

## Experimental evaluation of robot controllers

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## Experimental Evaluation of Robot Controllers

Bram de Jager, Jos Banens  
Faculty of Mechanical Engineering  
Eindhoven University of Technology  
P.O. Box 513, 5600 MB Eindhoven, The Netherlands  
Email: jag@wfw.wtb.tue.nl Fax: +31 40 447355.

### Abstract

In the last decade an abundance of control laws for nonlinear robotic systems was proposed. A careful evaluation of several classes of these controllers has been made, to overcome some drawbacks of previous evaluations. Experiments were done on a simple robotic system, with prismatic joints only and with low cost controller hardware. A flexible and effective real-time software environment, using object oriented programming techniques, was developed, easing the implementation and the evaluation of many control laws. The experience gained leads to the recommendation to develop as good a model as possible combined with adaptive control for tuning of the model parameters.

### 1. Introduction

The evaluation of controllers is a venture. Advocates of a specific class of control laws are, rightfully, sceptical if their favorite control scheme is judged unfavorable, and are ready to point out flaws in the evaluation. These flaws are easy to identify and often unavoidable. Simulations are flexible and can guide the developer during the initial stages of control law design, e.g., to isolate problem areas, but are inadequate for assessing robustness and performance of control systems. A real-time environment imposes constraints that influence the performance, and this influence is not necessarily the same for every control system and control law. Furthermore, it is not always possible to tune the control system parameters until the last bit of performance is obtained. In practice simple tuning rules are advocated and used, to finish the commissioning of the control system within the deadline and with low costs. This means that an evaluation after careful but endless tuning of parameters is not a realistic goal, although it may be current practice.

Therefore, a practice oriented evaluation should be performed on a realistic experimental system and with simple tuning rules in mind. If these rules are not available for a specific class of controllers, this disadvantage will necessarily be reflected in the performance and robustness obtained if limited effort can be spent in tuning the parameters with trial and error. So, the flaws mentioned before are not real flaws but inherent and necessary constituents of a faithful evaluation of controllers, having in mind their application in harsh industrial environments. This does not mean that one can carelessly implement the controllers and plug in some controller parameters. The evaluation should be artfully done: careful, accurate, repeatable, and unbiased. In this way only can we complete the square connecting theory to practice, and back.

Because we will concentrate on an experimental evaluation, we will not review the literature on control schemes for nonlinear robotic systems and/or on using those schemes in simulations, but concentrate on research that concerns itself with reality, and not a virtual one that is. The experimental evaluation and comparison of advanced robot controllers in real-time systems have been performed by several researchers.

An *et al.* [1] present experiments with feedforward and computed torque control on a serial link direct drive arm with rotary joints. Both feedforward and computed torque control performed similar when equal sampling rates are used. They remark that model errors, e.g., motor dynamics and friction, which are normally no big issues in direct drive robots, yield larger trajectory errors. Berghuis [2] investigates a series of adaptive controllers on a two rotary joint robot. He tries to get past known problems with adaptive control, like parameter drift, and verifies his theoretical findings to some degree. The control is implemented on a specific hardware platform, based on transputers. Bühler *et al.* [3] use nonlinear feedback for their juggling robot with one rotary joint and equipped with a DC direct drive servo actuator, while local controllers based on linear design failed. The controller is implemented on a transputer based platform. Koshla *et al.* show in a series of papers [4-6] that computed-torque control performs better than independent joint control, that high sampling rates are profitable because they allow high gains, and that feedback and feedforward schemes perform comparably. All results refer to experiments with a direct drive robot arm with three rotary joints, equipped with special purpose controller hardware. Leahy *et al.* [7,8] evaluate controllers for the first three links of a PUMA-560 robot. The links are connected by rotary joints and controlled by a specific hardware setup, bypassing the built-in control. All controllers were model based utilizing among others (adaptive) feedforward compensation, sliding mode like feedback, and feedback based on Quantitative Feedback Theory. The last method received the highest praise, mainly for its well developed tuning rules. Niemeyer and Slotine [9-11]

emphasize the use of adaptive control and test direct and composite versions of it with a recursive implementation for a whole arm manipulator, a human arm like robot with four rotary joints. This robot is controlled by a multiprocessor control system. They conclude that while the performance of both direct and composite adaptive control is comparable, the prediction error converges with the composite variant only. Stoten and Hodgson [12] compare several adaptive robot controllers on a planar robot with two rotary joints. They advocate the use of the minimal control synthesis algorithm, co-developed by one of them, based mainly on noise sensitivity and parameter convergence arguments and not on performance. Tarn *et al.* [13-15] include the effect of actuator dynamics in the model used for control, by that improving performance. They also compare several control schemes and emphasize the use of high sampling rates and accurate models. All their experiments are with a PUMA-560 robot using specialized and powerful controller hardware. Whitcomb *et al.* [16] give an extensive evaluation of several adaptive controllers, and conclude that differences between them are not so significant to allow a recommendation. They also state that the adaptive controllers always performed better than their nonadaptive counterparts, but that the performance improvement of model based control is limited by errors in the model. Their results were obtained on two different robots, both with rotary joints only.

The main drawbacks of these studies are that they

- consider robots with revolute joints only
- compare mostly controllers inside a class, not between different classes,
- often use specialized and costly controller hardware, probably requiring complete re-coding by hand of the control algorithms.

