

Stochastic resonance in soft-glassy materials

Citation for published version (APA):

Benzi, R., Bernaschi, M., Perlekar, P., Sbragaglia, M., Succi, S., & Toschi, F. (2014). *Stochastic resonance in soft-glassy materials*. (arXiv.org; Vol. 1403.2213 [cond-mat.soft]). s.n.

Document status and date:

Published: 01/01/2014

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

Stochastic Resonance in Soft-Glassy Materials

R. Benzi¹, M. Bernaschi², P. Perlekar³, M. Sbraglia¹, S. Succi² and F. Toschi^{2,4}

¹ Department of Physics and INFN, University of “Tor Vergata”, Via della Ricerca Scientifica 1, 00133 Rome, Italy

² Istituto per le Applicazioni del Calcolo CNR, Via dei Taurini 19, 00185 Rome, Italy

³ TIFR Centre for Interdisciplinary Sciences, 21 Brundavan Colony, Narsingi, Hyderabad 500075, India

⁴ Department of Physics and Department of Mathematics and Computer Science, Eindhoven University of Technology, 5600 MB Eindhoven

Flow in soft-glasses occurs via a sequence of reversible elastic deformations and local irreversible plastic rearrangements. Yield events in the material cause kicks adding up to an effectively thermal noise, an intuition that has inspired the development of phenomenological models aiming at explaining the main features of soft-glassy rheology. In this letter, we provide a specific scenario for such mechanical activation, based on a general paradigm of non-equilibrium statistical mechanics, namely *stochastic resonance*. By using mesoscopic simulations of emulsion droplets subject to an oscillatory strain, we characterize the response of the system and highlight a resonance-like behavior in the plastic rearrangements. This confirms that the synchronization of the system response to an external time-dependent load is triggered by the mechanical noise resulting from disordered configurations (polydispersity).

PACS numbers: 47.50.Cd, 47.11.St, 87.19.rh, 83.60.Rs

The effective temperature is emerging as an essential ingredient in theories of non-equilibrium phenomena for amorphous and soft-glassy materials [1–7]. Evidence for the existence of such temperature is sometimes indirect [4, 6, 7], but nevertheless compelling. For example, the Shear-Transformation-Zone (STZ) theory [8] of plastic deformations in glass-forming materials has been recently reformulated since an effective disorder temperature emerges as a key internal-state variable [1]. The notion of effective temperature has also proven very fruitful to describe complex and non-linear rheology of soft-glassy materials [4, 5, 9]. The mechanical noise induced by the flow can also be seen as a source of non-locality [6, 7, 9]: the extent of fluidization (determined by local fluctuations) correlates on length-scales which increase as strain rates decrease, as first found in experiments [9, 11] and then confirmed by mesoscopic models [7, 10, 12]. A one-to-one mapping between the effective temperature and the dynamic rheological properties is elucidated in the SGR (Soft-Glassy Rheology) model, proposed years back by Sollich *et al.* [4, 5] and further developed by other authors [6, 7]. The SGR builds on Bouchaud’s trap model [13] and is based on the idea of an activated escape from free-energy barriers due to mechanical noise, under the assumption that temperature alone is unable to achieve a complete structural relaxation. The SGR model has turned out to be very successful in explaining many features of soft-glasses, including aging and non-linear Herschel-Bulkley rheology. Like for the STZ theory [1], it has been recently argued that the effective temperature can be regarded as a genuine thermodynamic temperature [5].

Despite its remarkable success, the SGR model remains inherently phenomenological in nature, and consequently

a microscopic derivation or justification of that model remains highly desirable. In the present letter, we support the notion of effective temperature as the mechanism of activated escape from free-energy barriers via mechanical noise, with no need of heuristic assumptions on the statistical distribution of the stress within the material. To that purpose, we use a Lattice version of Boltzmann (LB) kinetic equation with competing short-range attraction and mid-range repulsion (see supplementary material) developed in the recent years [14–17]. More in detail, we provide a specific scenario for mechanical activation, based on a general paradigm of non-equilibrium statistical mechanics, namely *stochastic resonance* [18], a widespread phenomenon in nonlinear systems, wherein weak periodic signals are amplified under the assistance of noise.

