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Beta Working Paper series 456

BETA publicatie	WP 456 (working paper)
ISBN	
ISSN	
NUR	804
Eindhoven	June 2014

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Abstract

We consider a maintenance shop that is responsible for the availability of a fleet of assets, e.g., trains. Unavailability of assets may be due to active maintenance time or unavailability of spare parts. Both spare assets and spare components may be stocked in order to ensure a certain percentage of fleet readiness (e.g., 95%), i.e., having sufficient assets available for the primary process (e.g., running a train schedule). This is different from guaranteeing a certain average availability, as is typically done in the literature on spare parts inventories. We analyse the corresponding system, assuming continuous review and base stock control. We propose an algorithm, based on a marginal analysis approach, to solve the optimization problem of minimizing holding costs for spare assets and spare parts. Since the problem is not item separable, even marginal analysis is time consuming, but we show how to efficiently solve this. Using a numerical experiment, we show that our algorithm generally leads to a solution that is close to optimal, and we show that our algorithm is much faster than an existing algorithm for a closely related problem.

Keywords: Maintenance · Inventory · Fleet sizing

1 Introduction

Many important services and (military) operations depend on the availability of a sufficiently large fleet of assets. An airline, for example, depends on a fleet of aircraft to service all planned flights, while railway companies depend on a fleet of rolling-stock to make the train schedule work. Other examples exist in the defense and maritime industries. In all such cases, the availability of assets (the fraction of time that they are available to operate) is not the most appropriate measure of fleet performance. A more accurate measure of performance is the fraction of time that sufficient assets are available to fulfill the function of the fleet, i.e., the probability that sufficient assets are available at an arbitrary moment in time. We refer to this performance measure as fleet readiness.

A fleet readiness of 100% cannot be achieved, because assets are subject to failures and need maintenance. The maintenance time of an asset consists of two main parts: the *active maintenance time* in which the actual maintenance operations occurs (usually the replacement of line replaceable units) and the *maintenance delay time* which is the waiting time for maintenance resources to become available. (Some authors call it time to support.) A major culprit for maintenance delay is a lack of spare parts needed for replacement.

High fleet readiness can be achieved by a combination of the following: (1) Buying assets in addition to what is necessary to run daily operations; (2) Reducing the maintenance delay time by stocking spare parts; (3) Reducing the required number of maintenance actions by increasing asset reliability; Or (4) improving the speed of maintenance/replacement operations. This paper focusses on the first two options as these amount to investment decisions of a logistical nature. The last two options can usually only be achieved by making asset engineering modifications that are specific to the technology of the asset.

Buying as many assets as a given budget allows is a popular method to increase fleet readiness but it is not always effective. The money needed to buy assets and spare parts usually comes from the same budget. In the last decades of the previous century, the Dutch defense engaged in what has come to be called "carcass politics"[1]. Under carcass politics, the available budget to establish a fleet is spent as much as possible on buying complete assets, and the remainder is spent on spare parts. Spare parts become short in supply soon after this and as a result, technicians start using parts from complete assets leaving only a "carcass" behind. This practice is often referred to as cannibalization. Clearly, this practice does not necessarily lead to high fleet readiness. There is a trade-off in investing in assets and spare parts to meet a certain fleet readiness and this paper explores this trade-off.

The trade-off between investing in assets or spare parts to realize a certain fleet readiness objective is non-trivial. In general, this problem is non-convex and the analysis cannot be separated into an analysis per spare part type and asset: Its evaluation requires the convolution of backorder distributions per spare part type. This is in stark contrast with many spare part inventory problems in which the resulting optimization problems are convex and separable per item (see, e.g., Sherbrooke, 2004; Muckstadt, 2005; Basten and Van Houtum, 2014). Assets and spare parts achieve a certain fleet readiness jointly and so their analysis cannot be separated. In fact, we will show that their joint analysis is mathematically a generalization of multi-echelon inventory theory, even though we consider only a single stock point. Unfortunately, this generalization is not susceptible to standard tools such as Clark and Scarf decomposition (Clark and Scarf, 1960) and METRIC type (Sherbrooke, 1968) inventory models.

Our main contributions in this paper are the following: We consider the problem of deciding on asset investment and spare part investment jointly, whereas previous work consider them separately; see also Section 2. This is also what we often see in practice. However, both are sizeable investments that serve a common purpose in the end: achieving high fleet readiness. Fleet readiness is usually not used as service measure in this setting because it is untractable. Indeed, we show that this problem of deciding *jointly* on asset and spare part investment to meet a fleet readiness requirement is in general non-convex and non-separable, and enumeration is required to guarantee finding the optimal solution. Enumeration is impractical for several reasons. One of those reasons is also a problem for heuristic algorithms: Evaluating the fleet readiness for a given investment requires the computation of $O(n)$ convolutions, where n is the number of different spare part types. We develop a greedy heuristic for this problem that is

[1] The Dutch word is "rompenpolitiek", see, e.g., Tjepkema (2010)

computationally efficient, not only because it is greedy, but especially because it involves a novel technique that reduces the number of convolutions required to compute the readiness in any iteration. Our technique reduces the number of convolutions that need to be computed from $O(n)$ to $O(\log n)$ after an initial evaluation that still takes $O(n)$ convolutions. Furthermore, we provide simple bounds that our heuristic uses to decrease the size of the search neighborhood. In a numerical experiment, we compare our heuristic with enumeration on small instances and find that our heuristic finds the optimal solution of 51% of our test instances and has an average optimality gap on the other instances of 3.7%. Our algorithm is 50 times faster on medium size instances than an existing algorithm that was developed for a related problem. (The existing algorithm takes too much time to perform a comparison on large instances.)

The remainder of this paper is organized as follows. We discuss related literature in Section 2 and position our work with respect to previous work. In Section 3, we explain the system that we model and the optimization problem that we focus on. We analyse the system in Section 4; we show that the problem is not convex, but we can prove some other properties. We use those to construct an algorithm to solve the optimization problem in Section 5. In Section 6 we perform a numerical experiment, and we conclude in Section 7.

2 Related literature

We indicated that our main contributions are the combination of the fleet sizing and spare part investment decisions subject to a service level constraint that is not often used. Accordingly, this literature review is structured as follows: We discuss the fleet readiness measure in Section 2.1, fleet sizing in Section 2.2 and spare parts optimization in Section 2.3. In Section 2.3, we specifically focus on a closely related paper by De Smidt-Destombes et al. (2011).

2.1 Fleet readiness

Fleet readiness as a performance measure is not as common as availability. Some authors however, already noted that in many instances the readiness is a more appropriate performance measure. Safaei et al. (2011), for instance, consider a deterministic maintenance scheduling problem subject to a manpower constraint and a fleet readiness constraint. Jin and Wang (2012) use the fleet readiness measure in the context of performance based contracting. They approximate this measure by using the availability as the probability that a vehicle is available at an arbitrary moment in time and then use the binomial distribution to compute the fleet readiness. This approximation is more tractable than actual fleet readiness but it assumes that the availability of different vehicles is uncorrelated at any particular time point. A similar approach has been followed by Costantina et al. (2013) in a multi-echelon, multi-indenture spare parts inventory setting. Some authors use the term fleet readiness as the average number of vehicles of a fleet that are available, e.g., Sherbrooke (1971) and Salman et al. (2007). That is, these authors consider the availability times the size of the fleet rather than the fleet readiness as we define it.

A closely related concept from the reliability engineering literature is the availability of a k-out-of-N system (e.g., De Smidt-Destombes et al., 2004). In this setting, a system consists of N components and only functions if k out of those N components are operational. The availability is then defined as the probability that k out of the N components are operational. In our setting, we would say that a fleet is ready if at least k out of N assets are operational, or alternatively, if not more that $N-k$ assets are unavailable. Thus these measures are equivalent.

2.2 Fleet sizing

Fleet sizing for vehicles has been studied in different settings. Hoff et al. (2010) and Pantuso et al. (2014) provide a review of these models in the general and maritime setting, respectively. Most of these models are deterministic and are concerned with calculating the minimum fleet size necessary to perform daily operations. Our model takes this minimum number of vehicles needed as an input and supports the investment decision in additional vehicles (or other assets) and spare parts to make sure that the fleet is operationally ready with a certain probability at any moment in time. Hoff et al. (2010) already mention that dealing with uncertainty is an important aspect to incorporate when making the fleet sizing decision. Our work partially fills this gap by providing a model that deals with the uncertainty in the number of vehicles down for maintenance or lack of a spare part.

