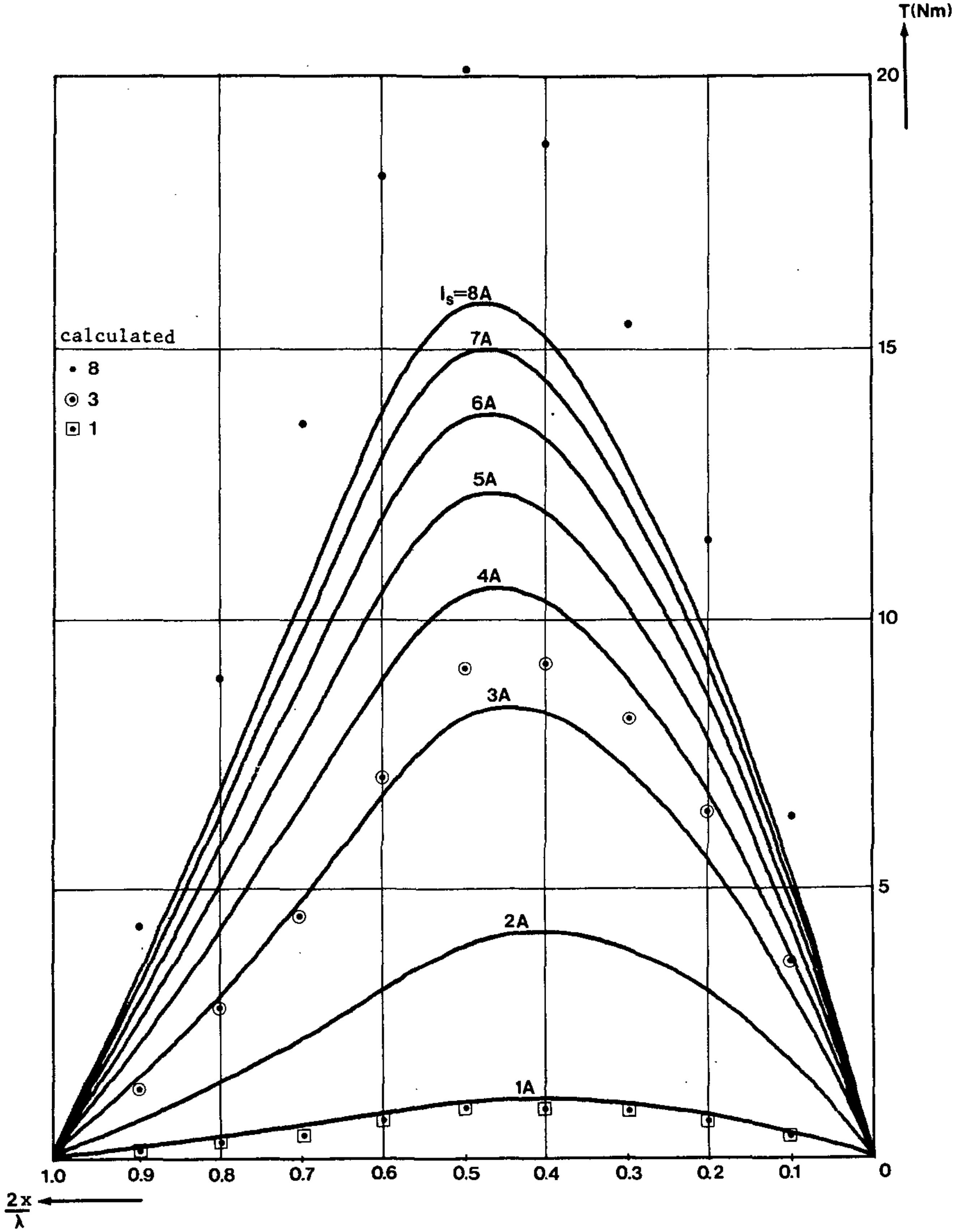


Fig. 47 $\lambda/g = 10$, $t = 1/4\ \lambda$

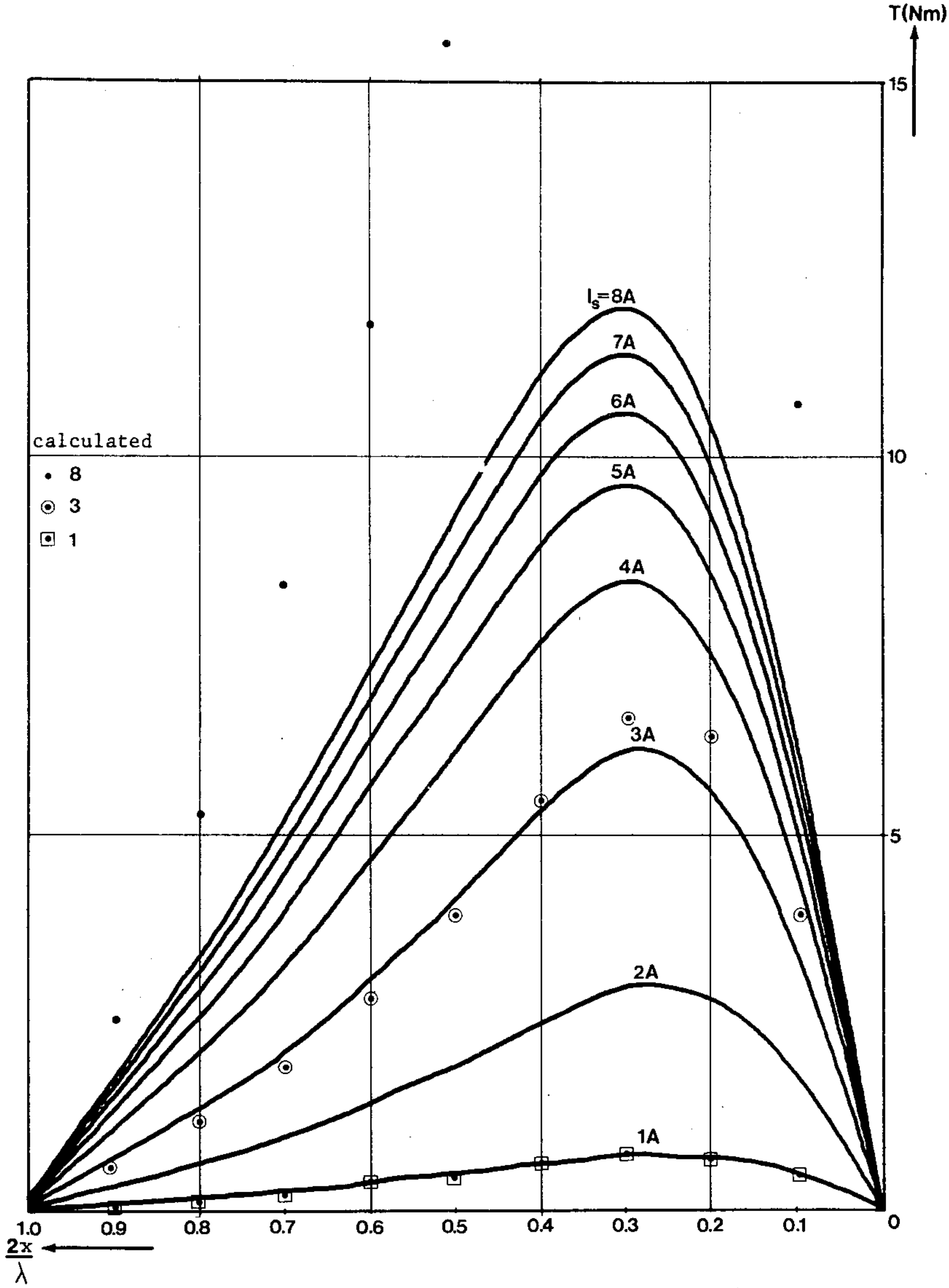


Fig. 48 $\lambda/g = 10$, $t = 1/8\ \lambda$

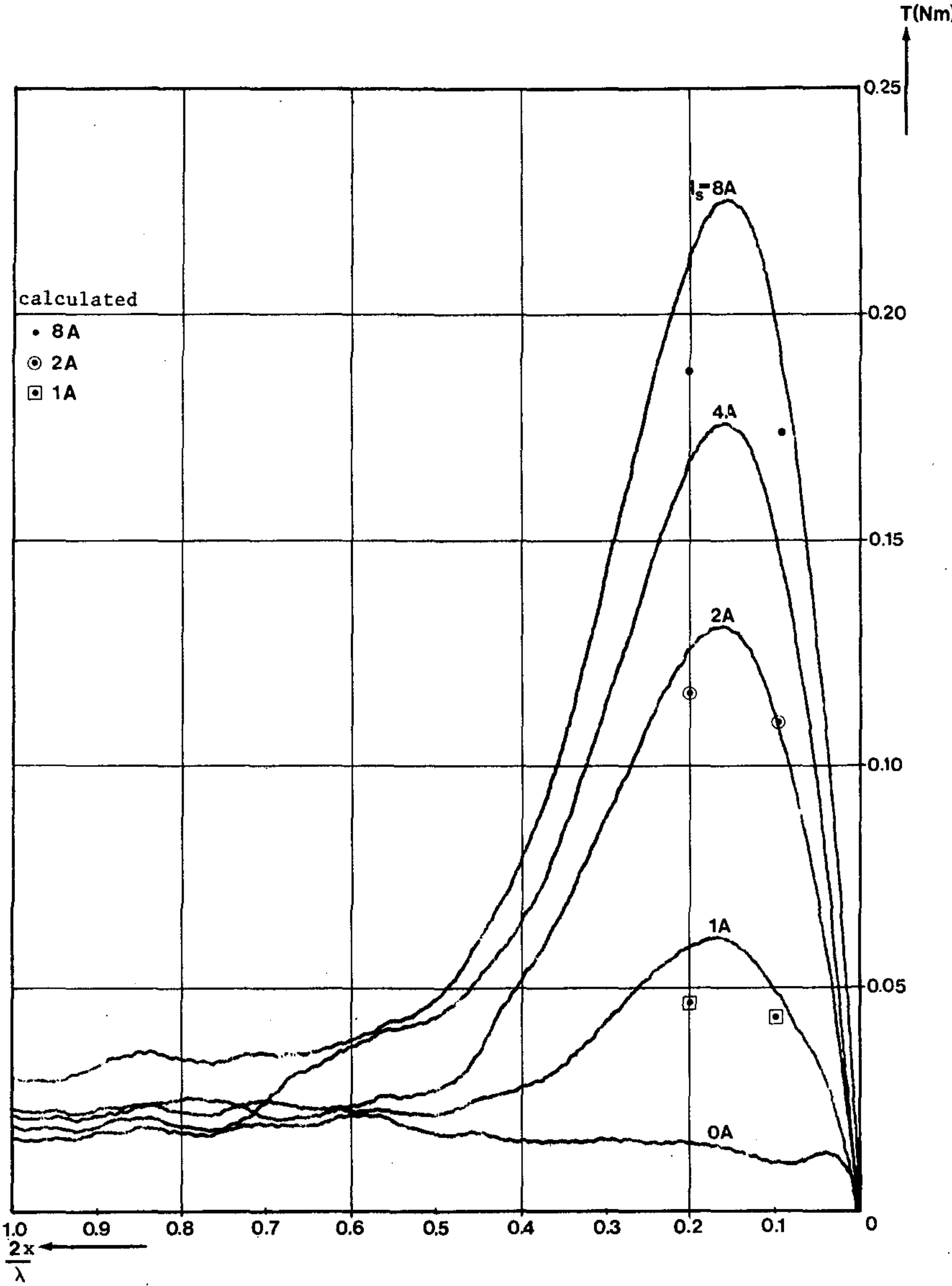


Fig. 49 $\lambda/g = 8.05$, $t = 7/8\ \lambda$