A collection of closely packed droplets is simulated by using a two-component (*A* and *B*) LB model [14] (see figure 1). Within this framework, the free parameters of the model permit the tuning of the basic physical and rheological properties, namely the packing fraction, the surface tension and the disjoining pressure, as well as the polydispersity of the binary mixture [15, 17]. The packing fraction of the dispersed (*A*) phase in the continuum (*B*) phase is approximately equal to 90% in all the simulations presented in this letter. The degree of polydispersity is defined as the standard deviation of the area of the droplets, $\sigma(A)$, expressed in units of its mean, i.e. $d = \sigma(A)/\langle A \rangle = 0.3$ in the present simulations. The system is confined between two solid walls, with no-slip boundary conditions. With the parameters used, the model has been shown to reproduce most of the known results obtained in laboratory experiments, such as yield stress and jamming, Herschel-Bulkley rhe-

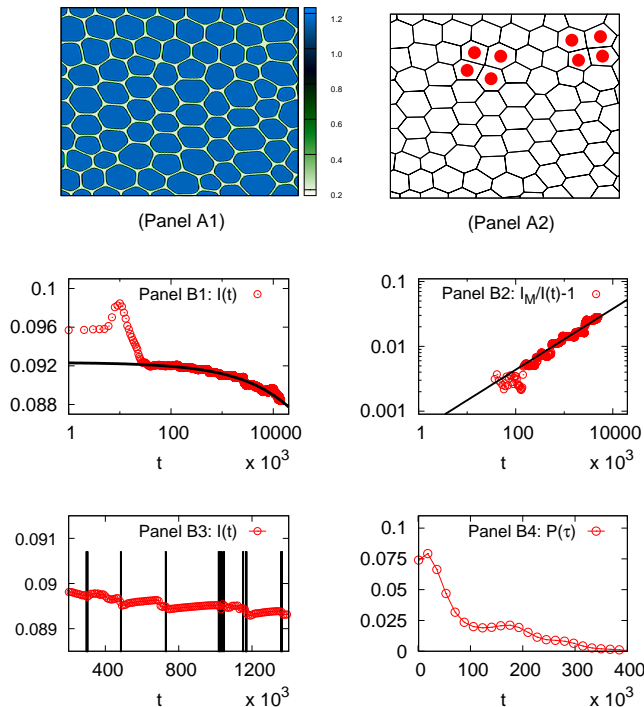


FIG. 1: A collection of closely packed droplets is simulated by using a two-component (A and B) lattice Boltzmann (LB) method [14]. Panel A1 reports the color map of the A component. Blue/green (dark/light) color refers to A -rich/ B -rich regions. Panel A2 reports the corresponding Voronoi tessellation of the centers of mass of the droplets. The involved Voronoi cells in the plastic event are labeled by a central dot. Panel B1: Behavior of the interface length indicator $I(t)$ (see equation (1)) as a function of time. The function $I(t)$ is a measure of the interface length in the system. Panel B2: behavior of $I_M/I(t) - 1$ where I_M is the maximum value of $I(t)$. The figure shows that $I(t)$ is very well approximated by a scaling law of the form $I(t) = I_M/(1 + (t/t_0)^\alpha)$ with $\alpha \sim 0.5$ and $t_0 \sim 10^8$. Panel B3: magnified view of panel B1 during the period $t \in [200 - 1400] \times 10^3$. The vertical spikes indicate when plastic events occur. Note that there is a clear correlation between plastic events and the decrease in $I(t)$. Panel B4: probability density distribution $P(\tau)$ of the time interval τ between two successive plastic events. $P(\tau)$ is qualitatively close to an exponential distribution with a peak value at around $\tau_0 \approx 3 \times 10^4$.

ology [16] and cooperativity [15, 17]. We are interested in the dynamics of the system at low frequency and sub-yield strains $\gamma < \gamma_Y$. The main goal is to investigate the coupling between internal rearrangements and external time-periodic forcing. To that purpose, we start by considering the system at $\gamma = 0$, i.e., when no external forcing is imposed. Even if no energy is available from external forcing, the system shows aging and undergoes a very slow relaxation process towards equilibrium [4, 19]. The reason is that the interface between the two fluids is

spatially disordered, and there exists a non-homogeneous pressure difference in the system, which drives a very complex interface dynamics. In particular, local rearrangements are observed and take place in the form of plastic events. At each instant of time, the system is in a metastable configuration, characterized by a comparatively small growth rate in the unstable directions, a mechanism common to many soft-glassy materials. In figure 1, panel B1, we show the time behavior of the following interface length indicator:

$$I(t) = \frac{1}{L^2} \int |\vec{\nabla} \phi(\vec{x})|^2 d\vec{x} \quad (1)$$

where L is the characteristic size of the system and $\phi = \rho_A - \rho_B$ is the density difference. In figure 1, panel B2, we report $(I_M/I(t) - 1)$ (where I_M is the maximum value of $I(t)$) which shows that the interface length decreases in time as $(1 + (t/t_0)^\alpha)$ with $\alpha \sim 0.5$ and $t_0 \approx 10^8$ [20].