2.3 Spare parts optimization

The optimization of spare part inventory decisions has a long history that started with the work of Feeney and Sherbrooke (1966) and Sherbrooke (1968). This line of research has led to a large stream of literature that has been consolidated in the books of Sherbrooke (2004), Muckstadt (2005) and the review papers by Kennedy et al. (2002), Guide Jr. and Srivastava (1997), and Basten and Van Houtum (2014). We already mentioned some work that includes the optimization of spare part inventories in Section 2.1. Here, we focus on the most closely related work that has been done by De Smidt-Destombes et al. (2011). In that paper, the authors consider a fleet that is taken on a mission with a package of spare parts. The objective is to minimize the investment in this spare parts package subject to a constraint on the probability that the fleet remains ready throughout the mission. We extend their model in two ways: (1) We also consider the size of the fleet as a decision variable and (2) we account for the fact that maintenance itself requires time and renders an asset unavailable. We will show that their constraint on the probability of readiness at the end of a mission is mathematically equivalent to fleet readiness as used in this paper. We also show that, even for a fixed fleet size, optimizing the spare parts package is not a separable and convex problem. Despite this, De Smidt-Destombes et al. (2011) use a marginal analysis approach and we pursue a similar approach. As a new contribution, we benchmark this approach with respect to the optimal solution found by enumeration. We find that our algorithm yields high quality solutions. Furthermore, we provide results that make algorithms based on marginal analysis more tractable by giving easy to compute bounds so that gradients do not need to be computed for every direction of ascent. In addition, we provide an

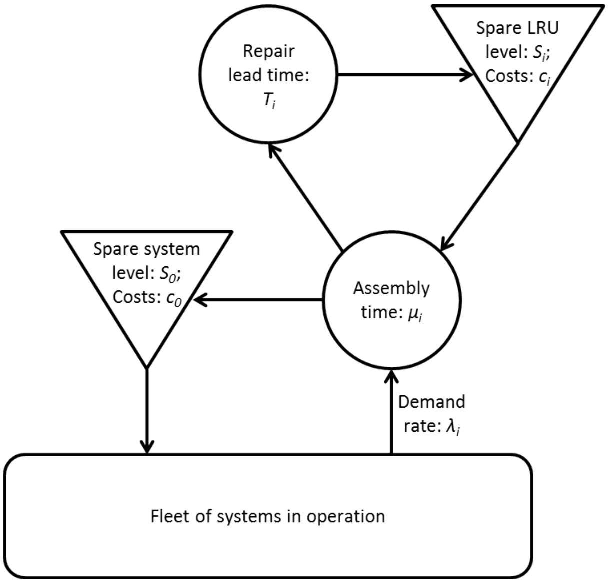

Figure 1: Modeled system

algorithm that computes the gradient in $O(\log n)$ time instead of $O(n)$ time, with n being the number of distinct spare part types.

3 Model description

The system that we analyze is shown in Figure 1. We consider a fleet of assets that are composed of line replaceable units (LRUs). We let I denote the set of LRUs. Assets fail randomly due to a failure in exactly one LRU $i \in I$; such failures occur according to a Poisson process with intensity λ_i and the total intensity over all LRUs is denoted by $\lambda_0 = \sum_{i \in I} \lambda_i$. We reserve the index 0 for the assets and we denote the set of LRUs plus assets by $I_0 = I \cup \{0\}$.

An asset is repaired by replacement of the failed part by a functioning spare part. In the remainder of this paper, if we refer to (spare) parts, components, or items of LRU type i, we say parts of LRU i. We assume that disassembly of the failed part takes zero time (i.e., is instantaneous); assembly of the functioning spare part into the asset takes exactly μ_i time units for LRU $i \in I$ if a spare part is available immediately from stock (i.e., μ_i is deterministic). After being repaired, the asset is sent to the pool of stand-by assets. We also refer to this pool as the stock of spare assets.

The failed part of LRU $i \in I$ is sent to the repair shop; its repair lead time is generally distributed with mean T_i time units. Repair times of parts of the same LRU are independent and identically distributed (i.i.d.) and repair times of parts of different LRUs are independent of each other. In other words, we assume that the repair shop has an infinite number of servers, or that the repair shop is able to schedule repairs and hire capacity such that it can guarantee a certain average repair time (we have made an analogous assumption for the maintenance shop). After being repaired, a part is returned to stock. Repairs may be performed either at an internal repair shop, or they may be outsourced to an external repair shop. In fact, the model can also be used if parts are discarded and replaced by new parts. In that case, repair lead time should be read as supply lead time or order-and-ship time.

All stock points are controlled using a continuous review $(S_i - 1, S_i)$ base stock policy (i.e., one-for-one replenishment) with S_i being the base stock level for the asset (0) or LRU $i \in I$. Under such a policy, the dynamics of the system can be described as follows: Let $D_i(t', t)$ denote the demand for LRU i (or, equivalently, the number of failures in parts of LRU i) between time t' and t. Let $X_i(t)$ denote the number of parts of LRU $i \in I$ in repair, also called the pipeline of LRU i, at time t. Then, if the repair lead time is deterministic, it is easily seen that $X_i(t) = D_i(t - T_i, t)$. Due to Palm's theorem (Palm, 1938), this equality still holds in distribution if the repair lead time is not deterministic. The number of backorders for LRU $i \in I$ is denoted by $B_i(t, S_i)$ and satsifies $B_i(t, S_i) = [X_i(t) - S_i]^+$. We denote by $Y_0(t)$ the number of assets in the maintenance shop that are actively being maintained at time t (i.e., the assets that are waiting for a spare part are not included): $Y_0(t) = \sum_{i \in I} D_i(t - \mu_i, t)$. (Remember that the active maintenance time, i.e., assembly time, is assumed to be deterministic). For notational convenience, we introduce $\mathbf{S}$ as the vector of all base stock levels S_i for $i \in I$. The pipeline $X_0(t, \mathbf{S})$ of assets in the maintenance shop at time t is:

$$X_0(t, \mathbf{S}) = Y_0(t) + \sum_{i \in I} B_i(t - \mu_i, S_i) = \sum_{i \in I} D_i(t - \mu_i, t) + \sum_{i \in I} \left[D_i(t - \mu_i - T_i, t - \mu_i) - S_i\right]^+ ,$$

while the number of assets short is denoted by $B_0(t, \mathbf{S_0}) = [X_0(t, \mathbf{S}) - S_0]^+$, with $\mathbf{S_0}$ being the vector of all base stock levels S_i for $i \in I_0$. (This can also be interpreted as the number of backordered assets.) The readiness, $R(\mathbf{S_0})$, is equal to the probability of not being any assets short in steady state: $R(\mathbf{S_0}) = \lim_{t \to \infty} \mathbb{P}\{B_0(t, \mathbf{S_0}) = 0\}$.

Remark 3.1. If the asset consists of one LRU only, our system simplifies to a two-echelon serial inventory system. Specifically, when $|I| = 1$, $Y_0(t)$ can be interpreted as the number of orders in transit from the upstream stock point to the downstream stock point, while $B_1(t - \mu_1, S_1)$ represents the orders from the downstream stock point that are backordered at the upstream stock point. By allowing $|I| > 1$, we are dealing with a generalization of a two-echelon serial inventory system under base-stock control.

Remark 3.2. When $\mu_i = 0$ and $T_i = T$ for all $i \in I$, then $R(\mathbf{S_0})$ can also be interpreted as the probability that the fleet remains ready during a mission of length T when a spare parts package of size $\mathbf{S}$ is brought on the mission. (Note that $T_i = T$ for all $i \in I$ implies that spare parts cannot be repaired during the mission.) For a fixed fleet size S_0, this is the setting that De Smidt-Destombes et al. (2004) consider.

The costs of holding spare assets and LRUs are linear in their base stock level: c_i per unit for asset or LRU $i \in I_0$. Our goal is to find the base stock levels that minimize the total costs $C(\mathbf{S_0}) = \sum_{i \in I_0} c_i S_i$, such that the target readiness R^{obj} is achieved. Formally, our optimization problem, Problem (P), is:

$$\begin{aligned} \min_{\mathbf{S_0} \in \mathbb{N}_0^{|I_0|}} \quad & C(\mathbf{S_0}) \\ \text{subject to} \quad & R(\mathbf{S_0}) \geq R^{\text{obj}}, \end{aligned}$$

with $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ being the set of non-negative integers. We emphasize that problem (P) is not separable per item because $R(\mathbf{S_0})$ cannot be written as a sum of terms that depend on one S_i only.