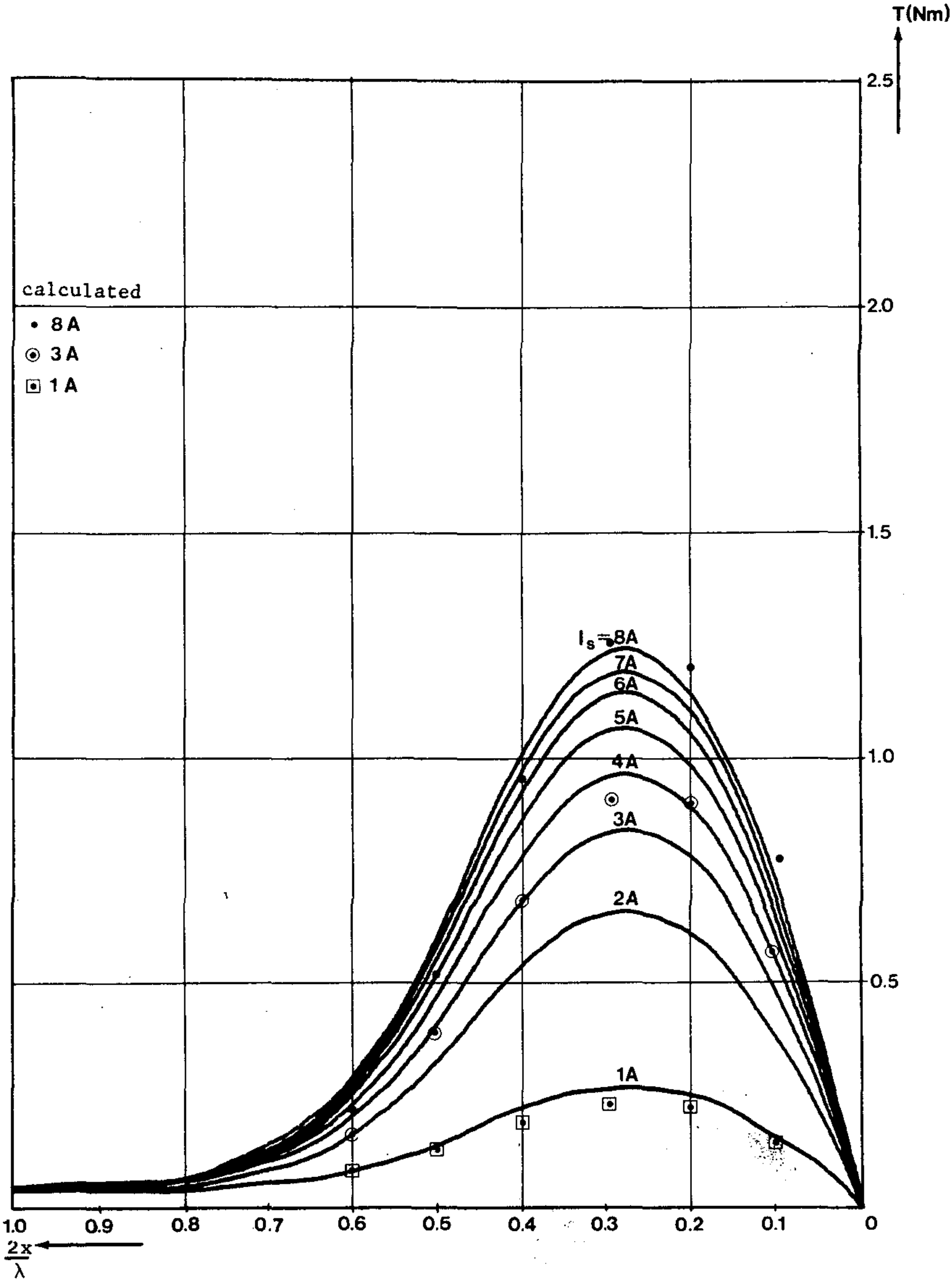


Fig. 50 $\lambda/g = 8.05$, $t = 3/4\ \lambda$

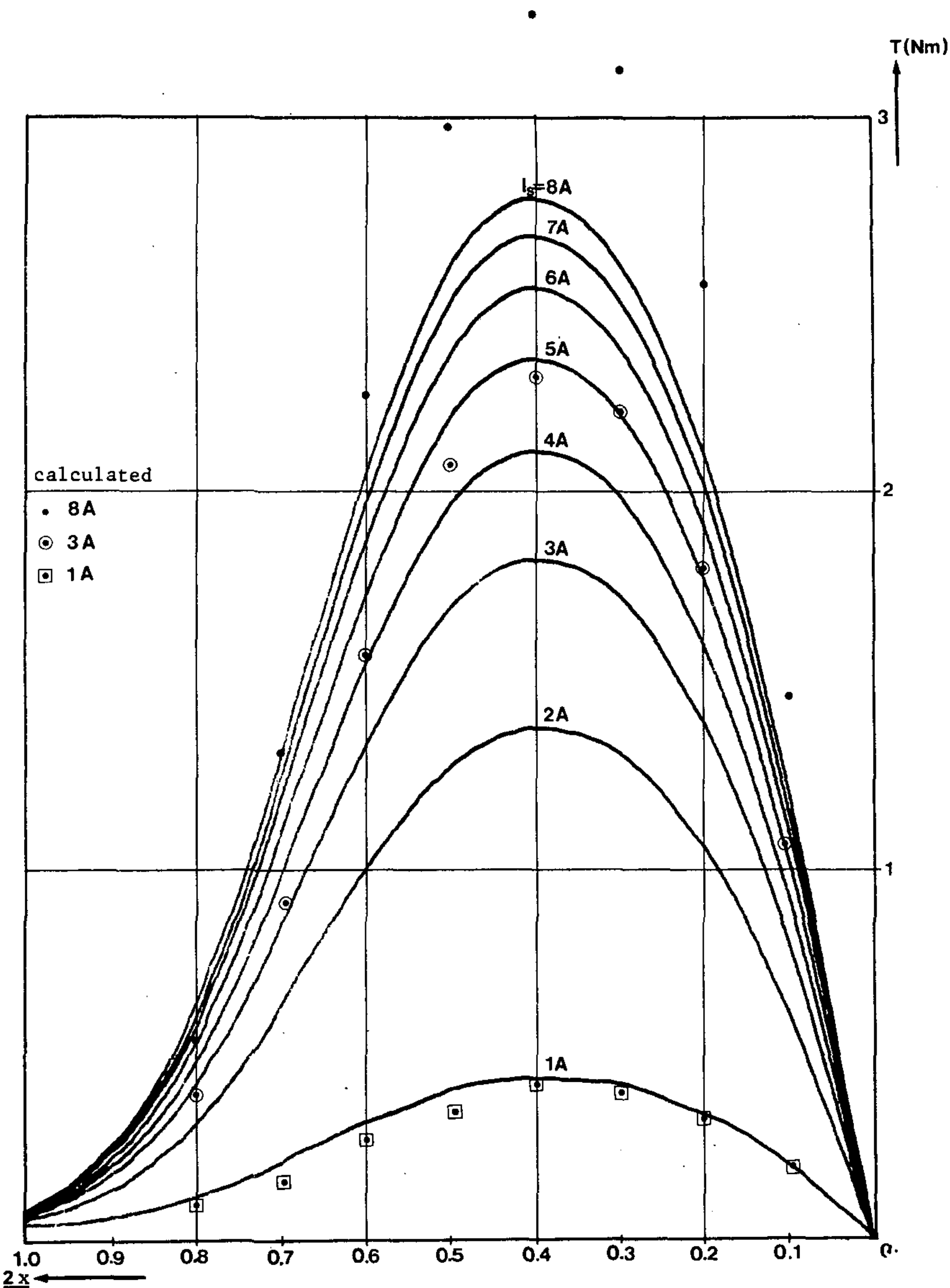


Fig. 51 λ/g = 8.05, t = 5/8 λ

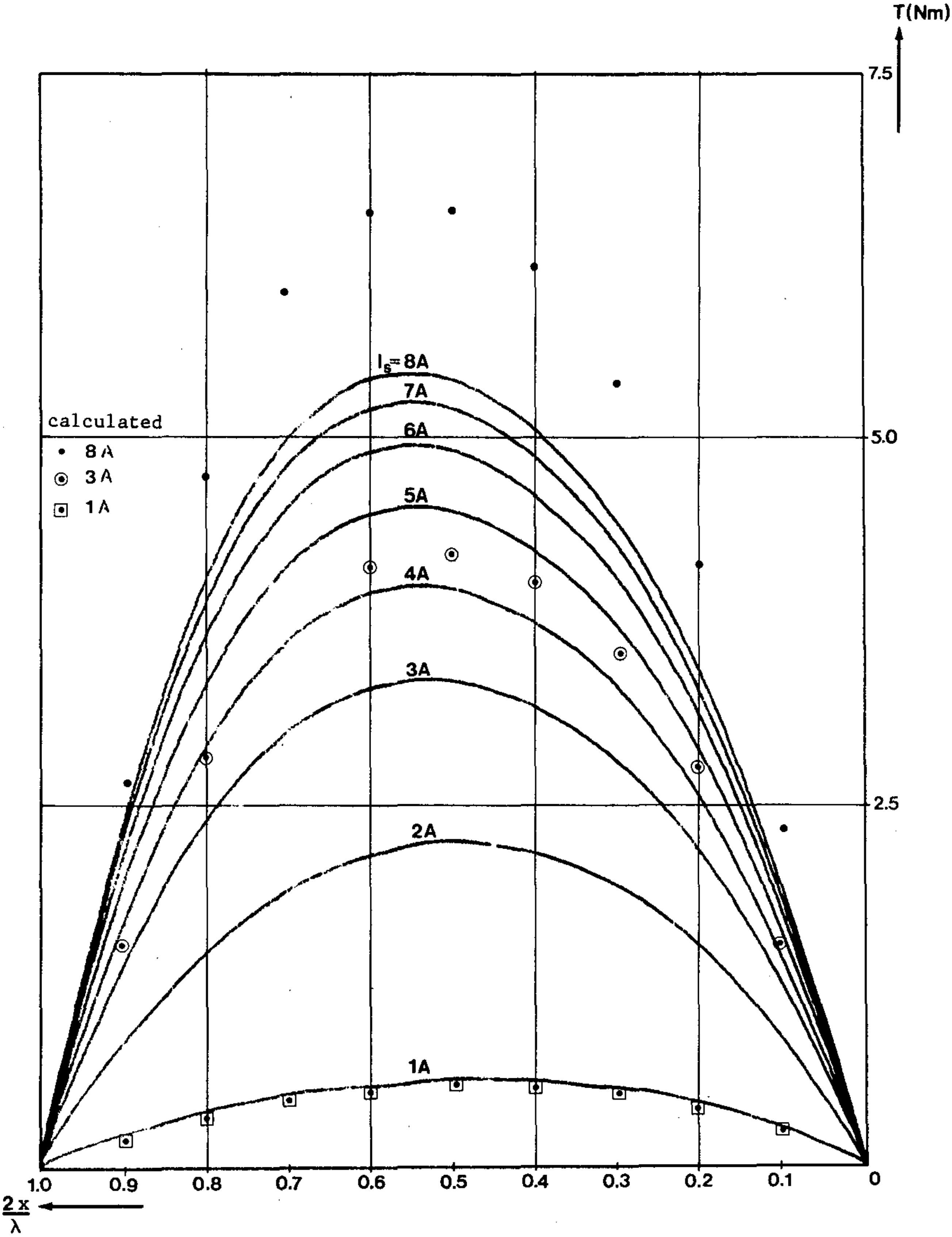


Fig. 52 λ/g = 8.05, t = 1/2 λ.