A closer inspection into the dynamics shows that the (very slow) decrease of the interface length is associated with localized plastic events. A direct computation of the plastic events has been performed by using a Voronoi reconstruction of the emulsion interface [21] at each time step and by identifying plastic events as sudden topological changes in the Voronoi tessellation (see panel A2 in figure 1). In figure 1, panel B3, we show a magnified view of the behavior of $I(t)$, with the indication of the plastic events observed during the evolution (black spikes). From these observations, we realize that, due to spatial disorder, rearrangements occur at random instants and random space locations. The basic question we wish to address is whether such mechanism can be regarded as a non-thermal activated process, induced by a suitable inherent “noise”. To that purpose, we consider the quantity $\tau \equiv t_i - t_{i-1}$, where t_i is the time at which the i -th plastic event occurs. In figure 1, panel B4, we show the probability density function (pdf) $P(\tau)$ of τ . The figure shows that $P(\tau)$ can be assumed to be exponentially distributed in time, as if plastic events were triggered by an activated process. We estimate a characteristic time scale of the system at the peak value of the pdf to be close to $\tau_0 = 3 \times 10^4$.

To gain further insight, we consider the case of an external oscillatory strain at $\gamma(t) = \gamma_P \sin(\omega t)$ with frequency ω and peak amplitude γ_P smaller than the yield strain γ_Y . For small γ_P , we can assume that the external strain provides an elastic energy input in the system of the order $E\gamma_P^2/2$, to be compared to the energy barrier for a plastic event, which can be estimated of the order of $E\gamma_Y^2/2$, with E the load modulus. In figure 2, we report the number $N(\omega) = \langle \tau \rangle / T(\omega)$ defined as the ratio of the average time $\langle \tau \rangle$ between two consecutive plastic events and the forcing period $T(\omega)$. Note that, $1/N(\omega)$ is just the number of plastic events per cycle. In the figure, we plot $N(\omega)$ in log scale versus $(\gamma_P/\gamma_Y)^2$, for two oscilla-

tory strains with period $T(\omega) = 4\tau_0 = 12 \times 10^4$ (blue triangles) and $T(\omega) = 2\tau_0 = 6 \times 10^4$ (red dots) as a function of $(\gamma_P/\gamma_Y)^2$. Since $(\gamma_P/\gamma_Y)^2$ represents the ratio of the energy induced by the external strain with respect to the energy barrier, in an activated process we expect a linear decrease of $\log(N(\omega))$ as $(\gamma_P/\gamma_Y)^2$ increases. This is exactly what figure 2 shows for both $T(\omega) = 12 \times 10^4$ and $T = 6 \times 10^4$. Figures 1 and 2 support the idea that the internal (deterministic) dynamics acts like an effective noise in the system. We argue that the strength of this effective noise should be related to the spatial disorder of the system [4–6]. Under the assumption that this interpretation of plastic events as noise-induced activated processes is correct, we argue that there should exist a specific frequency at which the mechanism of Stochastic Resonance (SR) [18] is observed. To explore such a non trivial conjecture, we begin with a quick review of the main distinctive features of SR. In figure 3, we show the classic signature of SR, as obtained for the (scalar) model stochastic differential equation:

$$\frac{dx(t)}{dt} = x(1 - x^2) + A \sin(\omega t) + \sqrt{\epsilon} W(t) \quad (2)$$

where $W(t)$ is a white noise delta-correlated in time. When $A = 0$ (zero external forcing), the system shows transitions between the two minima of the double-well potential, exponentially distributed in time with an average time $\tau_{x,0} = \frac{\pi}{\sqrt{2}} \exp(2\Delta V/\epsilon)$, where, for the present case, the potential barrier is $\Delta V = 1/4$. A value of $A \neq 0$ induces a variation in the potential barrier, $\Delta V = 1/4 + A \sin(\omega t)$. For $T = \frac{2\pi}{\omega} = 2\tau_{x,0}$, the system shows a nearly periodic transition between the two minima, even at relative small values of A . SR is a general mechanism for non-linear systems; it has been observed for both stochastic and deterministic chaotic systems (where the chaotic dynamics plays the role of the noise). Going back to our system, the results shown in figures 1 and 2 suggest that for an external oscillatory strain with period $T = \frac{2\pi}{\omega} = 2\tau_0$, we should observe SR, whereas for periods $T(\omega)$ different from $2\tau_0$ (smaller or larger), this should not be the case. SR in our case should be detected in two different ways, namely *i*) plastic events should be in phase with the strain and *ii*) the probability distribution of τ should be peaked around $\tau = 0.5 T(\omega)$.

