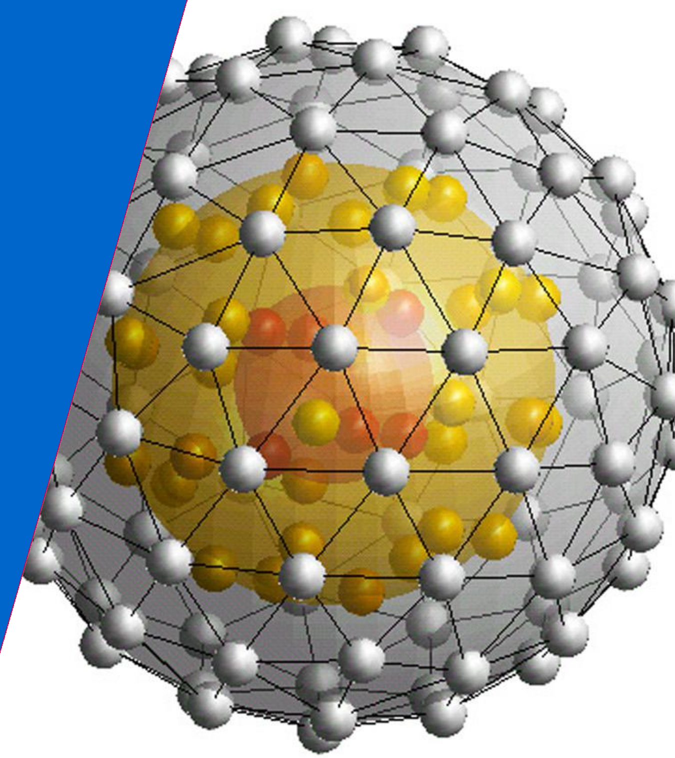
Finite range effects in two-body and three-body interactions

Finite range effects in two-body and three-body interactions

Critical Stability, Santos (2014)

Servaas Kokkelmans



Technische Universiteit
Eindhoven
University of Technology

Where innovation starts

The Efimov effect and Three-body recombination

In 1970 predicted by Efimov

- **Study of nuclear physics problem: tritium**

Universal description

- **Discrete scaling symmetry**
- **Insensitive to microscopic details**

Not easy to change the nuclear forces..

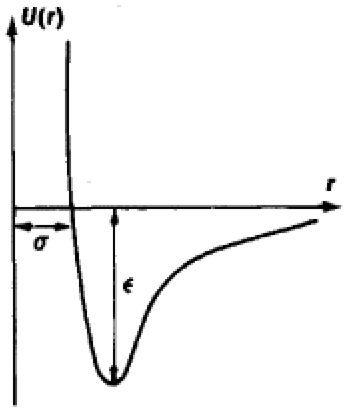
- **Interaction strength can be changed in atomic physics**
- **Observed via three-body recombination**
- **Several different species, mixtures**

Efimov states

- **Short range two-body interactions**

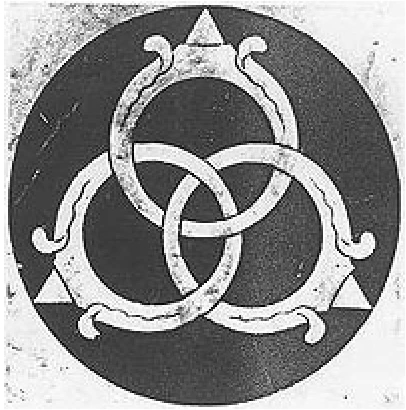
  ➡ **s-wave scattering length**

$$a = -\lim_{k \to 0} \frac{\tan \delta(k)}{k}$$



Potential *just* unbound ➡ $a \to -\infty$

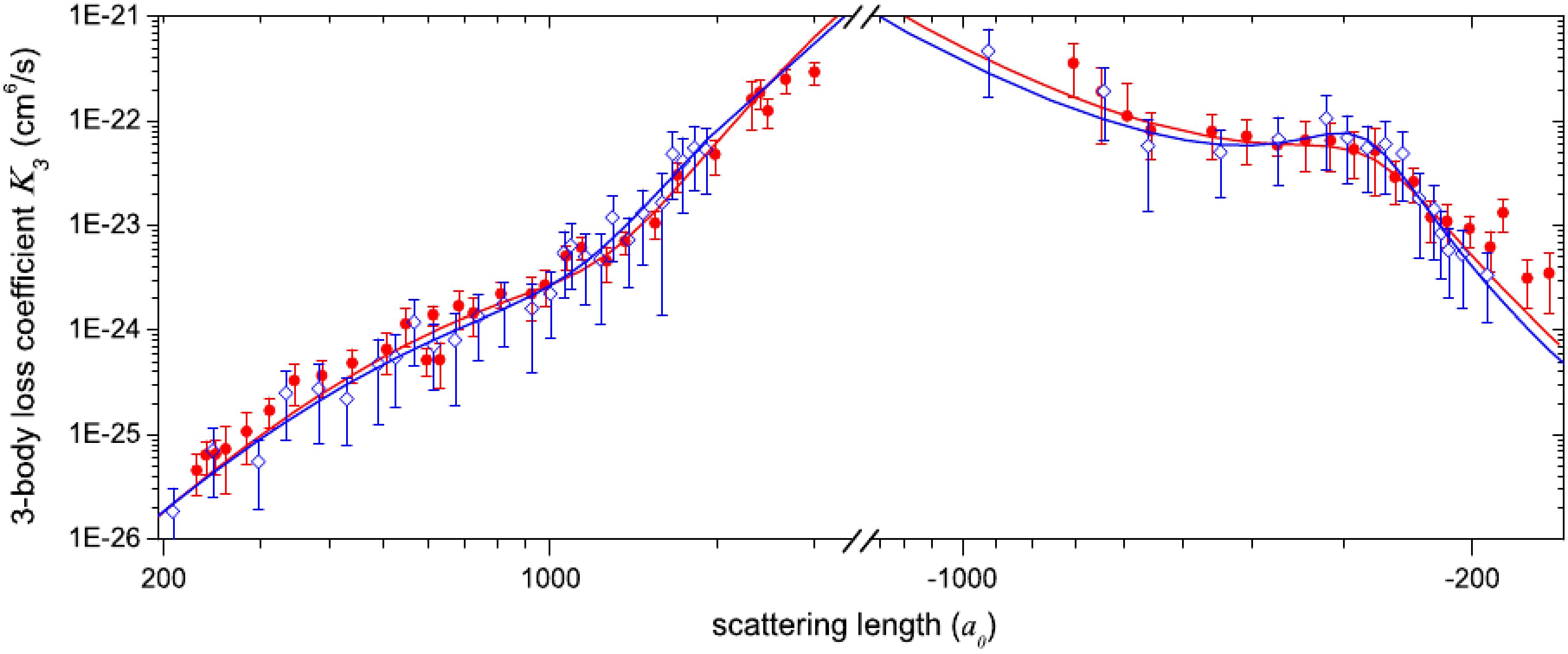
- **No dimer state**
- **...but an infinite number of trimer states!**



Borremean rings

Three-body recombination rate for ^{7}Li

- **Recombination rate:** $K_3 = 3C(a)\hbar a^4/m$



Two different spin states

Ref.: [N. Gross, Z. Shotan, S. Kokkelmans, L. Khaykovich, PRL **105**, 103203 (2010)]

Universality

- **Universal Few body physics: interactions insensitive to microscopic details interaction**
- **Efimov spectrum: depends only on two generic two-body parameters**

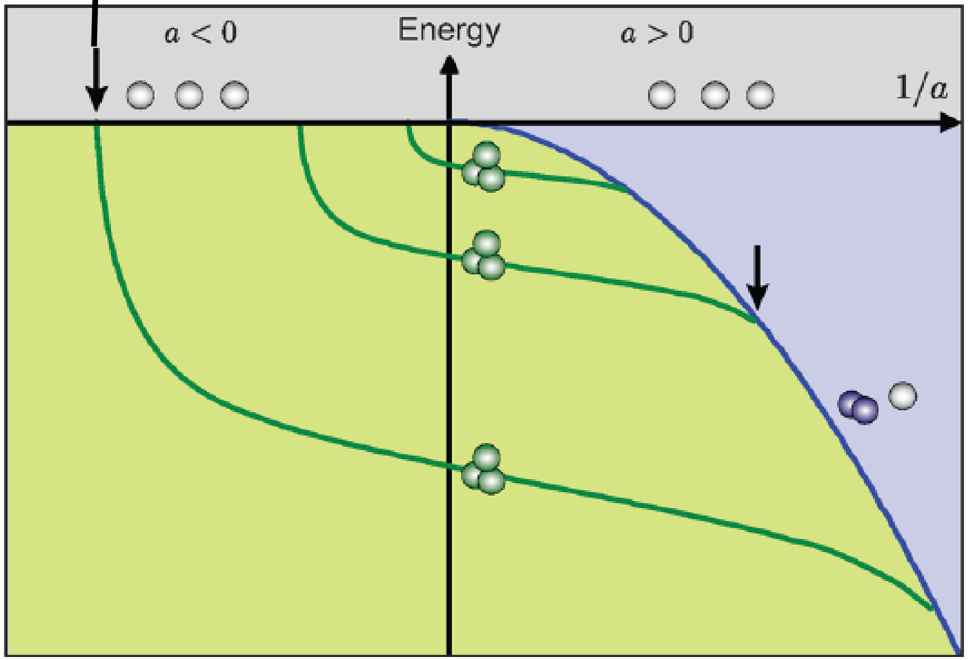
$a_- \approx -9.8\, a_{vdW}$

difference in scattering length:

$e^{\pi/s_0} \approx 22.7$

spacing bound states:

$E_{n+1}/E_n \approx e^{-2\pi} \approx 1/515$



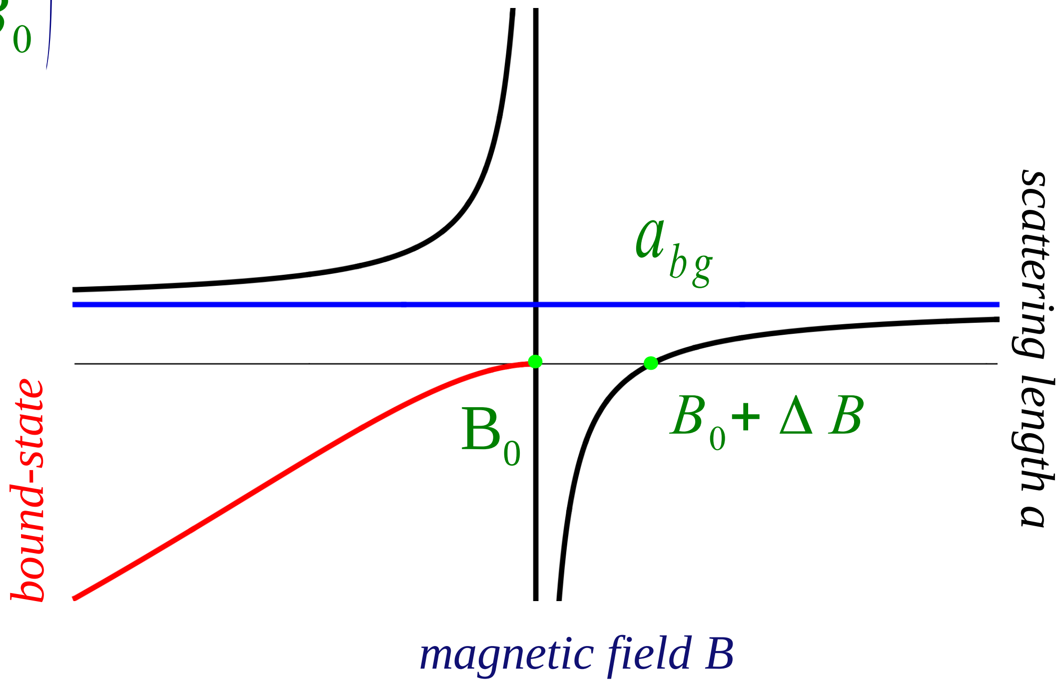
$E_b = \frac{-\hbar^2}{m a^2}$

Overview: [F. Ferlaino and R. Grimm, Physics 3, 9 (2010); C. Greene, Physics Today, march 2010]

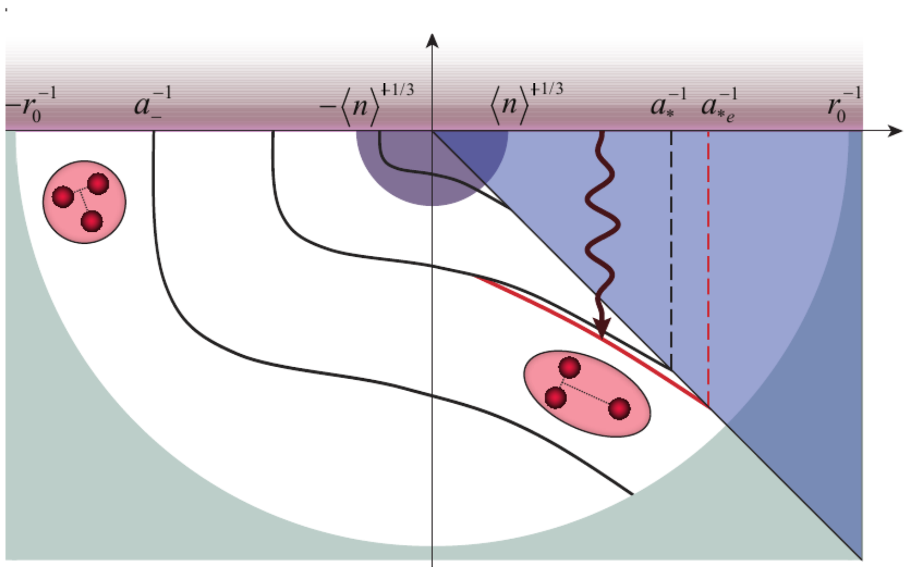
Feshbach resonance

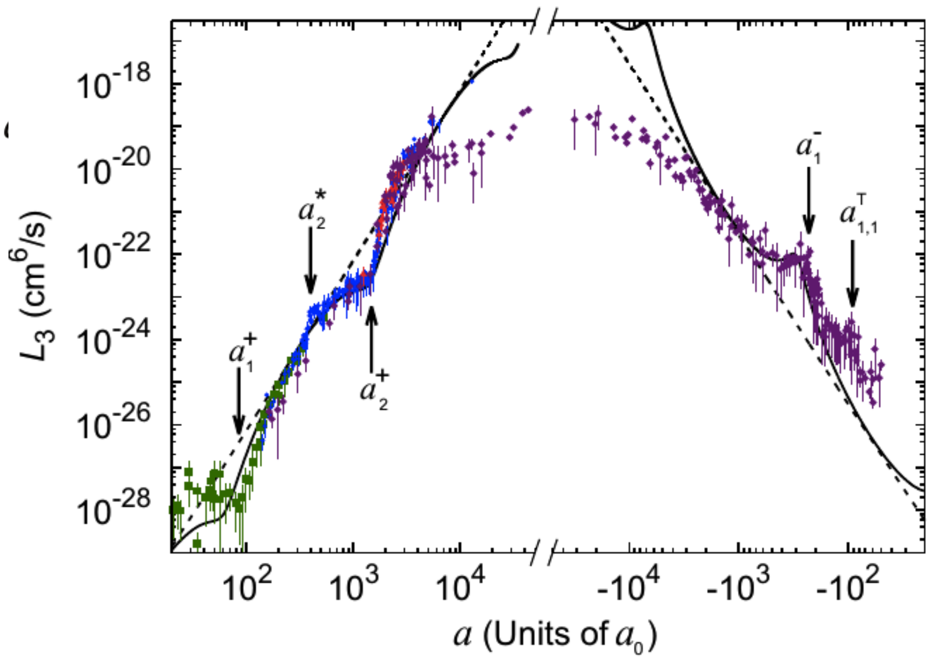
- **Control over scattering length with magnetic field**

$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



Deviations from universality





Shift $a_* = 288\, a_0$ → $a_* = 180\, a_0$

[Zav Shotan, Noam Gross, Lev Khaykovich
Phys. Rev. Lett. 108, 210406 (2012)]

P. Dyke, S. E. Pollack, and R. G. Hulet
Phys. Rev. A **88**, 023625 (2013)

- Finite range effects?
- Extreme non-universal limit: $a=0$
- What should replace $K_3 = 3C(a)\hbar a^4/m$?

Non-universal corrections

- **Theoretical non-universal extensions**
- **Effective range R_e as additional parameter**

$$k\cot\delta(k) = \frac{-1}{a} + \frac{1}{2} R_e k^2$$

- **Connected to inverse width of resonance:**

$$R^* = \hbar^2 / (m\,\delta\mu\,\Delta B\, a_{bg})$$

- **How do we obtain this expression?**

Width: Feshbach as a Breit-Wigner resonance

Forget about background effects:

$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right) \approx -\frac{a_{bg}\,\Delta B}{B - B_0}$$

Width: Feshbach as a Breit-Wigner resonance

➡ **Forget about background effects:**

$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right) \approx -\frac{a_{bg}\Delta B}{B - B_0}$$

$$= -\frac{\delta\mu\,\Delta B\,a_{bg}}{\delta\mu\left(B - B_0\right)} = -\frac{\delta\mu\,\Delta B\,a_{bg}}{E_{res}}$$

$$E_{res} = \delta\mu\left(B - B_0\right)$$

Width: Feshbach as a Breit-Wigner resonance

$$a \approx -\frac{\delta\mu \, \Delta B \, a_{bg}}{E_{res}}$$

- **Breit-Wigner resonance determined by two quantities:**
- **Position and Width**

$$\tan\delta(k) = \frac{\Gamma/2}{E - E_{res}}$$

$$\boxed{\Gamma = \delta\mu \, \Delta B \, a_{bg} \, k}$$

Width: Feshbach as a Breit-Wigner resonance

$$a \approx -\frac{\delta\mu\,\Delta B\,a_{bg}}{E_{res}}$$

- **Breit-Wigner resonance determined by two quantities:**
- **Position and Width**

$$\tan\delta(k) = \frac{\Gamma/2}{E - E_{res}}$$

$$\Gamma = \delta\mu\,\Delta B\,a_{bg}\,k$$

- **Effective range formula:**

$$k\cot\delta(k) = \frac{-2\mathrm{k}(E_{res} - E)}{\Gamma} = \frac{-2\,E_{res}}{\Gamma/k} + \frac{\hbar^2}{m\Gamma/k}k^2 = \frac{-1}{a} + \frac{1}{2}R^* k^2$$

$$R^* = \hbar^2/(m\,\delta\mu\,\Delta B\,a_{bg})$$

Non-universal corrections

- **Theoretical non-universal extensions**
- **Effective range R_e as additional parameter**

$$k\cot\delta(k)=\frac{-1}{a}+\frac{1}{2}R_e k^2$$

- **Connected to inverse width of resonance:**

$$R^*=\hbar^2/(m\,\delta\mu\,\Delta B\,a_{bg})$$

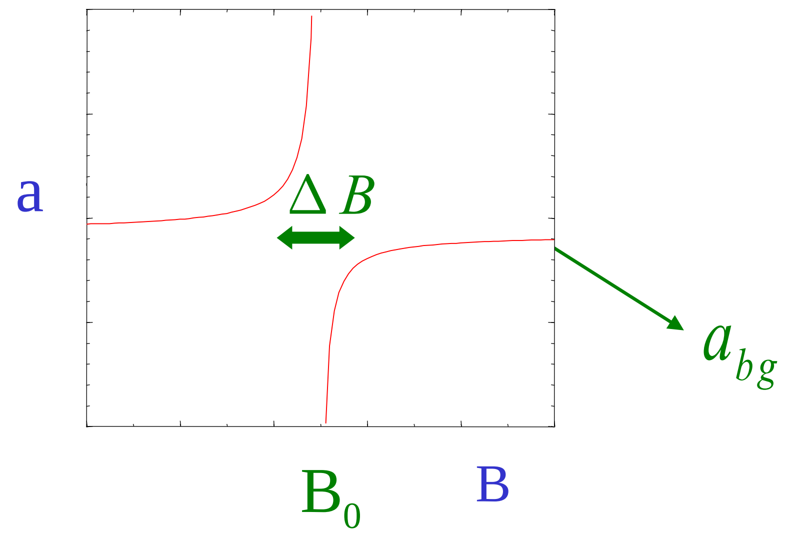
- **Narrow Feshbach resonances:** $R_e=-2R^*$

- **But there are some problems with this length scale**

What other length scales are important?

