Formal specification of a generic separation kernel

D31.1
Formal Specification of a Generic Separation Kernel

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Abstract:	We introduce a theory of intransitive non-interference for separation kernels with control. We show that it can be instantiated for a simple API consisting of IPC and events.

Keywords:	separation kernel with control, formal model, instantiation, IPC, events, Isabelle/HOL

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Executive Summary

Intransitive noninterference has been a widely studied topic in the last few decades. Several well-established methodologies apply interactive theorem proving to formulate a noninterference theorem over abstract academic models. In joint work with several industrial and academic partners throughout Europe, we are helping in the certification process of PikeOS, an industrial separation kernel developed at SYSGO. In this process, established theories could not be applied. We present a new generic model of separation kernels and a new theory of intransitive noninterference. The model is rich in detail, making it suitable for formal verification of realistic and industrial systems such as PikeOS. Using a refinement-based theorem proving approach, we ensure that proofs remain manageable.

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Contents

1 Introduction

Separation kernels are at the heart of many modern security-critical systems [23]. With next generation technology in cars, aircrafts and medical devices becoming more and more interconnected, a platform that offers secure decomposition of embedded systems becomes crucial for safe and secure performance. PikeOS, a separation kernel developed at SYSGO, is an operating system providing such an environment [12, 2]. A consortium of several European partners from industry and academia works on the certification of PikeOS up to at least Common Criteria EAL5+, with "+" being applying formal methods compliant to EAL7. Our aim is to derive a precise model of PikeOS and a precise formulation of the PikeOS security policy.

A crucial security property of separation kernels is *intransitive noninterference*. This property is typically required for systems with multiple independent levels of security (MILS) such as PikeOS. It ensures that a given security policy over different subjects of the system is obeyed. Such a security policy dictates which subjects may flow information to which other subjects.

Intransitive noninterference has been an active research field for the last three decades. Several papers have been published on defining intransitive noninterference and on unwinding methodologies that enable the proof of intransitive noninterference from local proof obligations. However, in the certification process of PikeOS these existing methodologies could not be directly applied. Generally, the methodologies are based on highly abstract generic models of computation. The gap between such an abstract model and the reality of PikeOS is large, making application of the methodologies tedious and cumbersome.

This paper presents a new generic model for separation kernels called CISK (for: Controlled Interruptible Separation Kernel). This model is richer in details and contains several facets present in many separation kernels, such as *interrupts*, *context switches* between domains and a notion of *control*. Regarding the latter, this concerns the fact that the kernel exercises control over the executions as performed by the domains. The kernel can, e.g., decide to skip actions of the domains, or abort them halfway. We prove that any instantiation of the model provides intransitive noninterference. The model and proofs have been formalized in Isabelle/HOL [21] which are included in the subsequent sections of this document.

We have adopted Rushby's definition of intransitive noninterference [24]. We first present an overview of our approach and then discuss the relation between our approach and existing methodologies in the next section.

Overview

Generally, there are two conflicting interests when using a generic model. On the one hand the model must be sufficiently abstract to ensure that theorems and proofs remain manageable. On the other hand, the model must be rich enough and must contain sufficient domain-knowledge to allow easy instantiation. Rushby's model, for example, is on one end of the spectrum: it is basically a Mealy machine, which is a highly abstract notion of computation, consisting only of state, inputs and outputs [24]. The model and its proofs are manageable, but making a realistic instantiation is tedious and requires complicated proofs.

We aim at the other side of the spectrum by having a generic model that is rich in detail. As a result, instantiating the model with, e.g., a model of PikeOS can be done easily. To ensure maintainability of the theorems and proofs, we have applied a highly modularized theorem proving technique.

Figure 1 shows an overview. The initial module "Kernel" is close to a Mealy machine, but has several facets added, including interrupts, context switches and control. New modules are added in such a way that each new module basically inserts an adjective before "Kernel". The use of modules allows us to prove, e.g., a separation theorem in module "Separation Kernel" and subsequently to reuse this theorem later on when details on control or interrupts are added.

The second module adds a notion of separation, yielding a module of a Separation Kernel (SK). A security policy is added that dictates which domains may flow information to each other. Local proof