To investigate the existence of SR, we performed simulations with an external periodic strain for three different periods $4\tau_0$, $2\tau_0$ and τ_0 . The external strain amplitude is such that $\gamma_P/\gamma_Y = 0.4$. To draw a comparison with the double well potential of equation (2), our choice $\gamma_P/\gamma_Y = 0.4$ corresponds to $A = 0.04$ (see also figure 2). Our choice of γ_P is close to the smallest amplitude in the external strain for which SR can be observed in the system. In figure 4, we show a snapshot of 10 cycles for the three different periods. The solid line corresponds to $|\sin(\omega t)|$, whereas the red dots indicate the occurrence of plastic events. In the top panel of figure 5,

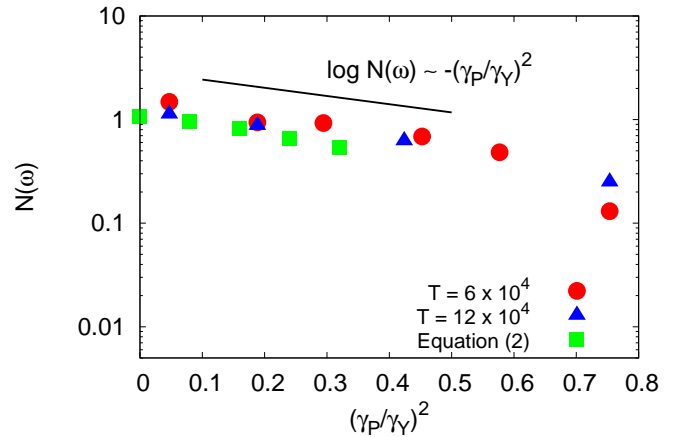


FIG. 2: Behavior of $N(\omega) = \langle \tau \rangle / T(\omega)$ as a function of $(\gamma_P/\gamma_Y)^2$, where $T(\omega)$ is the period of the external strain forcing, $\langle \tau \rangle$ is the average time observed between two consecutive plastic events, γ_P is the peak value of the external strain applied to the system and γ_Y is the yield strain. Two values of $T(\omega)$ are considered, $T(\omega) = 6 \times 10^4$ (red circles) and $T(\omega) = 12 \times 10^4$ (blue triangles). In both cases the ratio $\langle \tau \rangle / T(\omega)$ decays exponentially with $(\gamma_P/\gamma_Y)^2$ (the solid line is a guide for the eye). The green squares correspond to $N(\omega)$ observed in the model reported in equation (2).

we show the pdf of τ , as obtained from 75 cycles of the external forcing. Both figures clearly show that SR occurs with the right signature: plastic events are nearly in phase with the forcing and the pdf of τ is peaked around $\tau = 0.5 T(\omega)$. Note that at $T(\omega) = \tau_0$ the pdf shows two clear peaks at $\tau = 0.5 T(\omega)$ and $\tau = T(\omega)$. In fact, we may qualitatively think of our system as a collection of different spatial regions where the probability for a plastic event to occur reaches a maximum at the largest strain (in absolute value), i.e. at times $T_n \equiv (n + 1)T(\omega)/2$. Thus we expect that, for frequency higher than the resonance one, the probability distribution of $P(\tau)$ should show well defined peaks at $\tau = T_n$, in agreement with the results shown in figures 4 and 5. Finally, it is interesting to observe that the green squares shown in figure 2 correspond to $N(\omega)$ (the number of transitions per cycle) observed in the naive double well potential model equation (2), with $\omega = 2\pi/\tau_{x,0}$, i.e. at the SR frequency, and different values of A dependently on the value of γ_P/γ_Y . As one can see, the slope of $\log(N(\omega))$ is close to the one observed in our simulations, lending further support to the idea that the internal dynamics of the binary fluid mixture can be regarded as a form of intrinsic noise.

Another signature of SR is given by the disappearance of the narrow peak at $\tau = 0.5 T(\omega)$, upon increasing or decreasing of the noise amplitude away from the resonance value. This effect also occurs in our case: we repeated the simulation at the resonance frequency by