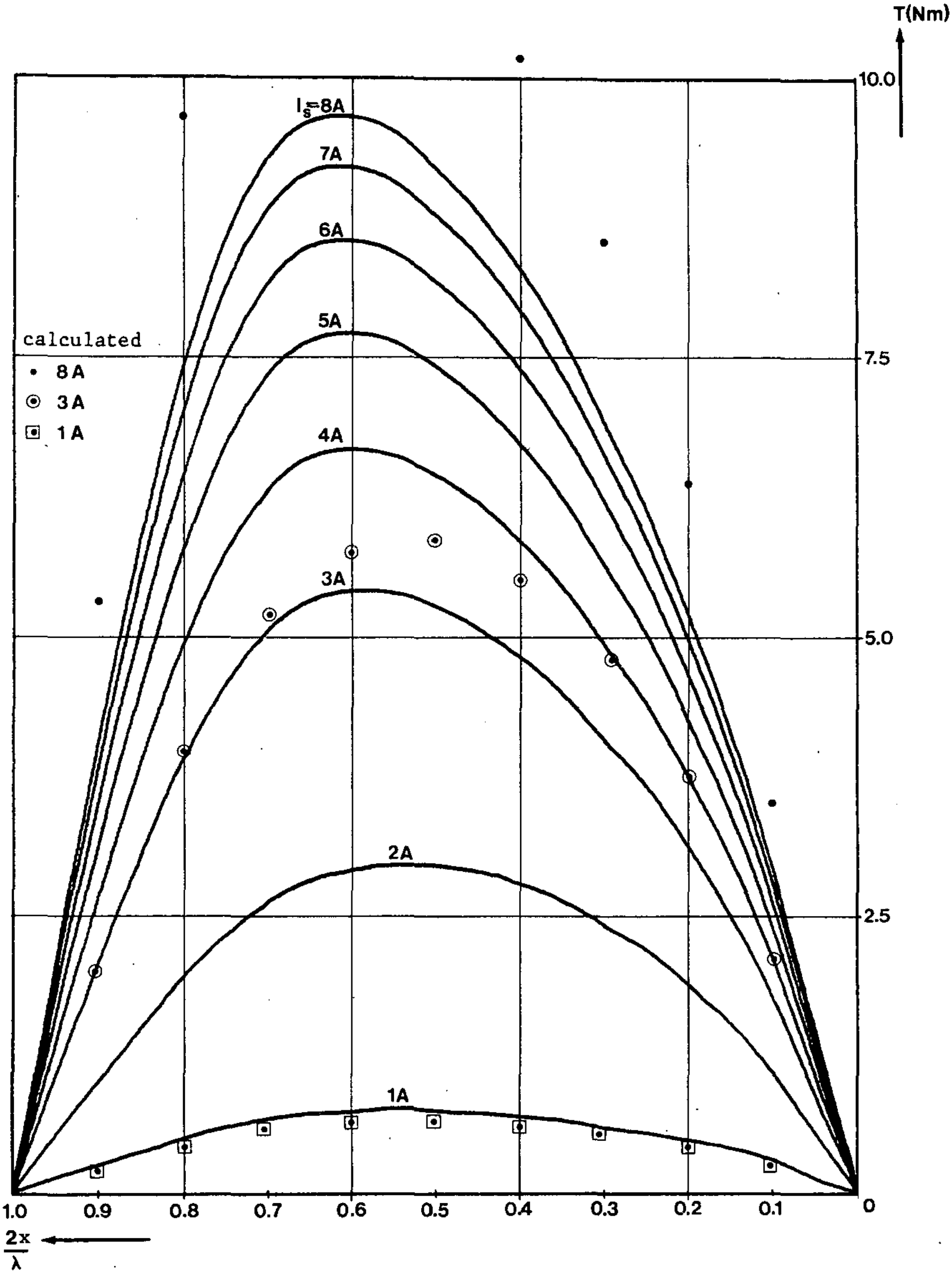


Fig. 53 $\lambda/g = 8.05, t = 3/8\,\lambda$

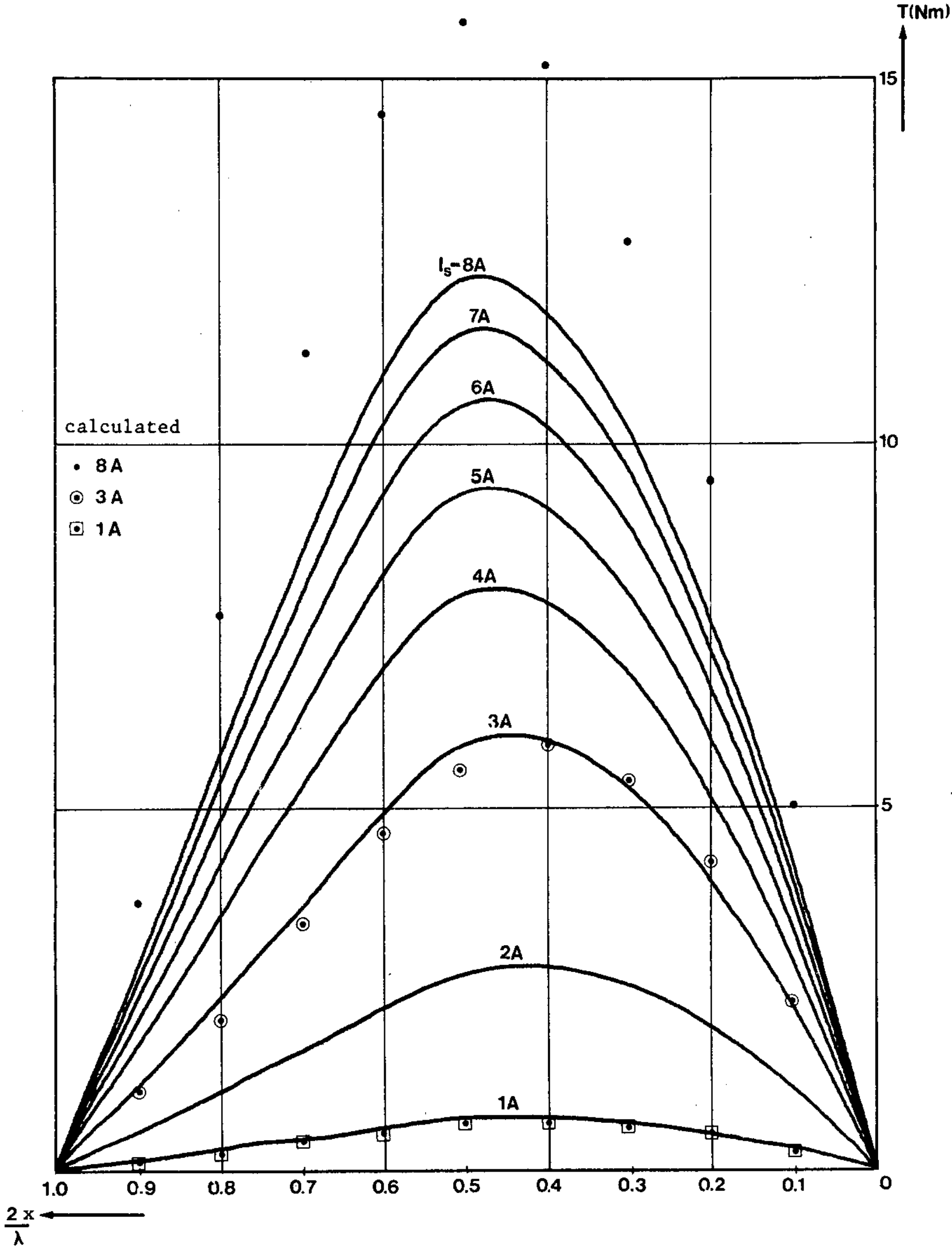


Fig. 54 λ/g = 8.05, t=1/4 λ

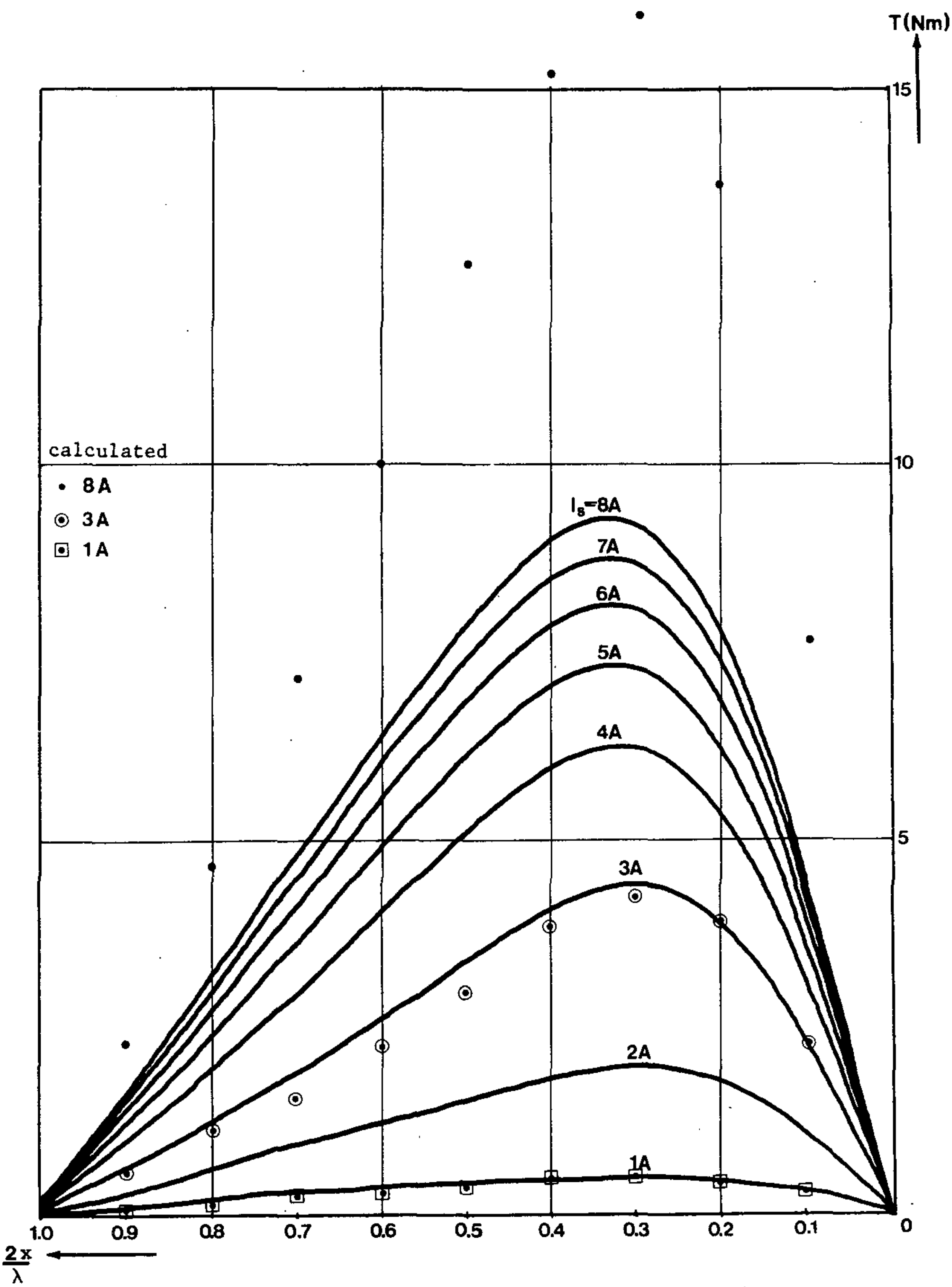


