

The influence of a latent heat storage on the heat flux through a brick wall

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Opleiding Wiskunde voor de Industrie Eindhoven

STUDENT REPORT 91-05

THE INFLUENCE OF A LATENT HEAT STORAGE
ON THE HEAT FLUX THROUGH A BRICK WALL

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February 1991

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Contents

Summary	2
Information on this report	2
Section 1: Introduction	3
1.1 General introduction to the problem	3
1.2 Typical data	4
1.3 The results	4
Section 2: The instationary 1 – D model	5
2.1 Mathematical formulation	5
2.2 Dimensionless model	7
2.3 Formal analytical solution	8
2.4 Initial and boundary conditions	12
Section 3: Results from the 1 – D model	14
3.1 Data	14
3.2 Numerical results	15
Section 4: The stationary 2 – D model	21
4.1 The 2 – D problem	21
4.2 Mathematical formulation	21
Section 5: Results from the 2 – D model	25
5.1 Numerical analysis	25
5.2 Numerical results	25
Section 6: Discussion and conclusions	30
References	31

Summary

The problem of studying the influence of a latent heat storage on the expenditure of energy is considered. Two models are introduced based on the one-dimensional respectively two-dimensional heat equation. In the second model the solution is numerically obtained using the succesively overrelaxation method. Numerical results are given for circumstances in which latent heat storage is ensued. It is concluded, that latent heat storage has a very positive effect on the expenditure of energy, provided the climate is rather extreme, viz. with a maximal temperature of $70^{\circ} C$ during several hours a day.

Information on this report

In 1990, from August 20 until August 28, Dr. W.G. Eschmann from the University of Kaiserslautern (Germany) gave a course called "Cases in mathematical modelling" at the University of Technology Eindhoven (The Netherlands). This course was given to an audience of ECMI-students and faculty staff members. This report treats a problem that was presented to us by dr. W.G. Eschmann. We have been working on this problem during three months with good support from the staff members. We would like to thank Dr. Ir. S.J.L. van Eijndhoven, Dr. J. Molenaar and Dr. S.W. Rienstra for their useful contributions. We hope you enjoy reading this report.

§1 Introduction

§1.1 General introduction to the problem

In Germany the heating of houses gives a major contribution to the household energy costs. This fact motivated a producer of bricks to design a new type of brick using a special material named texxos. Figures 1.1.a and 1.1.b show this new brick, containing a cylinder filled with texxos, in more detail.

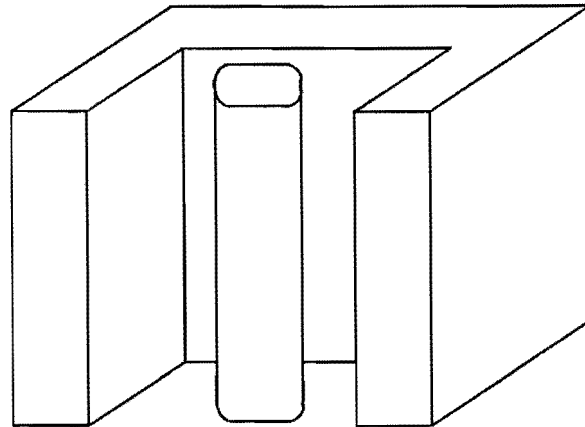


Figure 1.1.a: New brick containing a texxos cylinder

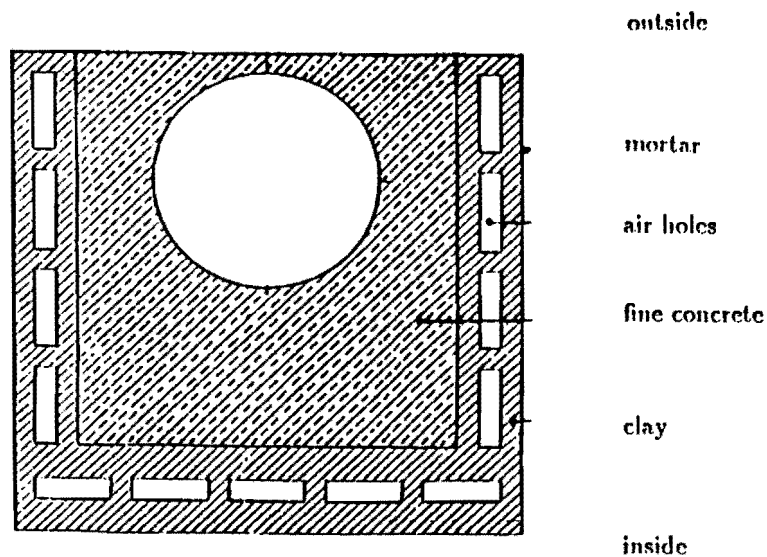


Figure 1.1.b: Cross section of the new brick

The texxos material has the interesting capability of storing heat, in the form of latent heat in a melting process, at a certain critical temperature, say T_{crit} . Below this temperature the cylinder temperature varies proportionally to the supplied heat (energy). At the critical temperature the supplied heat is used to start a melting process, which causes the cylinder temperature to stay at a constant level. After the melting process is completed the supplied energy is again used for heating up the cylinder. Evidently, texxos is not the only material with this behaviour. However, texxos has the convenient property of melting at a temperature just above room temperature. So at daytime, when the sun is shining, the bricks store the supplied energy of the sun to release it later to the room behind. Although the mechanism is known in principle, it is hard to estimate whether there will be any gain, because various effects interact in a subtle way. The present study is meant to analyse to what extent and under which circumstances the new brick reduces the heating costs.

§1.2 Typical data

In this section we summarize the typical data concerning this problem.

$L = 0.20$ [m]; thickness of the wall
 $T_{crit} = 300$ [K]; critical temperature of texxos

Material constants for fine concrete

$\alpha^2 = 8.75 \times 10^{-7}$ [m^2/sec] ; diffusivity constant
 $\rho = 1.2 \times 10^3$ [kg/m^3] ; mass density
 $c = 1.05 \times 10^3$ [$J/kg K$] ; specific heat
 $k = 1.1025$ [$W/m K$] ; thermal conductivity

§1.3 The results

The texxos cylinder has a very positive effect on the energy costs, if we are in a position that it functions during a long period of time. The texxos cylinder, however, will only function during a long period of time, when the climate is rather extreme, with enough sun radiation to yield a maximum outer wall temperature of $70^\circ C$ during several hours a day.

§2 The instationary 1 – D model

2.1 Mathematical formulation

To describe the heat flux through a brick wall, we have to formulate:

- heat diffusion in the material
- boundary conditions at the interfaces
- initial conditions

As we study the influence of latent heat storage on the heat flux through a wall we want the texxos layer to actually store supplied heat.

Therefore, we assume the texxos cylinder to remain at its critical temperature T_{crit} . Consequently, we do not need any equation to describe the effectively instantaneous heat diffusion in the texxos cylinder. The heat diffusion through the fine concrete in the wall is assumed to be described by the heat equation

$$(2.1) \quad \rho c \frac{\partial u}{\partial t} = \nabla \cdot (k \nabla u)$$

where $u(\mathbf{x}, t)$ is the temperature at position \mathbf{x} in the wall at time t .

Since the thermal conductivity k will only vary slightly in the range of temperatures considered, and since the fine concrete will be approximately homogeneous it seems reasonable to assume k to be constant in place and time. So Equation (2.1) reduces to

$$(2.2) \quad \frac{\partial u}{\partial t} = \alpha^2 \Delta u$$

where $\alpha^2 = k/\rho c$ is the diffusivity constant for fine concrete.

By neglecting geometrical effects, that we expect to have only small influence on the net heat flux through the wall, we are able to simplify this three dimensional equation further.

First, we note that the clay and the airholes within it only delay the heat diffusion. Therefore, we omit the clay parts of the wall, as its influence could well be modelled by a thicker wall of fine concrete. Next we assume the texxos to be contained in a layer instead of a cylinder as indicated in Figure 2.1.