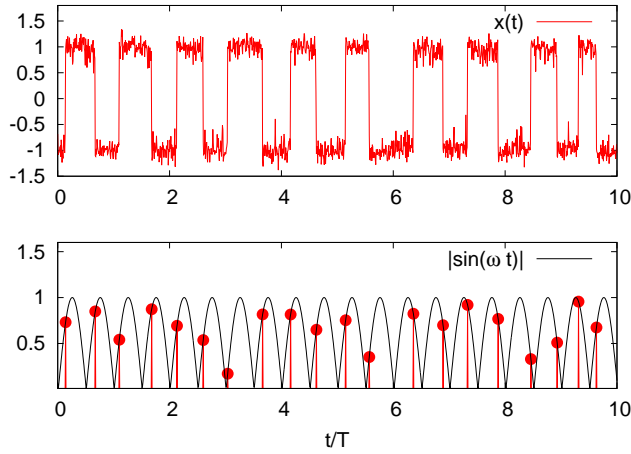


FIG. 3: Typical signature of stochastic resonance for the system described by equation (2) with $\Delta V = 1/4$, $\epsilon = 0.0625$, $A = 0.08$. The period $T = 2\pi/\omega = 12 \times 10^3$ of the applied forcing is twice the internal time scale of the system $\tau_{x,0} = \frac{\pi}{\sqrt{2}} \exp(2\Delta V/\epsilon) \approx 6 \times 10^3$. The system shows a nearly periodic transition between the two minima (upper panel). In the lower panel we plot the strength of the external forcing $|\sin(\omega t)|$ (the amplitude has been set to 1 for simplicity), whereas the red dots indicate the transition from one state to the other. In this conceptual picture, the red dots are equivalent to the plastic events.

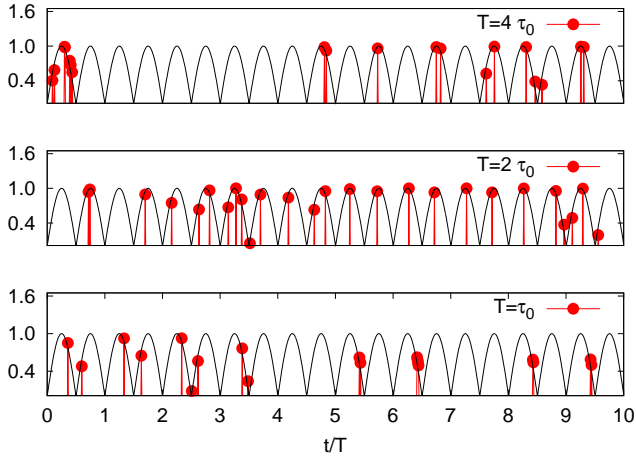


FIG. 4: Plastic events observed during 10 cycles for the strain $\gamma(t) = \gamma_P \sin(\omega t)$ with $\gamma_P/\gamma_Y = 0.4$ and three different periods: $T(\omega) = 4\tau_0$ (upper panel), $T(\omega) = 2\tau_0$ (middle panel) and $T(\omega) = \tau_0$ (lower panel). The characteristic time τ_0 represents the average value of the time between successive plastic events in unstrained conditions (see figure 1). The solid line represents $|\sin(\omega t)|$. For the period $T(\omega) = 2\tau_0$ we observe one of the hallmarks of stochastic resonance, i.e. a synchronization of the plastic events with the applied strain.

