Survey on the modelling of columns with internal ribbing and apertures

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SURVEY ON THE MODELLING OF COLUMNS WITH INTERNAL RIBBING AND APERTURES

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1. Introduction

In order to estimate the static and dynamic behaviour of machine tool structures a number of methods are used. Some methods well known are:

a. making a topological model of beamlike elements, including the use of rigid beams, hinges, springs etc.

b. dividing the structure into a great number of finite elements as beams, plate- and cubic elements.

Especially for machine tools built up out of parts which are rather slender the first method mentioned is often used succesfully (1), (2), (3), (4). The number of elements used for the model is small, so the cost of preparation and computations is low.
But many machine tools are built up out of boxtype parts with the hight of the same order or smaller as the dimensions of the cross-section. For these machine tools the beam method most times will lead to unsatisfactory results. The main causes for this are local deformations at connection points and the difficulty to obtain the right values for the properties of the beam element. Also the properties of the beam elements for columns with internal ribbing, apertures and transverse partitions are hard to obtain.

In order to overcome most of these difficulties the finite element method can be used. This method will lead to more accurate results, but the number of elements and certainly the number of degrees of freedom will be much higher and the cost of preparation and computing will rise even more.

An approach in between the two methods is the calculation of some difficult elements with the help of a finite element program after which the computed characteristics are fed into a beam-element program.

2. The beam element

In order to give values for the strength and the stiffness of elastic beams approximation theories are used. The theory of Bernoulli-Navier is well known for bending. For torsion the theory of Bredt is used for beams with closed cross-sections and the theory of De Saint-Venant for beams with open cross-sections.

In many beam-element programs only the displacements and rotations of the centre of gravity are calculated. In fig. 2.1 the local coordinates and forces are shown. The normal force N, shear-forces D_y and D_Z, the torsional moment M_x and bending moments M_y an M_z can act at either end of the element. Fig. 2.2 shows the definition of the local displacements and rotations.

From each element the following characteristics must be known :

l = length of the element

A = cross-sectional area

I_y = moment of inertia about the Y-axis

I_z = moment of inertia about the Z-axis

J = torsional constant for the cross-section

κ_y = shear distribution factor in Y-direction

κ_z = shear distribution factor in Z-direction.

and the material constants

E = Youngs modulus of elasticity

G = shear modulus.

For the calculation of A, the centre of gravity, I_y and I_z the formulae are well known for simple cross-sections. In the case of more intricate cross-sections one can use a program as described by Döpper (5).

According to De Saint-Venant the torsional constant for an open thin walled cross-section will be

$$J = \frac{1}{3} \int_s t^3 \, ds \qquad (2.1)$$

where t = the local wall thickness.

The torsional constant for a closed thin walled cross section is according to the theory of Bredt

$$J = \frac{4 A_t^2}{\oint \frac{ds}{t}} \qquad (2.2)$$

where A_t = the total area closed by the profile line.

In order to tell something about the influence of the shear, its distribution factor κ can be found from

$$\kappa = \frac{A}{D} \int_A \tau^2 \, dA \qquad (2.3)$$

where τ = shear tension.

Dreyer (6) gives an approximation for κ as follows

$$\kappa_y = \frac{A}{I_z^2} \int_A \frac{S_z^2(y)}{b^2(y)} \, dA \qquad (2.4)$$

where $S_z(y)$ = moment of area about the z-axis and b is the total thickness of the walls in z-direction.

About the Z-axis

$$\kappa_z = \frac{A}{I_y^2} \int_A \frac{S_y^2(z)}{b^2(z)} \, dA \qquad (2.5)$$

In the table below the value for κ is given for some cross sections :

full square	1.2
full circular	1.1
full elliptical	1.15
circular tube	1.9
square tube	2.4

Schlemper (7) wrote a program to compute the approximate value of κ for closed cross sections.

3. Calculation of the displacements of a beam element.

The following displacements can be calculated for a beam clamped at one end and loaded at the other end:

3.1. Elongation of the beam due to a normal force N

$$u = \frac{N\,1}{E\,F} \tag{3.1}$$

3.2. Deflections and rotations due to the moments M_y and M_z

$$w = \frac{-\,M_y\,1^2}{2\,E\,I_y} \tag{3.2}$$

$$\varphi_y = \frac{M_x\,1}{E\,I_y} \tag{3.3}$$

$$v = \frac{M_z\,1^2}{2E\,I_z} \tag{3.4}$$

$$\varphi_z = \frac{M_z\,1}{E\,I_z} \tag{3.5}$$

3.3. Deflections and rotations due to the shear forces D_y and D_z. The shear force D_y causes a deflection v_b due to bending and an additional deflection v_s due to shear

$$v = v_b + v_s = \frac{D_y\,1^3}{3\,E\,I} + \frac{\kappa_y\,D_y\,1}{G\,A} \tag{3.6}$$

The rotation φ_z will be $\varphi_z = \frac{D_y 1^2}{2\,E\,I}$ (3.7)

Due to the shear force D_z

$$w = \frac{D_z\,1^3}{3\,EI_2} + \frac{\kappa_z\,D_z\,1}{G\,A} \tag{3.8}$$

$$\varphi_y = -\,\frac{D_z 1^2}{2\,EI_z} \tag{3.9}$$

Fig. 3.1 shows for a certain shearfactor the influence of the length of the beam on the ratio between w_s and w. Shear only causes a displacement but no rotation, therefore the influence of shear will decrease with the length of the coupled element. Fig. 3.2 shows the ratio between w_s and w when an element of a certain length is coupled to the loaded element.