Mechanical systems with prismatic joints are interesting because (1) they are frequently used in high accuracy manufacturing, e.g., in measuring equipment and wafer steppers, (2) their model is relatively simple, so the evaluation can focus more on control system behavior and less on implementation aspects, (3) for the control system PC-class hardware can be used, limiting the expenditure and cost, (4) the type of system imperfections is different, less emphasis is on inertia related nonlinearities and uncertain parameters, more on friction and unmodeled dynamics (flexibility and vibration).

The availability of a mechanical system with translational joints in our laboratory makes an experimental evaluation for this type of systems possible. Results obtained previously on the same experimental system are scattered through the literature, see [17-21], and also study different aspects of control system performance, e.g., the profitability of extensive models versus the use of robust control. Furthermore, they cannot be compared due to drift in the experimental conditions. It has been observed that after maintenance service of the experimental system the tracking errors could be reduced by almost a factor of two! Therefore these results do not enable an evaluation following the strict requirements set out earlier. In this paper we try to remedy and improve on this. Some modifications of existing control laws and extensive experiments with a new one [22] are documented for the first time also. Because implementing robot controllers can be time consuming, an object oriented software environment was developed, making this implementation a snap.

The main goals and contributions of this paper, therefore, are to

- experimentally evaluate controllers on a prismatic joint robot,
- compare controllers of different classes in closely tied sessions to avoid drift in the experimental conditions,
- modify some control laws to improve the tracking error or to make the controller parameters easier to tune,
- present experimental results for a new type of controller that merges sliding mode and adaptive components of existing ones.

All experiments should be performed with the requirements of a careful, accurate, repeatable, and unbiased evaluation in mind and with two main aims: comparing controllers and uncovering their weak spots when they are applied in practice.

The paper is structured as follows. First, Section 2 takes off with the control laws under scrutiny. Section 3 shows the experimental system and evaluation setup, followed by the presentation of the experimental results in Section 4. Finally, Section 5 kicks the paper down with a discussion of the results, the conclusions, and directions for future research.

## 2. The Control Laws

The controllers investigated can be divided in four classes, with some overlap between them. The classes are acceleration feedback based controllers, robust controllers, adaptive controllers, and sliding mode controllers. We only document the controllers and some modifications that were profitable in practice, without extensively discussing them since they are discussed already in the indicated literature.

The system to be controlled is assumed to be a mechanical system with  $n$  DOF (degrees-of-freedom)  $q$  whose dynamics can be represented by the model

$$H(q, \theta)\ddot{q} + C(q, \dot{q}, \theta)\dot{q} + g(q, \dot{q}, \theta) = f \quad (1)$$

with  $\theta$  the model parameters and  $f$  the generalized force generated by the actuators. The inertia matrix  $H$  is positive definite, the matrix  $C$ , if chosen appropriately, has the property that  $\dot{H} - 2C$  is skew symmetric, the vector  $g$  contains gravity and friction terms.

In the sequel  $\hat{H} = H(q, \hat{\theta})$  etc. indicate estimates,  $e = q_d - q$  is the tracking error with  $q_d$  the desired trajectory,  $s = \dot{e} + \Lambda e$  is a measure of tracking accuracy, and  $\dot{q}_r = \dot{q}_d + \Lambda e = \dot{s} + \dot{q}$  is a virtual reference velocity.

For reference a standard (CTC) computed torque controller is used

$$f = \hat{H}(\ddot{q}_d + K_d \dot{e} + K_p e) + \hat{C}\dot{q} + \hat{g}. \quad (2)$$

The control parameter matrices  $K_d$  and  $K_p$  should be positive definite. Because model based control has been shown to perform consistently better than PD feedback, we did not deem it necessary to compare with a PD controller also.

### 2.1. Acceleration feedback controllers

Controllers using acceleration are an often neglected class of controllers. Because the acceleration in mechanical systems is relatively easy to measure at low costs, this is surprising. We consider three controllers using acceleration directly in the feedback loop, and two variants thereof. The first controller, proposed by Heeren [23], is

$$f(q, \dot{q}, \ddot{q}, t) = (1 + \alpha)f^*(q, \dot{q}, t) - \alpha(\hat{H}\ddot{q} + \hat{C}\dot{q} + \hat{g}) \quad (3)$$

with  $f^*$  the torque command of any controller not using acceleration. When using for this purpose the controller proposed by Slotine and Li [24, 25], without its adaptive component,

$$f = \hat{H}\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{g}(q, \dot{q}) + K_v s \quad (4)$$

the control law becomes

$$f = (1 + \alpha)(\hat{H}\ddot{q}_r + K_v s + \hat{C}s) + \hat{C}\dot{q} + \hat{g} - \alpha\hat{H}\ddot{q}. \quad (5)$$

The second one is also based on (4), modified by feedback of the term  $\dot{s}$  following a suggestion in [26]

$$f = \hat{H}\ddot{q}_r + \hat{C}\dot{q}_r + \hat{g} + K_v s + \alpha\hat{H}\dot{s}.$$

Using the relation  $\dot{s} = \ddot{q}_r - \ddot{q}$  this can be written as

$$f = (1 + \alpha)\hat{H}\ddot{q}_r + K_v s + \hat{C}\dot{q}_r + \hat{g} - \alpha\hat{H}\ddot{q}. \quad (6)$$

The third controller was proposed by Berlin and Frank [27] and modifies the standard computed torque control law to (using a simplified choice for the original parameters)

$$f = \hat{H}((1 + \alpha)(\ddot{q}_d + K_d \dot{e} + K_p e) - \alpha\ddot{q}) + \hat{C}\dot{q} + \hat{g}.$$

If the controller parameters satisfy  $K_v = \hat{H}K_d$  and  $K_p \Lambda = \hat{H}K_p$  this is equivalent to

$$f = (1 + \alpha)(\hat{H}\ddot{q}_d + K_v s) + \hat{C}\dot{q} + \hat{g} - \alpha\hat{H}\ddot{q}. \quad (7)$$

This control law can also be obtained by combining (3) with (2). For a more detailed discussion of these controllers see [18].