increasing or decreasing the polydispersity of the system

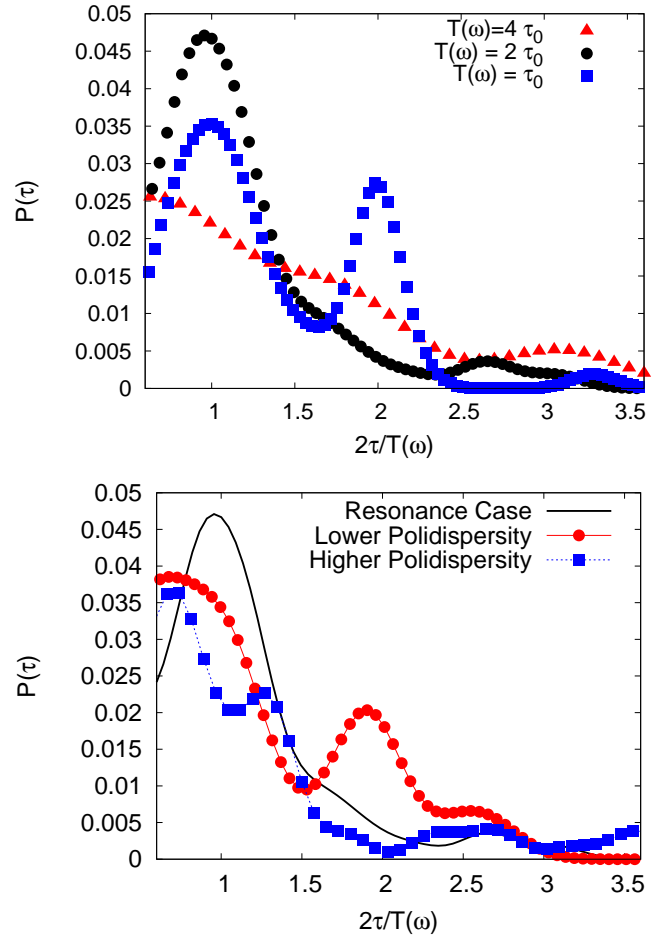


FIG. 5: Top Panel: probability density distribution $P(\tau)$ of the time interval τ between two successive plastic events at different periods. $P(\tau)$ is plotted as a function of $2\tau/(T(\omega))$ and the signature of stochastic resonance is a narrow peak for $\tau = 0.5T(\omega)$ when $T(\omega) = 2\tau_0$. Note that for $T(\omega) = \tau_0$, $P(\tau)$ shows two clearly defined peaks at $\tau = 0.5T(\omega)$ and $\tau = T(\omega)$. Bottom panel: probability density of τ for lower and higher polydispersity with respect to the case $T(\omega) = 2\tau_0$ reported in the top panel. By changing the polydispersity, we change the noise level and therefore the narrow peak at $\tau = 0.5T(\omega)$ observed in the resonant case disappears.

with all the other parameters being kept at the same value. With no external strain, the number of plastic events decreases (increases) by decreasing (increasing) the polydispersity by a factor 20%. In the bottom panel of figure 5 we show the probability distribution $P(\tau)$ for both cases of “weak” (lower polydispersity, $d = 0.22$) and “strong” (higher polydispersity, $d = 0.4$) noise in presence of the periodic strain. From this figure it is clear that upon increasing or decreasing the spatial disorder, SR fades away in accordance to the results expected from equation (2).

Summarizing, clear indications of a stochastic resonance

revealed by our lattice kinetic simulations, suggest that the intrinsic noise provides a mechanism responsible for activated processes in the form of plastic rearrangements. The intrinsic noise depends on spatial disorder and disappears when spatial order is restored (monodisperse droplets). For sub-yield strains and low-frequencies, the external forcing plays a non perturbative effect, which cannot be described by a linear response theory. Taken together, all these points lend strong support to several results and conjectures presented in the soft-glassy literature, particularly to the central idea of noise-induced activated escape from free energy random traps [4, 5]. Finally, we remark that our results do not depend on any interpretation of the internal fluctuations as an effective thermodynamic temperature.

The authors kindly acknowledge funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013)/ERC Grant Agreement no[279004].

-
- [1] E. Bouchbinder & J. S. Langer, *Phys. Rev. E* **80**, 031132 (2009); *Phys. Rev. E* **80**, 031131 (2009); *Phys. Rev. E* **80**, 031133 (2009)
- [2] L. F. Cugliandolo *et al.*, *Phys. Rev. E* **55**, 3898 (1997)
- [3] J.-L. Barrat & L. Berthier, *Phys. Rev. E* **63**, 012503 (2000); L. Berthier & J.-L. Barrat, *J. Chem. Phys.* **116**,

- 6228 (2002); L. Berthier & J.-L. Barrat, *Phys. Rev. Lett.* **89**, 095702 (2002); A. Nicolas *et al.*, *arXiv:14016340v1* (2014)
- [4] P. Sollich *et al.*, *Phys. Rev. Lett.* **78**, 2020 (1997); P. Sollich *Phys. Rev. E* **58**, 738 (1998)
- [5] P. Sollich & M. E. Cates, *Phys. Rev. E* **85**, 031127 (2012).
- [6] P. Hebraud & F. Lequeux, *Phys. rev. Lett.* **81**, 2934 (1998)
- [7] L. Bocquet *et al.*, *Phys. Rev. Lett.* **103**, 036001 (2009).
- [8] M. L. Falk & J. S. Langer, *Phys. Rev. E* **57**, 7192 (1998).
- [9] P. Jop *et al.*, *Phys. Rev. Lett.* **108**, 148301 (2012).
- [10] V. Mansard *et al.*, *Soft Matter* **7**, 5524 (2011).
- [11] J. Goyon *et al.*, *Nature* **454**, 84 (2008).
- [12] A. Nicolas & J.-L. Barrat, *Phys. Rev. Lett.* **110**, 138304 (2013)
- [13] J. P. Bouchaud, *J. Phys. I France* **2**, 1705-1713 (1992)
- [14] R. Benzi *et al.*, *J. of Chem. Phys.* **131**, 104903 (2009)
- [15] M. Sbragaglia *et al.*, *Soft Matter* **8** (41), 10773-10782 (2012)
- [16] R. Benzi *et al.*, *Europhys. Lett.* **91**, 14003 (2010)
- [17] R. Benzi *et al.*, *Europhys. Lett.* **104**, 48006 (2013)
- [18] R. Benzi *et al.*, *J. Phys. A: Math. Gen.* **14**, L453 (1981); L. Gammaitoni *et al.*, *Rev. Mod. Phys.* **70**, 223-287 (1998)
- [19] S. Vincent-Bonnieu *et al.*, *Europhys. Lett.* **74**, 533-539 (2006)
- [20] All quantities will be given in LB units (see also supplementary material)
- [21] The Voronoi tessellation were made by using the Voro++ library. <http://math.lbl.gov/voro++>