3.4. Rotation due to the torsional moment M_x

A moment M_x causes:

a) rotation

b) warping and

c) distortion of the cross-rection (see fig. 3.3).

ad a: The rotation of the cross-section S_x is given by

$$\varphi_x = \frac{M_x\, l}{G\, J} \qquad (3.10)$$

In this formule J is calculated by the formules (2.1) or (2.2) according to the theory of Bredt or De Saint-Venant. In these theories there is no axial normal tension and the cross section is free to warp. Further more it is assumed that the shape of the cross-section does not change.

ad.b: Warping does not seem to influence the rotation or displacement of the centre of gravity of the cross-section. Depending on the shape of the cross-section warping causes a relative axial displacement of one point to an other.

ad c: The distortion of the cross section is indicated by the change of the angle ψ between two adjacent walls. Distortion causes extra displacements of points within the cross-sectional area. Fig. 3.4. shows that for some points these displacements can be much higher in comparison with displacements without distortion.

4. Influence of ribbing and transverse partitions.

In order to research the influence of ribbing and transverse partitions, many practical and theoretical work has been done.
Dreyer (6) has measured these influences on a column as shown in fig. 4.1. The results of the measurements for bending are given in fig. 4.2. The figure shows the relative bending stiffness and the bending stiffness weight ratio. The results are also compared with the relative difference of I and the ratio I/A. From these results it can be seen that for this column there is a fair agreement between measurements and the bending theory of Bernoulli-Navier.
The measurements for the torsional moment do not give values for the rotation but for the displacements of one corner point. In those cases where there is a coverplate on the column, this displacement gives a good indication for the rotation of the cross-section. In fig. 4.3 and fig. 4.4 the results of the measurements are shown. The discrepancies between the theoretical value according to De Saint-Venant and the measurements is clear. This is due to the assumption that the cross-section is free to warp and particuraly that there is no distortion.
Based on the Vlasov theory Janssen and Veldpaus (8), (9), (10) analysed the strength and stiffness of rectangular box-ginders with transverse partitions. Veldpaus (11) evaluated this theory for all kinds of open and closed cylindrical thin walled cross-sections. With the help of a special program the characteristics can be calculated. In fig. 4.5 the analysis of the column of a milling machine is shown with and without a topplate. The influence of ribbing and topplate can be clearly seen.

5. Apertures.

In many column elements of machine tool structures apertures in walls and transverse partitions are present. To know the influence of these apertures Dreyer (6) and Bielefeld (12) did a number of experiments.
In fig. 5.1 the influence of an aperture in a transverse partition is shown. One can see that for $A'/A > 0,3$ the torsional stiffness decreases rapidly. The influence of apertures in the wall is shown in fig. 5.2. These apertures too have a remarkable influence on the stiffness even after been closed by a cover.

6. Finite elements.

To overcome the problems which occur at points with local deformations, torsion and bending of columns with internal ribbing, partitions and apertures the displacements can be calculated by dividing the column into a number of finite elements.
Typical basic elements include beam elements, thin plate elements of triangular, rectangular or general quadrilateral form and prismatic elements (see fig. 6.1). By connecting such finite elements to another at a definite number of nodal points a construction can be formed.
The deformation of the finite elements is constrained to a prescribed pattern which is expressed in mathematical form by a "displacement function". With these displacement functions the stiffness matrix of an element can be formed. For thin plate elements there are displacement functions describing separately the deformations of the plate under plane stress and the deformations of the plate when subjected to bending forces. These two situations are asumed to be independent.
In fig. 6.2 for a number of frequently used plate elements the displacement functions are given. It is obvious that for all these elements the displacement functions for the in plane deformations differ from the displacement functions for the out of plane displacements. This makes that the elements are not fully compatible when connected to each other under an out of plane angle and to calculate the characteristics of columns properly a fine mesh is necessary.
Hinduja and Cowley (13), (14), did compute the displacements of the column of fig. 4.1, which was used by Dreyer (6). A number of different plate elements were used to see the influence of the displacement functions and the division of the elements. In fig. 6.3 the meshes used to compute the column with rectangular elements are shown. Some of the computed and measured results are shown in fig. 6.4. By refining the mesh the results converge to a certain value.
Hinduja and Cowley (13) also computed the influence of the bending stiffness of the element on the total torsional stiffness of the column. They found that, depending of the hight of the column, there was a difference varying from 14 to 24 % between the deflections computed with elements having only a membrane (in plane) stiffness and elements having a membrane and flexural (bending) stiffness. So even for a thin walled column as used by Dreyer the bending stiffness of the plates have a remarkable influence.

Some examples of computing column structures are given by Noppen (15), Hoshi (16) and Sato (17) (see figs. 6.5, 6.6 and 6.7).
They all use in their programs beam and plate elements together. The number of elements used is large and with it the preparation time to make the computer input. The use of mesh generators seems to be necessary to reduce this preparation time and the possibility of making mistakes. Sata (18) has developped a system in which a construction can be built up with some basic elements (fig. 6.8) combined with modification by rib, window or massive volume (fig. 6.9). The basic elements are automatically divided into a number of finite elements. Fig. 6.10 and fig. 6.11 show the idealization and static deformations of a vertical jig boring machine built up out of these basic elements.

7. Conclusions.

Slender columns can be calculated by using beam elements. When shear has to be taken into account, the calculation of the shear distribution factor is approximatively done in most cases. There is a need for further investigation in this field.

Many experiments have been done in order to get insight into the problems of ribbing, transverse partitions and apertures in columns. The findings of these experiments can be succesfully used in applying beam elements for actual columns.

Finally, the finite elements method can be used for the calculation of columns. In order to diminish time and costs to an acceptable level, the use of mesh generators and standard elements is necessary.
Both subjects - mesh generators and standard elements - need further investigations.