Comparing the three expressions for the previous control laws, an obvious variation is

$$f = (1 + \alpha)\hat{H}\ddot{q}_d + K_v s + \hat{C}\dot{q} + \hat{g} - \alpha\hat{H}\ddot{q}. \quad (8)$$

This is equivalent with (6) using  $\ddot{q}_d$  and  $\hat{C}\dot{q}$  instead of  $\ddot{q}_r$  and  $\hat{C}\dot{q}_r$ .

The last controller in this class

$$f = \hat{H}\ddot{q}_d + (1 + \alpha)K_v s + \hat{C}\dot{q} + \hat{g} \quad (9)$$

does not use the acceleration at all, but increased feedback gains. The reason for including it will become clear in the discussion of the results.

### 2.2. Robust controllers

Several robust controllers are proposed by Spong [28]. These controllers have the advantage that only parameter error bounds are needed to design specific gains. Often, like in sliding mode control, the desired trajectory and manipulator state are also needed for this purpose. The controllers are based on (4), using the linear in the parameters form

$$f = Y(q, \dot{q}, \ddot{q}_r)\hat{\theta} + K_v s. \quad (10)$$

The first control law is given by

$$f = Y(\hat{\theta} + u) + K_v s. \quad (11)$$

where

$$u = \begin{cases} \rho \xi / \|\xi\| & \text{if } \|\xi\| > \varphi \\ \rho \xi / \varphi & \text{otherwise} \end{cases} \quad (12)$$

with  $\xi = Y^T s$ . The scalar  $\rho$  is a measure of the parameter mismatch  $\theta - \hat{\theta}$  and the scalar  $\varphi$  is a bound to prevent chattering.

The second is a variant of the previous one, using individual values of the column  $\xi$

$$u = \rho \text{sat}(\xi, \varphi) \quad (13)$$

with sat the saturation function, defined component wise by

$$\text{sat}(\xi_i, \varphi_i) = \begin{cases} \xi_i / |\xi_i| & \text{if } |\xi_i| > \varphi_i \\ \xi_i / \varphi_i & \text{otherwise.} \end{cases}$$

Here,  $\varphi$  is a column of positive values and  $\rho$  a diagonal matrix with components related to the individual parameter errors.

### 2.3. Adaptive controllers

Adaptive controllers have been studied extensively. It appeared that differences within this class are relatively small and that the adaptive controller proposed by Slotine and Li did perform quite well [2, 16]. We therefore use this controller and a distinct one proposed by Kelly.

The first one is given in (4) already. Its adaptive component is

$$\dot{\hat{\theta}} = \Gamma Y^T s$$

with  $Y$  from (10). Sometimes, due to initial position or velocity errors, the value of  $s$  can be very large at the beginning of a trajectory. Because this has no relation to parameter errors and can cause a large initial offset of the estimated parameters during the transient, leading to severe limitations on the adaptation gains to keep this offset within bounds,  $s$  is clipped before being used in the adaptation law. The controller is therefore implemented as

$$f = Y\hat{\theta} + K_v s \quad (14)$$

$$\dot{\hat{\theta}} = \Gamma Y^T \text{clip}(s, \epsilon)$$

and  $s_i$  is clipped if  $|s_i| > \epsilon_i$ .

The adaptive controller proposed by Kelly *et al.* [29, 30] is

$$f = \hat{H}(\ddot{q}_d + K_d \dot{e} + K_p e) + \hat{C}\dot{q} + \hat{g} + \hat{C}\dot{v} \quad (15)$$

$$\dot{\hat{\theta}} = \Gamma Y^T \text{clip}(\dot{v}, \epsilon)$$

with  $v$  a filtered version of the tracking error  $e$

$$\dot{v} + \lambda v = \dot{e} + K_d e + K_p \int_0^t e \, d\tau \quad (16)$$

and  $Y$  derived from a linear parameterization of the control law

$$f = Y(q, \dot{q}, \ddot{q} + \dot{v}, \ddot{q}_d + K_d \dot{e} + K_p e)\hat{\theta}.$$

In the adaptation law the value of  $\dot{v}$  is clipped, for the same reasons as in the adaptive controller (14). To ease the tuning of controller parameters a different parameterization of (16) was implemented, namely

$$\frac{1}{\lambda} \dot{v} + v = \dot{e} + K_d e + K_p \int_0^t e \, d\tau. \quad (17)$$