4 Analysis

In this section, we give results on the behavior of the fleet readiness as a function of the number of spare parts and spare assets. We use these results to explain, in Section 5, why we make certain choices in the algorithm that we use to solve Problem (P). Since we consider the system in steady state, we suppress the time parameter in the state variables from now on, and we show their distributions in Lemma 1. We further introduce additional notation that we require in the remainder of this section. We then get to the main part of this section: We give a counter example that shows that the fleet readiness is not in general jointly concave in S_0 and S_i. Therefore, we enumerate the asset stock levels in our algorithm and Proposition 2 gives bounds on the optimal asset stock levels. We next show that the fleet readiness is also not jointly concave in S_i and S_j for $i, j \in I$ with $i \neq j$. However, we are able to give some other convexity results: Lemma 3 states two difference functions and we use the second order difference function in Proposition 4, which gives convexity results for the fleet readiness as a function of S_i $(i \in I)$. Finally, Proposition 5 gives a result that we use in our algorithm to avoid performing unncessary calculations: Our algorithm uses a marginal analysis approach. In each iteration, an additional spare part is stocked of that LRU that gives the biggest 'bang for the buck'. The result in Proposition 5 gives an upper bound on how much this 'bang for the buck' may have changed for a certain LRU from one iteration to the next. That means that we can use the result to quickly check if the 'bang for the buck' of a certain LRU may be sufficiently high to perform exact calculations. If not, we do not need to perform these time consuming calculations.

The following lemma gives the distributions of the state variables in steady state. The proof follows directly from the discussion in Section 3 and is therefore omitted.

Lemma 1. *In steady state, the state variables are distributed as follows:*

(i) For $i \in I$, the pipeline, X_i, is Poisson distributed with mean $\lambda_i T_i$, i.e.:

$$\mathbb{P}\{X_i = x\} = \frac{(\lambda_i T_i)^x}{x!} e^{-\lambda_i T_i}, \forall x \in \mathbb{N}_0.$$

(ii) For $i \in I$, the distribution of the number of backorders, $B_i(S_i)$, is given by:

$$\mathbb{P}\left\{B_i(S_i) = b\right\} = \begin{cases} \sum_{x=0}^{S_i} \mathbb{P}\left\{X_i = x\right\} & , \text{ if } b = 0; \\ \mathbb{P}\left\{X_i = S_i + b\right\} & , \text{ if } b \in \mathbb{N}. \end{cases}$$

(iii) The number of assets in active maintenance, Y_0, is Poisson distributed with mean $\sum_{i \in I} \lambda_i \mu_i$, i.e.:

$$\mathbb{P}\left\{Y_0 = y\right\} = \frac{\left(\sum_{i \in I} \lambda_i \mu_i\right)^y}{y!} e^{-\sum_{i \in I} \lambda_i \mu_i}, \forall y \in \mathbb{N}_0.$$

(iv) The distribution of the pipeline, $X_0(\mathbf{S})$, is given by:

$$\mathbb{P}\left\{X_0(\mathbf{S}) = x\right\} = \sum_{y=0}^{x} \left[\mathbb{P}\left\{Y_0 = y\right\} \mathbb{P}\left\{\sum_{i \in I} B_i(S_i) = x - y\right\}\right], \forall x \in \mathbb{N}_0.$$

(v) The distribution of the number of assets short, $B_0(\mathbf{S_0})$, is given by:

$$\mathbb{P}\left\{B_0(\mathbf{S_0}) = b\right\} = \begin{cases} \sum_{x=0}^{S_0} \mathbb{P}\left\{X_0(\mathbf{S}) = x\right\} & , \text{ if } b = 0; \\ \mathbb{P}\left\{X_0(\mathbf{S}) = S_0 + b\right\} & , \text{ if } b \in \mathbb{N}. \end{cases}$$

We use additional notation in this section: Let $\mathbf{e}_i$ be a vector of length $|I_0|$ with all zeros, except at the location corresponding to the base stock level of spare assets ($i = 0$: S_0) or spare LRUs ($i \in I$: S_i). Furthermore, notice that concavity in $i \in I_0$ implies that $R(\mathbf{S_0}+\mathbf{e}_i)-R(\mathbf{S_0}) \geq R(\mathbf{S_0}+2\mathbf{e}_i)-R(\mathbf{S_0}+\mathbf{e}_i)$, while joint concavity in $i, j \in I_0$ implies that $R(\mathbf{S_0}+\mathbf{e}_j)-R(\mathbf{S_0}) \geq R(\mathbf{S_0}+\mathbf{e}_i+\mathbf{e}_j)-R(\mathbf{S_0}+\mathbf{e}_i)$.

The problem that we consider is not in general jointly concave in S_0 and S_i with $i \in I$. As a counter example, consider an asset consisting of one LRU, indexed 1, with $\lambda_1 = 2$ and $\mu_1 = T_1 = 1$. Evaluating $R(\mathbf{S_0})$ gives the following results: $R(0,0) \approx 0.1353$, $R(1,0) \approx 0.4061$, $R(0,1) \approx 0.2707$, and $R(1,1) \approx 0.6090$. It is easily seen that if either S_0 or S_1 is increased, the readiness increases. However, $R(0,1)-R(0,0) \approx 0.1353 < R(1,1)-R(1,0) \approx 0.2030$, and $R(1,0)-R(0,0) \approx 0.2707 < R(1,1)-R(0,1) \approx 0.3383$. This means that the problem is not jointly concave. For larger values of S_0, the problem does show concavity.

Because there exists no joint concavity in general, we are going to enumerate the number of spare assets; Proposition 2 gives bounds on its optimal value.

Proposition 2. *The optimal number of spare assets for problem P, denoted as S_0^*, is bounded as follows:*

(i) $S_0^* \geq S_0^{LB}$, with S_0^{LB} being the smallest integer S that satisfies $\mathbb{P}\left\{Y_0 \leq S\right\} \geq R^{obj}$.

(ii) $S_0^* \leq S_0^{UB}$, with S_0^{UB} being the smallest integer S for which it holds that there exists $\mathbf{S_0}' = \left(S_0', S_1', \ldots, S_{|I|}'\right)$ with $S_0^{LB} \leq S_0' \leq S$, $C(\mathbf{S_0}') < c_0(S+1)$ and $R(\mathbf{S_0}') \geq R^{obj}$.

Proof. For part (i): Since $X_0(\mathbf{S}) \overset{d}{=} Y_0 + \sum_{i\in I} B_i(S_i)$ by definition ($\overset{d}{=}$ denotes equality in distribution), we have that $\mathbb{P}\{X_0(\mathbf{S}) \geq x\} \geq \mathbb{P}\{Y_0 \geq x\}$ for all $x \in \mathbb{N}_0$ because $B_i(S_i)$ are non-negative random variables. The readiness constraint in Problem (P) requires $\mathbb{P}\{X_0(\mathbf{S}) \leq S_0\} \geq R^{\text{obj}}$, so a feasible S_0 must satisfy $\mathbb{P}\{Y_0 \leq S_0\} \geq R^{\text{obj}}$.