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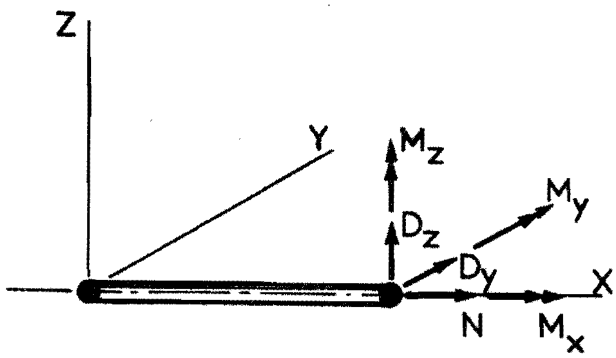


Fig. 2.1 Local coordinates and forces for a beam element

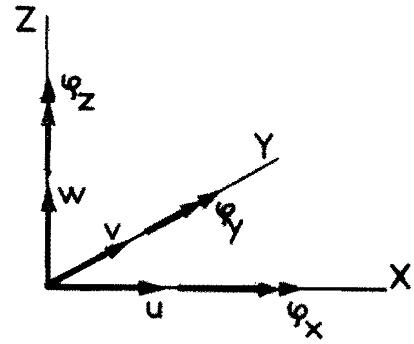


Fig. 2.2 Definition of the local displacements and rotations

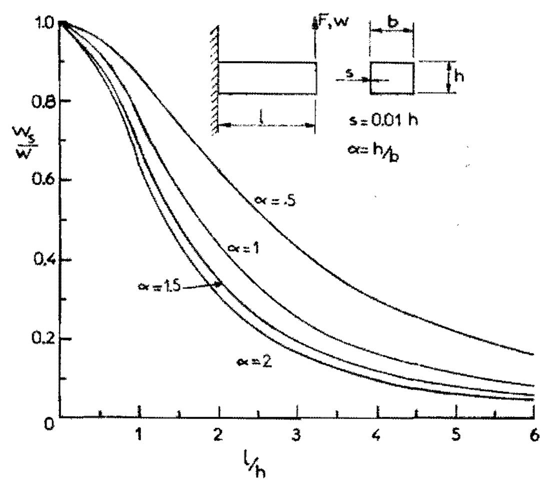


Fig. 3.1 Ratio w_s/w for a single beam

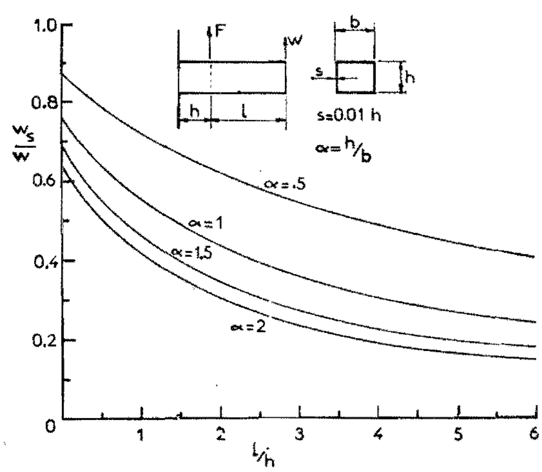


Fig. 3.2 Ratio w_s/w for a coupled beam

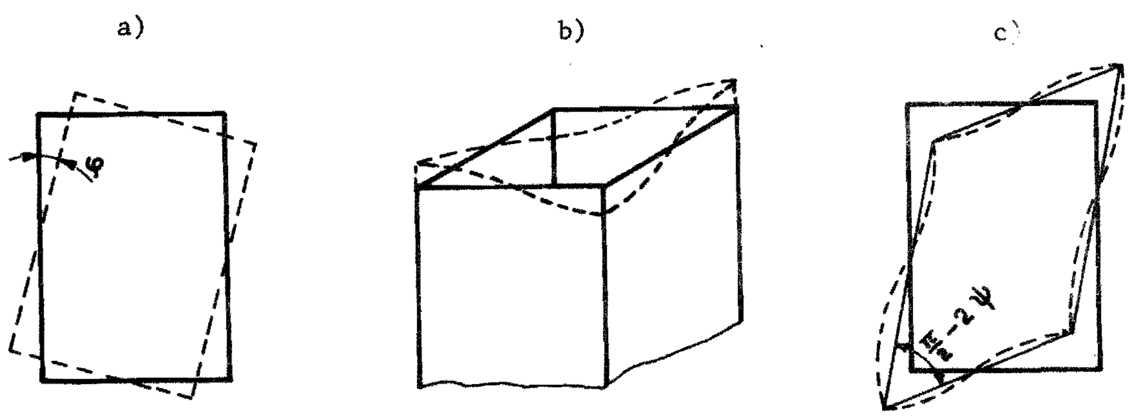


Fig. 3.3 Rotation (a), warping (b) and distortion (c) of the cross section

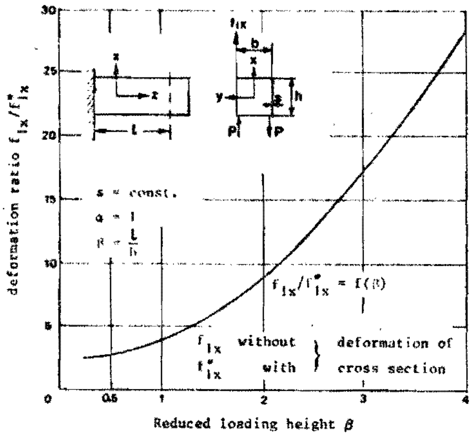


Fig. 3.4
Ratio of displacement
with and without topplate
Dreyer (6)

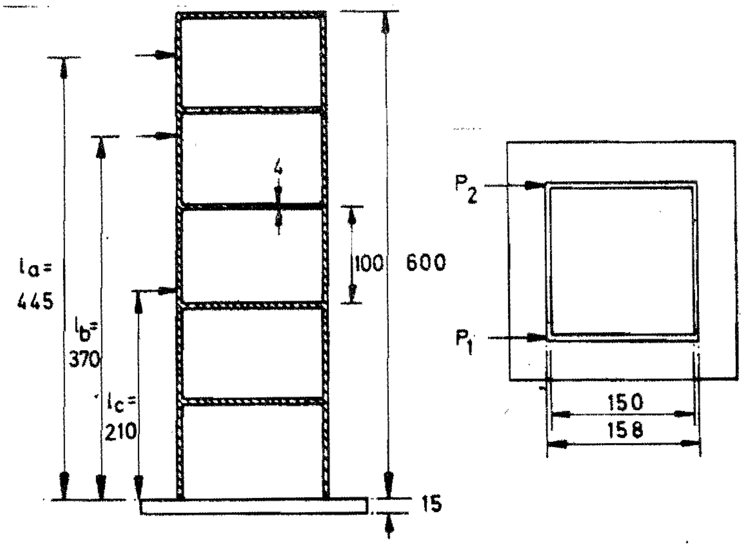
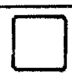
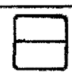
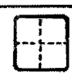
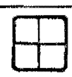
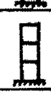