### 2.4. Sliding mode controllers

A SOSMC (second order sliding mode controller) for a more general nonlinear model was proposed in [31]. For the model (1) the equations are, see also [19],

$$f = \hat{H}(\ddot{q}_d + K_d \dot{e} + K_p e) + \hat{C}\dot{q} + \hat{g} + \hat{H}(-\Lambda \dot{s} + \Omega s + K_s \text{sgn } \dot{s}) \quad (18)$$

now with a different definition of  $s$ , namely

$$\dot{s} + \Lambda s = \dot{e} + K_d e + K_p \int_0^t e \, d\tau. \quad (19)$$

with  $K_s$  large enough to guarantee attractiveness of the sliding surface  $\dot{s} = 0$ . The term  $-\Lambda\dot{s}$  in (18) is a bit peculiar. It proved profitable in practice to remove this term. Besides simplifying the control law, it also enhances the stability, in the sense that for the original controller the time derivative of the Lyapunov function  $2V = \dot{s}^T \dot{s} + s^T \Omega s$  is equal to  $\dot{V} = -\dot{s}^T K_s \text{sgn } \dot{s}$ , while after modification this is  $\dot{V} = -\dot{s}^T (K_s \text{sgn } \dot{s} + \Lambda\dot{s})$ , so the Lyapunov function  $V$  will decrease faster. The definition of  $s$  in (19) is an instance of a more general modification of  $s = \dot{e} + \Lambda e$  proposed in [32]. They use definitions of  $s$  and  $q_r$  for (4) given by

$$s = F(p)e, \quad \dot{q}_r = \dot{q}_d + G(p)e, \quad F(p) = pl + G(p)$$

with  $p$  the Laplace variable.

To the SOSMC an adaptive component can be added, by combining it with the adaptive part of (15). To make the control schemes more similar  $\Lambda$  can be chosen as  $\lambda I$ . Then  $v$  can be identified with  $s$  and (16) and (19) are identical. Also at least the first few terms of (15) and (18) are the same. This comparison of both schemes leads to the proposal to use  $\dot{s}$  from (19) in the adaptive component for the second order sliding mode control scheme (18). So, the implemented controller is

$$f = Y\hat{\theta} = \hat{H}(\dot{q}_d + K_d\dot{e} + K_p e) + \hat{C}\dot{q} + \hat{g} + \hat{H}(\Omega s + K_s \text{sat}(\dot{s}, \varphi)) + \hat{C}\dot{s} \quad (20)$$

$$\hat{\theta} = \Gamma Y^T \text{clip}(\dot{s}, \epsilon)$$

where the sat function replaces the sgn function to avoid chattering and  $\dot{s}$  is clipped. The parameters in this control law have to be selected carefully to guarantee convergence of the tracking error  $e$ . Both  $e$  and  $\hat{\theta}$  must be integrated, so it is together with (15) the most dynamic one considered. A standard sliding mode controller is

$$f = \hat{H}(\dot{q}_d + K_d\dot{e} + K_s \text{sat}(s, \varphi)) + \hat{C}\dot{q} + \hat{g} \quad (21)$$

where  $K_s$  is chosen large enough to guarantee attractiveness of the sliding surface  $s = 0$ , and the sat function is used instead of the sgn function.

## 2.5. Overview

For a summary and tabulation of the controllers used, see Table 1.

Controller #	Name	Equations	Remarks
1	Computed torque	(2)	
2	Acc. feedb. Heeren	(5)	
3	Acc. feedb. Slotine	(6)	
4	Acc. feedb. Berlin/Frank	(7)	
5	Acc. feedb. new proposal	(8)	
6	Increased feedb. gains	(9)	
7	Robust Spong (scalar)	(12)	
8	Robust Spong (vector)	(13)	
9	Computed torque	(2)	
10	Nonadaptive Slotine/Li	(14)	$\Gamma = 0$
11	Adaptive Slotine/Li	(14)	$\Gamma \neq 0$
12	Nonadaptive Kelly	(15)	$\Gamma = 0$
13	Adaptive Kelly	(15)	$\Gamma \neq 0$
14	Nonadaptive SOSM	(20)	$\Gamma = 0$
15	Adaptive SOSM	(20)	$\Gamma \neq 0$
16	Sliding mode	(21)	
17	Computed torque	(2)	

Remark that the controllers labeled 1, 9, and 17 are the same. The additional experiments for the same controller are "guard" experiments. Because the experiments are performed in increasing sequence, the two redundant sets of experiments enable us to assess the drift in the experimental conditions, and the variation in the results. When the discrepancy between these results is too large, the complete set of results should be rejected.

## 3. The Experimental System and Evaluation Setup

In this section the experimental system, an XY-table, is described. The setup of the controller evaluation is discussed, including the definition of the reference trajectories used to set off the controllers against each other. The tuning of the controller parameters, a crucial point in a comparative study, is also elaborated.