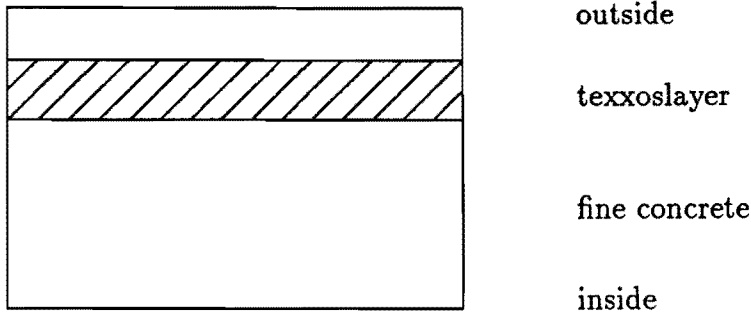


Figure 2.1: Cross section of the brick in top view

The validity of these assumptions compared with a 2 – D model will be discussed in Section 4. Since the heat diffusion in the texxos cylinder is effectively instantaneous the heat flux through the wall depends on the amount of fine concrete that is present in a brick. Therefore, we choose the layer volume to be equal to the cylinder volume. However, one can also argue to choose the surface of the layer equal to the surface of the cylinder. Because we are not interested in side effects at the corners of houses at the moment, we assume that the walls are infinitely long. In addition, we do not take into account the influence of wind and hot air circulation along the wall, because we think that these effects do not play an important role when comparing the net heat flux through a classical wall and through a wall consisting of new bricks. So we assume no convection of air at both interfaces. Owing to the assumptions made, Equation (2.3) reduces to the one-dimensional heat equation

$$(2.3) \quad \frac{\partial u}{\partial t}(x, t) = \alpha^2 \frac{\partial^2 u}{\partial x^2}(x, t).$$

It is clear that we need boundary conditions at both interfaces and an initial temperature distribution through the wall. At the outside we give the temperature as a function of time, $T_{out}(t)$, as we want to subject the wall to a known type of weather. In practice, the supplied or withdrawn heat will modify the inside temperature. For convenience we assume that these temperature changes do not occur owing to either heating or cooling the room properly. So we also give the desired inside temperature as a function of time, $T_{in}(t)$.

As the initial condition, we take some temperature distribution $\phi(x)$.

In conclusion our model is:

a wall of infinite length with given thickness, structure (with or without a texxos layer), prescribed time-dependent temperature profiles at both sides and a given initial temperature distribution.

As the texxos temperature is constant, the 1 – D brick with texxos is effectively split up into two parts which may be considered independently.

So the mathematical formulation of our model may be given generically as follows:

$$(2.4) \quad \left[\begin{array}{l} \frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \quad , \quad 0 < x < L \quad , \quad 0 < t < \infty \\ u(0, t) = T_{out}(t) \quad , \quad 0 < t < \infty \\ u(L, t) = T_{in}(t) \quad , \quad 0 < t < \infty \\ u(x, 0) = \phi(x) \quad , \quad 0 \leq x \leq L \end{array} \right.$$

where L denotes the thickness of each fine concrete part of the wall, so we deal with two different thicknesses L . As a matter of fact, we need to solve Equation (2.4) twice: once with $T_{in}(t) = T_{crit}$ and once with $T_{out}(t) = T_{crit}$.

By definition, the heat flux F at position x and at time t is given by

$$(2.5) \quad F(x, t) = -k \frac{\partial u}{\partial x} (x, t)$$

where k is the thermal conductivity of fine concrete (a measure for the ability to conduct heat).

§2.2 Dimensionless model

We non-dimensionalize Equation (2.4) by scaling as follows

$$(2.6) \quad \left[\begin{array}{l} x = Lx^* \quad , \\ t = Et^* \quad , \\ u(x, t) = \bar{u}_{out} + (\bar{u}_{out} - \bar{u}_{in}) u^*(x^*, t^*) \\ T_{out}(t) = \bar{u}_{out} + (\bar{u}_{out} - \bar{u}_{in}) T_{out}^*(t^*) \\ T_{in}(t) = \bar{u}_{out} + (\bar{u}_{out} - \bar{u}_{in}) T_{in}^*(t^*) \\ \phi(x) = \bar{u}_{out} + (\bar{u}_{out} - \bar{u}_{in}) \phi^*(x^*) \end{array} \right.$$

where E is the typical time scale of 24 hours, \bar{u}_{out} is the average outside temperature and \bar{u}_{in} is the average inside temperature. Substituting Equation (2.6) in Equation (2.4) and dropping the stars we get the following non-dimensional model:

$$(2.7) \quad \left[\begin{array}{l} \frac{\partial u}{\partial t} = \gamma^2 \frac{\partial^2 u}{\partial x^2} \quad , \quad 0 < x < 1 \quad , \quad 0 < t < \infty \\ u(0, t) = T_{out}(t) \quad , \quad 0 < t < \infty \\ u(1, t) = T_{in}(t) \quad , \quad 0 < t < \infty \\ u(x, 0) = \phi(x) \quad , \quad 0 \leq x \leq 1 \end{array} \right.$$

where $\gamma^2 = \frac{E\alpha^2}{L^2}$, the Fourier number [3]. In a physical sence, the Fourier number is the ratio of the rate of heat transfer by conduction to the rate of energy storage in the system. In order to see what solution we may expect, we consider the extreme values of the Fourier number.

– For $\gamma^2 \gg 1$ we get the quasi-stationary solution $(T_{in}(t) - T_{out}(t))x + T_{out}(t)$, as the heat diffusion is almost instantaneous. Consequently, the texxos cylinder will have hardly any effect on the heat flux through the wall.

– For $\gamma^2 \ll 1$ the heat diffusion through the wall is very slow and the solution will only vary in time in the small boundary layers of $x = 1 + O(\gamma)$ and $x = O(\gamma)$ at the interfaces.

Conclusion: for the texxos cylinder to function, $\gamma = O(1)$. In our case indeed $\gamma^2 \approx 2$.

§2.3 Formal analytical solution

It seems advantageous to write the solution as a perturbation of the quasi-stationary solution, since this yields homogeneous boundary conditions, which are slightly easier to handle.

So we write our solution as:

$$(2.8) \quad u(x, t) = S(x, t) + U(x, t)$$

where $S(x, t) = T_{out}(t)(1 - x) + T_{in}(t)x$ satisfies the boundary conditions.

Substituting Equation (2.8) in Equation (2.7) gives for the perturbation $U(x, t)$:

$$(2.9) \quad \left[\begin{array}{l} DU(x, t) = -\frac{\partial S}{\partial t}(x, t) \\ U(0, t) = 0 \\ U(1, t) = 0 \\ U(x, 0) = \tilde{\phi}(x) \end{array} \right.$$

where

$$\begin{cases} \frac{\partial S}{\partial t}(x, t) = \frac{dT_{\text{out}}(t)}{dt} (1-x) + \frac{dT_{\text{in}}(t)}{dt} x \\ \tilde{\phi}(x) = \phi(x) - S(x, 0) \\ DU(x, t) = \frac{\partial U}{\partial t}(x, t) - \gamma^2 \frac{\partial^2 U}{\partial x^2}(x, t). \end{cases}$$

Equation (2.9) is non homogeneous, has homogenous boundary conditions and a non homogeneous initial condition.

We will write the position dependent part of the solution in terms of a carefully chosen orthogonal basis. For the basisfunctions we take the eigenvectors of the position dependent part of the operator DU .

Solving the problem then reduces to finding the time dependent coefficients of the basisfunctions. First we determine the eigenvectors which are solutions of the Sturm-Liouville problem we get when solving (by separation of variables):

$$(2.10) \quad \begin{cases} DU(x, t) = 0 \\ U(0, t) = 0 \\ U(1, t) = 0. \end{cases}$$

More precisely we substitute $U(x, t) = X(x) T(t)$ in Equation (2.10) and then get for $X(x)$ the Sturm-Liouville problem

$$(2.11) \quad \begin{cases} X''(x) + \lambda^2 X(x) = 0 \\ X(0) = X(1) = 0. \end{cases}$$

Only for $\lambda = n\pi$, $n \in \mathbb{N}^+$ there exist non-trivial solutions to this problem, namely $\sin(n\pi x)$.

So we can take $X_n(x) = \sin(n\pi x)$, $n \geq 1$ as basisvectors, which form an $L_2[0, 1]$ -orthogonal basis [1].

Next we write $\frac{\partial S}{\partial t}(x, t)$ and the solution $U(x, t)$ in terms of the eigenvectors $X_n(x)$, i.e.

$$(2.12) \quad U(x, t) = \sum_{n=1}^{\infty} T_n(t) \sin(n\pi x)$$

$$(2.13) \quad \frac{\partial S}{\partial t}(x, t) = \sum_{n=1}^{\infty} s_n(t) \sin(n\pi x).$$

Using the orthogonality of the eigenvectors, the coefficients $s_n(t)$ can be calculated.

$$(2.14) \quad s_n(t) = 2 \int_0^1 \frac{\partial S}{\partial t}(x, t) \sin(n\pi x) dx .$$

Finally, we determine the coefficients $T_n(t)$, in Equation (2.12), which will give us the solution $U(x, t)$. Substituting Equation (2.12) and (2.14) in (2.9) we get

$$(2.15) \quad \begin{cases} \sum_{n=1}^{\infty} T_n'(t) \sin(n\pi x) = -\gamma^2 \sum_{n=1}^{\infty} (n\pi)^2 T_n(t) \sin(n\pi x) \\ \quad \quad \quad - \sum_{n=1}^{\infty} s_n(t) \sin(n\pi x) \\ \sum_{n=1}^{\infty} T_n(0) \sin(n\pi x) = \tilde{\phi}(x) . \end{cases}$$