For part (ii): $\mathbf{S_0}'$ represents a feasible solution, since $R(\mathbf{S_0}') \geq R^{\text{obj}}$. If the costs of that feasible solution are lower than the costs of storing $S+1$ spare assets (without any spare LRUs), and thus also of storing more than $S+1$ spare assets, then S is an upper bound on S_0^*. □

Problem P is not in general jointly concave in S_i and S_j with $i, j \in I$ and $i \neq j$. As a counter example, consider an asset consisting of two LRUs, indexed 1 and 2, with $S_0 = \mu_1 = \mu_2 = 0$. Joint concavity would imply that $R(0, \mathbf{S} + \mathbf{e}_1) - R(0, \mathbf{S}) \geq R(0, \mathbf{S} + \mathbf{e}_1 + \mathbf{e}_2) - R(0, \mathbf{S} + \mathbf{e}_2)$ and thus that

$$
\begin{aligned}
&(1 - \mathbb{P}\{X_1 \leq S_1 + 1\}\mathbb{P}\{X_2 \leq S_2\}) - (1 - \mathbb{P}\{X_1 \leq S_1\}\mathbb{P}\{X_2 \leq S_2\}) \\
&\qquad \geq (1 - \mathbb{P}\{X_1 \leq S_1 + 1\}\mathbb{P}\{X_2 \leq S_2 + 1\}) - (1 - \mathbb{P}\{X_1 \leq S_1\}\mathbb{P}\{X_2 \leq S_2 + 1\}), \\
&\mathbb{P}\{X_1 \leq S_1\}\mathbb{P}\{X_2 \leq S_2\} - \mathbb{P}\{X_1 \leq S_1 + 1\}\mathbb{P}\{X_2 \leq S_2\} \\
&\qquad \geq \mathbb{P}\{X_1 \leq S_1\}\mathbb{P}\{X_2 \leq S_2 + 1\} - \mathbb{P}\{X_1 \leq S_1 + 1\}\mathbb{P}\{X_2 \leq S_2 + 1\}, \\
&(\mathbb{P}\{X_1 \leq S_1\} - \mathbb{P}\{X_1 \leq S_1 + 1\})\mathbb{P}\{X_2 \leq S_2\} \geq (\mathbb{P}\{X_1 \leq S_1\} - \mathbb{P}\{X_1 \leq S_1 + 1\})\mathbb{P}\{X_2 \leq S_2 + 1\}, \text{and} \\
&\mathbb{P}\{X_2 \leq S_2\} \geq \mathbb{P}\{X_2 \leq S_2 + 1\}.
\end{aligned}
$$

However, $\mathbb{P}\{X_2 = S_2 + 1\} > 0$ for all $S_2 \geq 0$, so that $\mathbb{P}\{X_2 \leq S_2 + 1\} > \mathbb{P}\{X_2 \leq S_2\}$, showing that this problem is not jointly concave.

In the two counter examples above, we have already used difference functions. Lemma 3 states two general difference functions formally. The second function, in part (ii), is used in Proposition 4, which gives convexity results.

Lemma 3. *The difference functions of the fleet readiness behave as follows.*

(i) The difference function for the number of spare assets is:

$$\Delta_0 R(\mathbf{S_0}) = R(\mathbf{S_0} + \mathbf{e}_0) - R(\mathbf{S_0}) = \mathbb{P}\left\{Y_0 + \sum_{i\in I} B_i = S_0 + 1\right\}.$$

(ii) The difference function for the number of spare LRUs $i \in I$ is:

$$\Delta_i R(\mathbf{S_0}) = R(\mathbf{S_0} + \mathbf{e}_i) - R(\mathbf{S_0}) = \sum_{b=1}^{S_0+1} \mathbb{P}\{X_i = S_i + b\}\,\mathbb{P}\left\{Y_0 + \sum_{k\in I\setminus\{i\}} B_k = S_0 + 1 - b\right\}.$$

Proof. For part (i), the derivation is as follows:

$$\begin{aligned}
\Delta_0 R(\mathbf{S_0}) &= R(\mathbf{S_0}+\mathbf{e}_0) - R(\mathbf{S_0}) \\
&= \left(1 - \mathbb{P}\left\{Y_0 + \sum_{i\in I} B_i > S_0 + 1\right\}\right) - \left(1 - \mathbb{P}\left\{Y_0 + \sum_{i\in I} B_i > S_0\right\}\right) \\
&= \mathbb{P}\left\{Y_0 + \sum_{i\in I} B_i > S_0\right\} - \mathbb{P}\left\{Y_0 + \sum_{i\in I} B_i > S_0 + 1\right\} \\
&= \mathbb{P}\left\{Y_0 + \sum_{i\in I} B_i = S_0 + 1\right\}.
\end{aligned}$$

For notational convenience, let $Z_i = Y_0 + \sum_{k\in I\setminus\{i\}} B_k$. Then, for part (ii), the derivation is as follows:

$$\begin{aligned}
\Delta_i R(\mathbf{S_0}) &= R(\mathbf{S_0}+\mathbf{e}_i) - R(\mathbf{S_0}) \\
&= \left(1 - \mathbb{P}\left\{Z_i + [X_i - S_i - 1]^+ > S_0\right\}\right) - \left(1 - \mathbb{P}\left\{Z_i + [X_i - S_i]^+ > S_0\right\}\right) \\
&= \mathbb{P}\left\{Z_i + [X_i - S_i]^+ > S_0\right\} - \mathbb{P}\left\{Z_i + [X_i - S_i - 1]^+ > S_0\right\} \\
&= \sum_{x=0}^{\infty} \mathbb{P}\left\{Z_i + [X_i - S_i]^+ > S_0 \mid X_i = x\right\} \mathbb{P}\{X_i = x\} \\
&\quad - \sum_{x=0}^{\infty} \mathbb{P}\left\{Z_i + [X_i - S_i - 1]^+ > S_0 \mid X_i = x\right\} \mathbb{P}\{X_i = x\} \\
&= \sum_{x=S_i+1}^{S_i+S_0+1} \mathbb{P}\left\{Z_i + [X_i - S_i]^+ > S_0 \mid X_i = x\right\} \mathbb{P}\{X_i = x\} \\
&\qquad - \sum_{x=S_i+1}^{S_i+S_0+1} \mathbb{P}\left\{Z_i + [X_i - S_i - 1]^+ > S_0 \mid X_i = x\right\} \mathbb{P}\{X_i = x\} \\
&= \sum_{x=S_i+1}^{S_0+S_i+1} \left[\mathbb{P}\{Z_i > S_0 + S_i - x\} - \mathbb{P}\{Z_i > S_0 + S_i + 1 - x\}\right] \mathbb{P}\{X_i = x\} \\
&= \sum_{x=S_i+1}^{S_0+S_i+1} \mathbb{P}\{Z_i = S_0 + S_i + 1 - x\} \mathbb{P}\{X_i = x\} \\
&= \sum_{b=1}^{S_0+1} \mathbb{P}\{X_i = S_i + b\} \mathbb{P}\{Z_i = S_0 + 1 - b\}.
\end{aligned}$$

The fifth equation holds because if $X_i < S_i + 1$, then $[X_i - S_i]^+ = [X_i - S_i - 1]^+ = 0$, and if $X_i > S_i + S_0 + 1$, then $\mathbb{P}\{Z_i + [X_i - S_i]^+ > S_0\} = \mathbb{P}\{Z_i + [X_i - S_i - 1]^+ > S_0\} = 1$. □

Proposition 4. *The second order difference function for the number of spare LRUs $i \in I$, $\Delta_i^2 R(\mathbf{S_0}) = \Delta_i R(\mathbf{S_0}+\mathbf{e}_i) - \Delta_i R(\mathbf{S_0})$, behaves as follows:*

(i) If $S_0 + S_i < \lceil \lambda_i T_i \rceil - 2$, then $\Delta_i^2 R(\mathbf{S_0}) > 0$ and $R(\mathbf{S_0})$ is strictly convex in S_i.

(ii) If $S_i \geq \lceil \lambda_i T_i \rceil - 2$, then $\Delta_i^2 R(\mathbf{S_0}) \leq 0$ and $R(\mathbf{S_0})$ is concave in S_i.

Proof. For both parts (i) and (ii), we first require:

$$\begin{aligned}
\Delta_i^2 R(\mathbf{S_0}) &= \Delta_i R(\mathbf{S_0} + \mathbf{e}_i) - \Delta_i R(\mathbf{S_0}) \\
&= \sum_{b=1}^{S_0+1} \mathbb{P}\left\{X_i = S_i + 1 + b\right\} \mathbb{P}\left\{Y_0 + \sum_{k \in I \setminus \{i\}} B_k = S_0 + 1 - b\right\} \\
&\quad - \sum_{b=1}^{S_0+1} \mathbb{P}\left\{X_i = S_i + b\right\} \mathbb{P}\left\{Y_0 + \sum_{k \in I \setminus \{i\}} B_k = S_0 + 1 - b\right\} \\
&= \sum_{b=1}^{S_0+1} \left[\mathbb{P}\left\{X_i = S_i + 1 + b\right\} - \mathbb{P}\left\{X_i = S_i + b\right\}\right] \mathbb{P}\left\{Y_0 + \sum_{k \in I \setminus \{i\}} B_k = S_0 + 1 - b\right\}.
\end{aligned}$$