Fig. 4.1 Dimensions of the column used by Dreyer (6)

		Relative bending stiffness		Bending stiffness/weight ratio	
case	mode	theoretical	measured	theoretical	measured
A		100	100	100	100
B		100	100	100	100
A		110.5	113	82.8	90
B		110.5	117	82.8	94
A		111	114	75.2	76
B		111	114	75.2	76
A		115	119	86.5	90
B		115	121	86.5	90
A		128	132	79.2	83
B		128	132	79.2	81
A		100	91	93.5	85
A		100	85	88	75

A- model with cover plate
B- model without cover plate

Fig. 4.2 Relative bending stiffness of columns

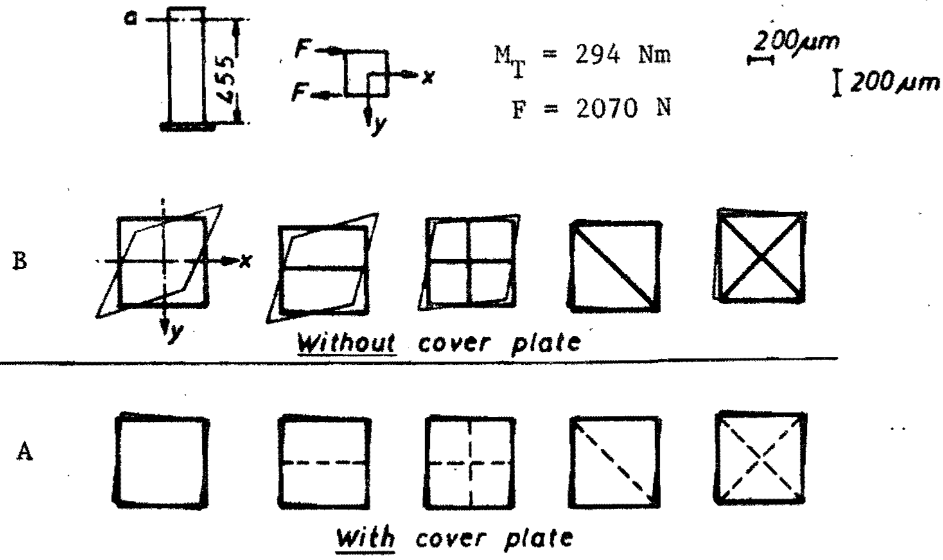


Fig. 4.3 *Cross-Sectional Deformation of Columns with Different Internal Ribs Loaded in Torsion*

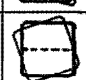
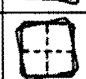
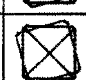
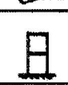
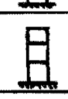
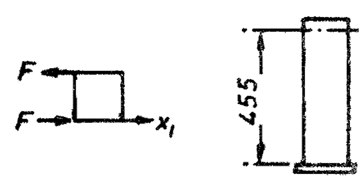
		Relative torsional stiffness	Torsional stiffness/ weight ratio
A		51.5%	51.5%
B		6.5%	6.5%
A		51.5%	42%
B		9%	7.5%
A		51.5%	36%
B		15%	10%
A		79.5%	60.5%
B		65%	48.5%
A		126%	79%
B		115%	70.5%
A		98%	92%
A		108%	95%