Supplementary Information

Stochastic Resonance in Soft-Glassy Materials

R.Benzi¹, M. Bernaschi², P. Perlekar³, M. Sbragaglia¹, S. Succi^{2,4} and F. Toschi^{2,4}

¹ Department of Physics and INFN, University of “Tor Vergata”, Via della Ricerca Scientifica 1, 00133 Rome, Italy

² Istituto per le Applicazioni del Calcolo CNR, Via dei Taurini 19, 00185 Rome, Italy

³ TIFR Centre for Interdisciplinary Sciences, 21 Brundavan Colony, Narsingi, Hyderabad 500075, India

⁴ Department of Physics and Department of Mathematics and Computer Science and J.M. Burgerscentrum, Eindhoven University of Technology, 5600 MB Eindhoven

We provide technical details about the two-component mesoscopic Lattice Boltzmann (LB) model with competing interactions used in the numerical simulations.

Our Lattice Boltzmann model can be seen as a lattice transcription of a continuum model for a binary fluid with phase-segregating interactions [1], augmented by the introduction of competing interactions providing an effect of *frustration* at the non ideal interface [2]. This lattice kinetic model has been shown to be in good quantitative agreement with many distinctive features of soft-glassy materials, such as aging [3] and non-linear rheology [4]. Specifically, it makes possible, with an affordable computational cost, the simulation of a collection of closely packed droplets, with variable polydispersity and packing fraction, at changing load conditions [5].

The mesoscopic kinetic model considers a binary mixture of fluids A and B , each described by a discrete kinetic Boltzmann distribution function $f_{\zeta i}(\mathbf{x}, \mathbf{c}_i, t)$, measuring the probability of finding a representative particle of fluid $\zeta = A, B$ at position \mathbf{x} and time t , with discrete velocity \mathbf{c}_i , where the index i runs over the nearest and next-to-nearest neighbors of \mathbf{x} in a regular two-dimensional lattice [3]. By definition, the mesoscale particle represents all molecules contained in a unit cell of the lattice. The distribution functions of the two fluids evolve under the effect of free-streaming and local two-body collisions, described, for both fluids, by a relation of momentum-relaxation to a local equilibrium ($f_{\zeta i}^{(eq)}$) on a time scale τ_{LB} :

$$f_{\zeta i}(\mathbf{x} + \mathbf{c}_i, \mathbf{c}_i, t + 1) - f_{\zeta i}(\mathbf{x}, \mathbf{c}_i, t) = -\frac{1}{\tau_{LB}} \left(f_{\zeta i} - f_{\zeta i}^{(eq)} \right) (\mathbf{x}, \mathbf{c}_i, t) + F_{\zeta i}(\mathbf{x}, \mathbf{c}_i, t). \quad (1)$$

The equilibrium distribution is given by

$$f_{\zeta i}^{(eq)} = w_i \rho_{\zeta} \left[1 + \frac{\mathbf{u} \cdot \mathbf{c}_i}{c_s^2} + \frac{\mathbf{u} \mathbf{u} : (\mathbf{c}_i \mathbf{c}_i - c_s^2 \mathbf{1})}{2c_s^4} \right] \quad (2)$$

with w_i a set of weights known a priori through the choice of the quadrature and c_s the lattice speed of sound. Coarse grained densities are defined for both species

$$\rho_{\zeta}(\mathbf{x}, t) = \sum_i f_{\zeta i}(\mathbf{x}, t)$$

as well as a global momentum for the whole mixture

$$\mathbf{j}(\mathbf{x}, t) = \rho(\mathbf{x}, t) \mathbf{u}(\mathbf{x}, t) = \sum_{\zeta, i} \mathbf{c}_i f_{\zeta i}(\mathbf{x}, t) \quad (3)$$

with $\rho = \sum_{\zeta} \rho_{\zeta}$. The term $F_{\zeta i}(\mathbf{x}, \mathbf{c}_i, t)$ is just the i -th projection of the total internal force which includes a variety of interparticle forces. First, a repulsive (r) force with strength parameter \mathcal{G}_{AB} between the two fluids

$$F_{\zeta}^{(r)}(\mathbf{x}, t) = -\mathcal{G}_{AB} \rho_{\zeta}(\mathbf{x}, t) \sum_{i, \zeta' \neq \zeta} w_i \rho_{\zeta'}(\mathbf{x} + \mathbf{c}_i, t) \mathbf{c}_i \quad (4)$$