The second equality follows from Lemma 3. Furthermore, by Lemma 1, X_i is a Poisson distributed random variable with mean $\lambda_i T_i$ so that we may express its probability mass function recursively as $\mathbb{P}\{X_i = k\} = \frac{\lambda_i T_i}{k} \mathbb{P}\{X_i = k-1\}$, for $k > 0$. Consider part (i): if $S_0 + S_i < \lceil \lambda_i T_i \rceil - 2$, then $\mathbb{P}\{X_i = S_i + 1 + b\} > \mathbb{P}\{X_i = S_i + b\}$ for $b \in \{1, \ldots, S_0 + 1\}$, so that $\Delta_i^2 R(\mathbf{S_0}) > 0$. And part (ii): if $S_i \geq \lceil \lambda_i T_i \rceil - 2$, then $\mathbb{P}\{X_i = S_i + 1 + b\} \leq \mathbb{P}\{X_i = S_i + b\}$ for $b \in \{1, \ldots, S_0 + 1\}$, so that $\Delta_i^2 R(\mathbf{S_0}) \leq 0$. $\square$

Additionally, notice that:

- A similar result as part (ii) has been shown by Rustenburg (2000, p.41).
- Equality in part (ii) of Proposition 4 occurs if and only if $S_0 = 0$ and $S_i = \lambda_i T_i - 2$.
- The behavior of $\Delta_i^2 R(\mathbf{S_0})$ is not clear beforehand in all cases that are not covered by Proposition 4 (i.e., if $S_i < \lceil \lambda_i T_i \rceil - 2$ and $S_0 + S_i \geq \lceil \lambda_i T_i \rceil - 2$).

Proposition 5 gives a result that we use in our algorithm to avoid performing unncessary calculations, as explained above. Since the proof is long and does not give insight into the problem, it is deferred to A

Proposition 5. *If $S_i \geq \lceil \lambda_i T_i \rceil - 2$ and $S_j \geq \lceil \lambda_j T_j \rceil - 2$, with $i, j \in I$, then:*

$$\Delta_i R(\mathbf{S_0} + \mathbf{e}_j) - \Delta_i R(\mathbf{S_0}) < \mathbb{P}\{X_j = S_j + 1\} \mathbb{P}\{X_i = S_i + 1\}.$$

5 Algorithm

We give the pseudo code of our algorithm in Figure 2 and we explain the complete algorithm in Section 5.1. Next, we focus on how to compute the convolutions in Line 11 of our algorithm in Section 5.2. This is a very time consuming step in the algorithm and we propose a novel way to do this efficiently.

1: $S_0 \leftarrow \min\{S \in \mathbb{N}_0 \mid \mathbb{P}\{Y_0 \le S\} \ge R^{\text{obj}}\}$
2: Calculate the probability mass function of Y_0
3: **while** $c_0 S_0 \le C^{\text{best}}$ **do**
4: $S_i \leftarrow \max\{0, \lceil \lambda_i T_i \rceil - 2\}$ for all $i \in I$
5: Calculate the probability mass functions of B_i for all $i \in I$, and of $Y_0 + \sum_{i \in I} B_i$
6: $R^{\text{cur}} \leftarrow R(\mathbf{S_0})$; $\Gamma^{\text{best}} \leftarrow 0$
7: $i^{\text{best}} \leftarrow -1$; $\mathbb{P}\{X_{-1} = S_{-1}\} \leftarrow 1$; $\Gamma_i \leftarrow 1/c_i$ for all $i \in I$
8: **while** $R^{\text{cur}} < R^{\text{obj}}$ **do**
9: **for** $i \in I$ **do**
10: $\Gamma_i \leftarrow \Gamma_i + \frac{\mathbb{P}\{X_{i^{\text{best}}} = S_{i^{\text{best}}}\}\mathbb{P}\{X_i = S_i + 1\}}{c_i}$
11: **if** $\Gamma_i \ge \Gamma^{\text{best}}$ or $i = i^{\text{best}}$ **then**
12: $\Gamma_i \leftarrow \frac{R(\mathbf{S_0} + \mathbf{e}_i) - R^{\text{cur}}}{c_i}$
13: $\Gamma^{\text{best}} \leftarrow \max\{\Gamma_i, \Gamma^{\text{best}}\}$
14: **end if**
15: **end for**
16: $i^{\text{best}} \leftarrow \arg\max_{i \in I} \Gamma_i$; $S_{i^{\text{best}}} \leftarrow S_{i^{\text{best}}} + 1$; $R^{\text{cur}} \leftarrow R(\mathbf{S_0})$
17: **end while**
18: **if** $C(\mathbf{S_0}) < C^{\text{best}}$ **then**
19: $C^{\text{best}} \leftarrow C(\mathbf{S_0})$
20: **end if**
21: **end while**

Figure 2: Greedy algorithm for Problem (P)

5.1 Overview

The algorithm functions as follows. We enumerate the asset base stock level between a lower bound (Line 1) and an upper bound (Line 3), based on Proposition 2. For each asset base stock level, we initialize each LRU base stock level at a lower bound based on Proposition 4 (Line 4). Notice that this lower bounds guarantees that the readiness is convex in each LRU base stock level. It is not guaranteed that the optimal base stock level is above this lower bound. Although in practice it typically is, it is easy to give an example where it is not.

We then compute the probability mass functions of B_i for $i \in I$, and of $Y_0 + \sum_{i \in I} B_i$. We use a smart way of ordering and storing these computation in order to reduce the number of computations that we need to perform per iteration of the marginal analysis approach that we use to stock additional spare parts (Lines 12 and 16). We explain this in detail below.

Using the result in Proposition 5, we are able to further reduce the number of computations that we perform per iteration (Lines 10 and 11). In our numerical experiment (Table 3), we find that in this way, we save over 50% of computation time for problem instances with 256 components and that the relative savings increase with an increasinng problem size.

Note that Line 7 ensures that in the first iteration of the while loop (Lines 8 to 17) the first condition of the if-clause on Line 11 is always true. The second condition of that if-clause is required because Proposition 5 holds only for $i \neq j$.

As soon as the target readiness is reached, the marginal analysis approach is stopped, and the asset base stock level is increased if its upper bound has not been reached yet. The upper bound that we use is straightforward. Still, we find in our numerical experiment (Tables 3 and 4) that typically, the number of asset base stock levels that we consider in our algorithm is small, i.e., the difference between the lower bound and the upper bound on the asset base stock level is small.

5.2 Convolutions

The computationally most demanding step in Algorithm 2 is in Line 11: the computation of $\Gamma_i = (R(\mathbf{S}+\mathbf{e}_i) - R^{\text{cur}})/c_i$. The difficult computation here lies in the evaluation of $R(\mathbf{S}+\mathbf{e}_i) = \mathbb{P}\left\{B_0(\mathbf{S_0}) = 0\right\} = \mathbb{P}\left\{Y_0 + \sum_{i\in I} B_i(S_i) \leq S_0\right\}$ because it requires computing the probability mass function of $U(\mathbf{S}) := Y_0 + \sum_{i\in I} B_i(S_i)$ by convolution. In this sub-section, we provide an algorithm to compute the probability mass function of $U(\mathbf{S}+\mathbf{e}_i)$ using results that have already been computed for $U(\mathbf{S})$.

We require some additional notation. Let $\mathbf{B}_i(S_i)$ be a vector containing the probability mass function of $B_i(S_i) = (X_i - S_i)^+$ up to S_0, i.e., $\mathbf{B}_i(S_i) = (\mathbb{P}\left\{B_i(S_i) = 0\right\}, \mathbb{P}\left\{B_i(S_i) = 1\right\}, \ldots, \mathbb{P}\left\{B_i(S_i) = S_0\right\})$. Similarly, let $\mathbf{Y}_0 = (\mathbb{P}\left\{Y_0 = 0\right\}, \ldots, \mathbb{P}\left\{Y_0 = S_0\right\})$ and $\mathbf{U}(\mathbf{S}) = (\mathbb{P}\left\{U(\mathbf{S}) = 0\right\}, \ldots, \mathbb{P}\left\{U(\mathbf{S}) = S_0\right\})$. Furthermore we let $\mathbf{a} * \mathbf{b}$ denote the convolution of the vectors $\mathbf{a}$ and $\mathbf{b}$ of equal length. Specifically, if $\mathbf{c} = \mathbf{a} * \mathbf{b}$ then $\mathbf{c}$ has the same length as both $\mathbf{a}$ and $\mathbf{b}$ and the i-th element of $\mathbf{c}$ is given by $c_i = \sum_{j=0}^{i} a_{i-j} b_j$. (Note that we start numbering elements in a vector starting from 0 since this is notationally convenient in this context.) The convolution operator satisfies commutativity ($\mathbf{a} * \mathbf{b} = \mathbf{b} * \mathbf{a}$) and associativity ($(\mathbf{a} * \mathbf{b}) * \mathbf{c} = \mathbf{a} * (\mathbf{b} * \mathbf{c})$). Finally, we let $\mathbf{B}_{a,b}(\mathbf{S}) = \mathbf{B}_a(S_a) * \mathbf{B}_{a+1}(S_{a+1}) * \cdots * \mathbf{B}_{b-1}(S_{b-1}) * \mathbf{B}_b(S_b)$ for $a \leq b$. Now observe that