### 3.1. XY-table

The experimental system is an XY-table with three degrees-of-freedom, moving in the horizontal plane. Two of the degrees-of-freedom are coupled by a spindle with a stiffness that can be varied. This spindle is assumed completely stiff in the nominal model used in the model based controllers and for the tuning of the controller parameters. Therefore, we present a model with two degrees-of-freedom only. See Fig. 1 for a sketch of the experimental system. The two DOF model of the system, using the previous notation, is given by

$$q = \begin{bmatrix} x \\ y \end{bmatrix}, \quad H = \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix}, \quad g = \begin{bmatrix} \theta_3 \text{sgn } \dot{q}_1 + \theta_5 \dot{q}_1 \\ \theta_4 \text{sgn } \dot{q}_2 + \theta_6 \dot{q}_2 \end{bmatrix}$$

$C = 0$  for there is no coupling between  $x$  and  $y$  direction. Remark that all rotary movements of the motors etc. are lumped to the planar movement of

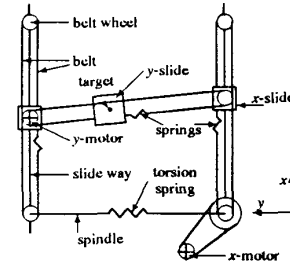


Figure 1: Sketch of XY-table

the slides and the coordinates  $x$  and  $y$  correspond with the position of the "target" point in the plane. Because the manipulator moves in the horizontal plane, the column  $g$  only contains Coulomb and viscous friction terms. The regressor matrix  $Y$  used in the robust and adaptive controllers is different for each type, because it depends on the controller equations. It can easily be derived for the simple model presented above. The model parameters used in the model based controllers and for the control design and tuning are listed in Table 2.

Table 2: Model parameters

Parameter	Value	Unit
$\theta_1$	34.0	kg
$\theta_2$	2.3	kg
$\theta_3$	40.0	N
$\theta_4$	13.0	N
$\theta_5$	35.0	$\text{N s m}^{-1}$
$\theta_6$	5.0	$\text{N s m}^{-1}$

Some characteristics of the XY-table are

- reachable area  $1.07 \times 0.774$  [m],
- working area  $0.9 \times 0.62$  [m] (smaller due to safety constraints),
- the spindle between the two slide-ways in  $x$ -direction partly consists of a replaceable torsion spring,
- the system is powered with two current amplifiers that feed the  $x$  and  $y$  permanent magnet DC motors,
- position measurements for the three DOF are available by three encoder wheels, two of which are mounted on the  $x$  and  $y$  motor shafts, the third is on the side of the spindle away from the  $x$ -motor,
- velocity measurements by tacho generators incorporated in the motors,
- two accelerometers mounted on the  $y$ -slide, lined up with the  $x$  and  $y$  axis, measure its acceleration,
- a vision system is available for detecting the  $y$ -slide position, therefore an LED is mounted at the "target" point,
- dedicated hardware and software for edge detection and computation of  $y$ -slide position makes operation at high sampling rates possible,
- the control system software is implemented on commodity PC-class hardware,
- interfacing with the experiment is performed by a data acquisition board containing analog and digital IO terminals, by the parallel, and by the serial port,
- C++ equipped with a specialized matrix class object library is the programming environment.

### 3.2. Reference trajectories

Two completely different sets of trajectories were chosen for the controller evaluation. The first set consists of several curved segments, smoothly connected to each other, where inertia and friction forces dominate in certain parts because both low acceleration-high speed and high acceleration-low speed regimes are present. Trajectories from this class are called rich. The second set of trajectories consists of several basically straight segments, often lined up with the slide-ways, traveled at constant speed along the segments, connected with discontinuous changes in the components of the desired velocity at corner points. Trajectories from this class are called poor.

Both sets are believed to represent common tasks performed by robots with prismatic joints employed in industry. The speed of traveling along the trajectories is chosen so the computed  $f$  does not exceed the DA converter input range or saturate the current amplifiers during transients of short duration only, but not during steady operation. Representative examples of these classes of trajectories are in Figs. 2-3.

The first set is aimed at illuminating the advantage of computed torque like controllers with adaptation. The trajectories are persistently exciting, so parameter adaptation should present no problems.

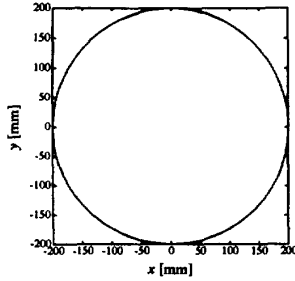


Figure 2: Circular reference trajectory

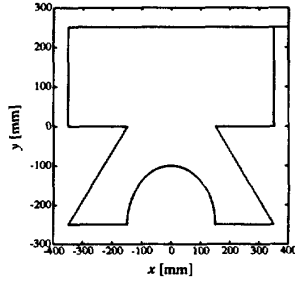


Figure 3: Straight lines dominated reference trajectory

The second set challenges the adaptive controllers: the desired acceleration is often zero and the velocity constant, so the inertia parameters are almost impossible to estimate, and Coulomb and viscous friction parameters cannot be adapted separately. These difficulties can cause parameter drift in the adaptive controllers. Several measures are available to prevent this, e.g., bound the parameters to a physical reasonable range or stop the adaptation if  $s$ ,  $\dot{v}$ , or  $\dot{s}$  is small. Especially the estimate of the inertia parameter is susceptible to drift. This is due to the tracking error term in the entries of the regressor matrix  $Y$  that correspond with inertia parameters. Even if the desired acceleration  $\ddot{q}_d$  is zero, the presence of a static tracking error  $e$ , e.g., due to imperfect Coulomb friction compensation, causes adaptation of the inertia parameters, without any real physical meaning. Stopping the inertia parameter update if  $\dot{q}_d$  is small compared with the tracking error related terms will prevent this. In our experiments excessive parameter drift was not observed, mainly because the experiments were of short duration, so these measures were not implemented, with the intent to not unduly increase the number of parameters to be tuned. For prolonged adaptation on trajectories from the poor set these measures will undoubtedly prove to be necessary, and therefore other adaptive controllers that address this problem directly [2] deserve attention.