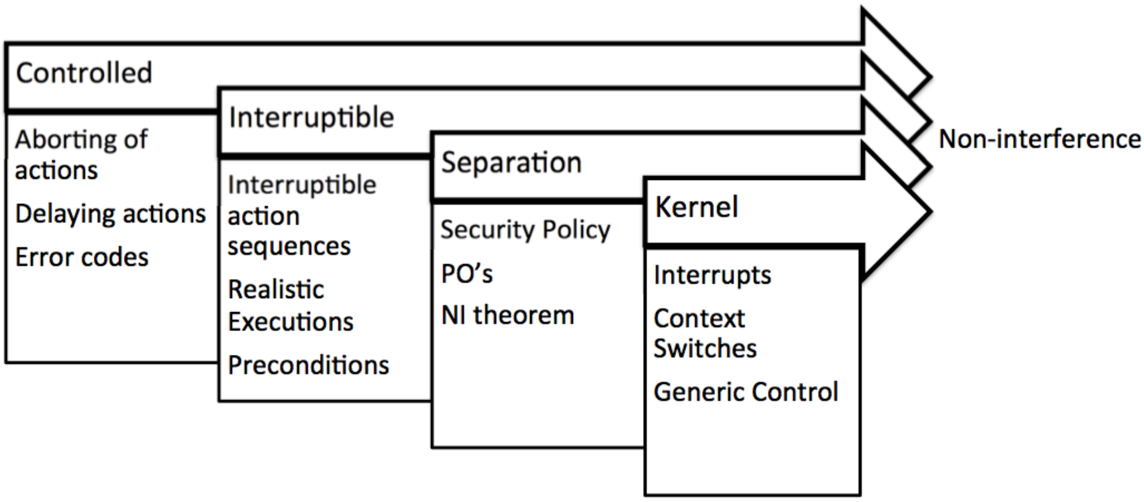

Figure 1: Overview of CISK modular structure

obligations are added from which a global theorem of noninterference is proven. This global theorem is the *unwinding* of the local proof obligations.

In the third module calls to the kernel are no longer considered atomic, yielding an Interruptible Separation Kernel (ISK). In this model, one call to the kernel is represented by an *action sequence*. Consider, for example, an IPC call (for: Inter Process Communication). From the point of view of the programmer this is one kernel call. From the point of view of the kernel it is an action sequence consisting of three stages IPC_PREP, IPC_WAIT, and IPC_SEND. During the PREP stage, it is checked whether the IPC is allowed by the security policy. The WAIT stage is entered if a thread needs to wait for its communication partner. The SEND stage is data transmission. After each stage, an interrupt may occur that switches the current context. A consequence of allowing interruptible action sequences is that it is no longer the case that any execution, i.e., any combination of atomic kernel actions, is realistic. We formulate a definition of *realistic execution* and weaken the proof obligations of the model to apply only to realistic executions.

The final module provides an interpretation of control that allows atomic kernel actions to be aborted or delayed. Additional proof obligations are required to ensure that noninterference is still provided. This yields a Controlled Interruptible Separation Kernel (CISK). When sequences of kernel actions are aborted, error codes can be transmitted to other domains. Revisiting our IPC example, after the PREP stage the kernel can decide to abort the action. The IPC action sequence will not be continued and error codes may be sent out. At the WAIT stage, the kernel can delay the action sequence until the communication partner of the IPC call is ready to receive.

In Section 3 we introduce a theory of intransitive non-interference for separation kernels with control, based on [31]. We show that it can be instantiated for a simple API consisting of IPC and events (Section 4). The rest of *this* section gives some auxiliary theories used for Section 3.

2 Preliminaries

2.1 Binders for the option type

theory *Option-Binders*
imports *Option*
begin

The following functions are used as binders in the theorems that are proven. At all times, when a result is None, the theorem becomes vacuously true. The expression "$m \rightharpoonup \alpha$" means "First compute m, if it is None then return True, otherwise pass the result to α". B2 is a short hand for sequentially doing two independent computations. The following syntax is associated to B2: "$m_1 \| m_2 \rightharpoonup \alpha$" represents "First compute m_1 and m_2, if one of them is None then return True, otherwise pass the result to α".

definition $B :: {}'a\ option \Rightarrow ({}'a \Rightarrow bool) \Rightarrow bool$ (**infixl** $\rightharpoonup$ 65)
where $B\ m\ \alpha \equiv case\ m\ of\ None \Rightarrow True \mid (Some\ a) \Rightarrow \alpha\ a$

definition $B2 :: {}'a\ option \Rightarrow {}'a\ option \Rightarrow ({}'a \Rightarrow {}'a \Rightarrow bool) \Rightarrow bool$
where $B2\ m1\ m2\ \alpha \equiv m1 \rightharpoonup (\lambda\ a\ .\ m2 \rightharpoonup (\lambda\ b\ .\ \alpha\ a\ b))$

syntax $B2 :: [{}'a\ option, {}'a\ option, ({}'a \Rightarrow {}'a \Rightarrow bool)] => bool\ ((\text{-} \parallel \text{-} \rightharpoonup \text{-})\ [0, 0, 10]\ 10)$

Some rewriting rules for the binders

lemma *rewrite-B2-to-cases*[*simp*]:
shows $B2\ s\ t\ f = (case\ s\ of\ None \Rightarrow True \mid (Some\ s1) \Rightarrow (case\ t\ of\ None \Rightarrow True \mid (Some\ t1) \Rightarrow f\ s1\ t1))$
using *assms* **unfolding** *B2-def B-def* **by**(*cases s*,*cases t*,*simp+*)
lemma *rewrite-B-None*[*simp*]:
shows $None \rightharpoonup \alpha = True$
unfolding *B-def* **by**(*auto*)
lemma *rewrite-B-m-True*[*simp*]:
shows $m \rightharpoonup (\lambda\ a\ .\ True) = True$
unfolding *B-def* **by**(*cases m*,*simp+*)
lemma *rewrite-B2-cases*:
shows $(case\ a\ of\ None \Rightarrow True \mid (Some\ s) \Rightarrow (case\ b\ of\ None \Rightarrow True \mid (Some\ t) \Rightarrow f\ s\ t))$
$= (\forall\ s\ t\ .\ a = (Some\ s) \wedge b = (Some\ t) \longrightarrow f\ s\ t)$
by(*cases a*,*simp*,*cases b*,*simp+*)