Note, that our choice of the basisfunctions gives expressions in terms of $\sin(n\pi x)$ only. In addition, it causes the homogeneous boundary conditions to be satisfied for any coefficients $T_n(t)$, provided $\sum_{n=1}^{\infty} T_n(t) \sin(n\pi x)$ converges uniformly. Again using the orthogonality of the basisfunctions we see that $T_n(t)$ will satisfy the initial value problem

$$(2.16) \quad T_n'(t) + (n\pi\gamma)^2 T_n(t) = -s_n(t)$$

$$T_n(0) = 2 \int_0^1 \tilde{\phi}(\xi) \sin(n\pi\xi) d\xi$$

with solution:

$$(2.17) \quad T_n(t) = T_n(0) \exp\{-(n\pi\gamma)^2 t\} \\ - \int_0^t \exp\{-(n\pi\gamma)^2 (t - \tau)\} s_n(\tau) d\tau , \quad n \geq 1 .$$

Hence the solution to problem (2.9) is

$$(2.18) \quad U(x, t) = \sum_{n=1}^{\infty} T_n(0) \exp\{-(n\pi\gamma)^2 t\} \sin(n\pi x) \\ - \sum_{n=1}^{\infty} \sin(n\pi x) \int_0^t \exp\{-(n\pi\gamma)^2 (t - \tau)\} s_n(\tau) d\tau$$

The solution to or original problem, Equation (2.7) is:

$$(2.19) \quad U(x, t) = T_{out}(t) + x(T_{in}(t) - T_{out}(t)) \\ + \sum_{n=1}^{\infty} T_n(0) \exp\{(-n\pi\gamma)^2 t\} \sin(n\pi x) \\ - \sum_{n=1}^{\infty} \sin(n\pi x) \int_0^t \exp\{-(n\pi\gamma)^2 (t - \tau)\} s_n(\tau) d\tau$$

where

$$\left[\begin{array}{l} T_n(0) = 2 \int_0^1 \tilde{\phi}(\xi) \sin(n\pi\xi) d\xi \\ \tilde{\phi}(\xi) = \phi(\xi) - S(\xi, 0) \\ s_n(t) = 2 \int_0^1 \frac{\partial S}{\partial t}(x, t) \sin(n\pi x) dx \\ = \frac{2}{n\pi} [T'_{out}(t) - T'_{in}(t) \cos(n\pi)] . \end{array} \right.$$

The three different parts of the solution (2.19) can be interpreted as follows. The second term represents the effect of the initial condition, since it contains $T_n(0)$ and with that the initial temperature distribution ϕ . Note that this is only a transient phenomenon, because its influence fades away with time. As mentioned before the first term is the quasi-stationary solution: $S(x, t)$. This is the temperature distribution when taking constant (time independent) boundary conditions T_{out} and T_{in} together with a corresponding initial condition $u(x, 0) = S(x, 0)$. When the boundary temperatures do not change for a long time the third term tends to zero. Therefore, this term mainly represents the deviation from the quasi-stationary solution, caused by temperature changes at the boundaries. It includes delayed responses, since it contains an integration from the switch-on time to time t .

Finally we give an expression for the heat flux F .

$$(2.20) \quad F(x, t) = -k \frac{\partial u}{\partial x}(x, t) \\ = -k [T_{in}(t) - T_{out}(t)] \\ - k\pi \sum_{n=1}^{\infty} T_n(0) \exp\{-(n\pi\gamma)^2 t\} \cos(n\pi x) \\ + k\pi \sum_{n=1}^{\infty} n \cos(n\pi x) \int_0^t \exp\{-(n\pi\gamma)^2 (t - \tau)\} s_n(\tau) d\tau$$

In this equation we recognize the same effects as in the solution for the temperature, see Equation (2.19).

§2.4 Initial and boundary conditions

To investigate the model by numerical experiments we have to choose typical initial and boundary values ϕ , T_{out} and T_{in} . As already mentioned before, the second term in Equation (2.19) only affects the solution shortly after the switch-on time. When we assume for convenience this switch-on time at minus infinity, so $\phi(x) = u(x, -\infty)$ we can neglect this transient phenomenon. Of course we have to shift the lower integration boundary in the third term. So we get the solution

$$(2.21) \quad u(x, t) = T_{out}(t) + x[T_{in}(t) - T_{out}(t)] - \sum_{n=1}^{\infty} \sin(n\pi x) \int_{-\infty}^t \exp\{-(n\pi\gamma)^2 (t - \tau)\} s_n(\tau) d\tau .$$

Note, that we do not need to specify the initial condition ϕ anymore because it has disappeared out of the solution.

For the inside temperature T_{in} we choose a desired room temperature, that is obtained by either heating or cooling the room, independent of T_{out} .

Next we try to model some typical autumn type of weather in Germany. As mentioned before in Section 2.1, we assume the texxos layer to remain at its critical temperature T_{crit} .

Therefore, the outside temperature at day time is taken high enough to ensure an energy balance between the heat supplied during the day and withdrawn during the night from the texxos. It turns out that the outside temperature has to be quite high. Since there is absolutely no need to specify $T_{out}(t)$ and $T_{in}(t)$ in great detail, $T_{out}(t)$ and $T_{in}(t)$ are taken piecewise linear and periodic with a period equal to one day.

Both temperature profiles are shown in Figure 2.2a and 2.2b.

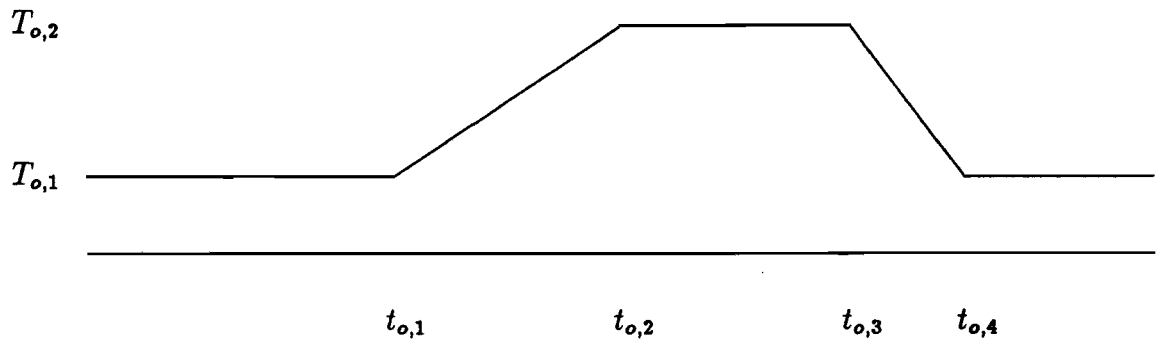


Figure 2.2a: Boundary condition $T_{out}(t)$

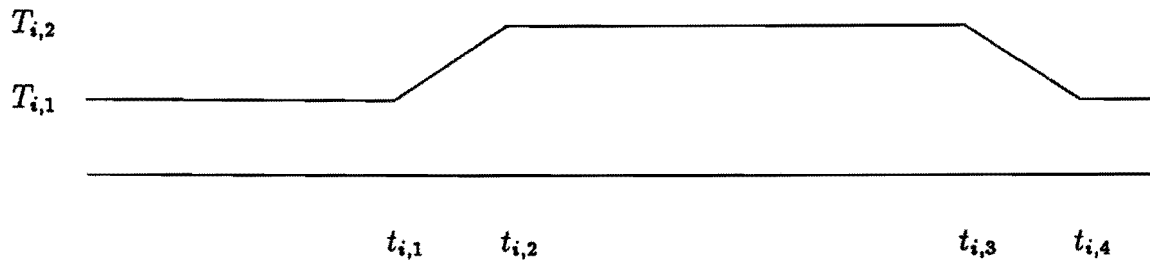


Figure 2.2b: Boundary condition $T_{in}(t)$

§3 Results from the 1 – D model

§3.1 Data

Before we give the results for the 1 – D model we summarize the data used in the calculations.

Parameter in the boundary conditions $T_{out}(t)$ and $T_{in}(t)$

$t_{0,1} = 7.00 h$	$t_{i,1} = 8.00 h$	$T_{0,1} = 5^\circ C$
$t_{0,2} = 12.30 h$	$t_{i,2} = 9.00 h$	$T_{0,2} = 70^\circ C$
$t_{0,3} = 17.30 h$	$t_{i,3} = 22.00 h$	$T_{i,1} = 15^\circ C$
$t_{0,4} = 20.30 h$	$t_{i,4} = 23.00 h$	$T_{i,2} = 20^\circ C$

Brick dimensions

Summarizing the previous discussion in §2.1 on the brick size, we have drawn in Figure 3.1a a brick including a texxos layer and in Figure 3.1b a common brick.