Fig. 4.4

Torsional stiffness of columns



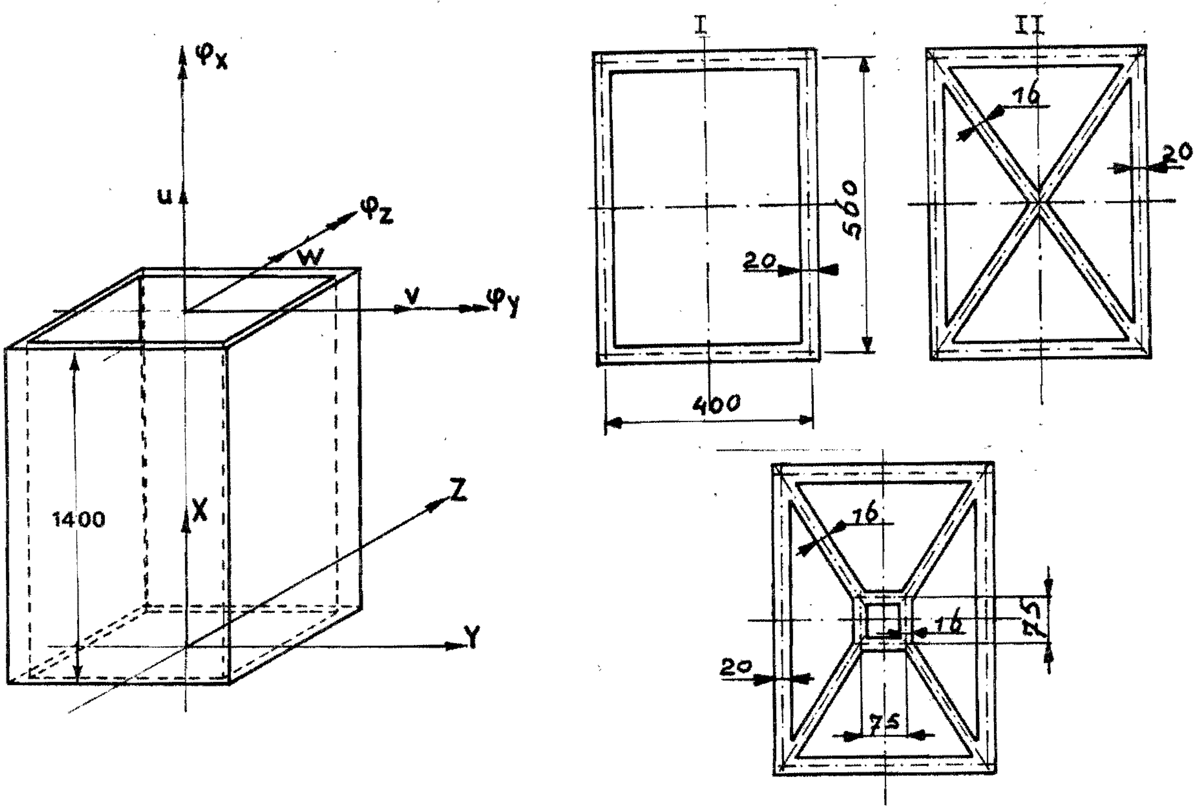


Fig. 4.5

		ϕ_x (rad)	V (mm)	ϕ_z (rad)	W (mm)	ϕ_y (rad)
F_y 1000N	I	0	$.1152 * 10^{-1}$	$.9815 * 10^{-5}$	0	0
	II	0	$.8837 * 10^{-2}$	$.7760 * 10^{-5}$	0	0
	III	0	$.8820 * 10^{-2}$	$.7745 * 10^{-5}$	0	0
F_z 1000N	I	0	0	0	$.7223 * 10^{-2}$	$-.5918 * 10^{-5}$
	II	0	0	0	$.5224 * 10^{-2}$	$-.4507 * 10^{-5}$
	III	0	0	0	$.5265 * 10^{-2}$	$-.4561 * 10^{-5}$
M_y 1000Nm	I	0	0	0	$-.5919 * 10^{-2}$	$.8454 * 10^{-5}$
	II	0	0	0	$-.4508 * 10^{-2}$	$.6439 * 10^{-5}$
	III	0	0	0	$-.4561 * 10^{-2}$	$.6514 * 10^{-5}$
M_z 1000Nm	I	0	$.9816 * 10^{-2}$	$.1402 * 10^{-5}$	0	0
	II	0	$.7761 * 10^{-2}$	$.1109 * 10^{-5}$	0	0
	III	0	$.7744 * 10^{-2}$	$.1105 * 10^{-5}$	0	0
M_x 1000Nm	IA*	$.1905 * 10^{-4}$	0	0	0	0
	IB*	$.1887 * 10^{-3}$	0	0	0	0
	IIA	$.1892 * 10^{-4}$	0	0	0	0
	IIB	$.1892 * 10^{-4}$	0	0	0	0
	IIIA	$.1876 * 10^{-4}$	0	0	0	0
	IIIB	$.2402 * 10^{-4}$	0	0	0	0

*) A with } top-plate
B without }

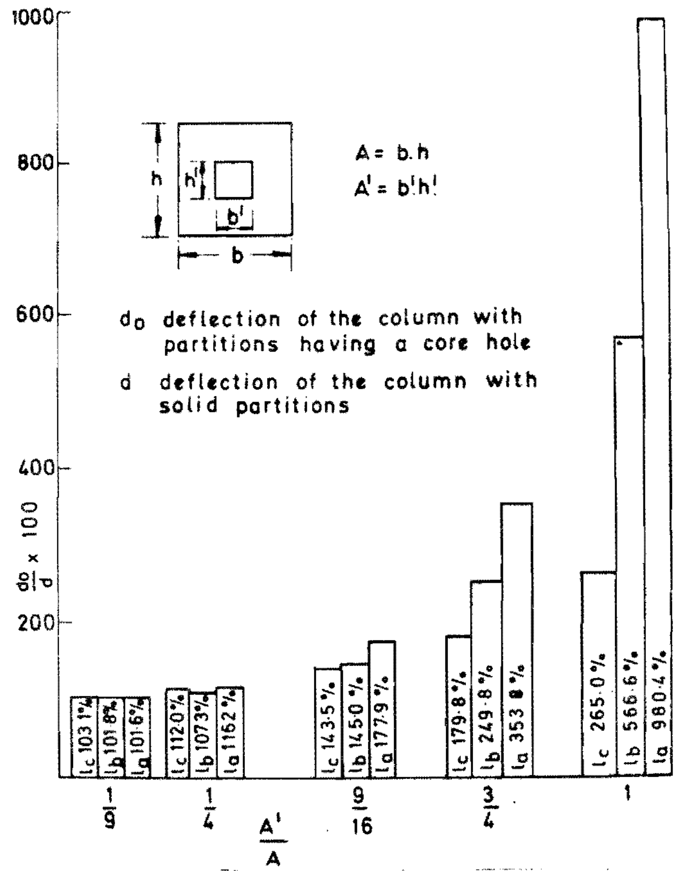


Fig. 5.1 Effect of varying the core hole area on the torsional stiffness of the column (14)

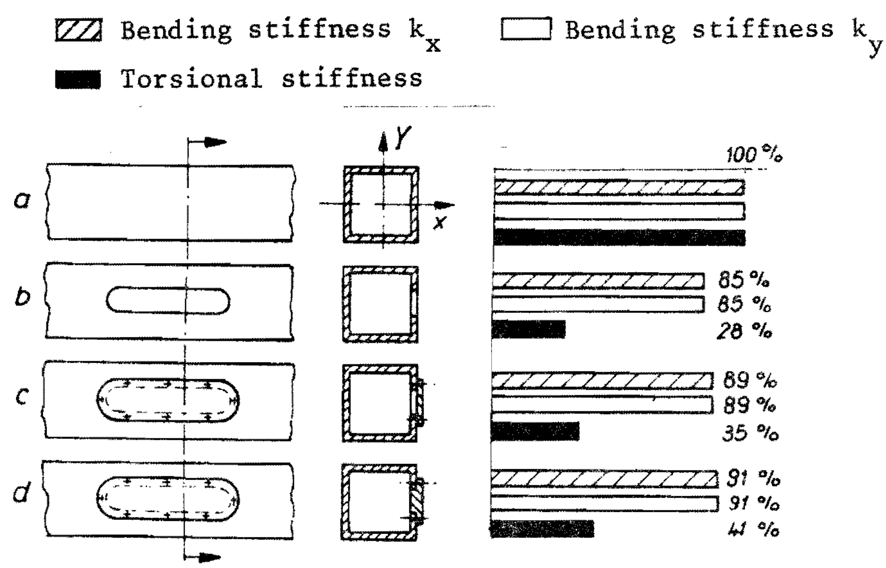


Fig. 5.2 Static stiffness of a box beam (12)