definition $strict\text{-}equal :: {}'a\ option \Rightarrow {}'a \Rightarrow bool$
where $strict\text{-}equal\ m\ a \equiv case\ m\ of\ None \Rightarrow False \mid (Some\ a') \Rightarrow a' = a$

end

2.2 Theorems on lists

theory *List-Theorems*
imports *List*
begin

definition $lastn :: nat \Rightarrow {}'a\ list \Rightarrow {}'a\ list$
where $lastn\ n\ x = drop\ ((length\ x) - n)\ x$

definition $is\text{-}sub\text{-}seq :: {}'a \Rightarrow {}'a \Rightarrow {}'a\ list \Rightarrow bool$
where $is\text{-}sub\text{-}seq\ a\ b\ x \equiv \exists\ n\ .\ Suc\ n < length\ x \wedge x!n = a \wedge x!(Suc\ n) = b$

definition $prefixes :: {}'a\ list\ set \Rightarrow {}'a\ list\ set$
where $prefixes\ s \equiv \{x\ .\ \exists\ n\ y\ .\ n > 0 \wedge y \in s \wedge take\ n\ y = x\}$

lemma *drop-one*[*simp*]:
shows $drop\ (Suc\ 0)\ x = tl\ x$ **by**(*induct x*,*auto*)
lemma *length-ge-one*:
shows $x \neq [] \longrightarrow length\ x \geq 1$ **by**(*induct x*,*auto*)
lemma *take-but-one*[*simp*]:
shows $x \neq [] \longrightarrow lastn\ ((length\ x) - 1)\ x = tl\ x$ **unfolding** *lastn-def*
using *length-ge-one*[**where** *x=x*] **by** *auto*
lemma *Suc-m-minus-n*[*simp*]:
shows $m \geq n \longrightarrow Suc\ m - n = Suc\ (m - n)$ **by** *auto*

lemma *lastn-one-less*:
shows $n > 0 \wedge n \leq length\ x \wedge lastn\ n\ x = (a\#y) \longrightarrow lastn\ (n - 1)\ x = y$ **unfolding** *lastn-def*
using *drop-Suc*[**where** n=*length* $x - n$ **and** *xs*=x] *drop-tl*[**where** n=*length* $x - n$ **and** *xs*=x]
by(*auto*)
lemma *list-sub-implies-member*:
shows $\forall\ a\ x\ .\ set\ (a\#x) \subseteq Z \longrightarrow a \in Z$ **by** *auto*
lemma *subset-smaller-list*:
shows $\forall\ a\ x\ .\ set\ (a\#x) \subseteq Z \longrightarrow set\ x \subseteq Z$ **by** *auto*
lemma *second-elt-is-hd-tl*:
shows $tl\ x = (a\ \#\ x') \longrightarrow a = x\ !\ 1$
by (*cases x*,*auto*)
lemma *length-ge-2-implies-tl-not-empty*:
shows $length\ x \geq 2 \longrightarrow tl\ x \neq []$
by (*cases x*,*auto*)
lemma *length-lt-2-implies-tl-empty*:
shows $length\ x < 2 \longrightarrow tl\ x = []$
by (*cases x*,*auto*)
lemma *first-second-is-sub-seq*:
shows $length\ x \geq 2 \Longrightarrow is\text{-}sub\text{-}seq\ (hd\ x)\ (x!1)\ x$
proof–
assume $length\ x \geq 2$
hence *1*: $(Suc\ 0) < length\ x$ **by** *auto*
hence $x!0 = hd\ x$ **by**(*cases x*,*auto*)
from *this 1* **show** $is\text{-}sub\text{-}seq\ (hd\ x)\ (x!1)\ x$ **unfolding** *is-sub-seq-def* **by** *auto*
qed
lemma *hd-drop-is-nth*:
shows $n < length\ x \Longrightarrow hd\ (drop\ n\ x) = x!n$
proof(*induct x arbitrary*: n)
case *Nil*
thus *?case* **by** *simp*
next
case (*Cons a x*)
{
have $hd\ (drop\ n\ (a\ \#\ x)) = (a\ \#\ x)\ !\ n$
proof(*cases n*)
case *0*
thus *?thesis* **by** *simp*
next
case (*Suc m*)
from *Suc Cons* **show** *?thesis* **by** *auto*
qed
}
thus *?case* **by** *auto*
qed

lemma *def-of-hd*:
shows $y = a\ \#\ x \longrightarrow hd\ y = a$ **by** *simp*
lemma *def-of-tl*:
shows $y = a\ \#\ x \longrightarrow tl\ y = x$ **by** *simp*
lemma *drop-yields-results-implies-nbound*:
shows $drop\ n\ x \neq [] \longrightarrow n < length\ x$
by(*induct x*,*auto*)
lemma *hd-take*[*simp*]:
shows $n > 0 \Longrightarrow hd\ (take\ n\ x) = hd\ x$
by(*cases x*,*simp*,*cases n*, *auto*)
lemma *consecutive-is-sub-seq*:
shows $a\ \#\ (b\ \#\ x) = lastn\ n\ y \Longrightarrow is\text{-}sub\text{-}seq\ a\ b\ y$