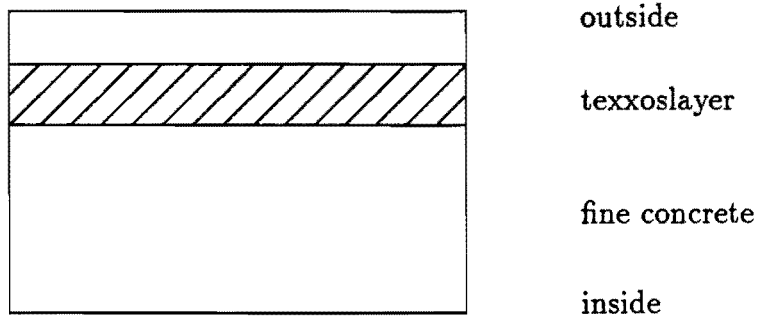


Figure 3.1a: New brick

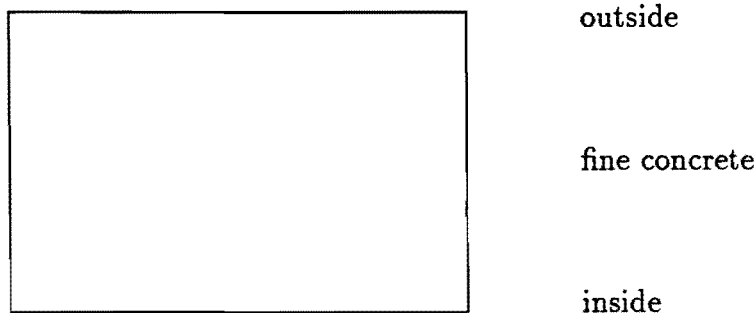


Figure 3.1b: Common brick

The brick dimensions we have used are:

$$\begin{aligned}
 L_1 &= 11 \text{ cm} \\
 L_2 &= 5 \text{ cm} \\
 L_3 &= 4 \text{ cm} \\
 L &= 20 \text{ cm}
 \end{aligned}$$

Further, we have used the typical data mentioned in Section 1.2.

§3.2 Numerical results

First we give some remarks on the (numerical) calculations

- (i) An integration interval of $(-72, t)$ turned out to be sufficient to approximate the infinite integral in (2.22).
- (ii) The infinite summation in (2.22) could be restricted to about 250 terms, depending on x and t .
- (iii) The $s_n(t)$'s as well as the integral in (2.22) could be calculated analytically as we have chosen piecewise linear boundary conditions.

Next we shall show several diagrams.

In Figure 3.2 (resp. Figure 3.3) the flux at the inside (resp. outside) of a normal wall is plotted against time, while in Figure 3.4 (resp. Figure 3.5) the flux is plotted against the position in the brick with (resp. without) a texxos layer. In Figure 3.6 (resp. Figure 3.7) the temperature is plotted as a function of the position in the brick. The numbers in Figures 3.4-7 correspond to different times in the following way:

Table I: Number-Time relation

Number	Time (h)
1	7.00
2	8.30
3	10.30
4	12.30
5	17.30
6	19.30
7	20.30
8	22.30
9	2.00