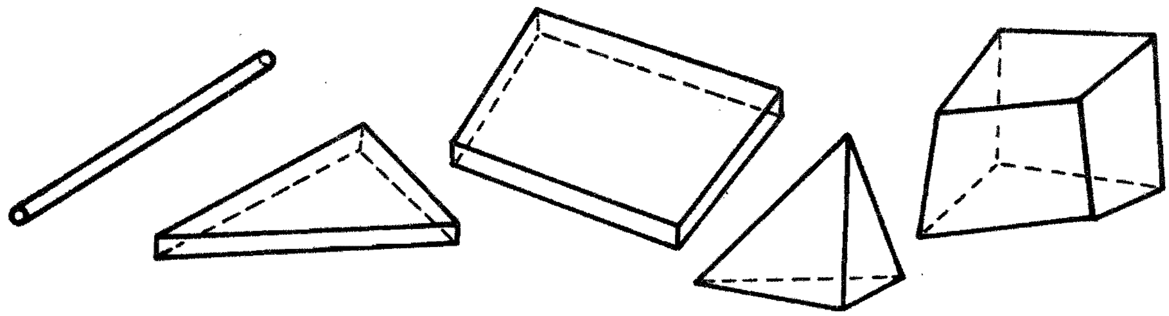


Fig. 6.1 Some basic elements

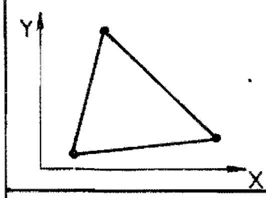
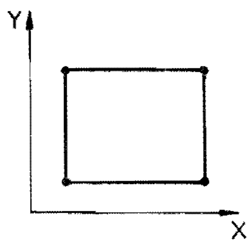
ELEMENT	IN-PLANE DISPLACEMENT FUNCTION	OUT-OF-PLANE DISPLACEMENT FUNCTION
	$u = a_1 + a_2x + a_3y$ $v = a_4 + a_5x + a_6y$	$w = a_1 + a_2x + a_3y + a_4xy + a_5x^2 + a_6y^2 + a_7x^3 + a_8y^3 + a_9xy^2$
	$u = a_1 + a_2x + a_3y + a_4xy$ $v = a_5 + a_6x + a_7y + a_8xy$	$w = a_1 + a_2x + a_3y + a_4xy + a_5x^2 + a_6y^2 + a_7x^2y + a_8xy^2 + a_9x^3 + a_{10}y^3 + a_{11}x^3y + a_{12}xy^3$
	$u = a_1 + a_2x + a_3y + a_4xy + (\frac{\nu}{1-\nu} a_4 - \frac{a_8}{2})y^2$ $v = a_5 + a_6x + a_7y + a_8xy + (\frac{\nu}{1-\nu} a_8 - \frac{a_4}{2})x^2$ (Cheung)	
	$u = a_1 + a_2x + a_3y + a_4xy + a_5xy^2 + a_6xy^3 + a_7y^2 + a_8y^3$ $v = a_9 + a_{10}x + a_{11}y + a_{12}xy + a_{13}x^2y + a_{14}x^3y + a_{15}x^2 + a_{16}x^3$	

Fig. 6.2 The displacement functions of some simple plate elements.

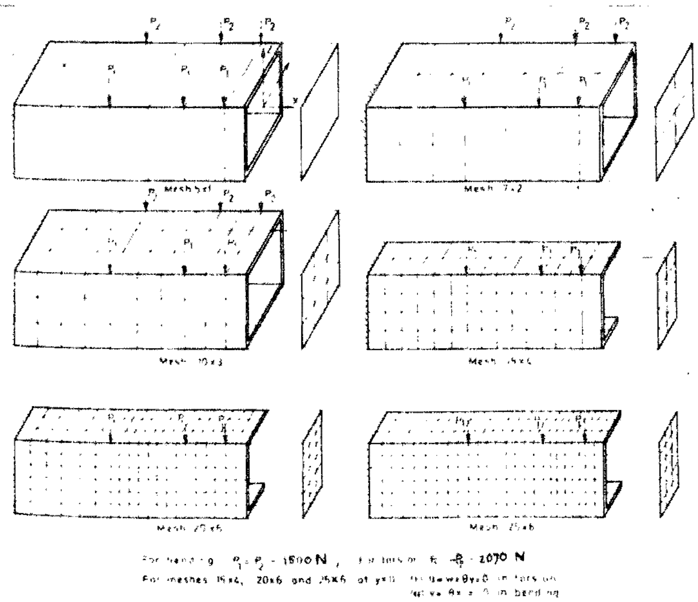
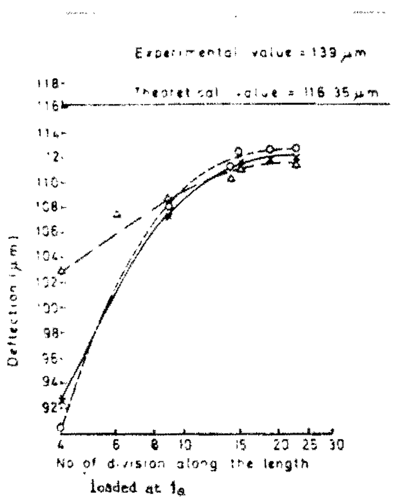


Fig. 6.3 Meshes used for column with/without end-plate (14)