More length scales beyond scattering length:

$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



- **Width of resonance**

$$R^* = \hbar^2 / (m\,\delta\mu\,\Delta B\,a_{bg})$$

- **Background scattering length**

$$a_{bg}$$

- **Range of the potential**

$$r_0 \approx r_{vdW}$$

➡ **Van der Waals length** $r_{vdW} = \frac{1}{2}\left(\frac{2\mu C_6}{\hbar^2}\right)^{1/4}$

Potential range: universal three-body parameter

- **Several systems: three-body recombination** $K_3 = 3C_+(a)\hbar a^4/m$
  - **Look for position first trimer resonance**
  - **Same three-body parameter**

$$a_- \approx -9.8\, a_{vdW}$$

  - **Experiments involved always Feshbach resonance**

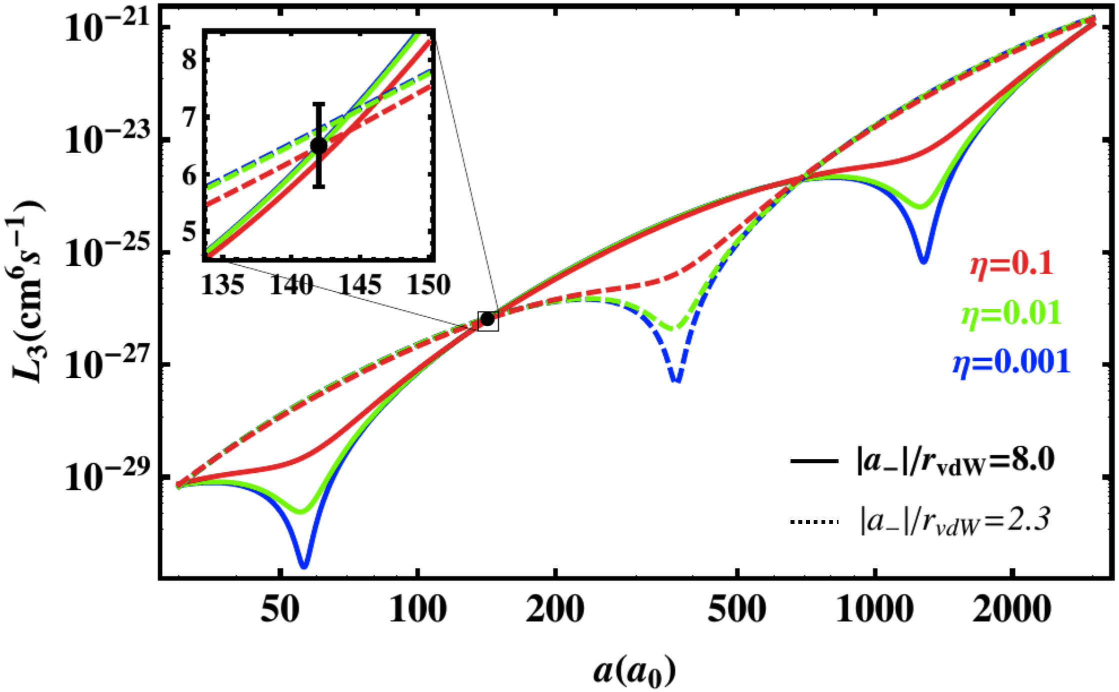
- **Different system:** $^4He^*$
  - **No Feshbach resonance**
  - **But large scattering length:** $a/a_{vdW} = 4.1$

[J. Wang, J. P. D'Incao, B. D. Esry, and C. H. Greene, Phys. Rev. Lett. 108, 263001 (2012).]
[P. Naidon, S. Endo, and M. Ueda, Phys. Rev. Lett. 112, 105301 (2014).]
[S. Knoop, J. S. Borbely, W. Vassen, and S. J. J. M. F. Kokkelmans, Phys. Rev. A 86, 062705 (2012).]

Analyze three-body recombination $^4He^*$

- **Scattering length $a>0$. Use** $a_+/a_- = -0.96$

$$C_+(a) = 67.1\, e^{-2\eta}\left(\cos^2[s_0 \ln a/a_+] + \sinh^2\eta\right) + 16.8\left(1 - e^{-4\eta}\right)$$

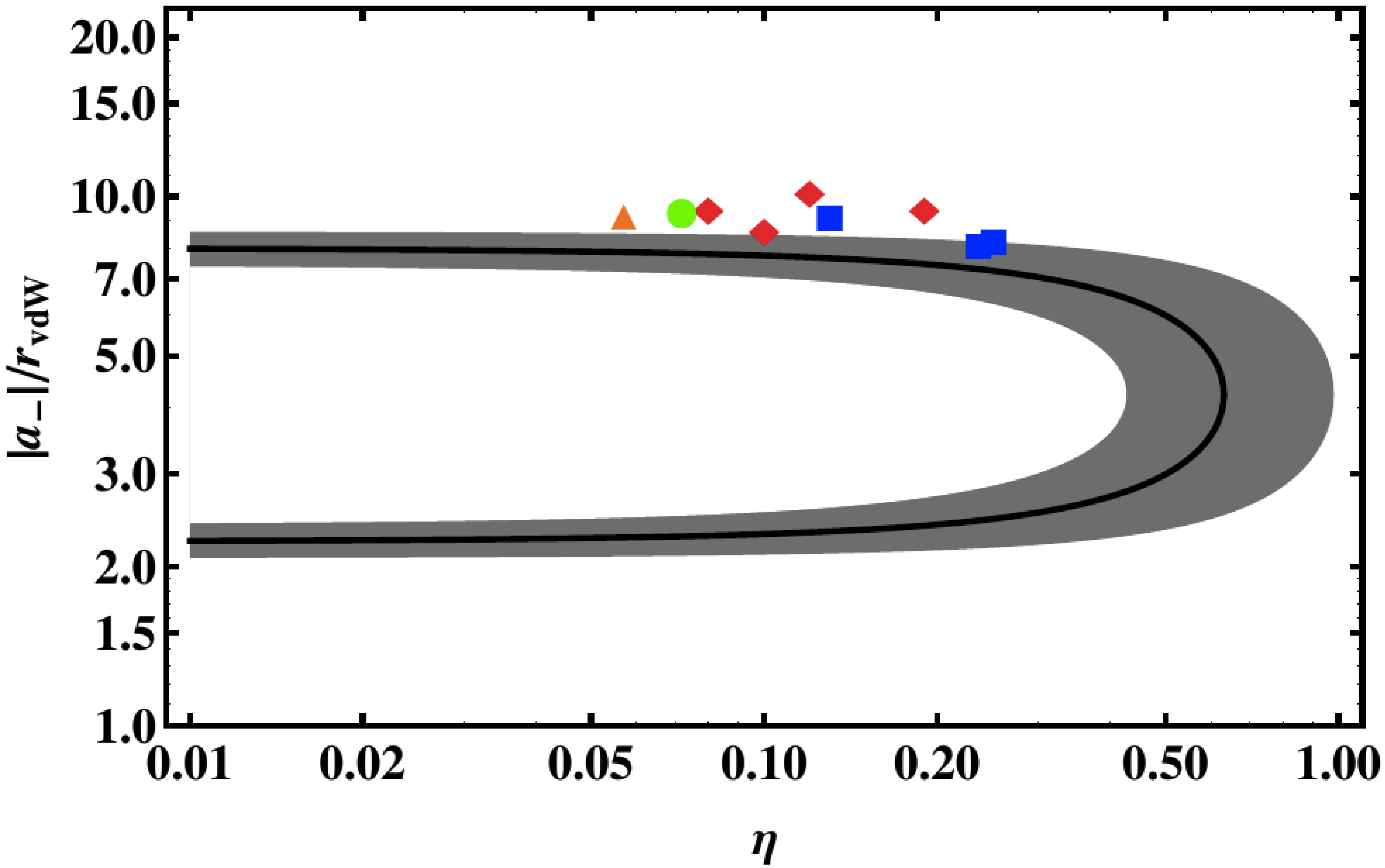


$$K_3 = 3\, C_+(a)\, \hbar a^4/m$$

[S. Knoop, J. S. Borbely, W. Vassen, and S. J. J. M. F. Kokkelmans, Phys. Rev. A 86, 062705 (2012).]