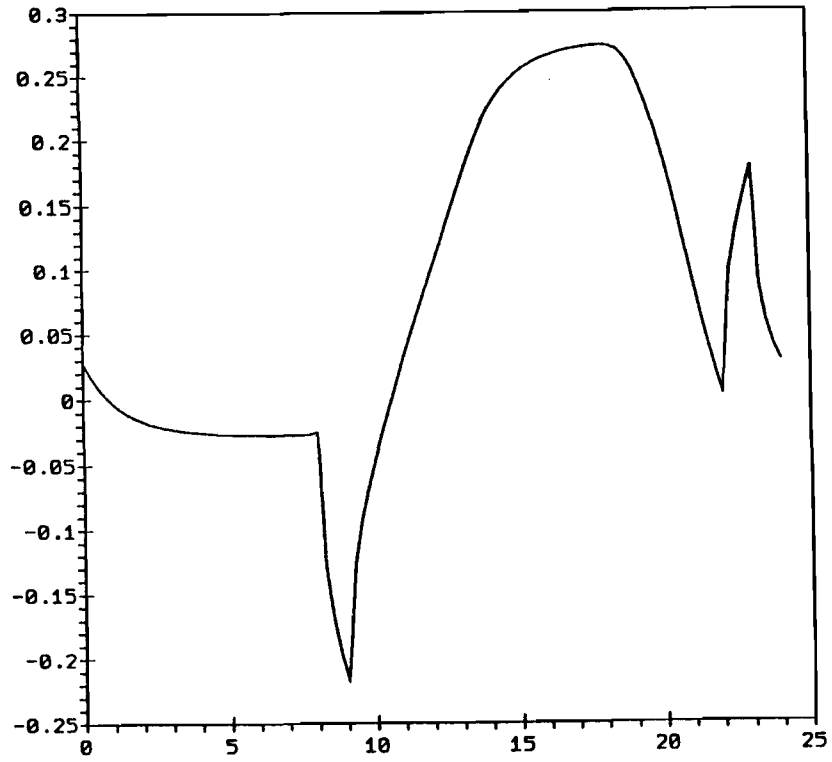


Figure 3.2: Flux on the inside of the wall during a day without a texxos layer

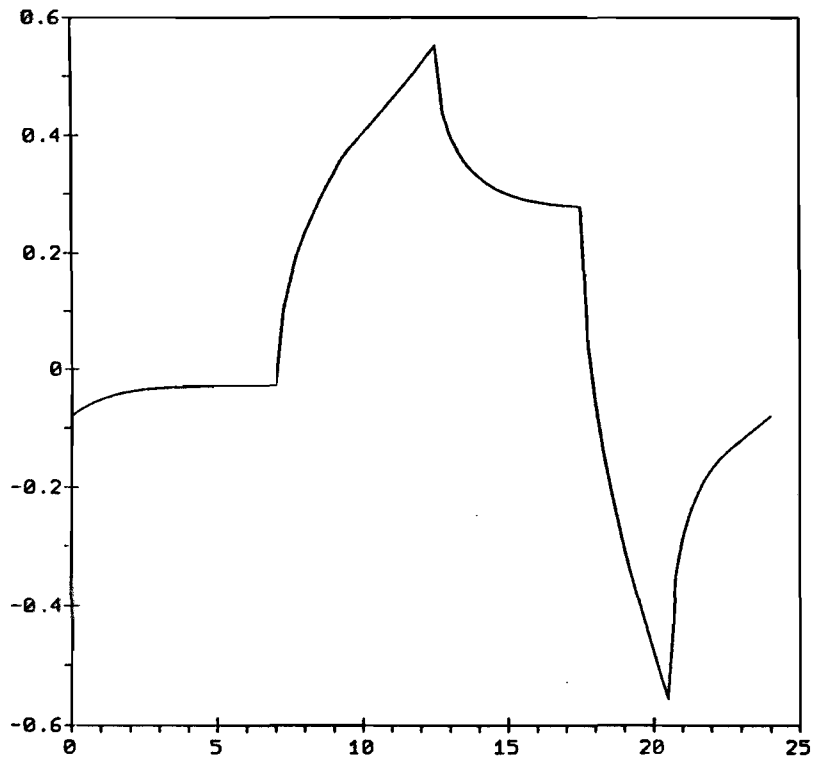


Figure 3.3: Flux on the outside of the wall during a day without a texxos layer

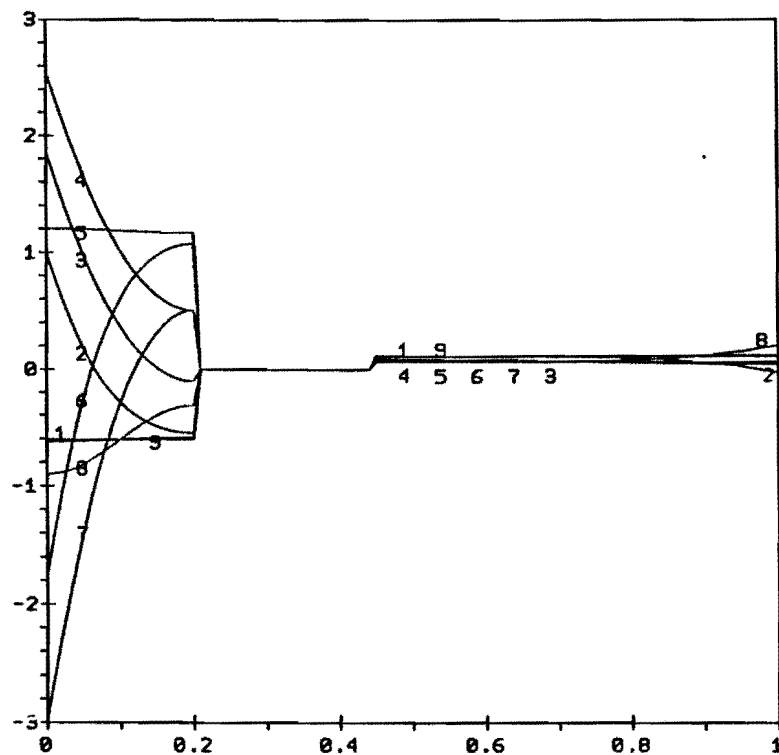


Figure 3.4: Flux in the brick with latent heat storage at some times

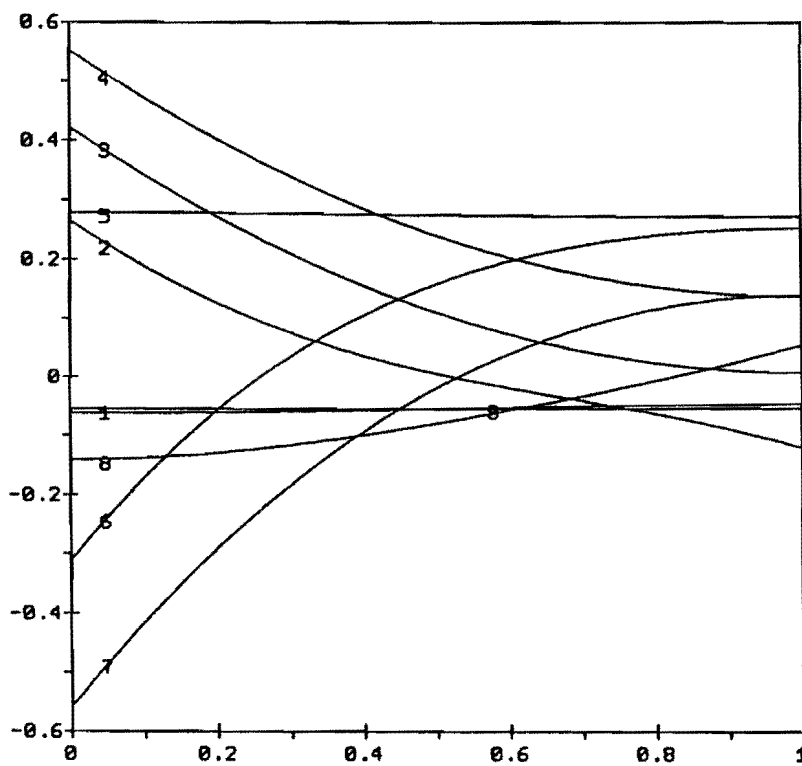


Figure 3.5: Flux in the brick without latent heat storage at some times

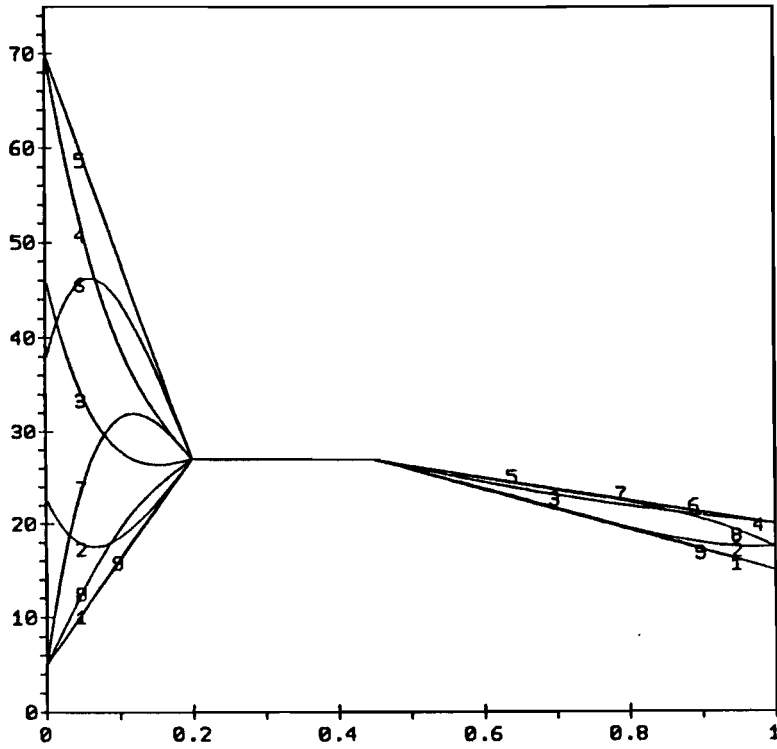


Figure 3.6: Temperature in the brick with latent heat storage at some times

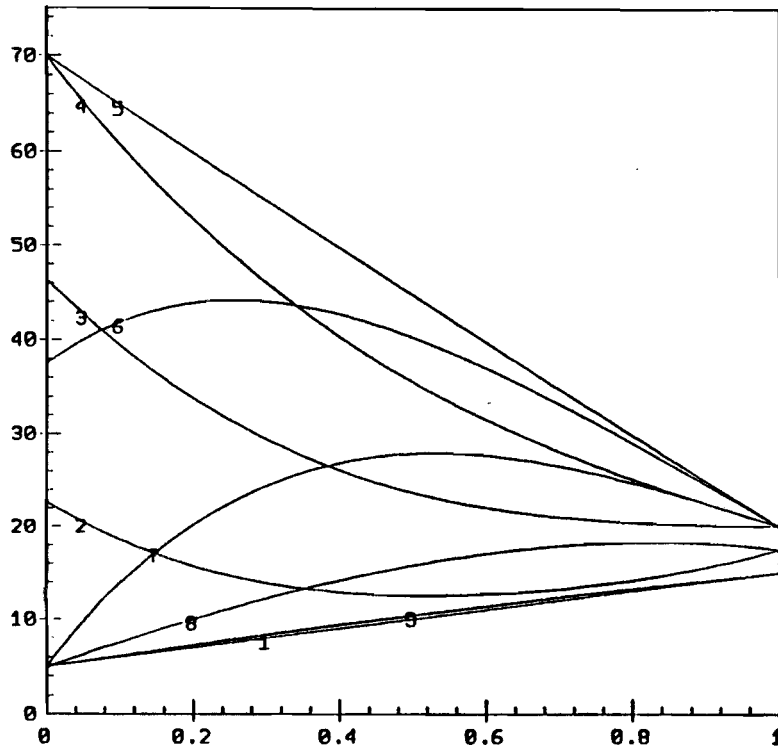


Figure 3.7: Temperature in the brick without latent heat storage at some times

Finally we have compared the amount of energy needed to heat up the room with or without latent heat storage. We have compared two situations as drawn in Figure 3.8 and Figure 3.9.

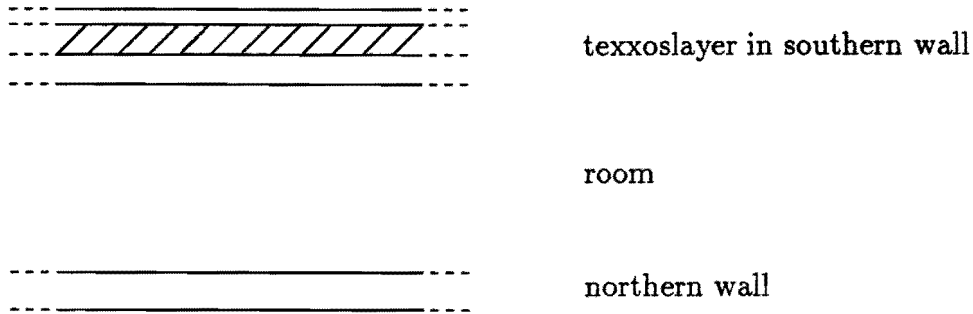


Figure 3.8: Situation 1

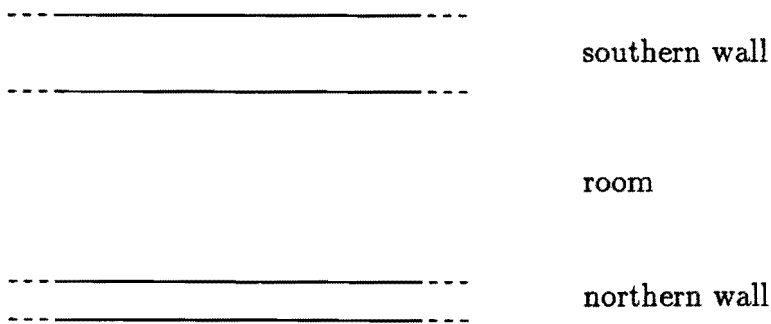


Figure 3.9: Situation 2

The temperature profile at the north side of the house is chosen at the same way as $T_{out}(t)$ except in this case $T_{0,1} = 10^\circ C$ and $T_{0,2} = 15^\circ C$.

The northern walls in Figure 3.8 and 3.9 are considerably thinner than the south ones. This is done to take into account the energy losses via the floor approximately, the ceiling and open doors to the neighbouring rooms and hall. One should realize that the heat flux through the northern walls is the same in both situations, so that comparison remains reliable. Since the inside temperature is kept fixed, it may also occur that we have to cool the room. This may happen when the loss of energy through the northern wall is less than the amount of energy entering the room through the south wall. In that case the house should be cooled by, for example, opening a window, so

that we may neglect costs for cooling.

Finally, we arrive, with the above assumption, at the following numbers:

The amount of energy needed in situation 1, *without* latent heat storage is 2.738 kWh/m^2 .

The amount of energy needed in situation 2, *with* latent heat storage is $1,054 \text{ kWh/m}^2$.