proof–
assume *1*: *a # (b # x) = lastn n y*
from *1 drop-Suc*[**where** *n=(length y) − n* **and** *xs=y*]
drop-tl[**where** *n=(length y) − n* **and** *xs=y*]
def-of-tl[**where** *y=lastn n y* **and** *a=a* **and** *x =b#x*]
drop-yields-results-implies-nbound[**where** *n=Suc (length y − n)* **and** *x=y*]
have *3*: *Suc (length y − n) < length y* **unfolding** *lastn-def* **by** *auto*
from *3 1 hd-drop-is-nth*[**where** *n=(length y) − n* **and** *x=y*] *def-of-hd*[**where** *y=drop (length y − n) y* **and** *x=b#x* **and** *a=a*]
have *4*: *y!(length y − n) = a* **unfolding** *lastn-def* **by** *auto*
from *3 1 hd-drop-is-nth*[**where** *n=Suc ((length y) − n)* **and** *x=y*] *def-of-hd*[**where** *y=drop (Suc (length y − n)) y* **and** *x=x* **and** *a=b*]
drop-Suc[**where** *n=(length y) − n* **and** *xs=y*]
drop-tl[**where** *n=(length y) − n* **and** *xs=y*]
def-of-tl[**where** *y=lastn n y* **and** *a=a* **and** *x =b#x*]
have *5*: *y!Suc (length y − n) = b* **unfolding** *lastn-def* **by** *auto*
from *3 4 5* **show** *?thesis*
unfolding *is-sub-seq-def* **by** *auto*
qed

lemma *sub-seq-in-prefixes*:
assumes *∃ y ∈ prefixes X. is-sub-seq a a′ y*
shows *∃ y ∈ X. is-sub-seq a a′ y*
proof–
from *assms* **obtain** *y* **where** *y*: *y ∈ prefixes X ∧ is-sub-seq a a′ y* **by** *auto*
then obtain *n x* **where** *x*: *n > 0 ∧ x ∈ X ∧ take n x = y*
unfolding *prefixes-def* **by** *auto*
from *y* **obtain** *i* **where** *sub-seq-index*: *Suc i < length y ∧ y ! i = a ∧ y ! Suc i = a′*
unfolding *is-sub-seq-def* **by** *auto*
from *sub-seq-index x* **have** *is-sub-seq a a′ x*
unfolding *is-sub-seq-def* **using** *nth-take* **by** *auto*
from *this x* **show** *?thesis* **by** *metis*
qed

lemma *set-tl-is-subset*:
shows *set (tl x) ⊆ set x* **by**(*induct x,auto*)
lemma *x-is-hd-snd-tl*:
shows *length x ≥ 2 ⟶ x = (hd x) # x!1 # tl(tl x)*
proof(*induct x*)
case *Nil*
show *?case* **by** *auto*
case (*Cons a xs*)
show *?case* **by**(*induct xs,auto*)
qed

lemma *tl-x-not-x*:
shows *x ≠ [] ⟶ tl x ≠ x* **by**(*induct x,auto*)
lemma *tl-hd-x-not-tl-x*:
shows *x ≠ [] ∧ hd x ≠ [] ⟶ tl (hd x) # tl x ≠ x* **using** *tl-x-not-x* **by**(*induct x,simp,auto*)

end

3 A generic model for separation kernels

This section defines a detailed generic model of separation kernels called CISK (Controlled Interruptible Separation Kernel). It contains a generic functional model of the behaviour of a separation kernel as a transition system,

definitions of the security property and proofs that the functional model satisfies security properties. It is based on Rushby's approach [25] for noninterference. For an explanation of the model, its structure and an overview of the proofs, we refer to the document entitled "A New Theory of Intransitive Noninterference for Separation Kernels with Control" [31].
The structure of the model is based on locales and refinement:

- locale "Kernel" defines a highly generic model for a kernel, with execution semantics. It defines a state transition system with some extensions to the one used in [25]. The transition system defined here stores the currently active domain in the state, and has transitions for explicit context switches and interrupts and provides a notion of control. As each operation of the system will be split into atomic actions in our model, only certain sequences of actions will correspond to a run on a real system. Therefore, the function run, which applies an execution on a state and computes the resulting new state, is partial and defined for realistic traces only. Later, but not in this locale, we will define a predicate to distinguish realistic traces from other traces. Security properties are also not part of this locale, but will be introduced in the locales to be described next.
- locale "Separation_Kernel" extends "Kernel" with constraints concerning non-interference. The theorem is only sensical for realistic traces; for unrealistic trace it will hold vacuously.
- locale "Interruptible_Separation_Kernel" refines "Separation_Kernel" with interruptible action sequences. It defines function "realistic_trace" based on these action sequences. Therefore, we can formulate a total run function.
- locale "Controlled_Interruptible_Separation_Kernel" refines "Interruptible_Separation_Kernel" with abortable action sequences. It refines function "control" which now uses a generic predicate "aborting" and a generic function "set_error_code" to manage aborting of action sequences.