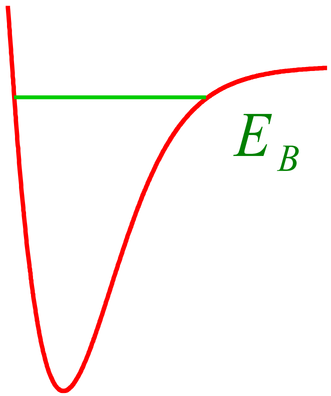
Compare to other atomic systems

Cs ^{6}Li ^{7}Li ^{85}Rb



$^{4}He^{*}$ Consistent with universal 3-body parameter!

Cold collisions and the highest bound state



Replace whole potential
by only a number

$$E_B = \frac{-\hbar^2 \kappa_B^2}{2\mu}$$

$$\psi(r) = \sqrt{\frac{\kappa_B}{2\pi}} \frac{e^{-\kappa_B r}}{r}$$

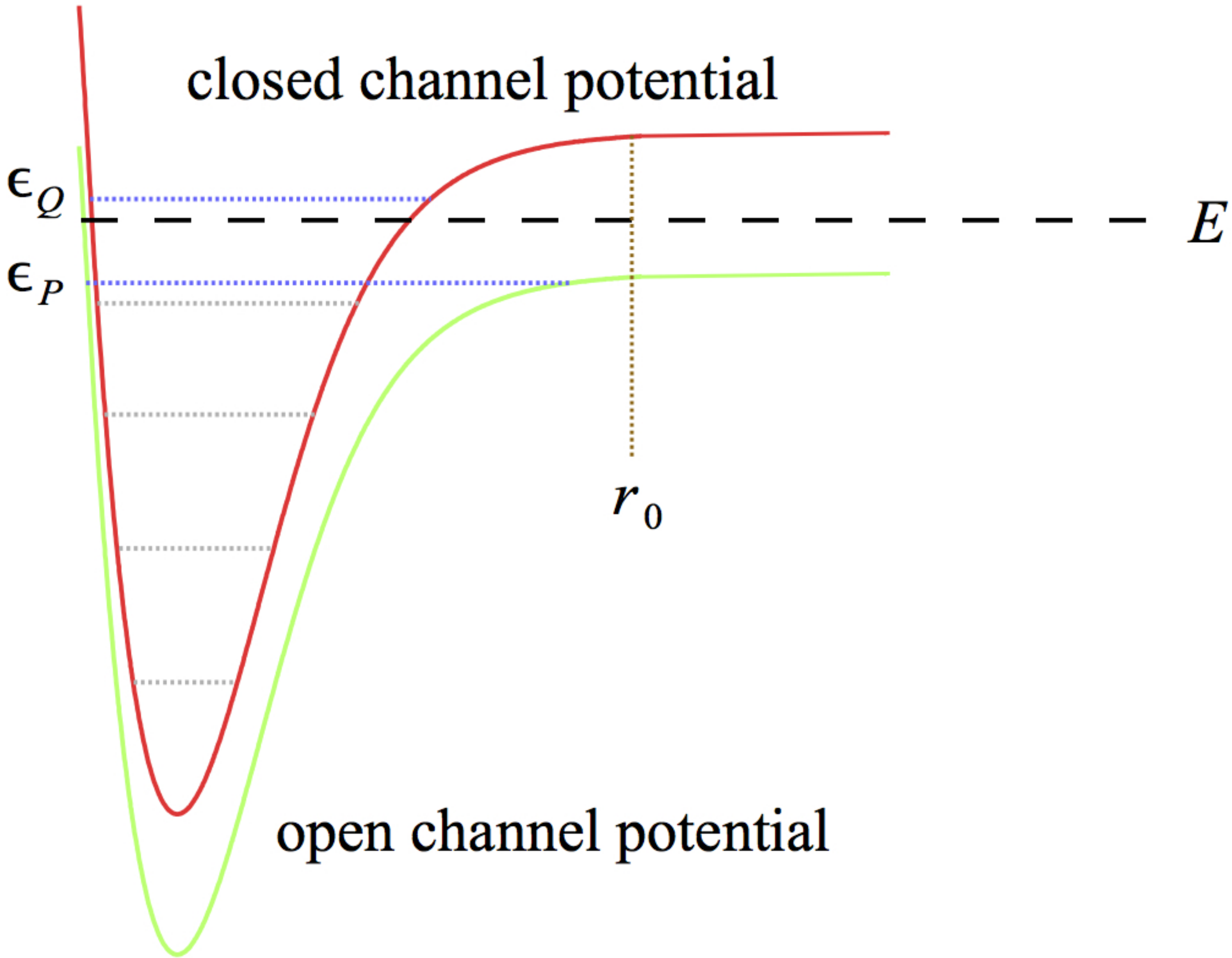
For the prediction and analysis of Feshbach resonances

- **Simple model: Asymptotic Bound-state Model**
- **based on highest bound state**

- **Introduce highest bound state for each potential in Feshbach projection formalism**

ABM model, see e.g.: [T. G. Tiecke et al, Phys. Rev. Lett. 104, 053202 (2010)]

Feshbach resonance: coupled-channels mechanism



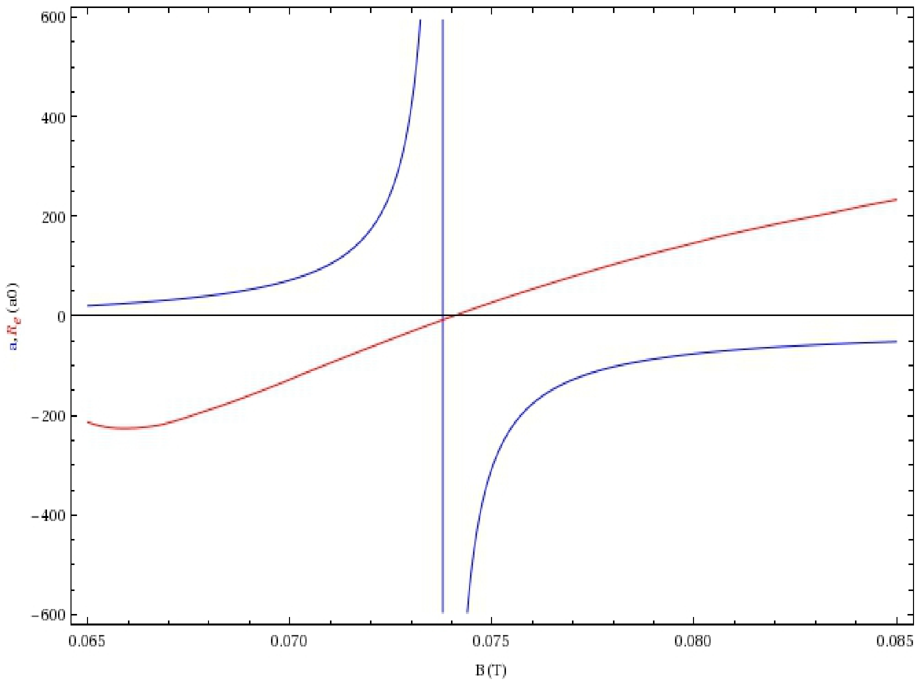
Non-universal physics in 2-body interactions: ^{7}Li

Non-typical behavior effective range

- **Broad resonance:** $R_e > 0$ **(potential resonance)**
- **Narrow resonance:** $R_e = -2R^* < 0$

➡ R^*, a_{bg}, r_0 **all needed to describe this**

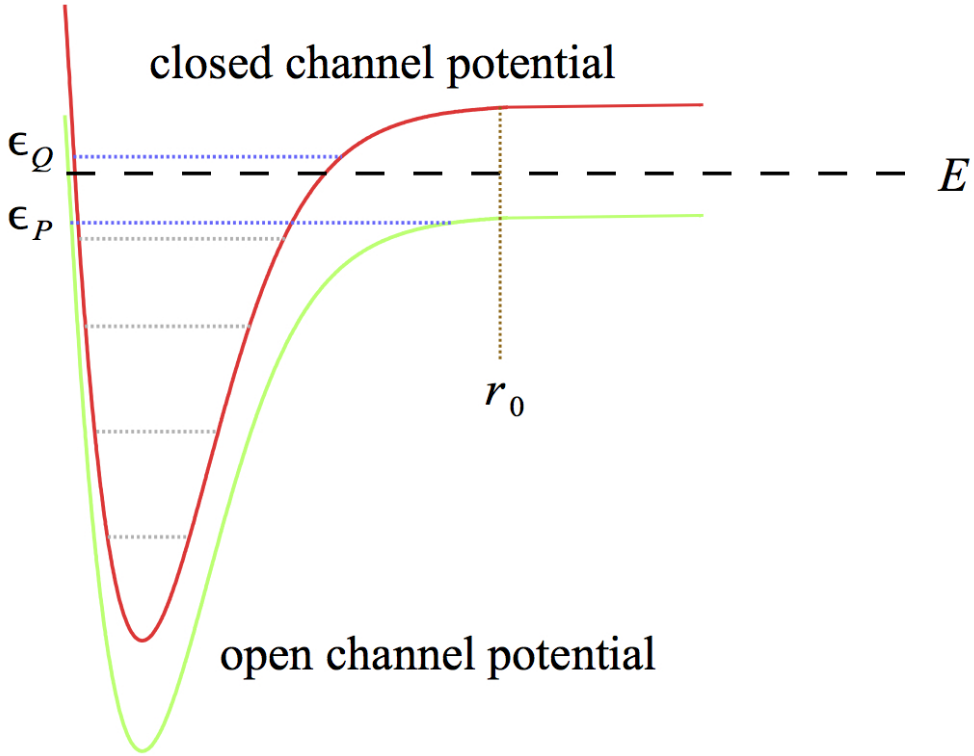
$a(a_0), R_e(a_0)$



$B\ (T)$

Double bound-state model

Account also for highest bound state in open channel:



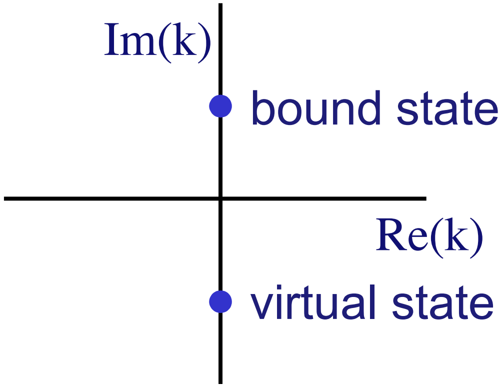
Direct (background) Scattering:

$$S^P(k) = e^{-2ik r_0} \frac{1 - ika^P}{1 + ika^P}$$

Resonant scattering:

$$S^{res}(k) = 1 - \frac{i\Gamma(E)}{E - \nu' - \Delta(E) + i\Gamma(E)/2}$$

resonance in complex k-plane:



Width of Feshbach resonance

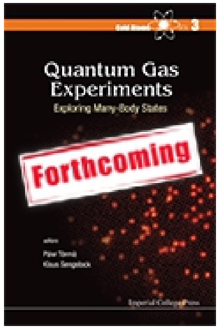
Shift and width of resonance:

Expansion into Gamow states

$$\Delta(E) - i\Gamma(E)/2 = \langle \varphi_b | H_{QP} \frac{1}{E - H_{PP}} H_{PQ} | \varphi_b \rangle$$

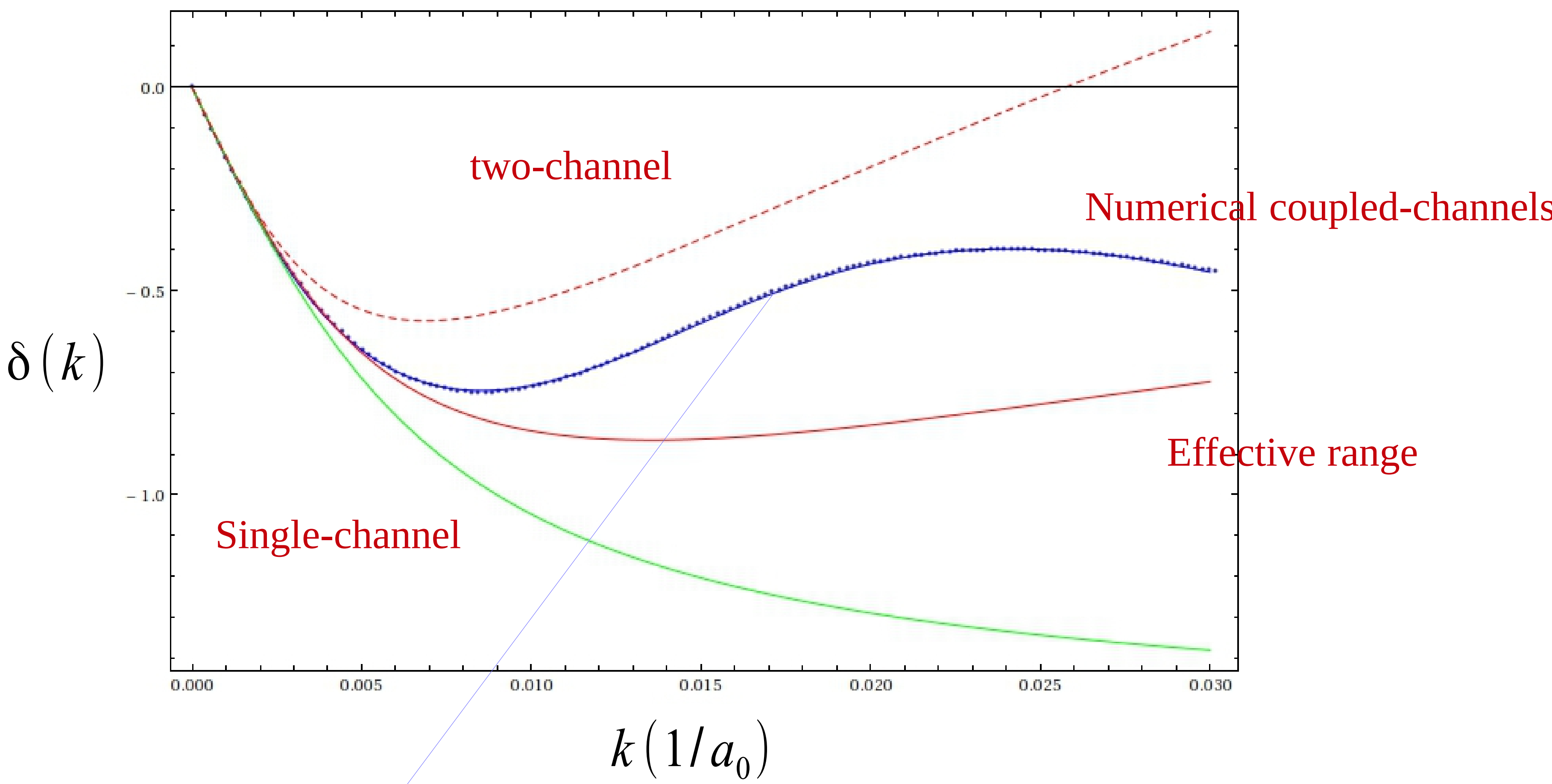
$$= \frac{-A/2}{k^2 + (1/a^P)^2} + i \frac{A k/2}{(k^2 + (1/a^P)^2)/a^P}$$

➡ **Non-trivial energy dependence**



See: [Feshbach resonances in ultracold gases, S. J. J. M. F. Kokkelmans, Chapter 4 in "Quantum gas experiments - exploring many-body states" (Imperial College Press, London, 2014)]

Scattering phase-shift

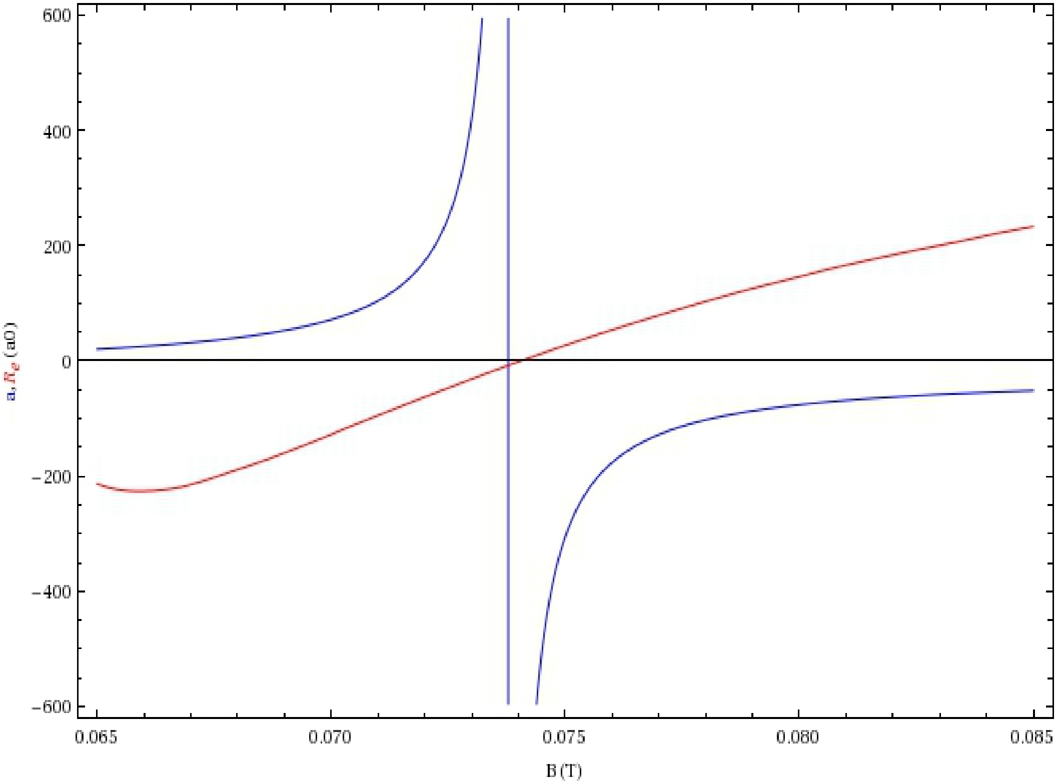


$$\delta_0(k) = -k\,r_0 - \arctan k\,a^P - arg\left[R^* k^2 + \frac{1}{a - a_{bg}} + \frac{ik}{1 + ika^P}\right]$$

^{7}Li – intermediate Feshbach resonance

- **Effective range depends on other length scales**

$a(a_0), R_e(a_0)$



$$R_e = -2R^*\left(1 - \frac{r_0}{R^*} - \frac{2a_{bg} - r_0^2/R^*}{a} + \frac{a_{bg}^2 - r_0^3/3R^*}{a^2}\right)$$

[Feshbach resonances in ultracold gases, S. J. J. M. F. Kokkelmans, Chapter 4 in "Quantum gas experiments - exploring many-body states" (Imperial College Press, London, 2014)]

What happens at zero crossing: *a=0*

- **Use effective range expansion?**

$$k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} R^{eff} k^2$$

- **Scattering length:** $a \rightarrow 0$
- **Effective range:** $R_e \rightarrow \infty$

What happens at zero crossing: *a=0*

- **Use effective range expansion?**

$$k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} R^{eff} k^2$$

- **Scattering length:** $a \rightarrow 0$
- **Effective range:** $R_e \rightarrow \infty$
- **Look at scattering phase shift directly!**

$$\delta(k) = -k\,a + k^3 V_e$$

$$= k^3 \left(R^* a_{bg}^2 - r_0^3/3 \right) \quad (a \rightarrow 0)$$

What happens at zero crossing: *a=0*

- **Use effective range expansion?**

$$k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} R^{eff} k^2$$

- **Scattering length:** $a \to 0$
- **Effective range:** $R_e \to \infty$
- **Look at scattering phase shift directly!**

$$\delta(k) = -k\,a + k^3 V_e$$

$$= k^3 \left(R^* a_{bg}^2 - r_0^3 / 3 \right) \quad (a \to 0)$$

$$= -k^3 R_e a^2 / 2 \quad (a \to 0)$$

Also noticed as relevant quantity for BEC near *a=0*, and in treatment of resonance approximations