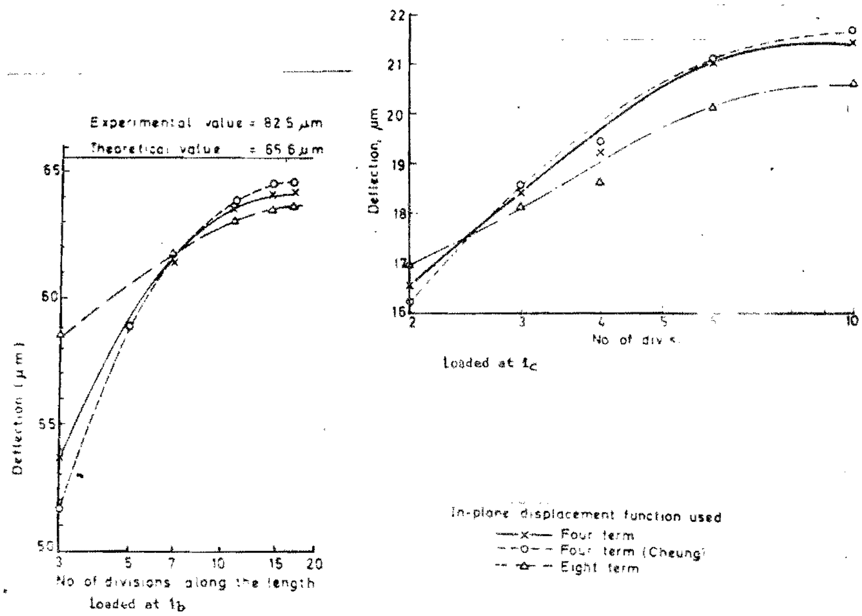


Fig. 6.4 Convergent curves for unribbed column loaded in torsion (14)

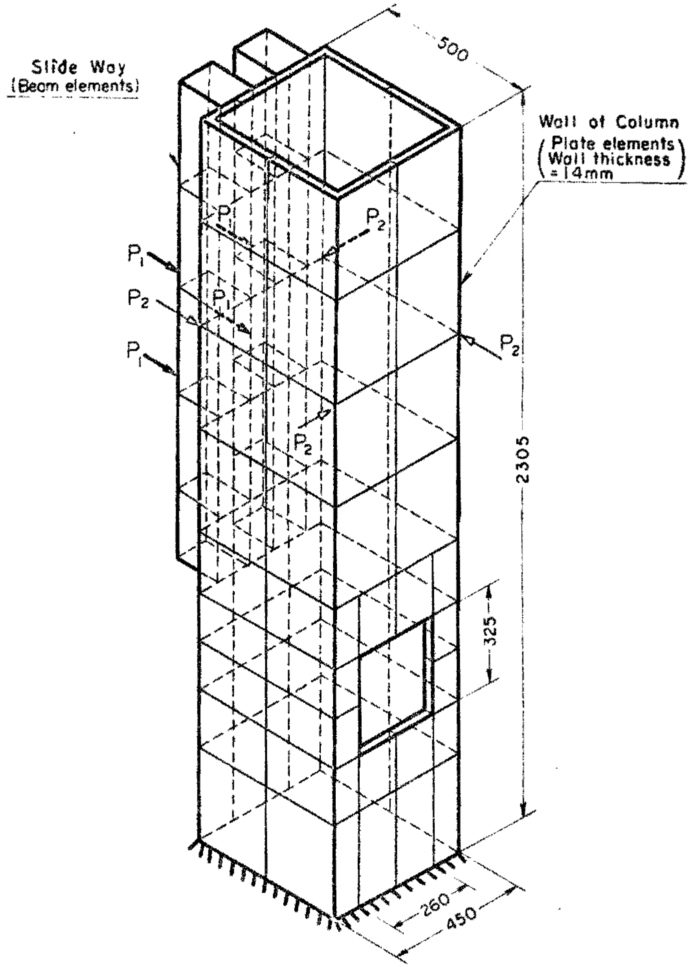


Fig. 6.5 Column model of NC Drill Press with a cut out (16)
Applied loads ; P_1 for bending
P_2 for torsion
Young's modulus =1.[illegible]x10^4kg/mm^2, Poisson's ratio =0.3
Mass density =7.25 [illegible]cm^3, No. of total plate elements=85
No. of total beam el[illegible]ts = 10

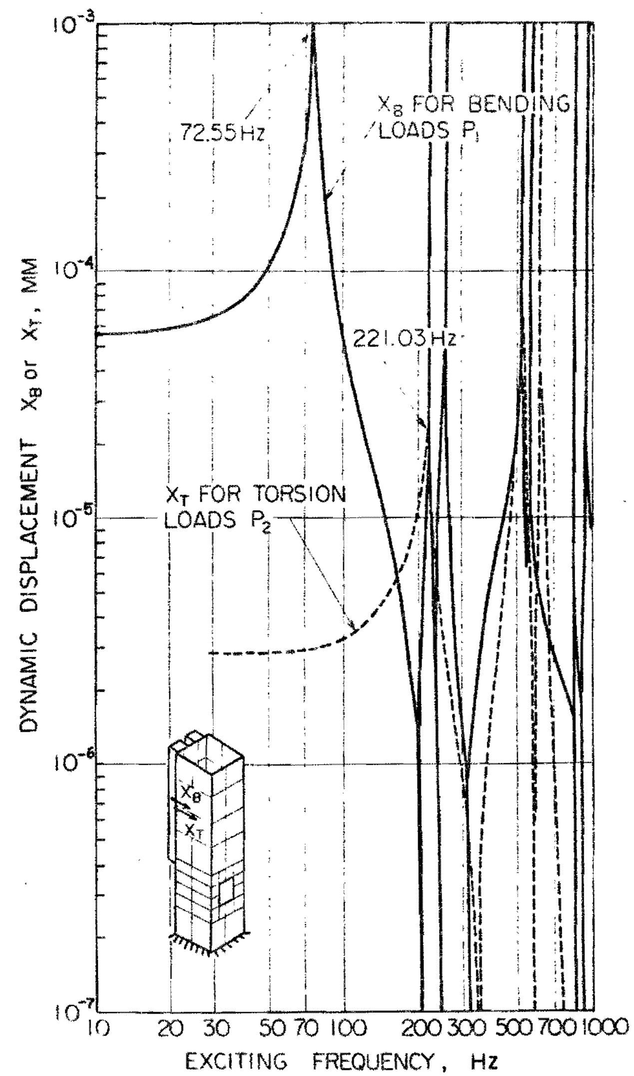


Fig. 6.6 Frequency responses for bending and torsional loads (16) (P_1=P_2=1N) on the column of NC boring machine