So we may conclude that the one-dimensional model shows that the influence of latent heat storage on the expenditure of energy is rather big.

Before we give the final conclusions we first want to exploit the two-dimensional case.

§4 The stationary 2 – D model

In this section we study the reliability of the 1 – D model by looking at a less reduced model.

§4.1 The 2 – D problem

There are two types of heat flux through a wall with a latent heat storage. There is heat flow through the wall via the cylinder filled with texxos. All the other heat flux, that is the heat flux directly from the outside to the inside or vice versa, we shall call leakage. It is this leakage that amongst other effects of minor importance we have excluded in our 1 – D model.

Our goal now is to find out to what extent the possibility of leakage influences the temperature distribution at the innerside of the wall and the heat flux through the wall. This will provide us a check on the reliability of the results from the 1 – D model.

§4.2 Mathematical formulation

We will restrict ourselves to a stationary temperature distribution in the wall because the temperature varies relatively slow in time.

In our 2 – D model for numerical convenience we model the texxos cylinder as a block as indicated in Figure 4.1.

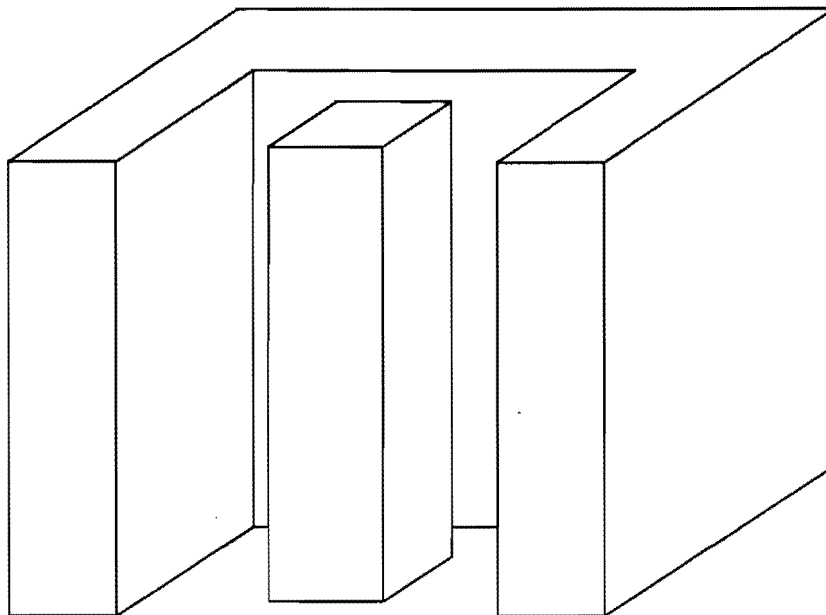


Figure 4.1: Brick with rectangular block filled with texxos

Further we make the following additional assumptions:

- (i) The intensity of the sunshine will be the same everywhere at the outer side of the wall. This implies that the flux into the wall is uniform over the outer surface, say F_{out} .
- (ii) The inside temperature will be kept on a constant level by either heating or ventilating the house.

The assumptions cause the temperature distribution to be independent of time and height. So the temperature T will satisfy the two-dimensional heat equation.

$$(4.1) \quad \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 .$$

Here the x and y axes are taken as indicated in Figure 4.2, where a horizontal cross section of the wall is drawn.

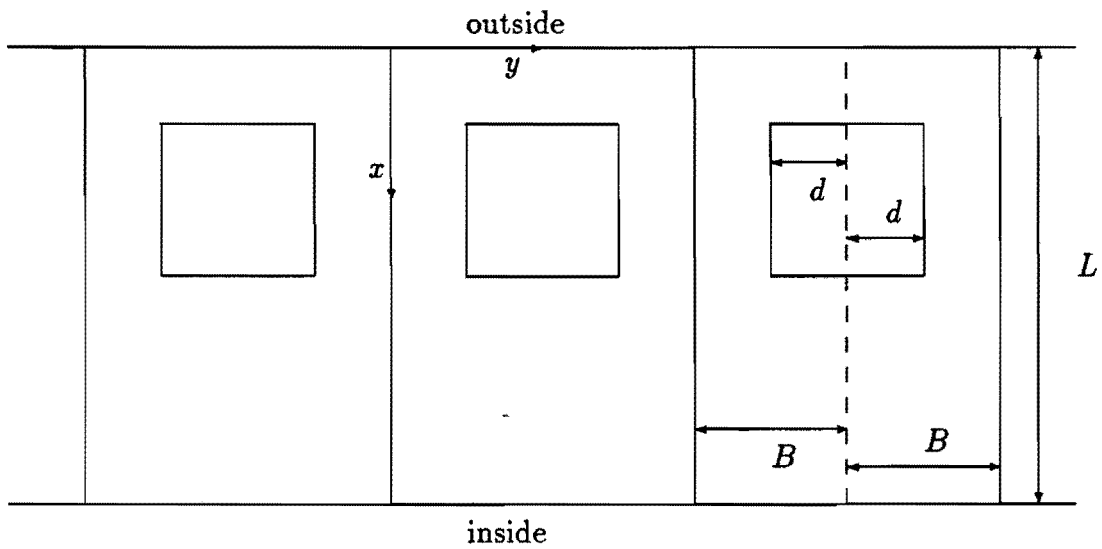


Figure 4.2: Horizontal cross section of the wall

Symmetry considerations tell us that the temperature distribution will be the same in each brick. Furthermore, the temperature distribution in a brick will be symmetric with respect to the dotted line in Figure 4.2. Therefore we may restrict ourselves to the following geometry:

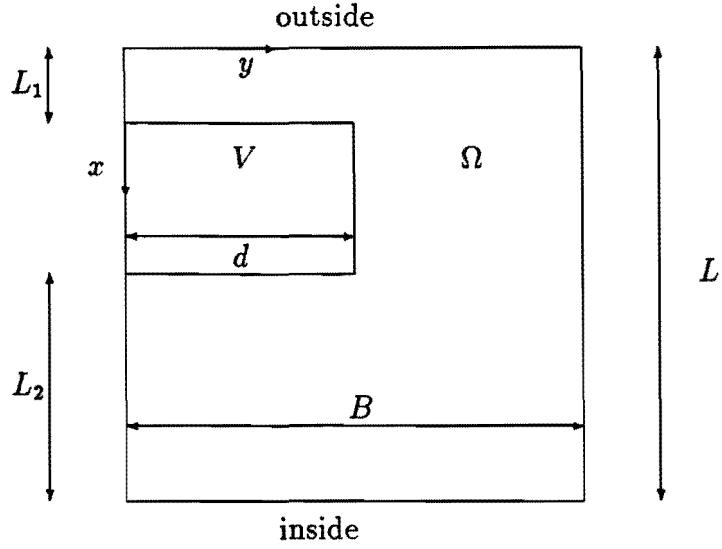


Figure 4.3: Used geometry with axes and parameters

Notice that we get a one-dimensional problem either if $d = 0$ (no texxos block) or if $d = B$, because in these cases the y dependence disappears.

Consequently, we have to solve Equation (4.1) on the domain Ω , specified in (4.6) with boundary conditions

$$(4.2) \quad \frac{\partial T}{\partial x} = -\frac{1}{k} F_{out}, \quad x = 0$$

$$(4.3) \quad T = T_{in}, \quad x = L$$

$$(4.4) \quad \frac{\partial T}{\partial y} = 0, \quad y = B \text{ and } (x, y) \in W$$

$$(4.5) \quad T = T_{crit}, \quad (x, y) \in V$$

where k is the thermal conductivity and

$$(4.6) \quad \Omega = \{(x, y) \mid 0 < x < L_1, \quad 0 < y < B\} \cup \\ \{(x, y) \mid L_1 \leq x \leq L_2, \quad d < y < B\} \cup \\ \{(x, y) \mid L_2 < x < L, \quad 0 < y < B\}$$

$$(4.7) \quad W = \{(x, 0) \mid 0 < x < L_1 \text{ or } L_2 < x < L\}$$

$$(4.8) \quad V = \{(L_1, y) \mid 0 < y \leq d\} \cup \\ \{(L_2, y) \mid 0 < y \leq d\} \cup \\ \{(x, d) \mid L_1 \leq x \leq L_2\} .$$

Remarks:

- (i) Boundary condition (4.4) follows from a symmetry argument.
- (ii) In the two-dimensional case (cf. Equation (2.5)) the heat flux \mathbf{F} is related to the temperature T as follows:

$$(4.9) \quad \mathbf{F} = -k \nabla T$$

with $\nabla = (\partial/\partial x \quad \partial/\partial y)$.

§5 The results for the 2 – D model

5.1 Numerical analysis

The equations in the previous section are discretized in a standard way and solved by the successively overrelaxation method (S.O.R.) which is a sophisticated Gauss-Seidel method. This standard numerical iteration method uses the newly computed values for the temperature as soon as they are available [2].