3.1 K (Kernel)

theory *K*
imports *Main List Set Transitive-Closure List-Theorems Option-Binders*
begin

The model makes use of the following types:

'state_t A state contains information about the resources of the system, as well as which domain is currently active. We decided that a state does *not* need to include a program stack, as in this model the actions that are executed are modelled separately.

'dom_t A domain is an entity executing actions and making calls to the kernel. This type represents the names of all domains. Later on, we define security policies in terms of domains.

'action_t Actions of type 'action_t represent atomic instructions that are executed by the kernel. As kernel actions are assumed to be atomic, we assume that after each kernel action an interrupt point can occur.

'action_t execution An execution of some domain is the code or the program that is executed by the domain. One call from a domain to the kernel will typically trigger a succession of one or more kernel actions. Therefore, an execution is represented as a list of *sequences* of kernel actions. Non-kernel actions are not take into account.

'output_t Given the current state and an action an output can be computed deterministically.

time_t Time is modelled using natural numbers. Each atomic kernel action can be executed within one time unit.

type-synonym ($'action\text{-}t$) *execution* = $'action\text{-}t$ *list list*
type-synonym *time-t* = *nat*

Function *kstep* (for kernel step) computes the next state based on the current state s and a given action a. It may assume that it makes sense to perform this action, i.e., that any precondition that is necessary for execution of action a in state s is met. If not, it may return any result. This precondition is represented by generic predicate *kprecondition* (for kernel precondition). Only realistic traces are considered. Predicate *realistic_execution* decides whether a given execution is realistic.

Function *current* returns given the state the domain that is currently executing actions. The model assumes a single-core setting, i.e., at all times only one domain is active. Interrupt behavior is modelled using functions *interrupt* and *cswitch* (for context switch) that dictate respectively when interrupts occur and how interrupts occur. Interrupts are solely time-based, meaning that there is an at beforehand fixed schedule dictating which domain is active at which time.

Finally, we add function *control*. This function represents control of the kernel over the execution as performed by the domains. Given the current state s, the currently active domain d and the execution α of that domain, it returns three objects. First, it returns the next action that domain d will perform. Commonly, this is the next action in execution α. It may also return None, indicating that no action is done. Secondly, it returns the updated execution. When executing action a, typically, this action will be removed from the current execution (i.e., updating the program stack). Thirdly, it can update the state to set, e.g., error codes.

locale *Kernel* =
  fixes *kstep* :: *'state-t* ⇒ *'action-t* ⇒ *'state-t*
    and *output-f* :: *'state-t* ⇒ *'action-t* ⇒ *'output-t*
    and *s0* :: *'state-t*
    and *current* :: *'state-t* => *'dom-t*
    and *cswitch* :: *time-t* ⇒ *'state-t* ⇒ *'state-t*
    and *interrupt* :: *time-t* ⇒ *bool*
    and *kprecondition* :: *'state-t* ⇒ *'action-t* ⇒ *bool*
    and *realistic-execution* :: *'action-t execution* ⇒ *bool*
    and *control* :: *'state-t* ⇒ *'dom-t* ⇒ *'action-t execution* ⇒
                    ((*'action-t option*) × *'action-t execution* × *'state-t*)
    and *kinvolved* :: *'action-t* ⇒ *'dom-t set*
begin

3.1.1 Execution semantics

Short hand notations for using function control.

definition *next-action*::*'state-t* ⇒ (*'dom-t* ⇒ *'action-t execution*) ⇒ *'action-t option*
where *next-action s execs* = *fst* (*control s* (*current s*) (*execs* (*current s*)))
definition *next-execs*::*'state-t* ⇒ (*'dom-t* ⇒ *'action-t execution*) ⇒ (*'dom-t* ⇒ *'action-t execution*)
where *next-execs s execs* = (*fun-upd execs* (*current s*) (*fst* (*snd* (*control s* (*current s*) (*execs* (*current s*))))))
definition *next-state*::*'state-t* ⇒ (*'dom-t* ⇒ *'action-t execution*) ⇒ *'state-t*
where *next-state s execs* = *snd* (*snd* (*control s* (*current s*) (*execs* (*current s*))))

A thread is empty iff either it has no further action sequences to execute, or when the current action sequence is finished and there are no further action sequences to execute.

abbreviation *thread-empty*::*'action-t execution* ⇒ *bool*
where *thread-empty exec* ≡ *exec* = [] ∨ *exec* = [[]]

Wrappers for function kstep and kprecondition that deal with the case where the given action is None.

definition *step* **where** *step s oa* ≡ *case oa of None* ⇒ *s* | (*Some a*) ⇒ *kstep s a*
definition *precondition* :: *'state-t* ⇒ *'action-t option* ⇒ *bool*
where *precondition s a* ≡ *a* ⇀ *kprecondition s*
definition *involved*
where *involved oa* ≡ *case oa of None* ⇒ {} | (*Some a*) ⇒ *kinvolved a*

Execution semantics are defined as follows: a run consists of consecutively running sequences of actions. These sequences are interruptable. Run first checks whether an interrupt occurs. When this happens, function cswitch may switch the context. Otherwise, function control is used to determine the next action a, which also yields a new state s'. Action a is executed by executing (step s' a). The current execution of the current domain is updated.