is responsible for phase separation [1]. Due to the effect of $F_{\zeta}^{(r)}(\mathbf{x})$, the physical domain is partitioned into two kinds of sub-domains, each occupied by a pure fluid, with the interface between the two fluids characterized by a positive surface tension Γ . The natural tendency of the interface to minimize its area (length, for the present case of 2d interfaces) via merger events is *frustrated* (F) [2] by the introduction of competing interactions which contribute to determine a positive disjoining pressure in the thin film separating two “droplets” of the same fluid [6]. In particular, we model

short range (nearest neighbor, NN) self-attraction (controlled by strength parameters $\mathcal{G}_{AA,1} < 0$, $\mathcal{G}_{BB,1} < 0$), and “long-range” (next to nearest neighbor, NNN) self-repulsion (regulated by strength parameters $\mathcal{G}_{AA,2} > 0$, $\mathcal{G}_{BB,2} > 0$)

$$F_{\zeta}^{(F)}(\mathbf{x}, t) = -\mathcal{G}_{\zeta\zeta,1}\psi_{\zeta}(\mathbf{x}, t) \sum_{i \in NN} w_i \psi_{\zeta}(\mathbf{x} + \mathbf{c}_i, t) \mathbf{c}_i - \mathcal{G}_{\zeta\zeta,2}\psi_{\zeta}(\mathbf{x}, t) \sum_{i \in NNN} w_i \psi_{\zeta}(\mathbf{x} + \mathbf{c}_i, t) \mathbf{c}_i \quad (5)$$

with $\psi_{\zeta}(\mathbf{x}, t) = \psi_{\zeta}[\rho(\mathbf{x}, t)]$ a suitable pseudopotential function [7, 8]. By a proper tuning of the phase separating interactions (4) and the competing interactions (5), the model simultaneously achieves small (positive) surface tension Γ and positive disjoining pressure Π_d . The emergence of a positive disjoining pressure $\Pi_d(h)$ can be controlled in numerical simulations by considering a thin *film* with two non-ideal flat interfaces developing along a given (say x) direction, separated by the distance h . Following Bergeron [9], we write the relation for the corresponding tensions

$$\Gamma_f(h) = 2\Gamma + \int_{\Pi_d(h=\infty)}^{\Pi_d(h)} h d\Pi_d$$

where Γ_f is the overall film tension, whose expression is known in terms of the mismatch between the normal (N) and tangential (T) components of the pressure tensor [10, 11], $\Gamma_f = \int_{-\infty}^{+\infty} (P_N - P_T(x)) dx$, where, in our model, $P_N - P_T(x) = p_s(x)$ can be computed analytically [12, 13]. From the relation $s(h) = \Gamma_f(h) - 2\Gamma$ it is possible to compute the disjoining pressure: a simple differentiation of $s(h)$ permits to determine the first derivative of the disjoining pressure, $\frac{ds(h)}{dh} = h \frac{d\Pi_d}{dh}$. This information, supplemented with the boundary condition $\Pi_d(h \rightarrow \infty) = 0$, allows to completely determine the disjoining pressure of the film [6].

-
- [1] S. Bastea & J. L. Lebowitz, *Phys. Rev. Lett.* **78** (1997)
 - [2] D. Seul & D. Andelman, *Science* **267**, 476 (1995)
 - [3] R. Benzi *et al.*, *J. of Chem. Phys.* **131**, 104903 (2009)
 - [4] R. Benzi *et al.*, *Europhys. Lett.* **91**, 14003 (2010)
 - [5] R. Benzi *et al.*, *Europhys. Lett.* **104**, 48006 (2013)
 - [6] M. Sbragaglia *et al.*, *Soft Matter* **8** (41), 10773-10782 (2012)
 - [7] X. Shan & H. Chen, *Phys. Rev. E* **47**, 1815 (1993)
 - [8] M. Sbragaglia & X. Shan, *Phys. Rev.* **E84**, 036707 (2011)
 - [9] V. Bergeron, *J. Phys.: Condens. Matter* **11**, R215 (1999)
 - [10] B. V. Derjaguin, *Theory of Stability of Colloids and Thin Films* (Consultants Bureau, New York, 1989)
 - [11] B. V. Toshev, *Current Opinion in Colloid & Interface Science* **13**, 100-106 (2008)
 - [12] X. Shan, *Phys. Rev.* **E77**, 066702 (2008)
 - [13] M. Sbragaglia & D. Belardinelli, *Phys. Rev. E* **88**, 013306 (2013).