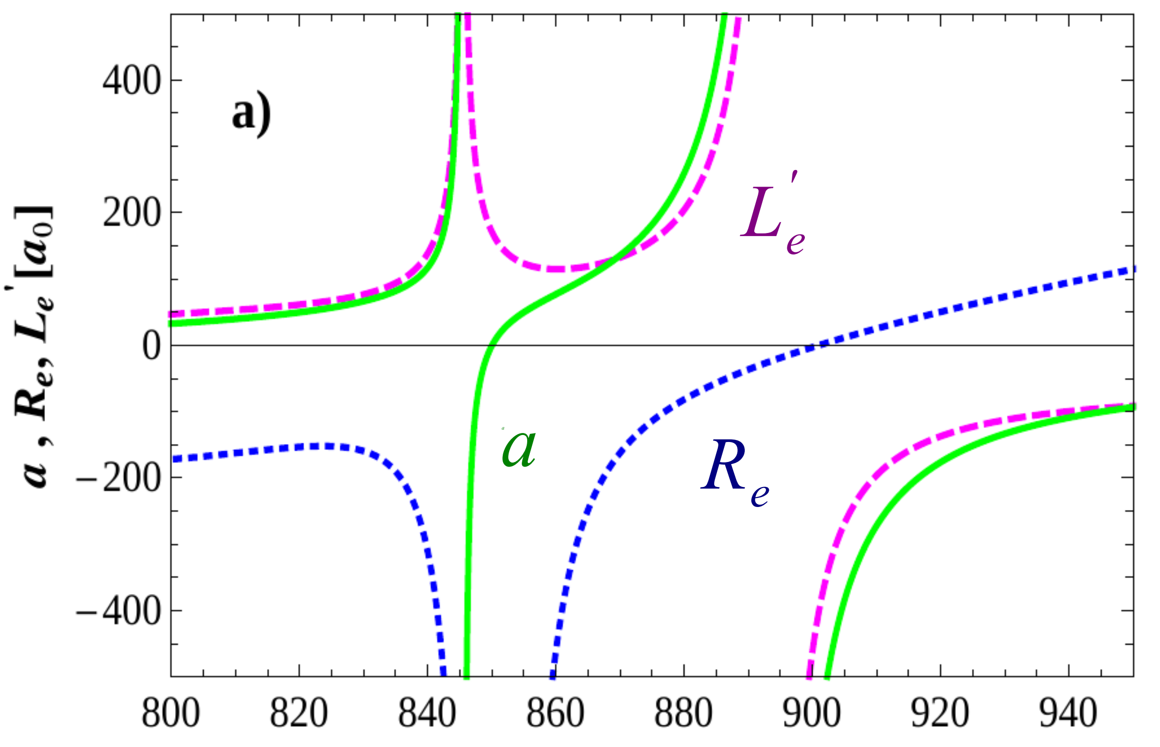
[Zav Shotan, Olga Machtey, Servaas Kokkelmans, Lev Khaykovich, PRL 113, 053202 (2014)]
[N. T. Zinner and M. Thøgersen, Phys. Rev. A 80, 023607 (2009).]
[M. Thøgersen, N. T. Zinner, and A. S. Jensen, Phys. Rev. A 80, 043625 (2009).]
[C. L. Blackley, P. S. Julienne, and J. M. Hutson, Phys. Rev. A 89, 042701 (2014).]

Define new length scale

- **Phase shift around *a=0*:** $\delta(k) = -ka - k^3 R_e a^2/2 + k^3 a^3/3$

➡ **Define effective length as:** $L_e' = \left(\frac{a^3}{3} - \frac{R_e a^2}{2}\right)^{1/3}$

Length scales from
Coupled channels calculation:
$f=1, m_f=0$ state



[Three-body recombination at vanishing scattering lengths in an ultracold Bose gas, Zav Shotan, Olga Machtey, Servaas Kokkelmans, Lev Khaykovich, PRL 2014PRL 113, 053202 (2014)]

Recombination rate at zero crossing

- **Now use same rate expression with new length scale**

$$K_3 = 3C \frac{\hbar}{m} L_e'^4$$

- **Rate is temperature-independent**
- **No fitting parameters**

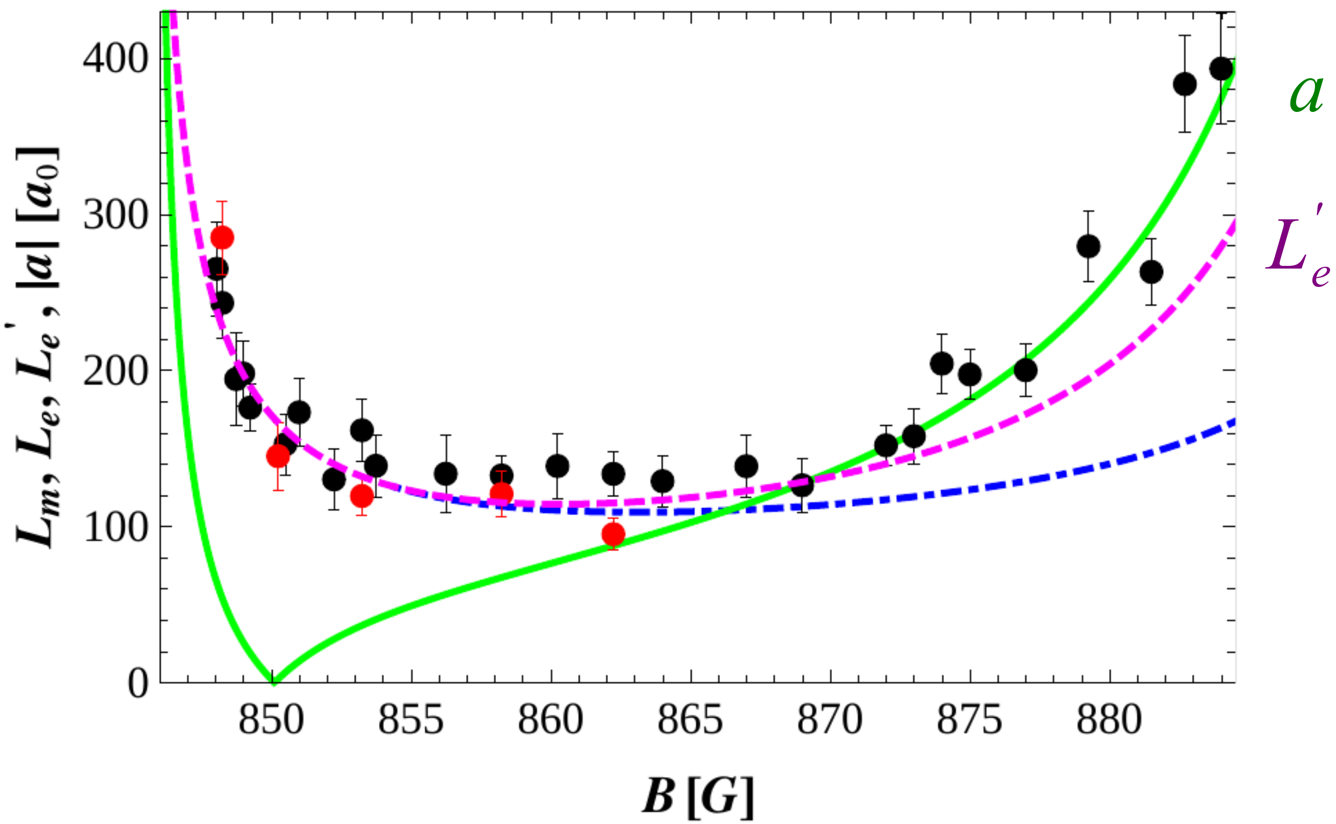
→ **Same value for *C* as measured for Efimov physics**