To compare the 1 – D and 2 – D results we use characteristic values for F_{out} and T_{in} in (4.2) and (4.3) from our 1 – D calculations in §3. To be specific, we take the values at 7.00 and 17.30 h . The 1 – D results vary only slightly in time around these time points so that a comparison of the dynamical 1 – D results and the stationary 2 – D results is possible.

For the value of F_{out} in the 2 – D calculations we cannot simply take the value from our 1 – D calculations.

A realistic choice, followed throughout our 2 – D calculation is,

$$F_{out} = -k(T_{out} - T_{crit}) / L_3$$

with

T_{out} the outside temperature,

T_{crit} the critical temperature of texxos and

L_3 the thickness of the wall between the texxos layer and the outside.

§5.2 Numerical results

In Figures 5.1a-c we give the temperature distribution through the wall for different values of d . In each of these figures we have taken $T_{out} = 5^\circ C$, $T_{in} = 15^\circ C$ and $T_{crit} = 27^\circ C$. The corresponding time in our 1 – D model is 7.00 h .

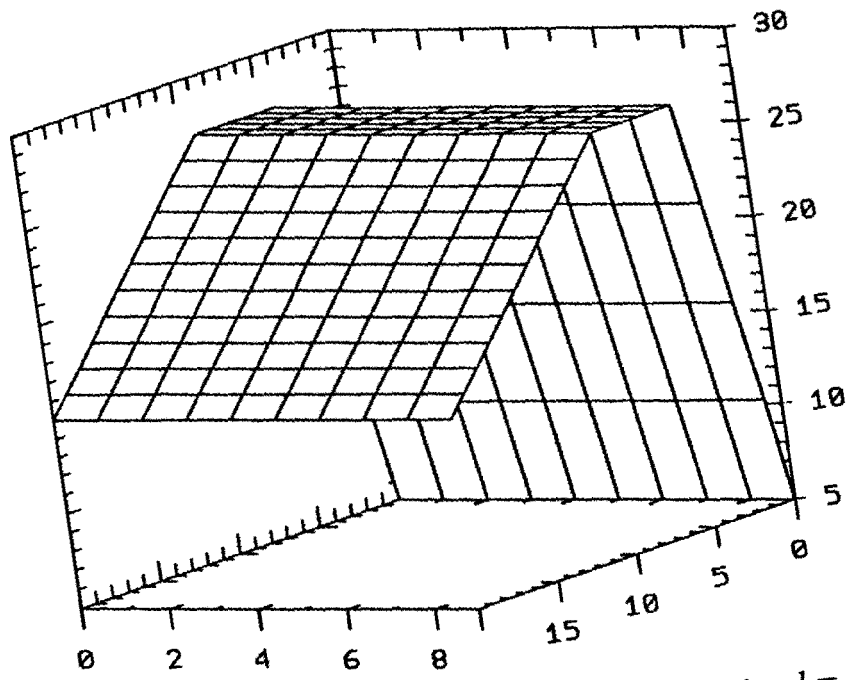


Figure 5.1a: Temperature distribution through the wall for $d = B$

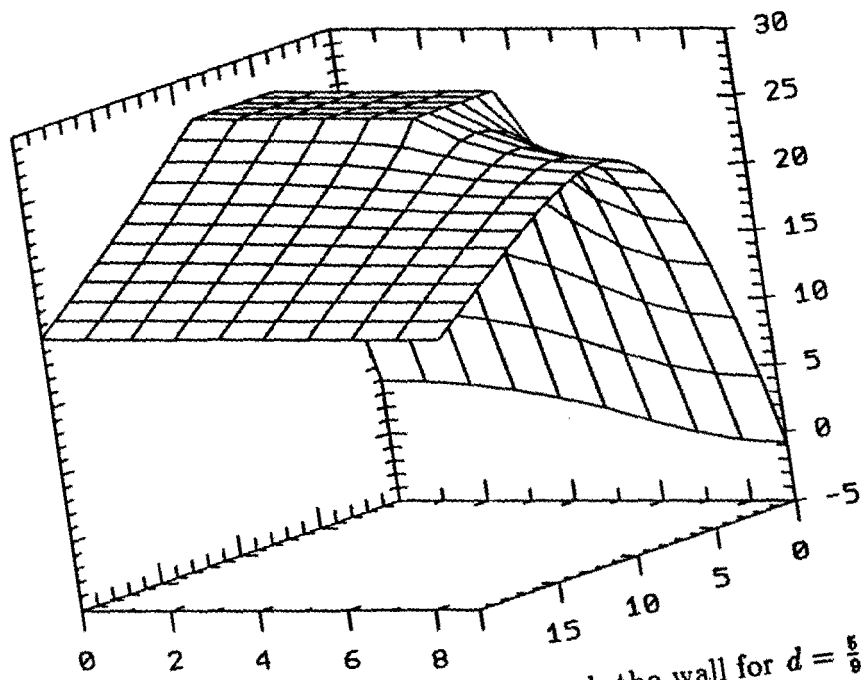


Figure 5.1b: Temperature distribution through the wall for $d = \frac{5}{9} B$

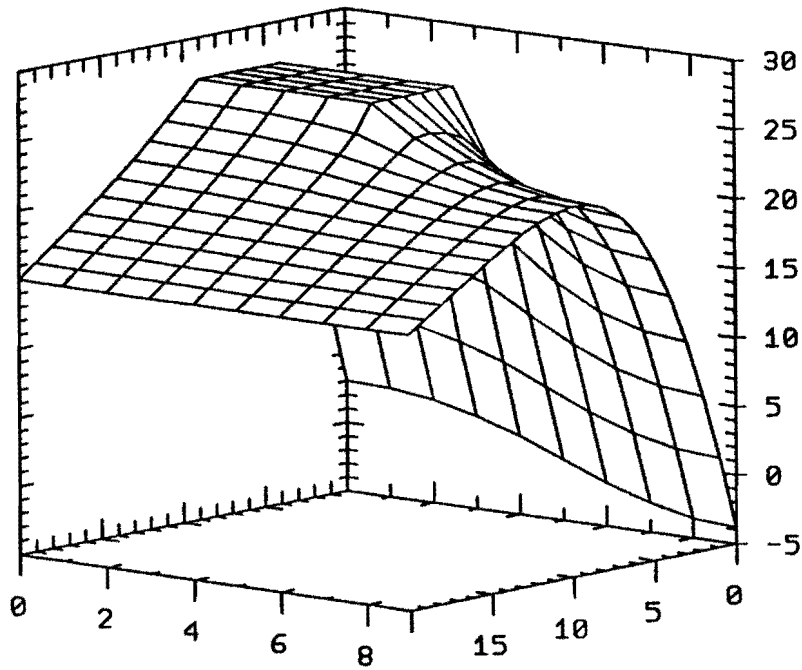


Figure 5.1c: Temperature distribution through the wall for $d = \frac{4}{9} B$

The total heat flux from the wall into the room is a function of d . In Table II we give the heat flux and the relative difference with respect to the flux for $d = B$ for some d 's that is:

$$(5.1) \quad \frac{|F(d = B) - F(d = \frac{i}{9} B)|}{F(d = B)} * 10^2, \text{ for } i = 2, \dots, 8.$$

The parameter values are the same as in Figure 5.1.

Table II: The heat flux for some d 's at 7.00 h ($T_{out} = 5^\circ C$, $T_{in} = 15^\circ C$)

d	flux [kW/m^2]	relative difference
B	54.1	-
$8/9 B$	54.0	0,2 %
$7/9 B$	53.5	1,1 %
$6/9 B$	52.2	3,5 %
$5/9 B$	49.6	8,3 %
$4/9 B$	45.2	16,5 %
$3/9 B$	38.7	28,5 %
$2/9 B$	29.6	45,3 %

In Figures 5.2a-c we give similar results as is Figures 5.1a-c but now for $T_{out} = 70^\circ C$, $T_{in} = 20^\circ C$ and $T_{crit} = 27^\circ C$. The correspondig time in our 1-D model is 17.30 h.