Note that run is a partial function, i.e., it computes results only when at all times the preconditions hold. Such runs are the realistic ones. For other runs, we do not need to – and cannot – prove security. All the theorems are formulated in such a way that they hold vacuously for unrealistic runs.

function *run* :: *time-t* ⇒ *'state-t option* ⇒ (*'dom-t* ⇒ *'action-t execution*) ⇒ *'state-t option*
where *run 0 s execs = s*
| *run* (*Suc n*) *None execs = None*
| *interrupt* (*Suc n*) ⟹ *run* (*Suc n*) (*Some s*) *execs = run n* (*Some* (*cswitch* (*Suc n*) *s*)) *execs*
| ¬*interrupt* (*Suc n*) ⟹ *thread-empty*(*execs* (*current s*)) ⟹ *run* (*Suc n*) (*Some s*) *execs = run n* (*Some s*) *execs*
| ¬*interrupt* (*Suc n*) ⟹ ¬*thread-empty*(*execs* (*current s*)) ⟹ ¬*precondition* (*next-state s execs*) (*next-action s execs*) ⟹ *run* (*Suc n*) (*Some s*) *execs = None*
| ¬*interrupt* (*Suc n*) ⟹ ¬*thread-empty*(*execs* (*current s*)) ⟹ *precondition* (*next-state s execs*) (*next-action s execs*) ⟹
run (*Suc n*) (*Some s*) *execs = run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*)
using *not0-implies-Suc* **by** (*metis option.exhaust prod-cases3,auto*)
termination by *lexicographic-order*
end

end

3.2 SK (Separation Kernel)

theory *SK*
imports *K*
begin

Locale Kernel is now refined to a generic model of a separation kernel. The security policy is represented using function *ia*. Function *vpeq* is adopted from Rushby and is an equivalence relation represeting whether two states are equivalent from the point of view of the given domain.

We assume constraints similar to Rushby, i.e., weak step consistency, locally respects, and output consistency. Additional assumptions are:

Step Atomicity Each atomic kernel step can be executed within one time slot. Therefore, the domain that is currently active does not change by executing one action.

Time-based Interrupts As interrupts occur according to a prefixed time-based schedule, the domain that is active after a call of switch depends on the currently active domain only (cswitch_consistency). Also, cswitch can *only* change which domain is currently active (cswitch_consistency).

Control Consistency States that are equivalent yield the same control. That is, the next action and the updated execution depend on the currently active domain only (next_action_consistent, next_execs_consistent), the state as updated by the control function remains in vpeq (next_state_consistent, locally_respects_next_state). Finally, function control cannot change which domain is active (current_next_state).

definition *actions-in-execution*:: *'action-t execution* ⇒ *'action-t set*
where *actions-in-execution exec* ≡ { *a* . ∃ *aseq* ∈ *set exec* . *a* ∈ *set aseq* }

locale *Separation-Kernel = Kernel kstep output-f s0 current cswitch interrupt kprecondition realistic-execution control kinvolved*
for *kstep* :: *'state-t* ⇒ *'action-t* ⇒ *'state-t*
and *output-f* :: *'state-t* ⇒ *'action-t* ⇒ *'output-t*