Fig. 55 $\lambda/g = 8.05$, $t = 1/8\ \lambda$

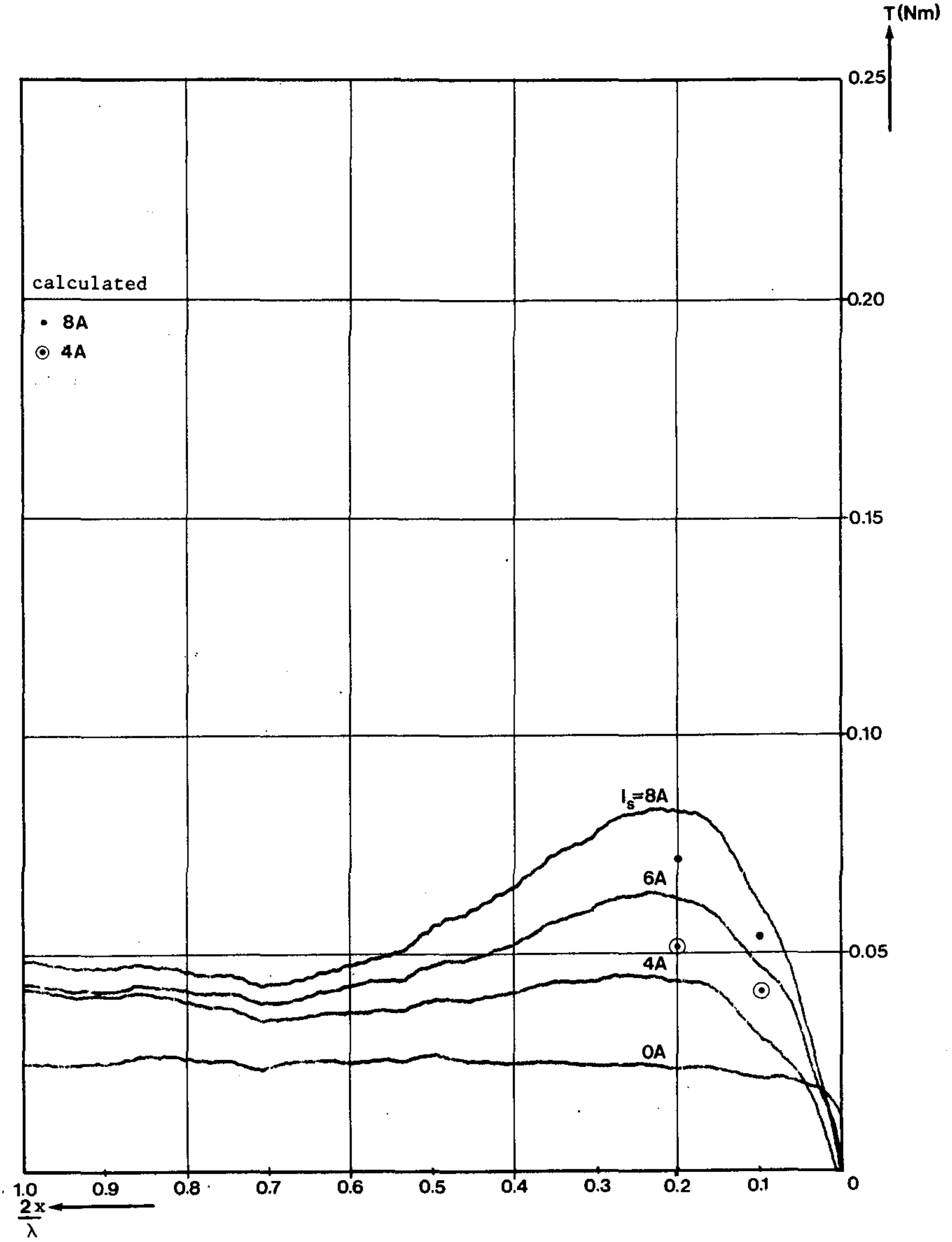


Fig. 56 $\lambda/g = 5$, $t = 7/8\ \lambda$

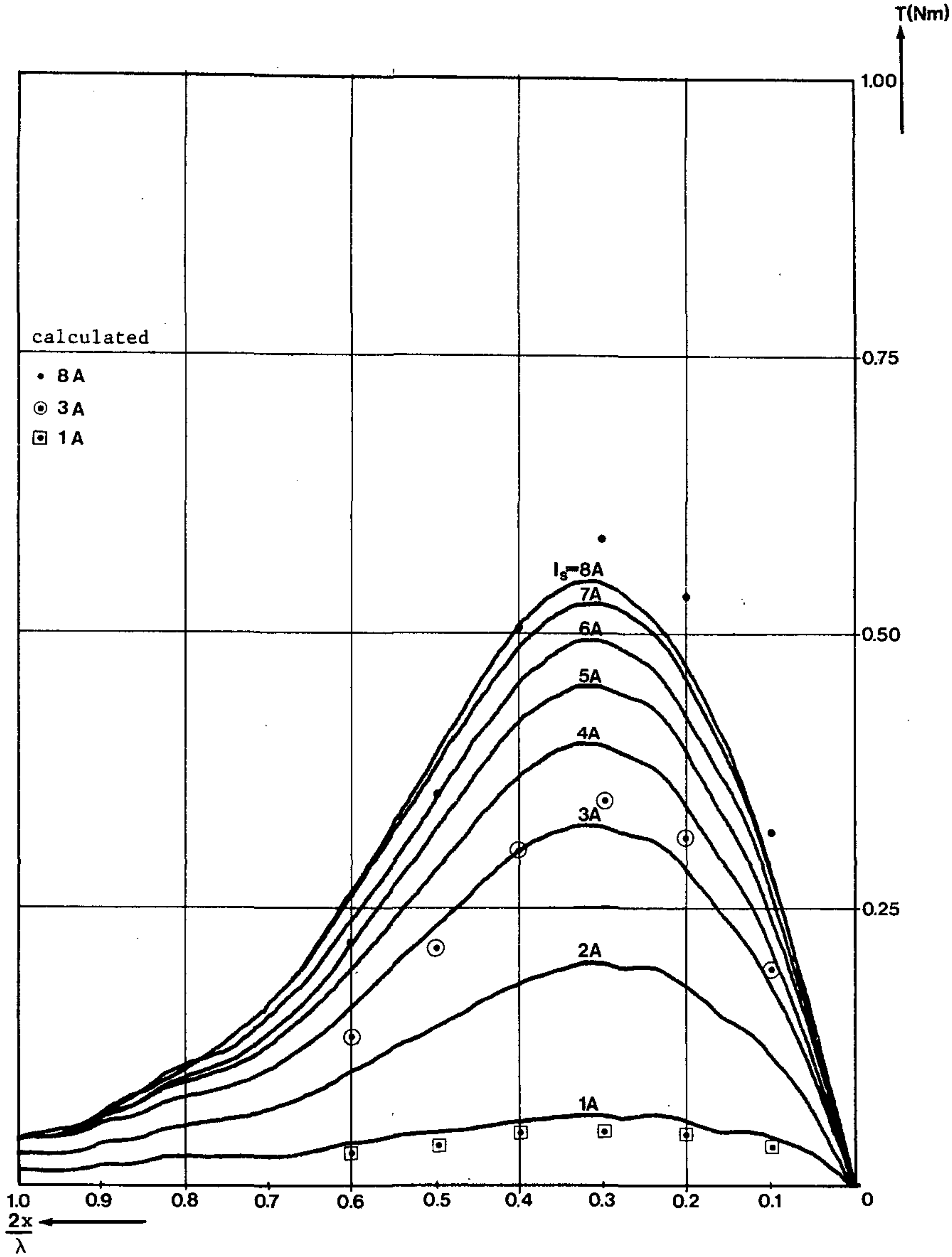