Furthermore, because the desired velocity changes at corner points the desired acceleration contains impulses. Because these impulses cannot be handled by the feedforward term of the controllers (the DA converter will clip the signal or the current amplifier will saturate during one time step and eventually almost nothing will happen) the desired acceleration at corner points is set to zero. This means that inertia forces are not compensated at the corners, eliminating the advantage of computed torque like controllers over a PD controller with friction compensation.

### 3.3. Feedforward versus feedback

One often has to face the question whether the model based part of the control will be implemented as a feedforward compensation using the reference trajectory  $q_d$  and its derivatives, with the advantages of off-line computation, no error prone measurements needed, and no stability problems, instead of a feedback implementation using the measured  $q$ , with the associated stability, noise and real-time issues. Motor saturation effects favor a feedback implementation, see [16]. In our case the answer is not difficult to find.

In the poor trajectories the use of feedforward is not an advantage because the Coulomb friction term is not compensated (the desired velocity is often zero), causing an appreciable tracking error. Using friction compensation based on the sign of the velocity  $\dot{q}$  is also not acceptable, because the signal is not error free and therefore its sign can change frequently without any underlying physical reason (this occurs in a low velocity regime as happens when the

desired velocity is zero), causing chattering in the control input that leads to a deterioration of the tracking error and increased wear of the XY-table. The solution chosen is to use feedback for the friction compensation with a saturation type of friction model instead of a sign type, where the width of the saturation is experimentally chosen to guarantee reasonable friction compensation without chatter. With this setup all experiments were performed. An alternative would be the approach advocated in [11] to base the Coulomb friction compensation on the virtual reference velocity  $\dot{q}_r = \dot{q}_d + \Lambda e$  that does not use the velocity signal and takes account of the tracking error. Besides avoiding problems in the friction compensation, this also improves the time derivative of the Lyapunov function used in the stability proof. Because  $\dot{q}_r$  is only used in (4) and the controllers based thereon, and not in the other control laws, we considered it artificial to base the Coulomb friction compensation on  $\dot{q}_r$ , and did not follow this suggestion.

### 3.4. Controller tuning

Because the tuning of controller parameters plays a crucial role in comparative studies, the rules followed and art used are documented in detail.

The similarity of certain terms in the controllers simplifies a tuning of parameters that set the controllers on an equal footing to start with. The parameters in (2) are chosen so the resulting error equations for the nominal model are a decoupled set of second order systems with undamped natural frequency  $\omega_0$  and damping coefficient  $\beta$ . All terms corresponding with the  $K_d$  and  $K_p$  parameter matrices in (2), i.e., the parameter  $K$ , and the  $\Lambda$  in the definition of  $s$ , are set to equivalent values, assuming nominal values for the inertia matrix  $H$ . The other controller parameters are chosen as follows.

The parameter  $\rho$  for the robust controller (13) is set to values based on the expected errors in the model parameters and on the range of values for the estimated model parameters of the adaptive controllers. The chattering preventing parameter  $\varphi$  was set to a fraction of the nominal parameters  $\theta$ , but the appropriate fraction was to be found experimentally. For the scalar version (12)  $\rho$  and  $\varphi$  are set to the norms of the corresponding vectors used in (13). It was however necessary to increase  $\varphi$  and this introduces additional differences between both controllers.

The adaptation gains  $\Gamma$  are set so a reasonable range of parameter values can be covered during the least exciting trajectory of the rich set. A final tuning of these settings proved to be necessary during the experiments, showing the need for more sophisticated tuning rules. Because the regressor matrices  $Y$  and the signals  $s$ ,  $\dot{v}$ , and  $\dot{s}$  used in the adaptation differ, the adaptation gains for the three adaptive controllers differ also. The clipping bounds  $\epsilon$  are set to values that slightly exceed the range of the signals to be clipped, after the initial transient is damped out, and are different for each of the adaptive controllers. The parameters  $\lambda$  and  $\Lambda$  in the controllers of Kelly and the SOSMC are set to achieve appropriate filtering of the error signals. The choice for  $\Omega$  is also based on bandwidth considerations, avoiding excitation of high frequency unmodeled dynamics. For more details for the tuning of the SOSMC see [19]. The parameters  $K_s$  in the sliding mode controllers were based on an analysis of the model errors and on the requirement to assure attractiveness of the boundary layer. The values for the first and second order controller differ.

The remaining controller parameters are set by trial and error. The parameters  $\alpha$  in the acceleration feedback controllers are all set to the same value, and are chosen so large that for none of the acceleration feedback controllers significant chattering occurs. The parameters  $\varphi$  used in the sliding mode controllers are experimentally determined and set to values avoiding chattering. Different values for both controllers were necessary.

Furthermore, all parameters of matrix type were chosen diagonal: because the nominal model is decoupled there was no evident need for off-diagonal terms. A summary of the main parameters is in Table 3.

Parameter	x	y	Unit
$\omega_0$	$4 \cdot 2\pi$	$4 \cdot 2\pi$	rad s <sup>-1</sup>
$\beta$	0.7	0.7	-
$\alpha$	0.4	0.3	-
$\lambda$	20.0	10.0	s <sup>-1</sup>
$\Omega$	63.2	63.2	s <sup>-2</sup>
$K_s$	8.0	8.0	m s <sup>-2</sup>

Due to the different dynamics for movements in x and y direction the control parameters may differ for these directions. The bandwidth for the CTC noise rejection loop is  $\approx 2\omega_0$  and is the same for both directions.