$$\mathbf{U}(\mathbf{S}) = \mathbf{Y}_0 * \mathbf{B}_{1,|I|}(\mathbf{S}) = \mathbf{Y}_0 * \mathbf{B}_1(S_1) * \mathbf{B}_2(S_2) * \cdots * \mathbf{B}_{|I|-1}(S_{|I|-1}) * \mathbf{B}_{|I|}(S_{|I|}),$$

which can be computed in a plethora of orders because of the associative and commutative properties of convolution. However, for our application, we already know that after computing $\mathbf{U}(\mathbf{S})$, we will also compute $\mathbf{U}(\mathbf{S}+\mathbf{e}_i)$ for some $i \in I$ in Line 11 of our greedy algorithm in order to evaluate $R(\mathbf{S}+\mathbf{e}_i)$. The complexity in computing $U(\mathbf{S})$ lies in the computation of $\mathbf{B}_{1,|I|}(\mathbf{S})$ because its complexity increases with the size of the instance as measured by $|I|$.

The straightforward way to compute $\mathbf{B}_{1,|I|}(\mathbf{S})$ is to first compute $\mathbf{B}_i(S_i)$ for all $i \in I$, and then successively compute as follows: $\mathbf{B}_{1,2}(\mathbf{S}) = \mathbf{B}_1(S_1) * \mathbf{B}_2(S_2)$, $\mathbf{B}_{1,3}(\mathbf{S}) = \mathbf{B}_{1,2}(\mathbf{S}) * \mathbf{B}_3(S_3), \ldots, \mathbf{B}_{1,|I|-1}(\mathbf{S}) = \mathbf{B}_{1,|I|-2}(\mathbf{S}) * \mathbf{B}_{|I|-1}(S_{|I|-1})$, $\mathbf{B}_{1,|I|}(\mathbf{S}) = \mathbf{B}_{1,|I|-1}(\mathbf{S}) * \mathbf{B}_{|I|}(S_{|I|})$. This requires performing $|I| - 1$ convolutions and this is what De Smidt-Destombes et al. (2011) do in their algorithm.

Alternatively, this can be done by building up a tree starting from its leaves. An example for $|I| = 8$ is shown in Figure 3. Formally the procedure works as follows: First, compute $\mathbf{B}_i(S_i)$ for all $i \in I$. Next, compute $\mathbf{B}_{1,2}(\mathbf{S}) = \mathbf{B}_1(S_1) * \mathbf{B}_2(S_2)$, $\mathbf{B}_{3,4}(\mathbf{S}) = \mathbf{B}_3(S_3) * \mathbf{B}_4(S_4), \ldots, \mathbf{B}_{|I|-1,|I|}(\mathbf{S}) = \mathbf{B}_{|I|-1}(S_{|I|-1}) * \mathbf{B}_{|I|}(S_{|I|})$. Then, compute $\mathbf{B}_{1,4}(\mathbf{S}) = \mathbf{B}_{1,2}(\mathbf{S}) * \mathbf{B}_{3,4}(\mathbf{S}), \ldots, \mathbf{B}_{|I-3|,|I|}(\mathbf{S}) = \mathbf{B}_{|I|-3,|I|-2}(\mathbf{S}) * \mathbf{B}_{|I|-1,|I|}(\mathbf{S})$. Continue in this manner until arriving at the root node of the tree: $\mathbf{B}_{1,|I|}(\mathbf{S})$. This procedure also requires $|I| - 1$ convolutions. However, computing $\mathbf{B}_{1,|I|}(\mathbf{S}+\mathbf{e}_i)$, for some $i \in I$, can now be done efficiently by reusing most

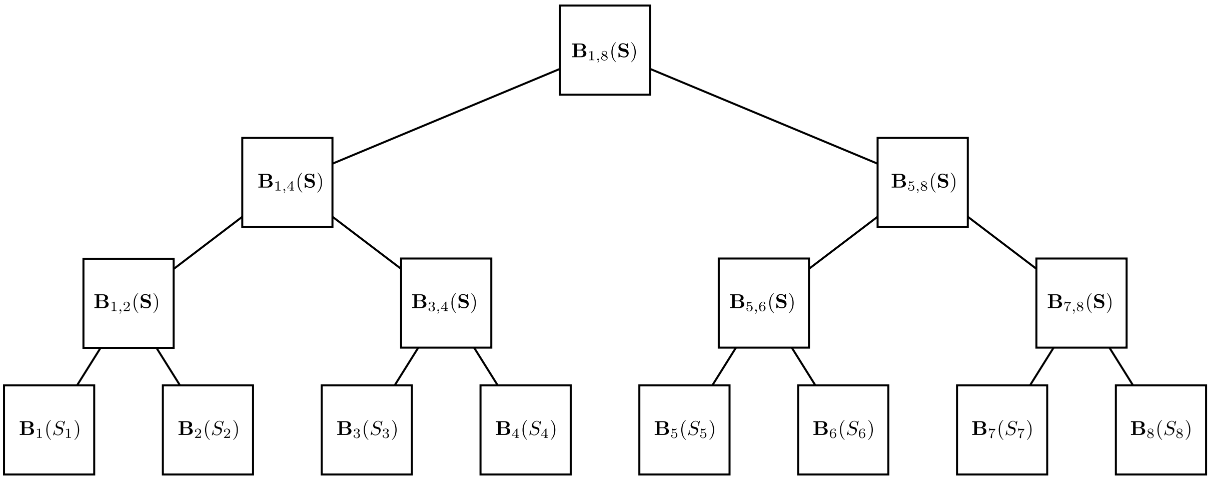

Figure 3: Computation of $\mathbf{B}_{1,|I|}(\mathbf{S})$ for $|I| = 8$ via a tree structure. Each non-leaf node in this tree is obtained by convolution of its two children nodes.

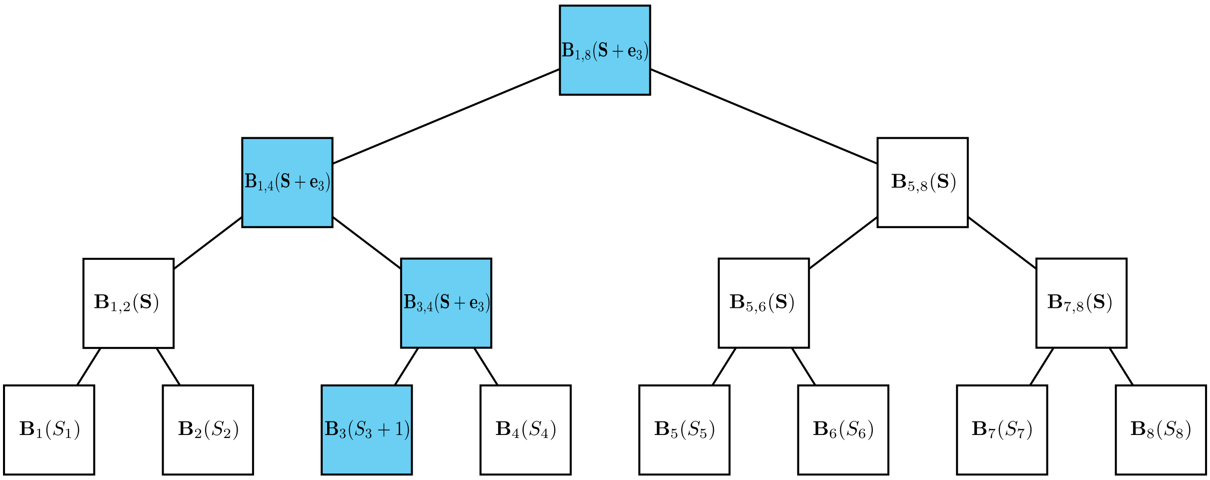

Figure 4: Computation of $\mathbf{B}_{1,|I|}(\mathbf{S} + \mathbf{e}_3)$ for $|I| = 8$ via a tree structure. This tree is identical to the tree for the computation of $\mathbf{B}_{1,|I|}(\mathbf{S})$ except in the filled nodes.