Fig. 57 λ/g = 5, t = 3/4 λ

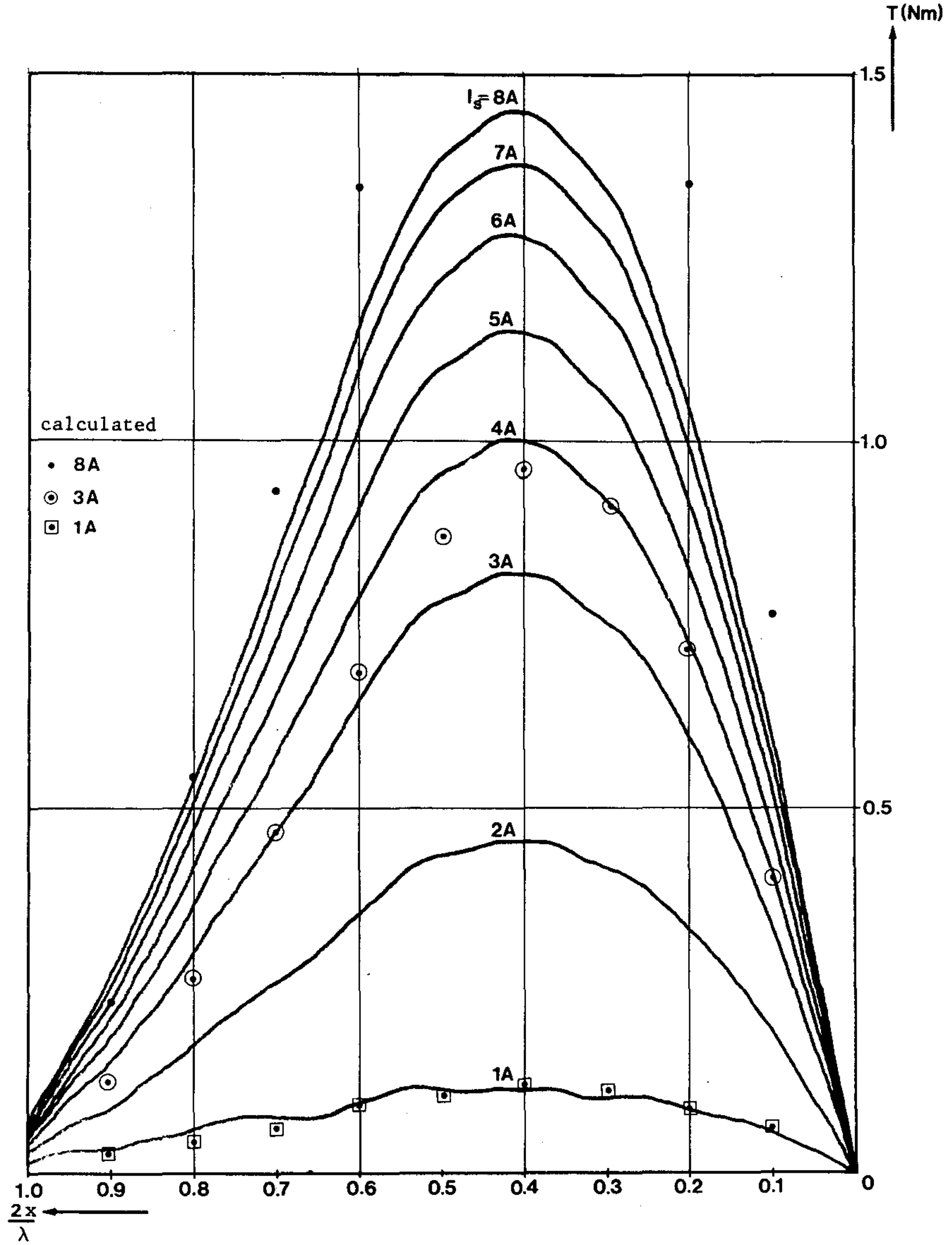


Fig. 58 $\lambda/g = 5$, $t = 5/8\ \lambda$

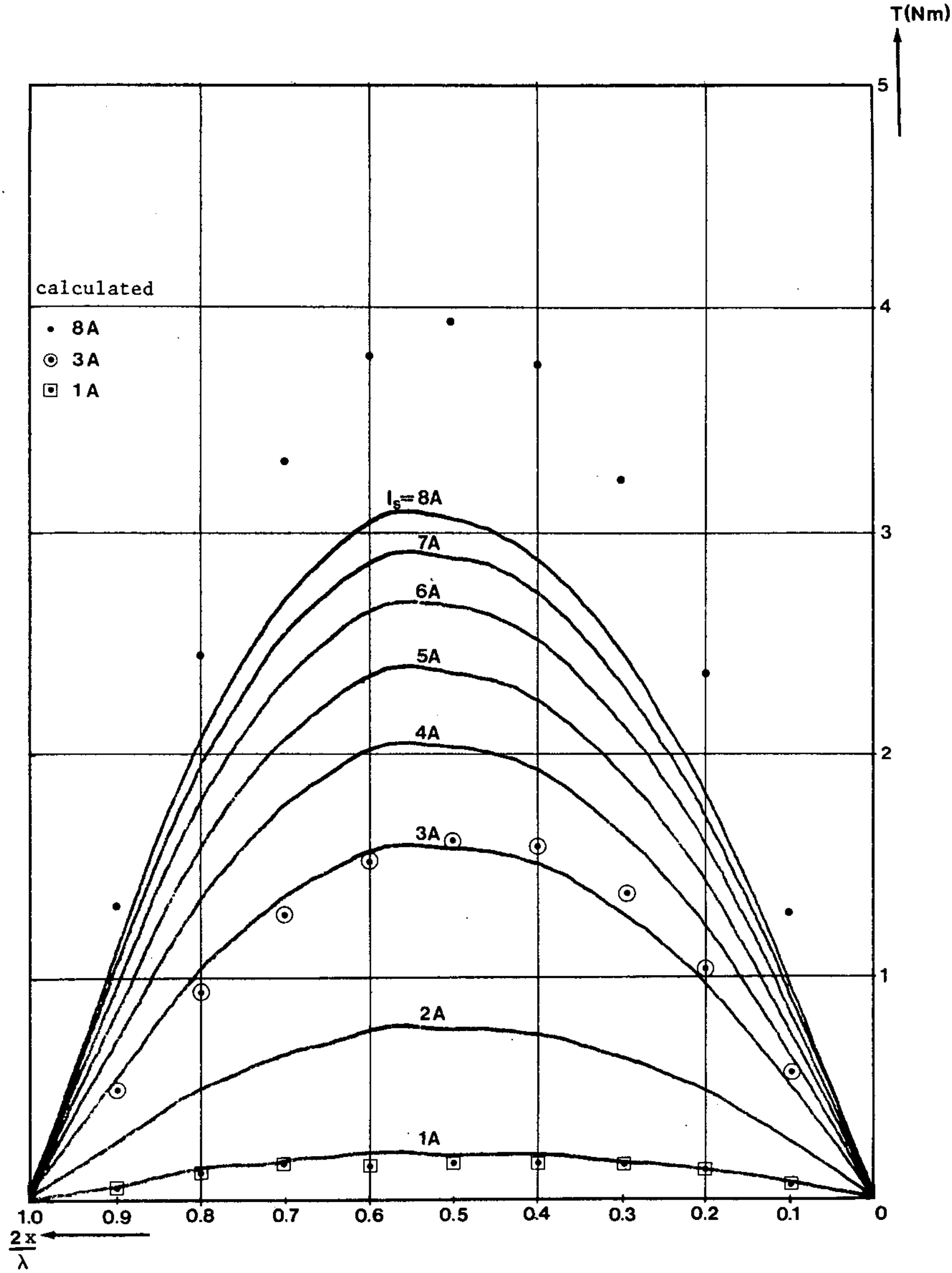


Fig. 59 $\lambda/g = 5$, $t = 1/2\ \lambda$.