- **Compare with experimental recombination length from atom number decay**

Measurement of recombination length

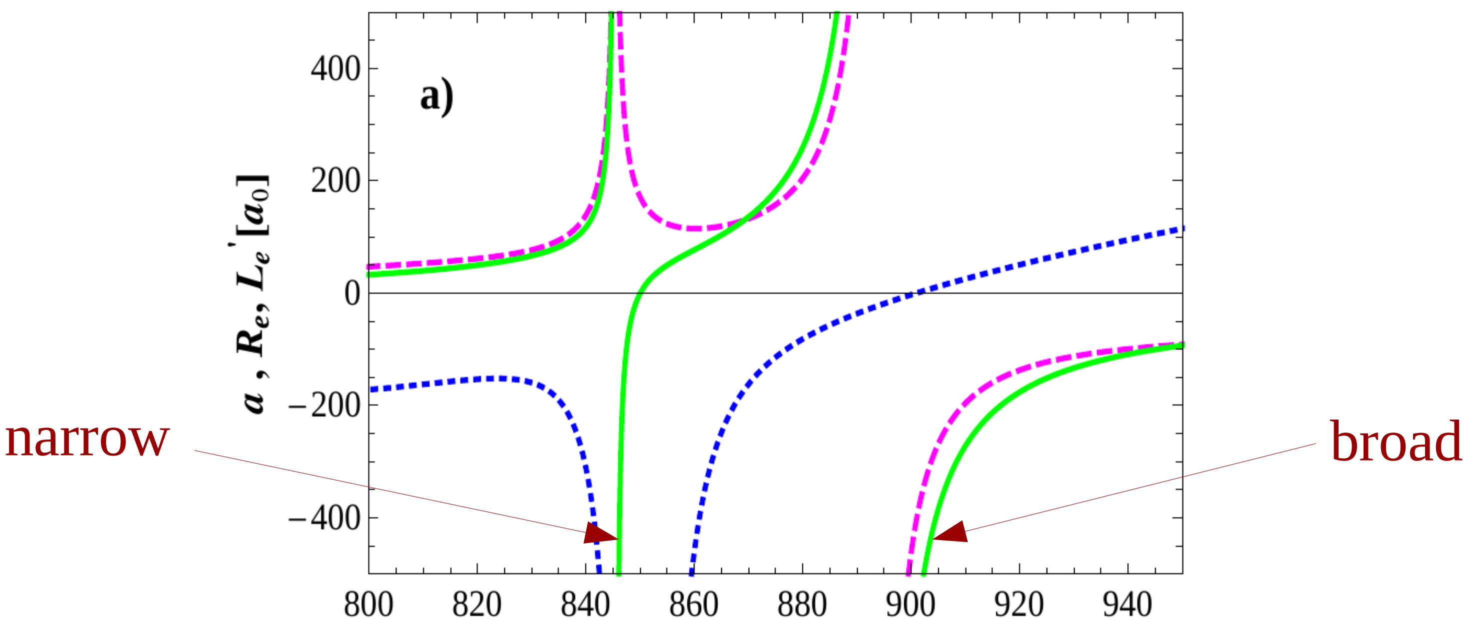
- **Comparison to calculated length scales**
- **Measurements for two different temperatures**

→ **No evidence for temp.-dependent recombination length**



Improvements analytic Feshbach model

- **Situation more complicated with 7Li**
- **Two resonances close together**



- **Numerics work fine, but...**

  ➡ **Analytic expressions better for understanding:**
- **Dependence on** R^*, a_{bg}, r_0

Double resonance system

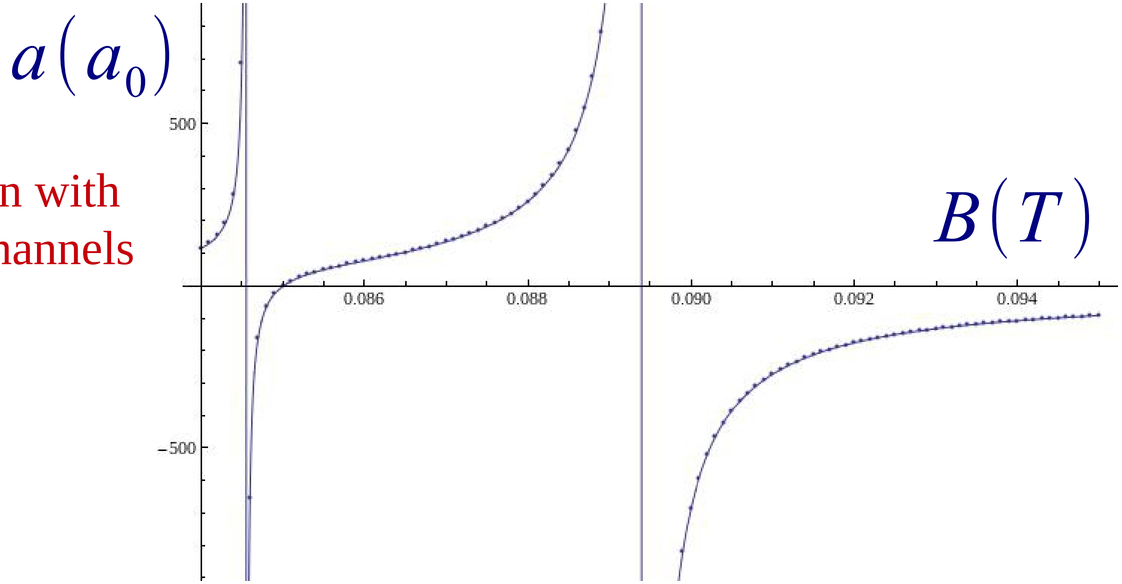
- **Go to a double resonance system!**
- **Apply Feshbach projection to two different molecular states**
- **Two resonant contributions to scattering length**

$$a = r_0 + a_P + a_1 + a_2$$

with

$$a_1 = a_P \frac{\Delta B_1}{B - B_1}, \quad a_1 = a_P \frac{\Delta B_2}{B - B_2}$$

Comparison with Coupled-channels result

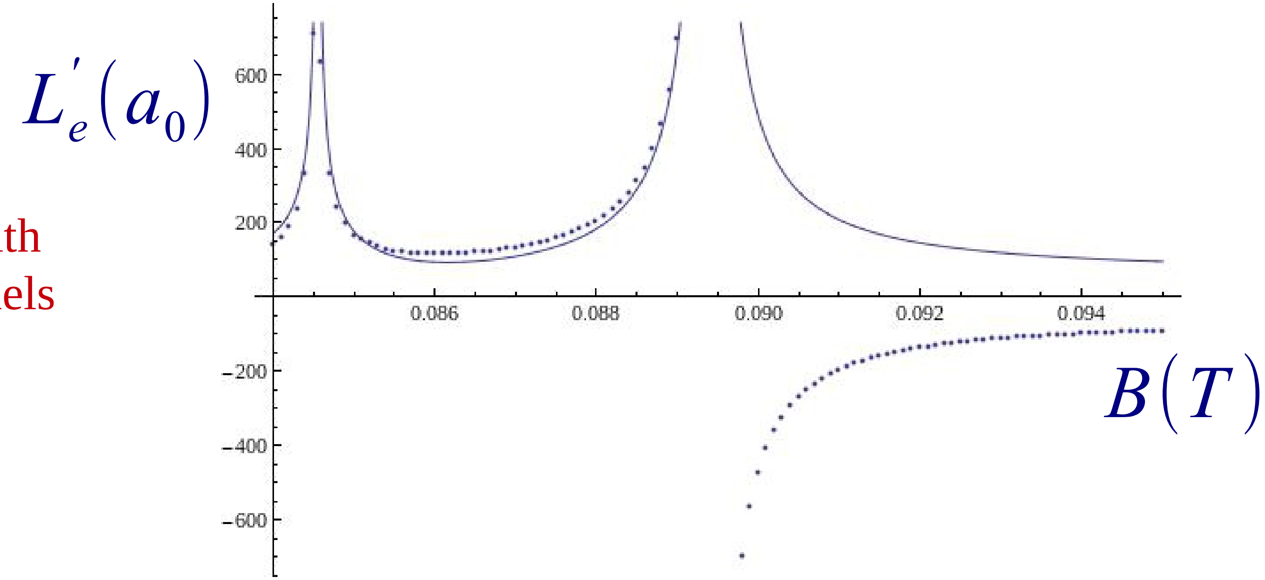


Now all parameters are determined

Analytical double-res. expression effective length

- **Derive effective volume from phase-shift**

$$\Rightarrow \quad V_e = \frac{(a_1 + a_2 + a_P)^3}{3} - 2 a_1 a_2 a_P + a_1^2 R_1^* + a_2^2 R_2^*$$