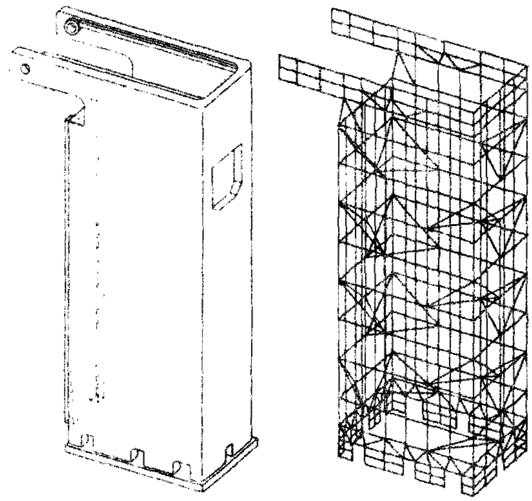


Fig. 6.7
Machine tool column and its model (15)

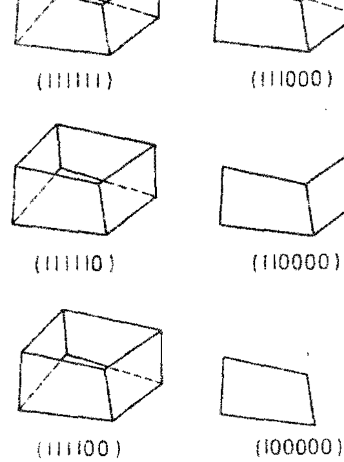
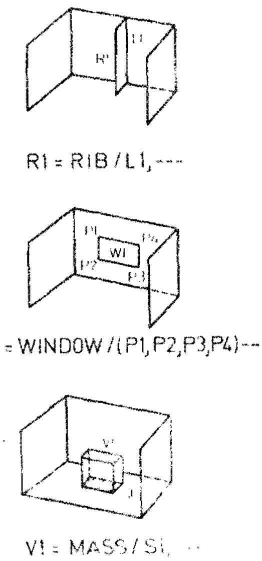


Fig. 6.8
Basic elements (18)

Fig. 6.9
Modified elements (18)



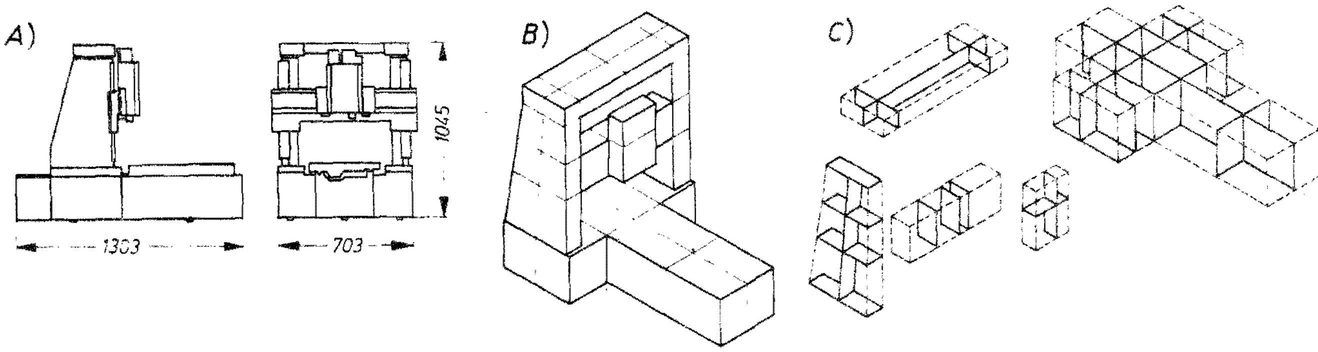


Fig. 6.10 Vertical boring machine (18)

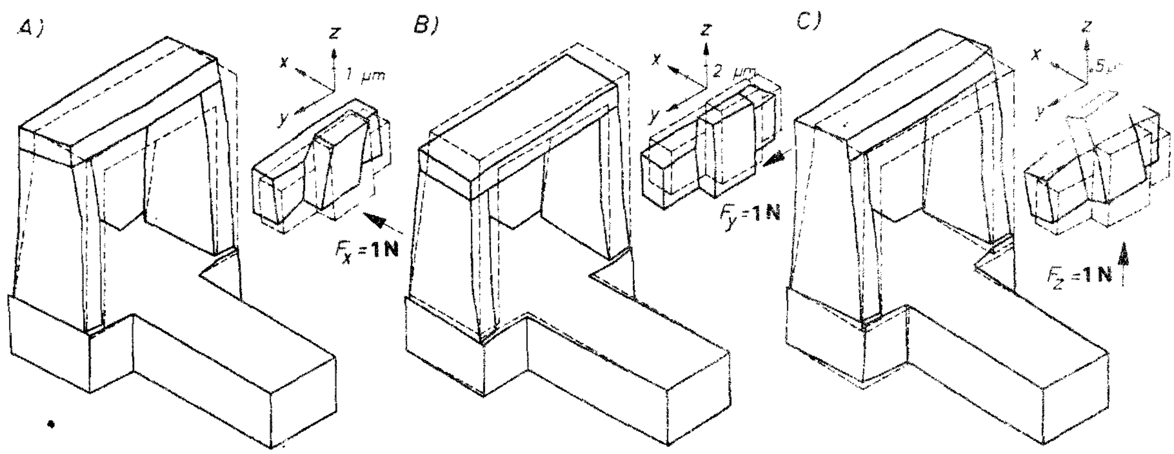


Fig. 6.11 Static loading of the vertical boring machine and the associated deflections (18)