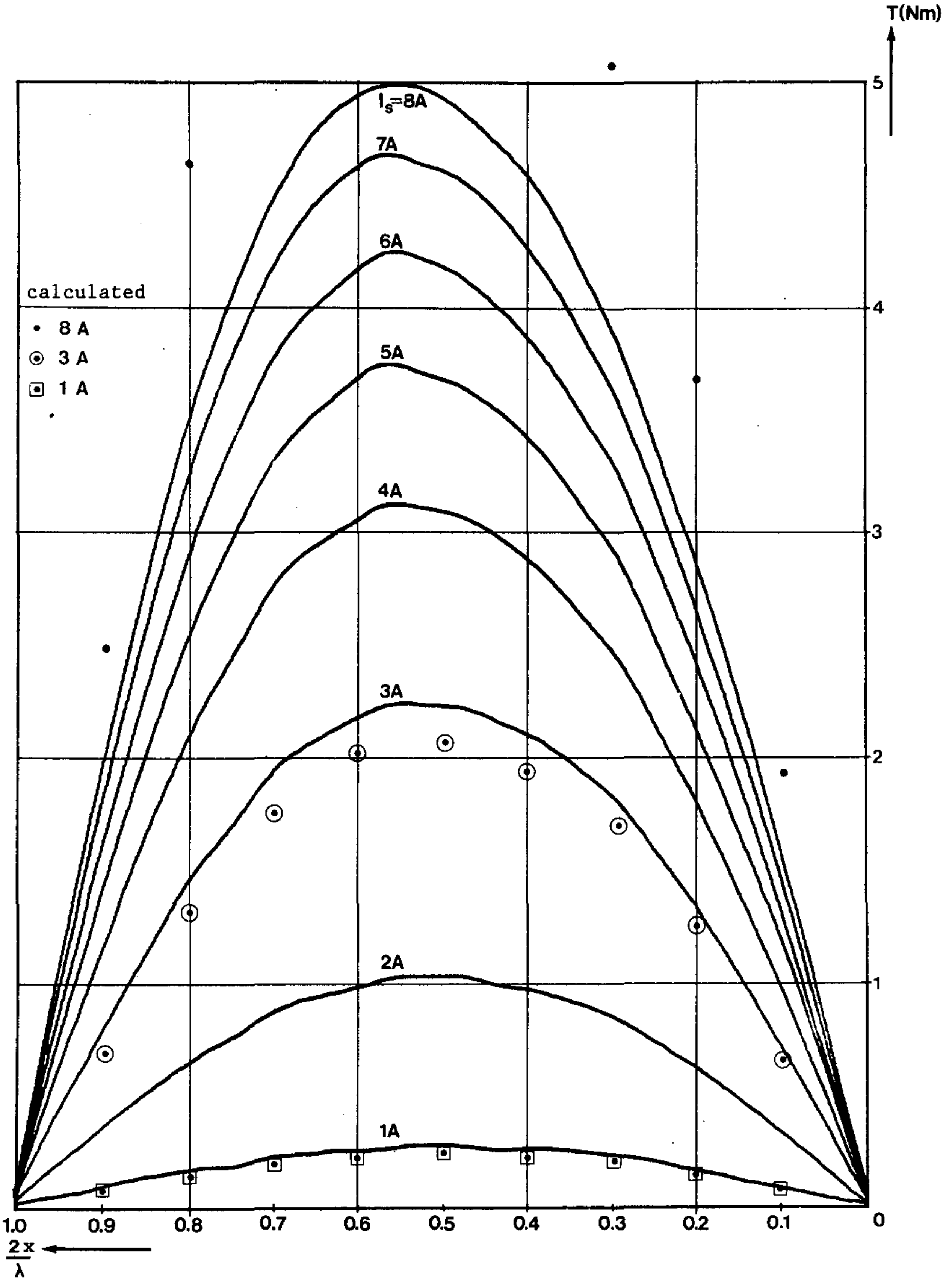


Fig. 60 $\lambda/g = 5, t = 3/8\ \lambda$

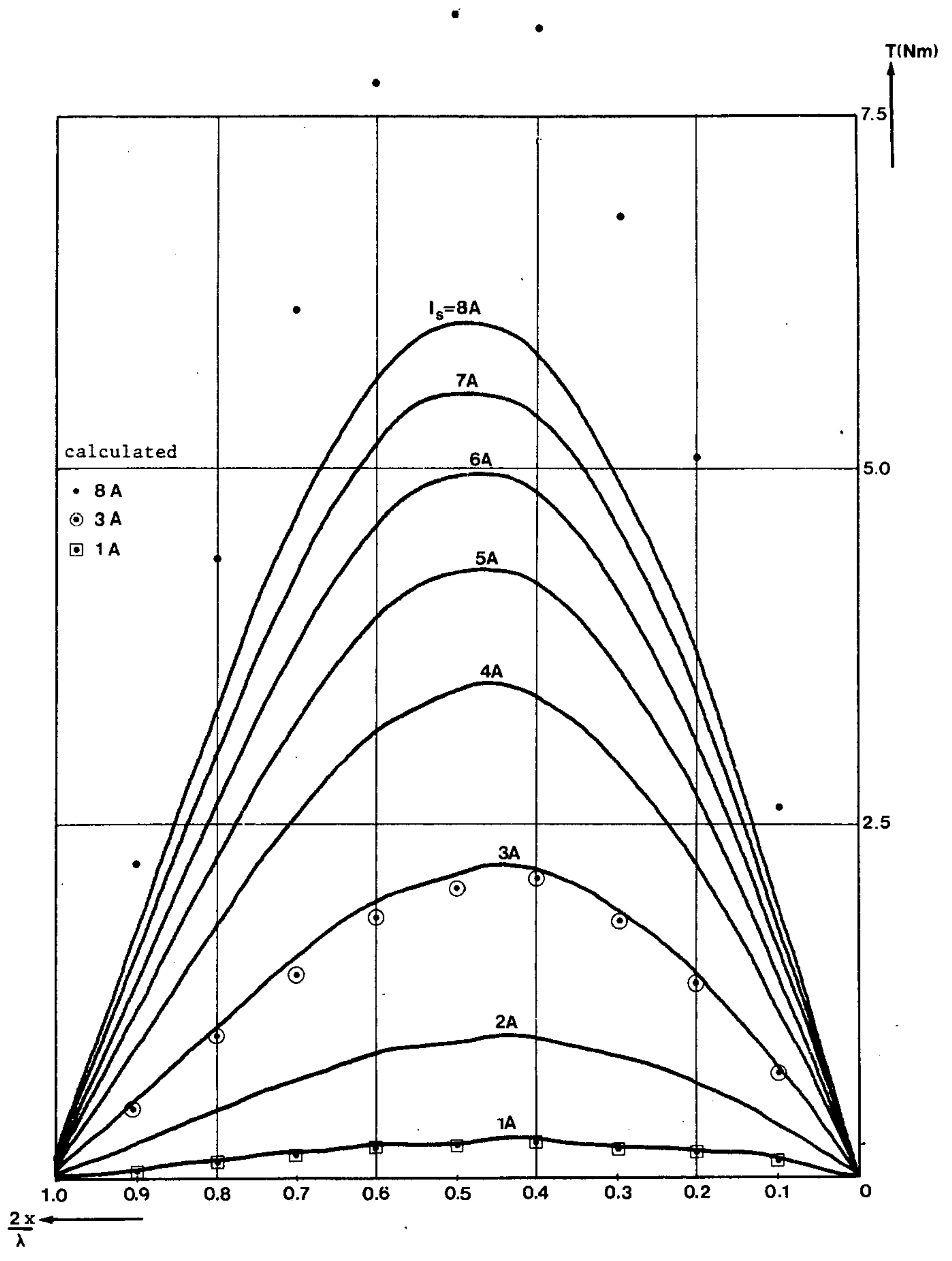


Fig. 61 λ/g=5, t=1/4 λ

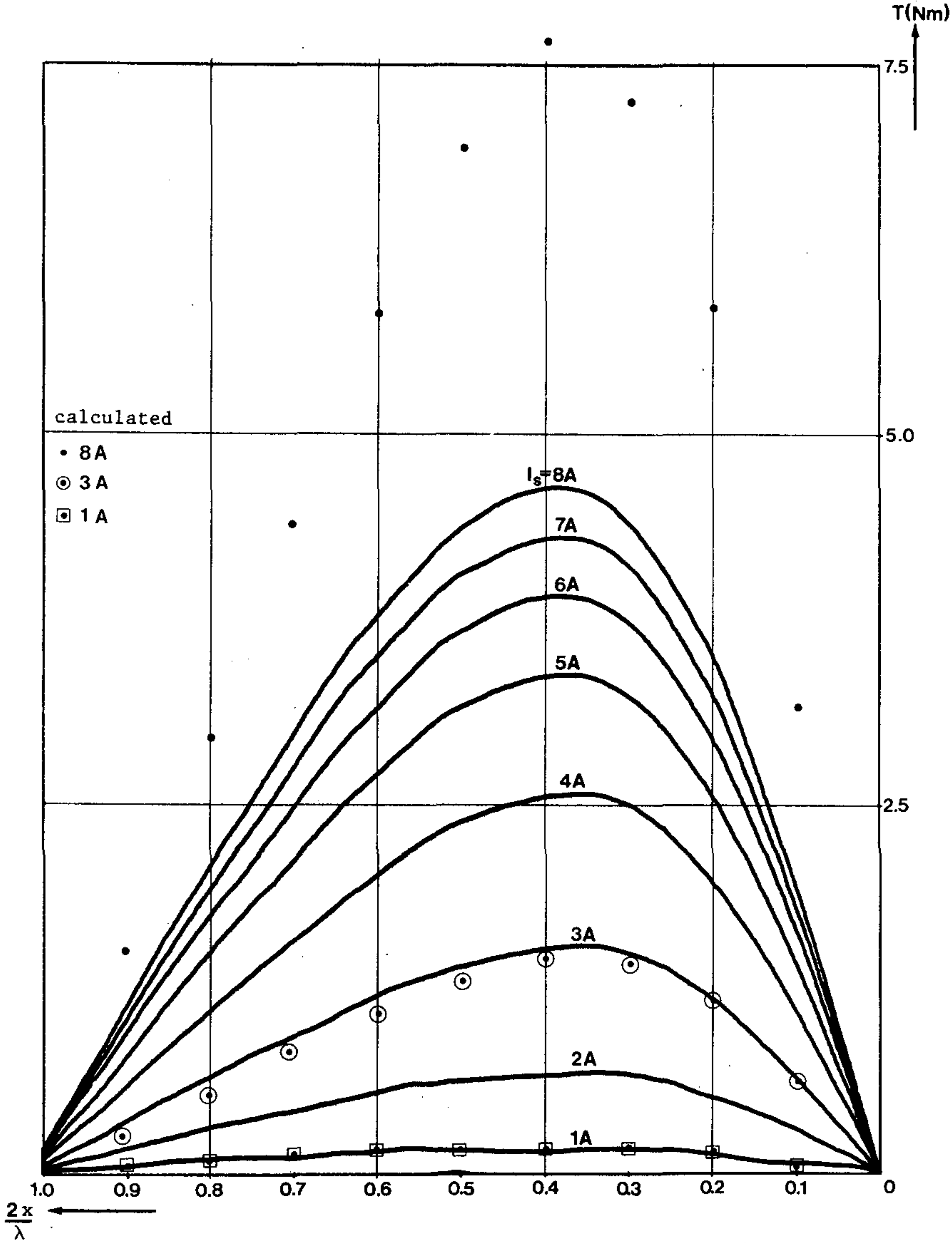


Fig. 62 $\lambda/g = 5$, $t = 1/8\ \lambda$

3. COMPARISON OF MEASUREMENTS AND CALCULATIONS AND CONCLUSIONS

In this chapter measurements and calculations are compared with each other and from these results conclusions are drawn.

3.1. Comparison of measurements and calculations

3.1.1. Idealised iron.

For very low values of the excitation current I_s, ($I_s \leq 1$ A), one can say that there is no saturation and that the iron behaves in an ideal way.
By calculating the torque for the measuring device from tables 3 - 7 for ideal iron and $I_s = 1$ A and comparing these with values calculated for normal iron in tables 14 - 18 for $I_s = 1$ A one can see that there is a very good agreement between the two.
Figures 28 - 62 show a very good agreement between calculated and measured values for $I_s = 1$ A.

3.1.2. Normal iron.

For values of the excitation current I_s larger than 1 A the influence of the iron begins to play a part. Comparison of measurements and calculations shows differences that, for decreasing toothwidths and decreasing λ/g keep increasing.
These differences are mainly attributed to the leakage fluxes that have been neglected in the calculations.
In [6] the influences of the leakage fluxes are estimated for the same kind of measuring device as was used for our investigations, for a toothwidth $t = \frac{1}{2}\lambda$. It was shown that for a decreasing λ/g ratio the influence of the leakage fluxes keeps increasing and gives decreasing force values. For, for instance, $\lambda/g = 8$, [6] gives a ratio of leakage flux to working flux of 13%, in the measuring device.

In our case we see that for a certain toothwidth the differences between calculations and measurements increase with decreasing λ/g values. This could indicate the influence of the leakage fluxes. These fluxes are detrimental to the exerted force because they contribute to the saturation of the rotorneck but not to the force. The influence of these leakage fluxes will be a case for further investigation.

3.1.3. Normal iron with teeth saturation.

The very large deviations between measurements and calculations (max. about 100%) for a toothwidth $t = \lambda/8$ and $\lambda/g = 40$, 20 and 10 are attributed to the occurrence of saturation of the teeth.