## 4. Experimental Results

The experimental results are very densely presented in two sets of figures, one for the rich and one for the poor trajectories. In each figure three indicators of tracking performance are used. The largest one is the MAX (MAXimum absolute value), the second one the RMS (Root Mean Square value), and the smallest one the MAV (Mean Absolute Value) of the tracking error. For the first set these three indicators are the averages over eight different trajectories from the rich class, for the other set over four trajectories from the poor class. The controller labels correspond with the numbers in Table 1.

Figures 4–5 give the indicators for the rich trajectories, respectively for  $x$  and  $y$ -direction, while Figs. 6–7 give them for the poor trajectories.

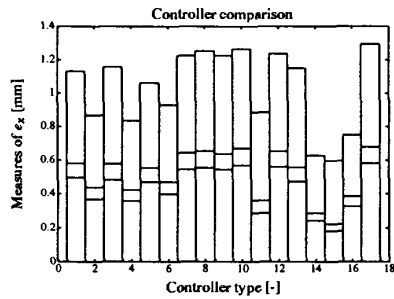


Figure 4: Indicators of tracking accuracy against type of controller for rich reference trajectories,  $x$ -direction

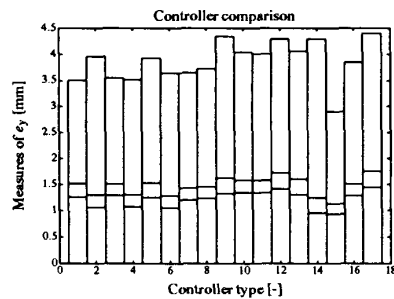


Figure 5: Indicators of tracking accuracy against type of controller for rich reference trajectories,  $y$ -direction

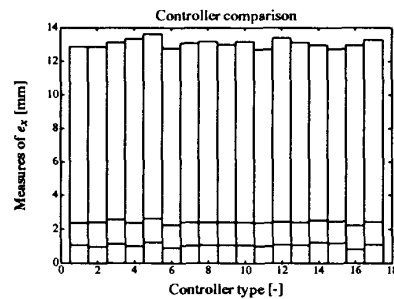


Figure 6: Indicators of tracking accuracy against type of controller for poor reference trajectories,  $x$ -direction

## 5. Discussion and Conclusions

In this section we will further detail our findings to help their dispersion. The observations lead to conclusions with respect to the choice for an appropriate controller and niches for future research.

### 5.1. Discussion

The results of the previous section lead to the observation that the absolute performance of the control schemes depend to a large extent on the class of trajectories. Therefore, we will discuss the results, later, as a function of the class, but will first give some remarks on the relative performance of the controllers within a single class of reference trajectories. First some general remarks.

The MAX is not a good indicator of performance for the rich trajectories. It is not always consistent with both the RMS and MAV indicators. A single glitch suffices to offset its value, although averaging over several trajectories is performed, while the other two indicators smooth this out more effectively. For the poor trajectories, the MAX indicator seems valuable, first, because the RMS and MAV average the large errors at the corners with the relatively small errors along the straight segments, second, because for tasks with this type of

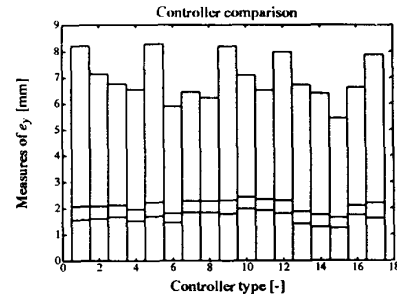


Figure 7: Indicators of tracking accuracy against type of controller for poor reference trajectories,  $y$ -direction

trajectory, e.g., torch burning, the maximum error is the relevant one. For the rich trajectories we therefore mainly refer to the RMS and MAV indicators, and for the poor trajectories to the MAX indicator.

#### 5.1.1 Rich trajectories

Starting with the rich trajectories, we see some drift in the result. Because for our system  $\hat{C} = 0$ , there are four controllers that implement the same algorithm, namely the ones labeled 1, 9, and 17, but also the nonadaptive version of the controller proposed by Kelly, # 12. There is a slight uphill trend in the data, but this trend looks linear and acceptable, and can be taken into account during the interpretation. The baseline performance is therefore established by lines connecting the results of these four controllers. There is no controller that sticks out above these lines, but several stick out below, # 2, 4, 6, 11, 13, 14, 15, 16 for the  $x$ -direction and # 2, 4, 6, 14, 15 for the  $y$ -direction.

The acceleration feedback controllers show a peculiar pattern, where # 2 and 4 consistently outperform # 3 and 5. Taking account of the differences between the controllers in this class, the suspicion arose that increased feedback gains, by the term  $(1 + \alpha)K_v s$ , were the agents of increased performance, and not the use of acceleration. This was the reason to include controller # 6, that indeed shows that increased feedback gains are an effective way to improve performance, but using acceleration offers additional benefits in  $x$ -direction, so the use of acceleration feedback is really effective.

The robust controllers of Spong did only show a slightly better than baseline performance in  $y$ -direction.

Adaptation for the  $x$ -direction proved to be very attractive, but for the  $y$ -direction the advantage was limited.