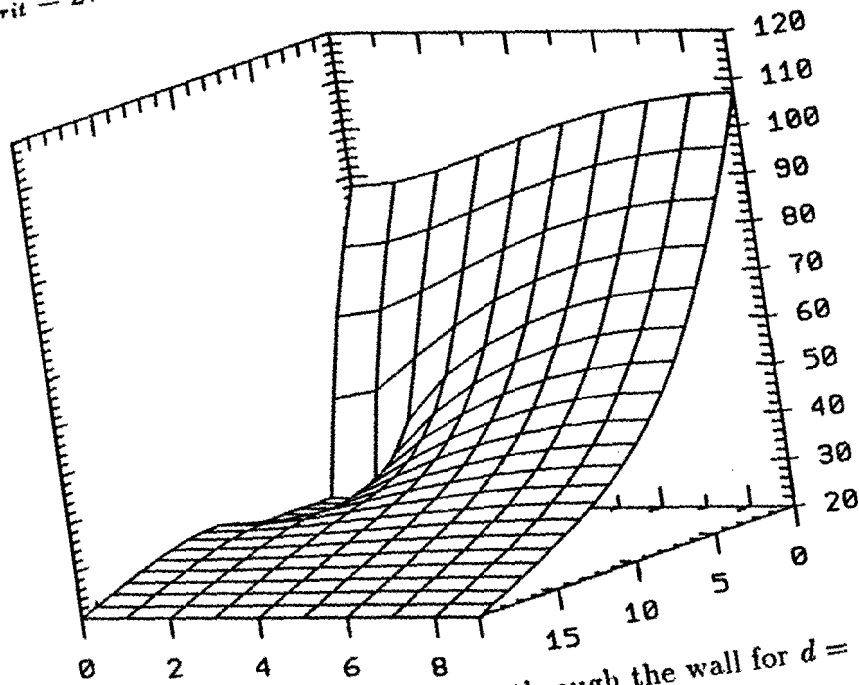


Figure 5.2a: Temperature distribution through the wall for $d = 1/9 B$

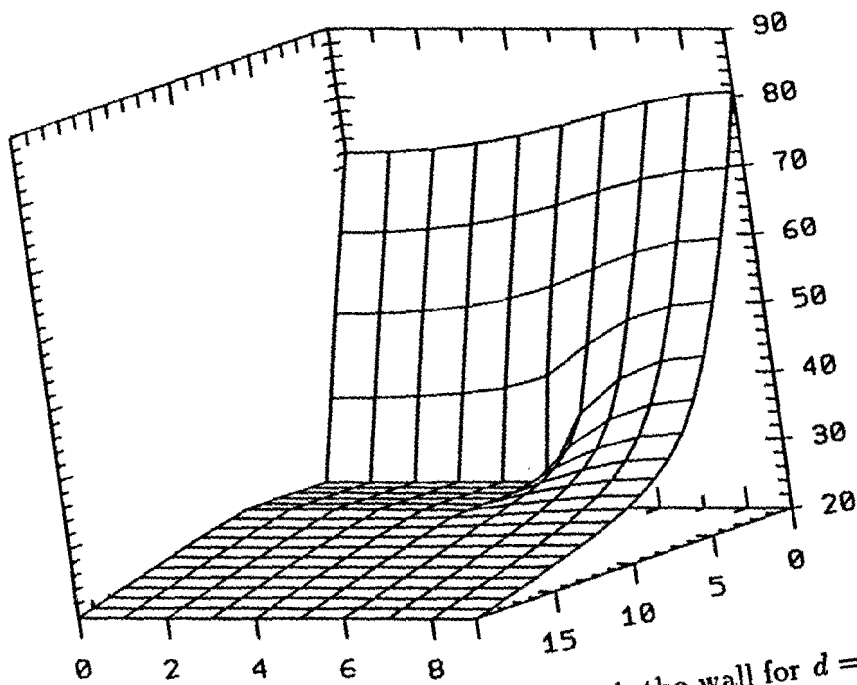


Figure 5.2b: Temperature distribution through the wall for $d = 5/9 B$

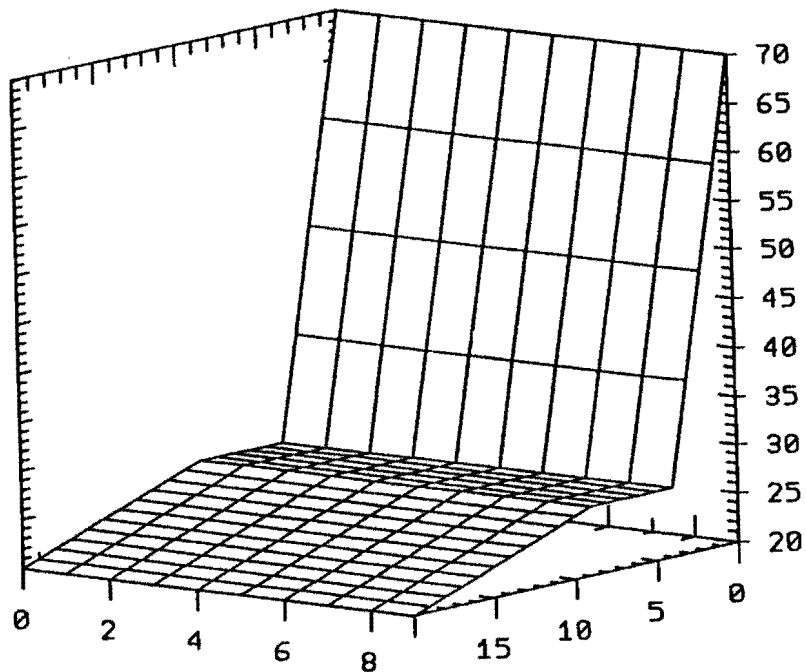


Figure 5.2c: Temperature distribution through the wall for $d = B$

In the same way as in Table II, we give in Table III the values of the heat flux and the relative difference with respect to the flux for $d = B$ for some d 's.

Table III: The heat flux for some d 's at 17.30 h ($T_{out} = 70^\circ C$, $T_{in} = 20^\circ C$)

d	flux [kW/m^2]	relative difference
B	31.6	–
$8/9 B$	31.5	0,3 %
$7/9 B$	31.4	0,6 %
$6/9 B$	31.9	0,9 %
$5/9 B$	34.1	7,9 %
$4/9 B$	38.9	23,1 %
$3/9 B$	47.0	48,7 %
$2/9 B$	59.0	86,7 %

So, we can conclude, from Table II en Table III that when the texxos block is not too small ($d \geq 2/3 B$), the results for the 1 – D model differ at most 3,5% from the results from the 2 – D model.

§6 Discussion and conclusions

We have examined the influence on the household energy costs of a texxos cylinder in the wall at the south side of a house. At a given critical temperature this texxos material is able to store and release heat without changing its temperature, as this heat is used for melting. Evidently, the capacity of the texxos cylinder is finite, since the melting process ends.

Our first thoughts concerned an extensive model in which the temperature of the livingroom is calculated for given outside temperatures. Hence we need estimates for the heat capacity of the air in the room, the heat exchange with other rooms and the heat leakage through the roof and floor.

On second thought this model is hardly realistic as the temperature in a livingroom is usually regulated by heating and ventilating the room.

So another model was set up with given temperatures at the outside as well as at the inside of the house. For the room temperatures we have chosen the following realistic values. At day-time we assume a room temperature of $20^{\circ} C$ and at night of $15^{\circ} C$. The outside temperature at night is estimated at $10^{\circ} C$. Furthermore the outside temperature at the northern wall is taken to be $15^{\circ} C$ at day-time.

The texxos cylinder behaves like a normal, though expensive, part of the wall when it is not at its critical temperature. So we only expect a significant reduction of the energy costs when the texxos cylinder remains at his critical temperature for a long time. This will be the case when there exists an energy balance between the supplied and withdrawn heat of the texxos cylinder.

Consequently, the question arises what outside temperature at day-time at the southern wall is needed to ensure this energy balance.

Together with the calculation of the corresponding energy costs this question can be answered by making some extra assumptions.

First we assume that we may change the geometry of a texxos brick as shown in Figure 2.1. Consequently, the texxos material divides the brick into two separate parts.

Furthermore, we assume that the temperature distributions in the wall depend only on the space-coordinate perpendicular to the wall. This leaves only one space-coordinate in the differential equation, describing the heat flux through a wall, which we have called the 1 - D model.

This allowed us to give an analytical expression for the temperature distribution in the wall.

For the texxos layer to function it turned out that the outside temperature at the southern wall needed to be $70^{\circ} C$ for several hours a day.

Moreover, we have calculated the heat fluxes entering the livingroom with classical walls at both sides of the house. These results have been compared with the results in case of a texxos layer in the southern wall. It appears that in the second case almost twice as much heat enters the livingroom during one day, which will cause a considerable saving of energy. Of course, this saving depends on the chosen temperature profiles, i.e. the climate, and the thickness of the wall, but the positive effect of the texxos layer on the energy costs is quite clear.

However, the needed intensity of the sun is rather unrealistic for most parts of Europe.

The results of our 1 - D model have been checked by a 2 - D model, in which the structure of the wall is given in more detail. In the 2 - D model the texxos cylinder is modelled as a rectangular block as indicated in Figure 4.1 and Figure 4.2.

When the texxos block is not too small ($d \geq 2/3 B$) the results from the 1 - D model differ at most 3,5% from the results obtained from the 2 - D model.

Summarizing we conclude:

- (1) The texxos cylinder has a very positive effect on the energy costs, if we are in a position that it functions during a long period of time.
- (2) The texxos cylinder will only function during a long period of time, when the climate is rather extreme, with enough sun radiation to yield a maximum outer wall temperature of $70^{\circ} C$ during several hours a day.

References

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2. J. Stoer, R. Bulirsch, Introduction to numerical analysis, Springer-Verlag New York.
3. F. Koeith, M.S. Bohn, Principles of Heat Transfer, Harper & Row New York, 4th ed. 1986.