and *s0* :: *'state-t*
and *current* :: *'state-t => 'dom-t* — Returns the currently active domain
and *cswitch* :: *time-t* ⇒ *'state-t* ⇒ *'state-t* — Switches the current domain
and *interrupt* :: *time-t* ⇒ *bool* — Returns t iff an interrupt occurs in the given state at the given time
and *kprecondition* :: *'state-t* ⇒ *'action-t* ⇒ *bool* — Returns t if an precondition holds that relates the current action to the state
and *realistic-execution* :: *'action-t execution* ⇒ *bool* — In this locale, this function is completely unconstrained.
and *control* :: *'state-t* ⇒ *'dom-t* ⇒ *'action-t execution* ⇒ ((*'action-t option*) × *'action-t execution* × *'state-t*)
and *kinvolved* :: *'action-t* ⇒ *'dom-t set*
+
fixes *ifp* :: *'dom-t* ⇒ *'dom-t* ⇒ *bool*
and *vpeq* :: *'dom-t* ⇒ *'state-t* ⇒ *'state-t* ⇒ *bool*
assumes *vpeq-transitive*: ∀ *a b c u*. (*vpeq u a b* ∧ *vpeq u b c*) ⟶ *vpeq u a c*
and *vpeq-symmetric*: ∀ *a b u*. *vpeq u a b* ⟶ *vpeq u b a*
and *vpeq-reflexive*: ∀ *a u*. *vpeq u a a*
and *ifp-reflexive*: ∀ *u* . *ifp u u*
and *weakly-step-consistent*: ∀ *s t u a*. *vpeq u s t* ∧ *vpeq* (*current s*) *s t* ∧ *kprecondition s a* ∧ *kprecondition t a* ∧ *current s* = *current t* ⟶ *vpeq u* (*kstep s a*) (*kstep t a*)
and *locally-respects*: ∀ *a s u*. ¬*ifp* (*current s*) *u* ∧ *kprecondition s a* ⟶ *vpeq u s* (*kstep s a*)
and *output-consistent*: ∀ *a s t*. *vpeq* (*current s*) *s t* ∧ *current s* = *current t* ⟶ (*output-f s a*) = (*output-f t a*)
and *step-atomicity*: ∀ *s a* . *current* (*kstep s a*) = *current s*
and *cswitch-independent-of-state*: ∀ *n s t* . *current s* = *current t* ⟶ *current* (*cswitch n s*) = *current* (*cswitch n t*)
and *cswitch-consistency*: ∀ *u s t n* . *vpeq u s t* ⟶ *vpeq u* (*cswitch n s*) (*cswitch n t*)
and *next-action-consistent*: ∀ *s t execs* . *vpeq* (*current s*) *s t* ∧ (∀ *d* ∈ *involved* (*next-action s execs*) . *vpeq d s t*) ∧ *current s* = *current t* ⟶ *next-action s execs* = *next-action t execs*
and *next-execs-consistent*: ∀ *s t execs* . *vpeq* (*current s*) *s t* ∧ (∀ *d* ∈ *involved* (*next-action s execs*) . *vpeq d s t*) ∧ *current s* = *current t* ⟶ *fst* (*snd* (*control s* (*current s*) (*execs* (*current s*)))) = *fst* (*snd* (*control t* (*current s*) (*execs* (*current s*))))
and *next-state-consistent*: ∀ *s t u execs* . *vpeq* (*current s*) *s t* ∧ *vpeq u s t* ∧ *current s* = *current t* ⟶ *vpeq u* (*next-state s execs*) (*next-state t execs*)
and *current-next-state*: ∀ *s execs* . *current* (*next-state s execs*) = *current s*
and *locally-respects-next-state*: ∀ *s u execs*. ¬*ifp* (*current s*) *u* ⟶ *vpeq u s* (*next-state s execs*)
and *involved-ifp*: ∀ *s a* . ∀ *d* ∈ (*involved a*) . *kprecondition s* (*the a*) ⟶ *ifp d* (*current s*)
and *next-action-from-execs*: ∀ *s execs* . *next-action s execs* ⇀ (λ *a* . *a* ∈ *actions-in-execution* (*execs* (*current s*)))
and *next-execs-subset*: ∀ *s execs u* . *actions-in-execution* (*next-execs s execs u*) ⊆ *actions-in-execution* (*execs u*)
begin

Note that there are no proof obligations on function "interrupt". Its typing enforces the assumptions that switching is based on time and not on state. This assumption is sufficient for these proofs, i.e., no further assumptions are required.

3.2.1 Security for non-interfering domains

We define security for domains that are completely non-interfering. That is, for all domains u and v such that v may not interfere in any way with domain u, we prove that the behavior of domain u is independent of the actions performed by v. In other words, the output of domain u in some run is at all times equivalent to the output of domain u when the actions of domain v are replaced by some other set actions.

A domain is unrelated to u if and only if the security policy dictates that there is no path from the domain to u.

abbreviation *unrelated* :: *'dom-t* ⇒ *'dom-t* ⇒ *bool*
where *unrelated d u* ≡ ¬*ifp*^** *d u*

To formulate the new theorem to prove, we redefine purging: all domains that may not influence domain u are replaced by arbitrary action sequences.

definition *purge* ::
 $('dom\text{-}t \Rightarrow 'action\text{-}t\ execution) \Rightarrow 'dom\text{-}t \Rightarrow ('dom\text{-}t \Rightarrow 'action\text{-}t\ execution)$
where $purge\ execs\ u \equiv \lambda\ d\ .\ (if\ unrelated\ d\ u\ then$
 $(SOME\ alpha\ .\ realistic\text{-}execution\ alpha)$
 $else\ execs\ d)$

A normal run from initial state $s0$ ending in state s_f is equivalent to a run purged for domain $(\text{current}s\_f)$.

definition *NI-unrelated* **where** *NI-unrelated*
 $\equiv \forall\ execs\ a\ n\ .\ run\ n\ (Some\ s0)\ execs \rightharpoonup$
 $(\lambda\ s\text{-}f\ .\ run\ n\ (Some\ s0)\ (purge\ execs\ (current\ s\text{-}f)) \rightharpoonup$
 $(\lambda\ s\text{-}f2\ .\ output\text{-}f\ s\text{-}f\ a = output\text{-}f\ s\text{-}f2\ a \wedge current\ s\text{-}f = current\ s\text{-}f2))$

The following properties are proven inductive over states s and t:

1. Invariably, states s and t are equivalent for any domain v that may influence the purged domain u. This is more general than proving that "vpeq u s t" is inductive. The reason we need to prove equivalence over all domains v is so that we can use *weak* step consistency.