results in the tree. Indeed, $\mathbf{B}_{a,b}(\mathbf{S} + \mathbf{e}_i) = \mathbf{B}_{a,b}(\mathbf{S})$ whenever $i < a$ or $b < i$, so that all nodes in the tree for which this condition is verified do not need to be recomputed. This is easily seen when we reconsider the example where $|I| = 8$. Suppose we wish to compute $\mathbf{B}_{1,8}(\mathbf{S}+\mathbf{e}_3)$. Figure 4 shows this tree. Note that the trees for the computation of $\mathbf{B}_{1,8}(\mathbf{S} + \mathbf{e}_3)$ (Figure 4) and $\mathbf{B}_{1,8}(\mathbf{S})$ (Figure 3) are identical for all nodes, except the nodes that are filled in Figure 4. Therefore, if we already evaluated $\mathbf{B}_{1,8}(\mathbf{S})$, the computation of $\mathbf{B}_{1,8}(\mathbf{S} + \mathbf{e}_3)$ only requires the evaluation of 4 nodes. Of those 4 nodes, one concerns the determination of $\mathbf{B}_3(S_3 + 1)$ and $3 = \log_2(8)$ require taking a convolution. The same reasoning can be applied for general $|I|$ and yields the following result.

Proposition 6. *After an initial evaluation of* $\mathbf{B}_{1,|I|}(\mathbf{S})$ *which requires* $O(|I|)$ *convolutions, all subsequent evaluations of* $\mathbf{B}_{1,|I|}(\mathbf{S}+\mathbf{e}_i)$ *with* $i \in I$ *require performing only* $O(\log |I|)$ *convolutions.*

The only thing that we have not explained yet is when to perform the convolution with Y_0.

		Set 1	Set 2
# Components	$\|I\|$	2; 4; 8	16; 64; 256; 1,024
Maximum assembly time	$\mu^{\max}$	0.001; 0.01	identical
Maximum resupply lead time	$T^{\max}$	0.01; 0.1	identical
Average costs per component	c^{average}	100; 1,000	identical
Relative costs of an asset	c^{relative}	0.5; 1; 2	identical
Target readiness	R^{obj}	0.9; 0.95; 0.975	identical

Table 1: Settings of the parameters that are varied in the numerical experiment

It would be straightforward to do this at the end (i.e., at the root of the tree), but that would mean that we would require an additional convolution each time that we increase an LRU stock level. Therefore, we have chosen to perform this convolution in the beginning: We first calculate $\mathbf{B}_{1,1}(\mathbf{S}) = \mathbf{Y}_0 * \mathbf{B}_1(S_1)$, and use that to calculate $\mathbf{B}_{1,2}(\mathbf{S}) = \mathbf{B}_{1,1}(\mathbf{S}) * \mathbf{B}_2(S_2)$.

6 Numerical experiment

We generate two sets of problem instances. Set 1 consists of smaller problem instances and is used to compare the solution of our algorithm with the optimal solution, found by enumeration. Set 2 consists of larger problem instances and is used to compare the computation times of our algorithm with that of De Smidt-Destombes et al. (2011) and to see how changes in parameters, e.g., the number of components, influence the computation times. We explain how we generate both sets, and the differences between the sets, in Section 6.1. The results and our analysis of the results are shown in Section 6.2. Notice that the solution found by our algorithm is identical to that found by the algorithm of De Smidt-Destombes et al. (2011).[2]

6.1 Set up

Table 1 shows the settings for the parameters that are varied in our numerical experiment for the two sets of problem instances. (We also vary λ_i when we vary the number of components; we explain this below.) We use a full factorial design per set and we generate ten problem instances per combination of parameters. As a result, Set 1 and Set 2 consist of 2,160 and 2,880 problem instances, respectively. The way in which we generate problem instances leads to instances that are realistic in practice, and to a wide range of combinations of parameter values in the sets.

We generate $|I|$ components and we draw a value μ from a uniform distribution on the range $[0, \mu^{\max}]$. Then, for each component $i \in I$ (see the explanation below):

- $\mu_i \leftarrow \mu$,
- T_i is drawn from a uniform distribution on the range $[0, T^{\max}]$,
- $\lambda_i \leftarrow \frac{128}{|I|}$ in Set 1 and $\lambda_i \leftarrow \frac{1{,}024}{|I|}$ in Set 2, and

[2]The experiment is implemented in Python 3.4 and performed on an Intel Xeon E5530 @ 2.4 GHz with 8 GB RAM, running Windows Server 2008 R2 Enterprise Service Pack 1.

# Components	2	4	8
# Spare assets in optimal solution	2.8	1.8	1.6
# Spare LRUs in optimal solution	5.2	7.6	12.3
− Divided by # components	2.6	1.9	1.5
% Problem instances with optimal solution	73%	55%	26%
Average additional costs in remaining instances	3.2%	3.9%	3.8%
Maximum additional costs in remaining instances	63%	40%	93%

Table 2: Set 1: Optimal solutions, and quality of the solutions found by our algorithm

- c_i is drawn from an exponential distribution with mean $\frac{1}{c^{\text{average}}}$. We add 10, which effectively means that there are no components with costs of less than 10, and the mean costs are $c^{\text{average}} + 10$.

We can use the same value μ for all $i \in I$, since it influences only the number of assets in active maintenance, Y_0.

λ_i is relevant only for calculating the number of components in resupply, X_i for $i \in I$, and the number of assets in active maintenance, Y_0. Since the average number of components in resupply is varied by varying T_i and since the average number of assets in the maintenance shop is varied by varying μ, we can keep λ_i constant in each problem instance. However, we do vary λ_i when we vary the number of components. Our aim is to get solutions in which the optimal number of spare assets and spare parts is realistic and higher than zero. We therefore show the average number of spare assets and spare parts in the solutions in the next section. The largest problem instances of Set 2 are the most realistic ones, with over 1,000 components and a demand rate per component of 1.

The costs of holding a spare asset, c_0, are equal to the summation of the costs of holding one spare of each of the spare parts, times c^{relative}. Finally, we vary the target readiness, R^{obj}.

6.2 Results

Table 2 gives the results on Set 1. If we look at the number of spare assets and LRUs in the optimal solution, we see that our choice for the demand rates has ensured that, even with these small unrealistic problem instances, we find optimal solutions that allow a meaningful analysis of the quality of the solutions that our algorithm finds.

Many problem instances, 51%, are solved to optimality by our marginal analysis approach, and the average difference with the optimal solution on the other instances is small: 3.7% on average. The maximum difference is large, 93%, but large differences for these small instances can be caused by stocking one additional spare part by our algorithm compared with the optimal solution. All in all, we believe that our algorithm typically finds good solutions.

Since the algorithm of De Smidt-Destombes et al. (2011) requires more computation time than our algorithm, we have only run the problem instances of up to 256 components using their algorithm. Table 3 shows the comparison of the computation times for an increasing number of components. Given the number of convolutions that both algorithms perform for each LRU

# Components	16	64	256
(1) Computation time (seconds) of our algorithm using bound	0.4	3.6	22.6
(2) Computation time (seconds) of our algorithm without using bound	0.5	5.5	48.9
(3) Computation time (seconds) of algorithm of De Smidt-Destombes et al.	1.4	41.8	1,146.8
Relative computation time (2)/(1)	1.2	1.5	2.2
Relative computation time (3)/(1)	3.3	11.6	50.7
Relative computation time (3)/(2)	2.7	7.6	23.5
# Spare asset levels enumerated	4.0	2.9	2.1
− maximum	12	8	5
# Spare assets in solution	5.6	5.2	5.1
# Spare LRUs in solution	67	151	396
− divided by # components	4.2	2.4	1.5

Table 3: Set 2: Computation times and solutions of our algorithm and that of De Smidt-Destombes et al. (2011), with 'bound' referring to the use (or not) of the results in Proposition 5