The second order sliding mode controller, with or without adaptation, did consistently outperform the other ones. It seems to be able to effectively filter unmodeled dynamics effects, and spend its increased control authority effectively, without causing unstable behavior. The standard sliding mode controller was really effective in  $x$ -direction.

The differences between results for  $x$  and  $y$  direction are due to the small effective inertia for the  $y$ -direction. Parameter  $\theta_2$  is more than an order of magnitude smaller than  $\theta_1$ . The friction forces are of the same order of magnitude. Also, the Coulomb friction model is not adequate for the  $y$ -direction [20]. The disturbance rejection for  $y$  is therefore, by design and due to the small inertia, less good, as the desired undamped natural frequencies  $\omega_0$  in the tuning of  $K_d$ ,  $K_p$ ,  $K_v$ , and  $\Lambda$  were chosen the same for both  $x$  and  $y$  direction.

#### 5.1.2 Poor trajectories

For the class of poor trajectories we observe that there is no perceptible drift in the experimental results. There is also not much distinction between the controllers for the  $x$ -direction, all perform approximately the same. The tracking error  $e_x$  shows spikes at corner points, and those spikes have approximately the same height, irrespective of the controller used. The tracking error along the straight segments is relatively small, showing the good disturbance rejection in  $x$ -direction.

As remarked before, it could be expected that model based control would not perform better than PD + friction compensation. Because the desired acceleration is set to zero at corners, the computed torque controller is equal to PD + friction compensation, except for curved segments, like the half circle shown in Fig. 3.

For the  $y$ -direction some differences between controllers surface. This is caused mainly by the relatively large Coulomb friction forces compared with inertia forces in  $y$ -direction. Some controllers handle incorrect friction compensation better, reducing the tracking error by relative high gain feedback. Here the increased feedback gain controller # 6 and again the SOSMCs # 14, 15 proved to be the most effective.

The acceleration feedback controllers have a disadvantage for this class of trajectories because the measured acceleration and the desired one are incon-

sistent at corner points. This also surfaced in slight chattering by controller # 2 during parts of one of the four trajectories used.

From the adaptive controllers # 13 obtained the smallest RMS and MAV errors. Initially the adaptive controllers presented some problems due to parameter drift, but after re-tuning the parameters these problems disappeared. During our relatively short experiments some not excessive parameter drift was observed. For tasks of longer duration this phenomenon may still present problems, necessitating the use of appropriate counter measures.

### 5.1.3 Between classes

It appeared that the trajectory used may play a crucial role in the absolute and relative performance obtained by the controllers. Adaptation is not always effective. The controllers that invariable showed better than baseline performance were # 6, 14, and 15, and therefore the only method to consistently improve tracking seems high feedback gains, making the controlled system sensitive for the effects of unmodeled dynamics. This looks like a dilemma that can be solved only by incorporating additional dynamics in the model based controllers, or by diminishing the effects of unmodeled dynamics while still using high gain feedback. The last approach seems to be handled effectively by the second order sliding mode controller that incorporates high gains at low frequencies due to integral action. This could not be achieved by other controllers, e.g., increasing  $\alpha$  in controller # 6 to increase the feedback caused instability before the performance reached that of controller # 15.

A "ranking" of the controllers in a top 3 is in Table 4.

Table 4: Ranking of controllers

Rank	Rich	Poor
1	15	15
2	14	6
3	4,6	4,14

One should not attach too much value to this ranking. Another set of trajectories or another choice of tuning guidelines may readily upset the ranking, although it is not expected that the difference will be more than a few places, due to the averaging over sets of trajectories.

### 5.2. Conclusions

Several classes of controllers were evaluated, using some modifications to allow easier control parameter tuning and to avoid large initial offsets in the estimated parameters. A second order sliding mode controller was modified to improve the tracking accuracy, and an adaptive component was added to this controller for which the implementation showed favorable results.

The question remains whether, based on the previous observations, we can make a motivated choice for the "best" controller. One could argue that the results have limited validity and are only relevant for the system under study. If the experimental system is similar to those systems for which the control law is intended, the results are at least indicative, and can be used as guidelines, how imperfect they may be: a mediated choice based on limited knowledge is better than a "blind dating" type one. For a thorough validation of the findings, evaluation on different experimental systems is advisable.

There are other issues that make a knowledgeable recommendation difficult. The choice for the "best" control scheme depends very much on the task at hand and an unequivocal recommendation cannot be made. It is clear from the results presented in the previous section and the observations made thereof that adaptive control is profitable for the rich trajectories, but not so much for the poor ones. It can be argued, however, that beforehand the type of trajectory is not known, so a controller should be used that can cope well with as large a problem set as possible. Then adaptation is recommended because in certain cases it will improve the tracking errors, and in other ones it will at least not make them worse when proper measures are used to avoid parameter drift during prolonged operation.

Based also on previous research [20] we recommend using as good a model as possible and including this model in controllers that can adapt the model parameters to better fit reality. In this class the adaptive second order sliding mode controller seems an attractive choice. When following this recommendation one will be able to achieve tracking accuracy compatible with the physical limitations of the controlled system, i.e., limited by measuring accuracy, generated torque accuracy, flexible mode mismatch, and a sampled data control implementation.

It is envisaged that unstructured nonlinear models, whose parameters have to be adapted in an easy and flexible manner without impairing stability, will be of some profit. They can be used instead of or alongside the highly structured manipulator models that are en vogue nowadays. Research in this direction is recommended.

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