Exact calculations for all positions of the rotors with respect to the stator were not carried out owing to very high costs of computation.

3.2. Conclusions

The endeavour of this investigation was to find a teeth configuration that was as good as possible.
It will be clear that, if the excitation permits, the airgap induction should be as high as possible. The tables and figures of chapter 2 show the performances with various parameters and, depending on the application, the most suitable set can easily be selected.
Important appears to be the permeance of the iron part of the magnetic circuit. If this sets the limit to the flux, one should aim at an induction in the teeth close to saturation, if necessary by designing with shorter teeth, if the leakage fluxes permit.

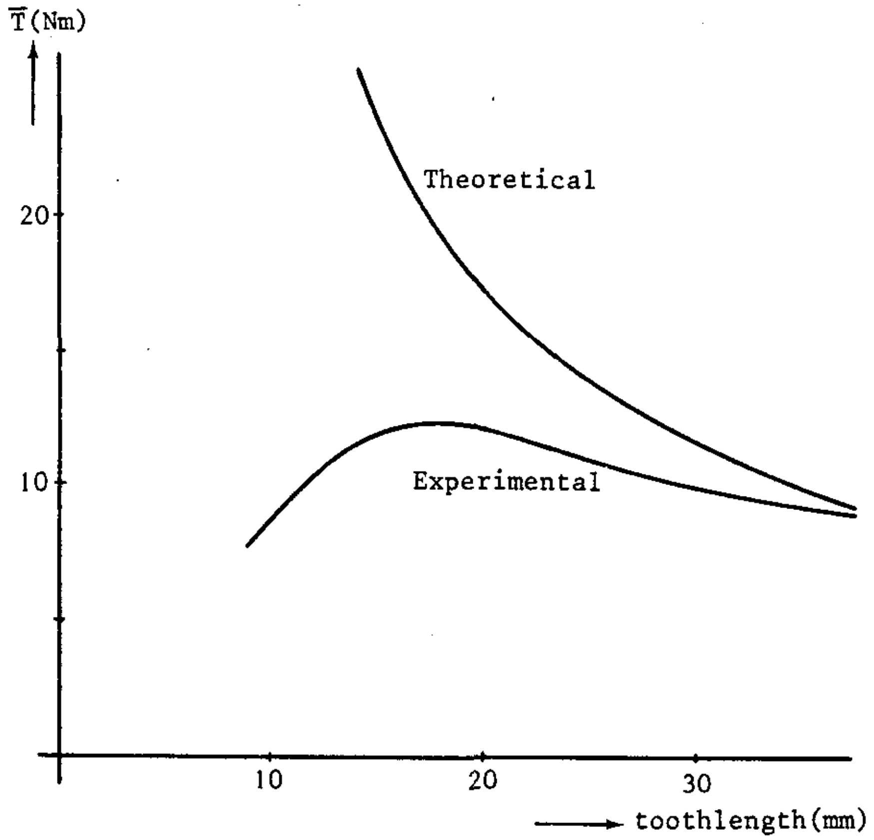


Fig. 63 Influence of toothlength.

As an example we refer to figure 63 from [11], showing the torque production in dependence of the toothlength, of a reluctance stepping motor when the total flux is kept constant. The difference of the calculated to the measured results must be attributed to the fact that the calculation did not take into account the leakage fluxes.
If the teeth have a given length, and the iron circuit, as well as the excitation is kept constant, one arrives at the following conclusions, keeping in mind the criteriums mentioned in the introduction.

a An as large as possible average force when varying the displacement from a position with maximum permeance to a position with minimum permeance.
The area of the measured torque-displacement curves for I_s = 8A was determined by means of a planimeter, thus giving the average values. These values are given in table 20 and are drawn in figure 64.

t/λ	λ/g				
	40	20	10	8.05	5
7/8	0.22	0.19	0.10	0.09	0.06
3/4	0.97	0.84	0.65	0.55	0.28
5/8	2.66	2.66	1.90	1.58	0.87
1/2	8.79	7.21	4.66	3.83	2.03
3/8	16.59	13.34	8.14	6.45	3.38
1/4	20.50	15.51	9.65	7.52	3.84
1/8	14.91	11.08	7.03	5.69	2.98

Table 20.

This figure shows that the maximum average torque (and the force) occurs for the smallest airgap (λ/g = 40) and a toothwidth of about 0.26λ.

b An as large as possible maximum force. The displacement value at which this maximum occurs can be of importance. The maximum torques were measured from figures 28 - 62 for I_s = 8A and can be found in table 21 (torque in Nm).