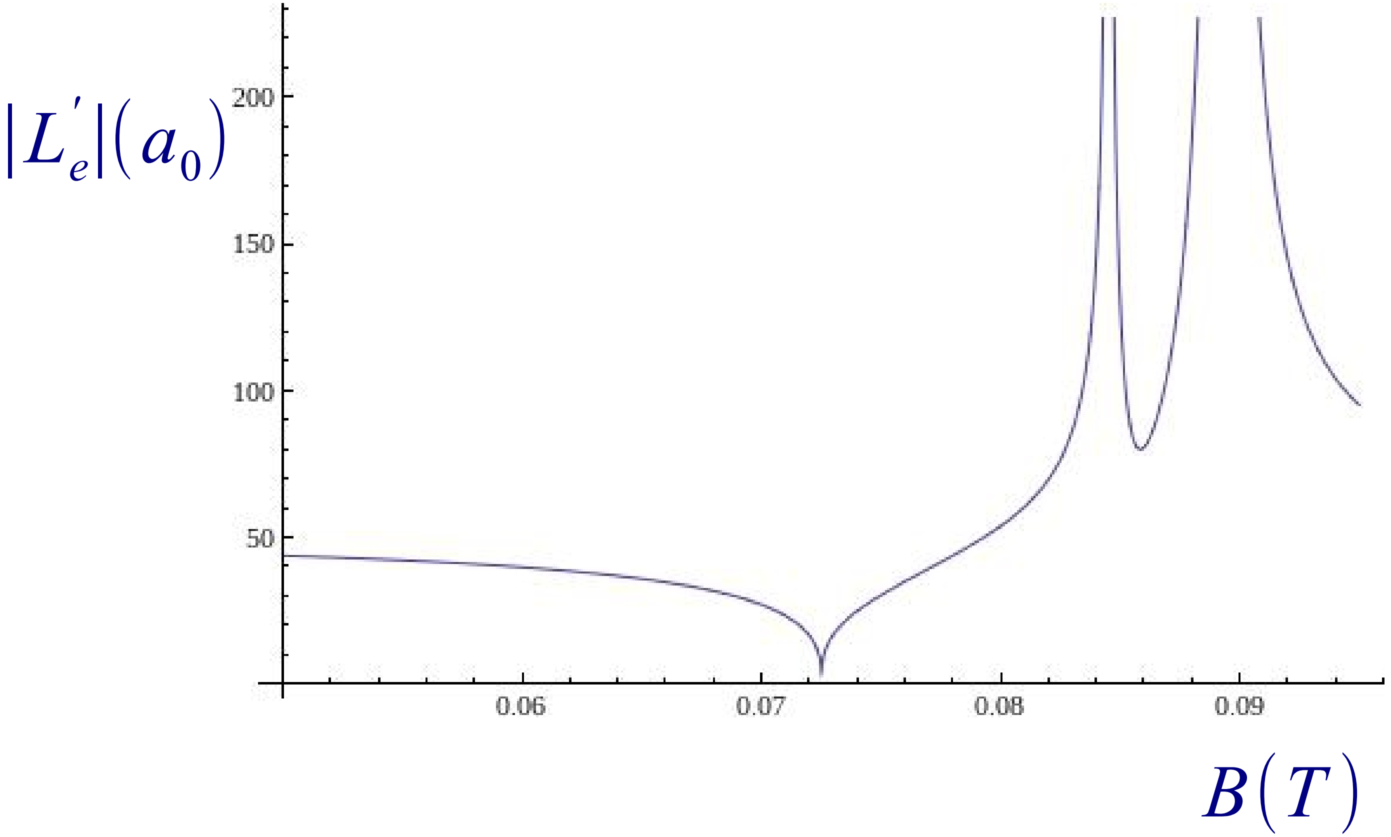


Comparison with Coupled-channels result

- **Could be improved by small variation in r_0**

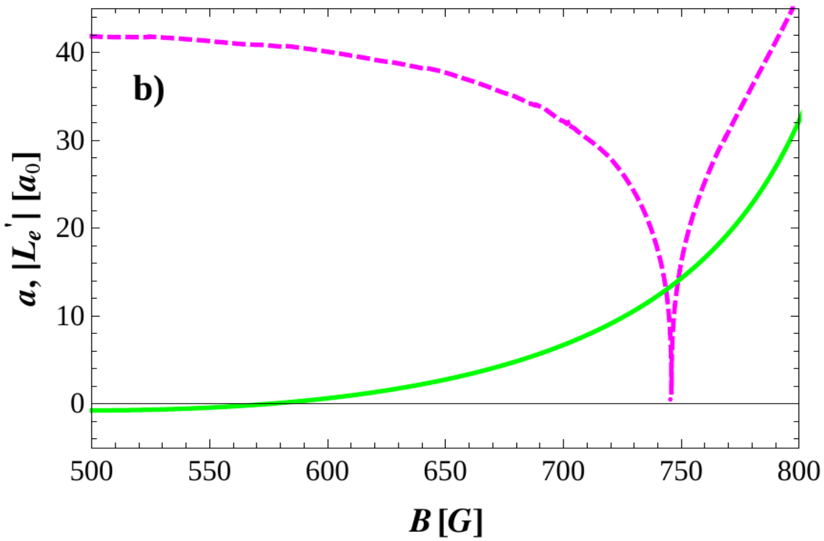
Effective length over large field range

- **Two resonances: one zero**
- **Does not coincide with zero in scattering length**



2nd zero crossing in scattering length

- **Smaller value L_e'**

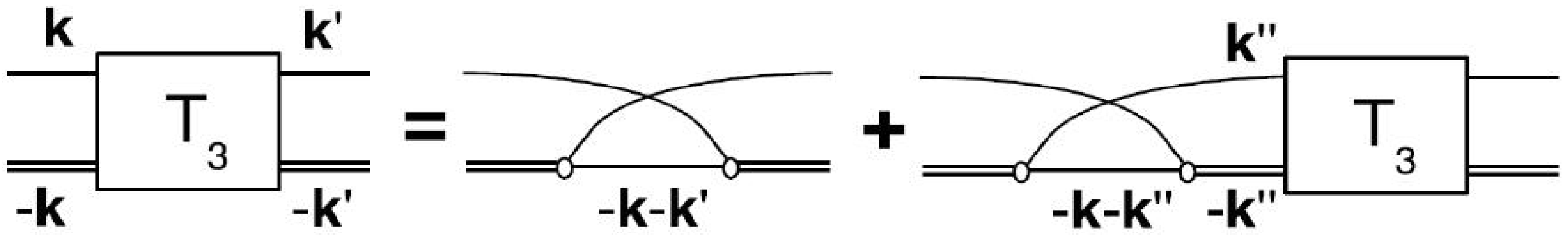


- **Recombination rate is two orders of magnitude smaller**

  Consistent with experiment (only upper limit)
- **Predict suppressed field value three-body recombination**

Theory of three-body recombination at a=0?

- Can K_3 be derived over full range of scattering length?
- Feshbach model is on-shell, well-behaved in k-space
- Possible approach: calculate three-body T-matrix



- Already interesting results obtained with separable potential for finite range effects
- Link between three-body parameter and potential range

[M. Jona-Lasinio and L. Pricoupenko, Phys. Rev. Lett **104**, 023201 (2010).]
[L. Pricoupenko and M. Jona-Lasinio, Phys. Rev. A **84**, 062712 (2011).]
Also, [J. Levinsen et al.]

Off-shell two-body T-matrix

- **Skorniakov Ter-Martirosian equation**

$$\frac{1}{t(E-3\epsilon_k/2)}T_3(k)=2\int\frac{d^3p}{(2\pi)^3}\frac{\xi(|\vec{k}-\vec{p}/2|)\xi(|\vec{p}-\vec{k}/2|)}{E-k^2/m-p^2/m-\vec{k}.\vec{p}/m}T_3(p)$$

- **Need off-shell T_2-matrix:easy with separable potential**

$$\langle k_f|T_2(E)|k_i\rangle=\xi(k_f)\xi(k_i)t(E)$$

- **Should satisfy on-shell condition**

$$\langle k|T_2(E)|k\rangle=\xi(k)\xi(k)t(E)=\frac{-1}{k\cot\delta(k)-ik}$$

- **Is it possible? Potential is non-local!**

Summary

Extreme limit non-universal regime three-body recombination

➡ **At vanishing scattering length**

Other length scales become important

- **Width resonance, potential range, background scattering length**

➡ R^*, a_{bg}, r_0

- **Can be expressed as combination of** a **and** R_e
- **Rate energy-independent**

Cover whole range from weak to strong two-body interactions

- **Predict non-trivial magnetic field value where three-body recombination is suppressed**

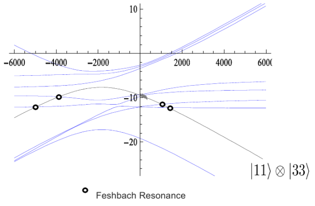
- **How to derive this quantity from three-body physics?**

[Three-body recombination at vanishing scattering lengths in an ultracold Bose gas, Zav Shotan, Olga Machtey, Servaas Kokkelmans, Lev Khaykovich, PRL 2014PRL 113, 053202 (2014)]

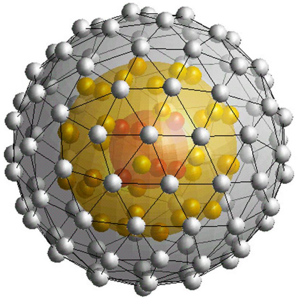
Acknowledgments

Feshbach res.

Maikel Goosen



Lev Khaykovich, Zav Shotan, Olga Machtey **(Bar-Ilan University)**



Rydberg lattices

Rick van Bijnen

Edgar Vredenbregt

Cornee Ravensbergen

Arjen Monden

Jaron Sanders

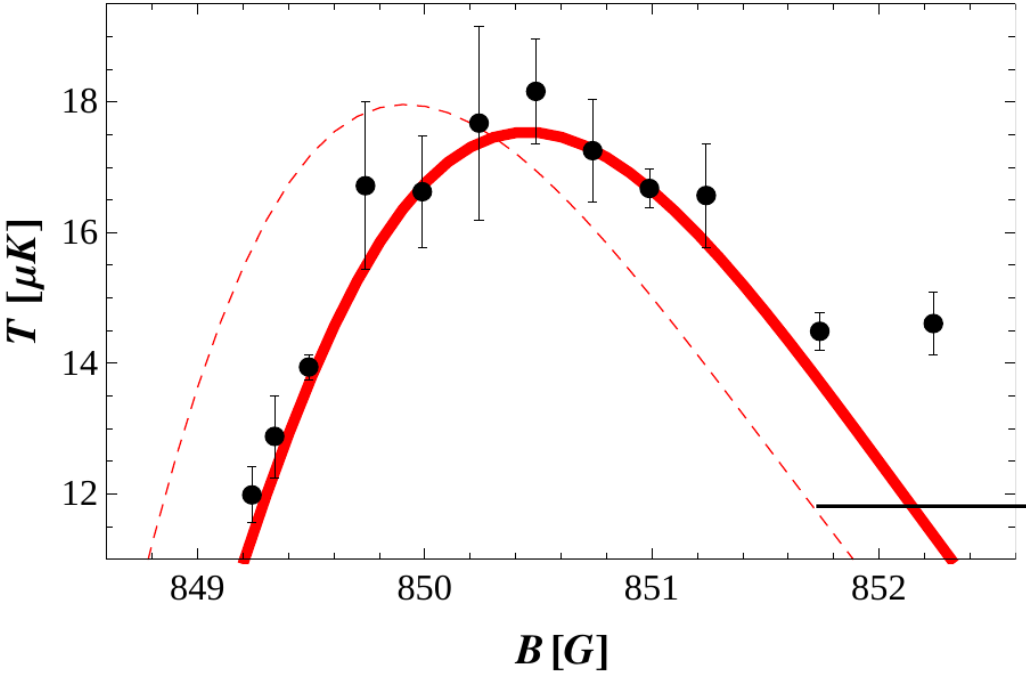
Rasmus Skannrup

Tarun Johri

Experimental determination zero crossing

- **Use evaporative cooling: not obvious**

➡ **Cross section strongly energy-dependent**



$k = 0$

$$\sigma(k) = 8\pi \frac{(V_e k^2 - a)^2}{1 + (V_e k^2 - a)^2 k^2}$$