Parameter	Setting	Computation time	# Spare asset levels enumerated	# Spare asset difference with LB	# Spares in solution assets	# Spares in solution LRUs
# Components	16	0.4	4.0	0.59	5.6	67
	64	3.6	2.9	0.20	5.2	151
	256	22.6	2.1	0.09	5.1	396
	1,024	229.9	1.8	0.04	5.0	1,251
Relative costs of an asset	0.5	95.4	4.2	0.55	5.6	445
	1	61.5	2.4	0.12	5.1	470
	2	35.5	1.5	0.02	5.0	483
Maximum resupply lead time	0.01	5.2	1.6	0.11	5.1	300
	0.1	123.1	3.8	0.35	5.3	632

Table 4: Set 2: Key results of our algorithm for relevant parameters

in each iteration of the marginal analysis approach, we would expect that for 16, 64, and 256 components, their algorithm would require 4, 10.67 and 32 times as much computation time, being $\frac{|I|}{\log_2 |I|}$. If we do not use the bound based on Proposition 5, we find that the relative performance of our algorithm is about 70% of what we expected. This is probably due to our algorithm requiring more storage and overhead. We further see that using the bound that is based on Proposition 5 saves a considerable amount of computation time, with the savings increasing with an increasing problem size. Finally, we see that the (average and maximum) number of spare asset levels that is enumerated, decreases when the number of components increases. For 1,024 components, the average and maximum number decrease even further, to 1.8 and 4, respectively. This is very positive for the computation times.

Table 4 shows the key results for our algorithm on Set 2 for three parameters that have a big influence on the computation times. The computation times increase if the number of components increases or if the maximum resupply lead time increases, which intuitively makes sense. Next, if the relative costs of an asset increase, then the computation times decrease. The key reason for this is that less spare asset levels need to be enumerated. Finally, the number of spare assets in the solution decreases a little when the asset price increases, while the number of spare LRUs sligtly increases. Also these results make sense intuitively.

It is further interesting to see that the number of spare assets in the solution that our algorithm finds is close to the lower bound (LB) that we use in our algorithm and that the gap becomes smaller when the problem size increases (to 0.04 on average for problem instances with 1,024 components). In fact, for more than 16 components, we never find a gap of more than 1 on this set. This suggests that the lower bound that we use is useful in practice to get an idea of the fleet size to acquire, while it is easy to calculate.

7 Conclusions and recommendations

We have considered the problem of jointly optimizing the number of spare LRUs and spare assets, i.e., the spare parts inventories and fleet size. This is a problem that needs to be solved by companies that use a fleet of assets, e.g., railway operaters, shipping companies or defence organizations. We have found that the optimization problem is challenging since it is not item-separable, nor jointly concave. However, we have shown some less strong results and we have used those to construct an algorithm. In a numerical experiment we have shown that this algorithm typically finds solutions that are close to optimal and that the algorithm is relatively fast due to the order in which we perform convolutions and a bound that we use to avoid performing unnecessary computations.

It would be interesting to extend our work by modelling the maintenance processes more realistically. We have now assumed that the repair shop that repairs failed component has ample servers and that repaired components are put back into a failed asset one by one, i.e., sequentially. The ample server assumption may be realistic in many settings, since it can represent lead time agreements with the repair shop, but in other settings it may not be. The assumption of sequential repairs of the asset can be relaxed to allow for parallel repairs as are often found in practice.

Another interesting extension would be to consider the optimization of the LRU level itself: In case of a failure in a certain component, it may be possible to exchange and repair that component, or a module in which the component is contained. This influences the exchange times, the required resources for the exchange, and the types and amounts of spare parts to stock. Some first results on that problem, without considering spare LRUs and spare assets, can be found in Parada Puig and Basten (2014).

Acknowledgements

The first author gratefully acknowledges the support of the Lloyd's Register Foundation (LRF). LRF helps to protect life and property by supporting engineering-related education, public engagement and the application of research.

A Proof of Proposition 5

For notational convenience, let $Z_{ij} = Y_0 + \sum_{k \in I \setminus \{i,j\}} B_k$. Then:

$$
\begin{aligned}
&\Delta_i R(\mathbf{S_0} + \mathbf{e}_j) - \Delta_i R(\mathbf{S_0}) \\
&= \sum_{b=1}^{S_0+1} \mathbb{P}\{X_i = S_i + b\} \\
&\quad \left[\mathbb{P}\left\{Z_{ij} + [X_j - S_j - 1]^+ = S_0 + 1 - b\right\} - \mathbb{P}\left\{Z_{ij} + [X_j - S_j]^+ = S_0 + 1 - b\right\}\right] \\
&= \sum_{b=1}^{S_0+1} \mathbb{P}\{X_i = S_i + b\} \sum_{z=0}^{S_0+1-b} \mathbb{P}\{Z_{ij} = z\} \\
&\quad \left[\mathbb{P}\left\{[X_j - S_j - 1]^+ = S_0 + 1 - b - z\right\} - \mathbb{P}\left\{[X_j - S_j]^+ = S_0 + 1 - b - z\right\}\right] \\
&= \sum_{b=1}^{S_0+1} \mathbb{P}\{X_i = S_i + b\} \sum_{z=0}^{S_0-b} \mathbb{P}\{Z_{ij} = z\} \\
&\quad \left[\mathbb{P}\left\{[X_j - S_j - 1]^+ = S_0 + 1 - b - z\right\} - \mathbb{P}\left\{[X_j - S_j]^+ = S_0 + 1 - b - z\right\}\right] \\
&\quad + \sum_{b=1}^{S_0+1} \mathbb{P}\{X_i = S_i + b\} \mathbb{P}\{Z_{ij} = S_0 + 1 - b\} \\
&\qquad \left[\mathbb{P}\left\{[X_j - S_j - 1]^+ = 0\right\} - \mathbb{P}\left\{[X_j - S_j]^+ = 0\right\}\right] \\
&= \sum_{b=1}^{S_0+1} \mathbb{P}\{X_i = S_i + b\} \sum_{z=0}^{S_0-b} \mathbb{P}\{Z_{ij} = z\} \\
&\quad [\mathbb{P}\{X_j = S_0 + S_j + 2 - b - z\} - \mathbb{P}\{X_j = S_0 + S_j + 1 - b - z\}] \\
&\quad + \mathbb{P}\{X_j = S_j + 1\} \sum_{b=1}^{S_0+1} \mathbb{P}\{X_i = S_i + b\} \mathbb{P}\{Z_{ij} = S_0 + 1 - b\}. \qquad (1)
\end{aligned}
$$

After the third equation, the case that $z = S_0 + 1 - b$ is considered separately. Furthermore, $\sum_{x=0}^{-1} x = 0$ by definition.

We now require two results, which we prove below, that we combine to prove Proposition 5. The first result is that the first of the two terms in Equation 1 is negative, while the second result is that the second term in that equation is smaller than $\mathbb{P}\{X_j = S_j + 1\} \mathbb{P}\{X_i = S_i + 1\}$. The summation of the two terms is then also smaller than $\mathbb{P}\{X_j = S_j + 1\} \mathbb{P}\{X_i = S_i + 1\}$.

1. X_j in $\mathbb{P}\{X_j = S_0 + S_j + 2 - b - z\}$ and $\mathbb{P}\{X_j = S_0 + S_j + 1 - b - z\}$ ranges from $S_j + 1$ and S_j, to $S_j + S_0 + 1$ and $S_j + S_0$, respectively. Since $S_j \geq \lceil \lambda_j T_j \rceil - 2$, the first term in Equation 1 must be negative (due to the properties of the Poisson distribution discussed in the proof of Proposition 4).

2. Since $S_i \geq \lceil \lambda_i T_i \rceil - 2$, it holds that:

$$\begin{aligned}
&\sum_{b=1}^{S_0+1} \mathbb{P}\left\{X_i = S_i + b\right\} \mathbb{P}\left\{Y_0 + \sum_{k \in I \setminus \{i,j\}} B_k = S_0 + 1 - b\right\} \\
&< \mathbb{P}\left\{X_i = S_i + 1\right\} \sum_{b=1}^{S_0+1} \mathbb{P}\left\{Y_0 + \sum_{k \in I \setminus \{i,j\}} B_k = S_0 + 1 - b\right\} \\
&< \mathbb{P}\left\{X_i = S_i + 1\right\}.
\end{aligned}$$

As a result, the second term in Equation 1 is smaller than $\mathbb{P}\left\{X_j = S_j + 1\right\} \mathbb{P}\left\{X_i = S_i + 1\right\}$.

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