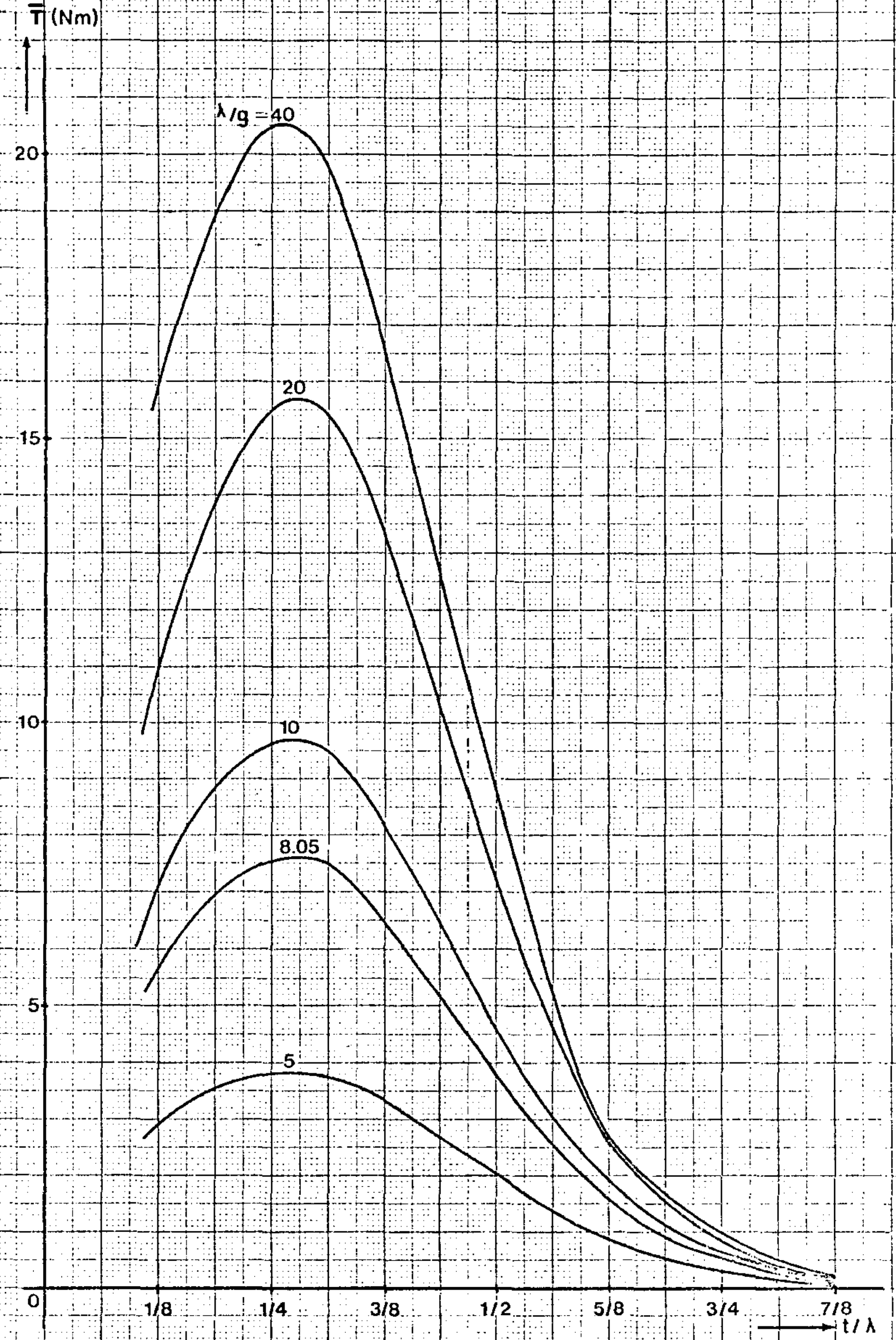


Fig 64 $\overline{T} = f(t/\lambda)$, $I_S = 8A$

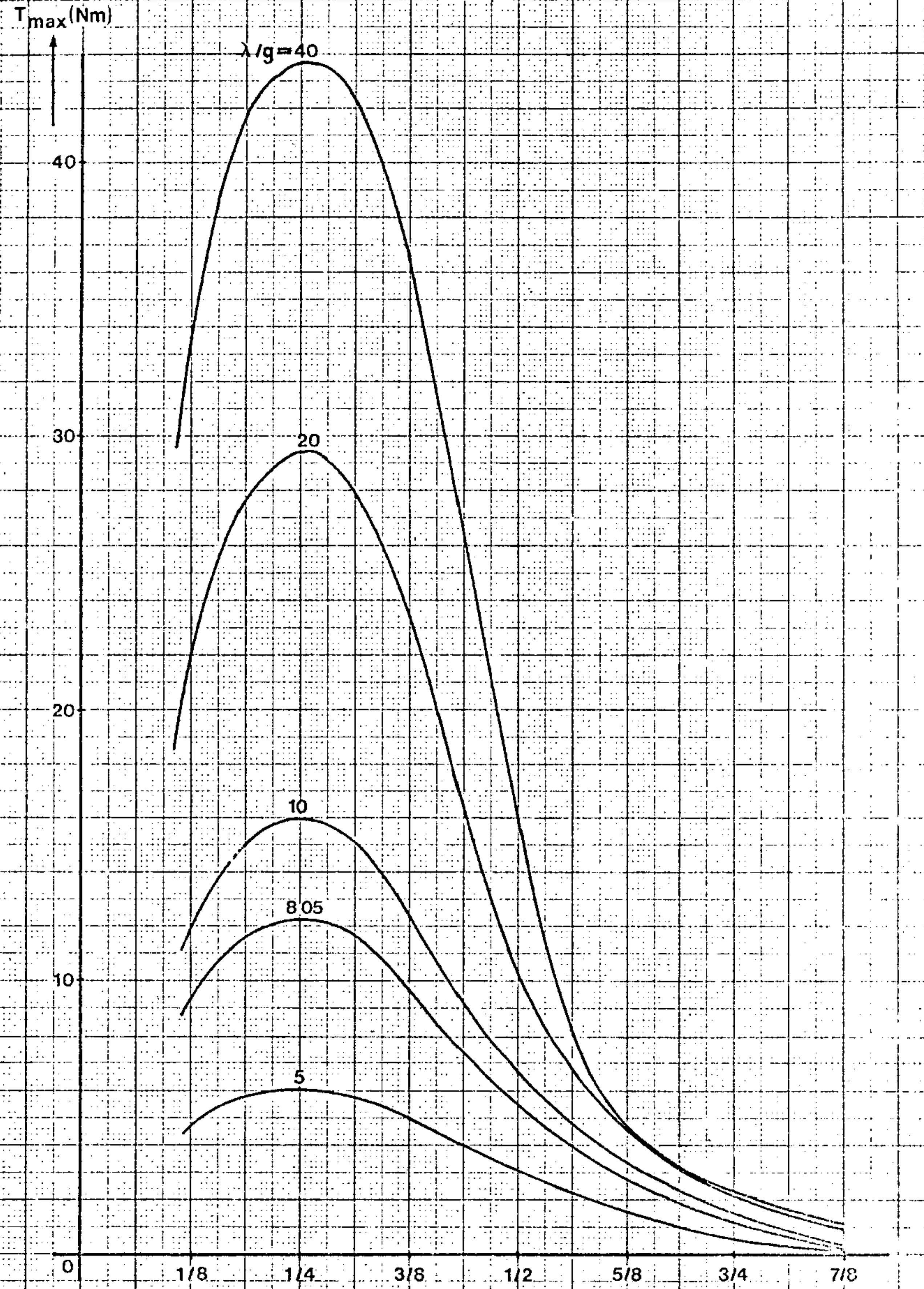


Fig 65 $T_{max} = f(t/\lambda)$, $I_s = 8$ A

t/λ	λ/g				
	40	20	10	8.05	5
7/8	1.01(0.14)	0.82(0.15)	0.32(0.16)	0.23(0.16)	0.08(0.20)
3/4	2.20(0.32)	2.32(0.24)	1.59(0.26)	1.24(0.28)	0.55(0.30)
5/8	4.62(0.55)	4.60(0.46)	3.43(0.41)	2.78(0.40)	1.45(0.41)
1/2	15.80(0.85)	10.40(0.73)	6.62(0.59)	5.45(0.55)	3.10(0.56)
3/8	36.40(0.72)	23.41(0.70)	12.38(0.64)	9.66(0.61)	5.00(0.55)
1/4	43.70(0.47)	29.35(0.49)	15.80(0.47)	12.25(0.47)	6.04(0.48)
1/8	33.60(0.25)	22.00(0.29)	12.00(0.29)	9.30(0.33)	4.65(0.38)

Table 21.

The values between brackets are the displacement values x/X where the maximums occur. The torque values are drawn in figure 65. This figure shows that the largest maximum torque (and force) occurs for λ/g = 40 and a toothwidth of 0.26λ. The table shows that these maximums occur for a displacement x/X = 0.47.

c A particular shape of the force-displacement curve that suits the dynamic behaviour of the investigated structure.
This was not looked into during this investigation.
It would be possible to look into this matter by simulating the dynamic behaviour of the moving parts of the apparatus and its load on an analogue computer, applying the various torque-displacement curves. Before that, however, there is to be found a consensus as to what is to be understood by "best dynamic performance".

List of symbols

A	area	m^2
B	magnetic inductance	Vs/m^2
F	force	N
g	airgap width	m
h	slotdepth	m
H	magnetic field intensity	A/m
I	current	A
J	current density	A/m^2
l_t	toothlength	m
$P(x)$	permeance of airgap space for position x	H
P_1	airgap permeance per toothpitch, position x = 0	H
P_2	airgap permeance per toothpitch, position x = -X	H
r	radius	m
s	slotwidth	m
t	toothwidth	m
T	torque	Nm
U	magnetic potential	A
W'_m	co-energy of magnetic field	J
x	position of one slotted surface relative to the other	m
X	half of toothpitch in central position	m
z	number of teeth	
ϕ	circulating flux	Vs
λ	toothpitch	m
Θ	excitation	A
μ_0	permeability in vacuum	Vs/Am
μ_r	relative permeability	

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