2. Invariably, states s and t have the same active domain.

abbreviation *equivalent-states* :: $'state\text{-}t\ option \Rightarrow 'state\text{-}t\ option \Rightarrow 'dom\text{-}t \Rightarrow bool$
where $equivalent\text{-}states\ s\ t\ u \equiv s \parallel t \rightharpoonup (\lambda\ s\ t\ .\ (\forall\ v\ .\ ifp\hat{}{*}{*}\ v\ u \longrightarrow vpeq\ v\ s\ t) \wedge current\ s = current\ t)$

Rushby's view partitioning is redefined. Two states that are initially u-equivalent are u-equivalent after performing respectively a realistic run and a realistic purged run.

definition *view-partitioned*::*bool* **where** *view-partitioned*
 $\equiv \forall\ execs\ ms\ mt\ n\ u\ .\ equivalent\text{-}states\ ms\ mt\ u \longrightarrow$
 $(run\ n\ ms\ execs\ \parallel$
 $run\ n\ mt\ (purge\ execs\ u) \rightharpoonup$
 $(\lambda\ rs\ rt\ .\ vpeq\ u\ rs\ rt \wedge current\ rs = current\ rt))$

We formulate a version of predicate view_partitioned that is on one hand more general, but on the other hand easier to prove inductive over function run. Instead of reasoning over execs and (purge execs u), we reason over any two executions execs1 and execs2 for which the following relation holds:

definition *purged-relation* :: $'dom\text{-}t \Rightarrow ('dom\text{-}t \Rightarrow 'action\text{-}t\ execution) \Rightarrow ('dom\text{-}t \Rightarrow 'action\text{-}t\ execution) \Rightarrow bool$
where $purged\text{-}relation\ u\ execs1\ execs2 \equiv \forall\ d\ .\ ifp\hat{}{*}{*}\ d\ u \longrightarrow execs1\ d = execs2\ d$

The inductive version of view partitioning says that runs on two states that are u-equivalent and on two executions that are purged_related yield u-equivalent states.

definition *view-partitioned-ind*::*bool* **where** *view-partitioned-ind*
 $\equiv \forall\ execs1\ execs2\ s\ t\ n\ u\ .\ equivalent\text{-}states\ s\ t\ u \wedge purged\text{-}relation\ u\ execs1\ execs2 \longrightarrow equivalent\text{-}states\ (run\ n\ s\ execs1)\ (run\ n\ t\ execs2)\ u$

A proof that when state t performs a step but state s not, the states remain equivalent for any domain v that may interfere with u.

lemma *vpeq-s-nt*:
 assumes *prec-t*: $precondition\ (next\text{-}state\ t\ execs2)\ (next\text{-}action\ t\ execs2)$
 assumes *not-ifp-curr-u*: $\neg\ ifp\hat{}{*}{*}\ (current\ t)\ u$
 assumes *vpeq-s-t*: $\forall\ v\ .\ ifp\hat{}{*}{*}\ v\ u \longrightarrow vpeq\ v\ s\ t$
 shows $(\forall\ v\ .\ ifp\hat{}{*}{*}\ v\ u \longrightarrow vpeq\ v\ s\ (step\ (next\text{-}state\ t\ execs2)\ (next\text{-}action\ t\ execs2)))$
proof–
 {
 fix *v*

assume *ifp-v-u*: *ifp^∗∗ v u*

from *ifp-v-u not-ifp-curr-u* **have** *unrelated*: *¬ifp^∗∗ (current t) v* **using** *rtranclp-trans* **by** *metis*
from *this current-next-state[THEN spec,THEN spec,***where** *x1=t]*
*locally-respects[THEN spec,THEN spec,THEN spec,***where** *x1=next-state t execs2] vpeq-reflexive*
prec-t **have** *vpeq v (next-state t execs2) (step (next-state t execs2) (next-action t execs2))*
unfolding *step-def precondition-def B-def*
by *(cases next-action t execs2,auto)*
from *unrelated this locally-respects-next-state vpeq-transitive* **have** *vpeq v t (step (next-state t execs2) (next-action t execs2))* **by** *blast*
from *this* **and** *ifp-v-u* **and** *vpeq-s-t* **and** *vpeq-symmetric* **and** *vpeq-transitive* **have** *vpeq v s (step (next-state t execs2) (next-action t execs2))* **by** *metis*
}
thus *?thesis* **by** *auto*
qed

A proof that when state s performs a step but state t not, the states remain equivalent for any domain v that may interfere with u.

lemma *vpeq-ns-t*:
assumes *prec-s*: *precondition (next-state s execs) (next-action s execs)*
assumes *not-ifp-curr-u*: *¬ ifp^∗∗ (current s) u*
assumes *vpeq-s-t*: *∀ v . ifp^∗∗ v u ⟶ vpeq v s t*
shows *∀ v . ifp^∗∗ v u ⟶ vpeq v (step (next-state s execs) (next-action s execs)) t*
proof–
{
fix *v*
assume *ifp-v-u*: *ifp^∗∗ v u*

from *ifp-v-u* **and** *not-ifp-curr-u* **have** *unrelated*: *¬ifp^∗∗ (current s) v* **using** *rtranclp-trans* **by** *metis*
from *this current-next-state[THEN spec,THEN spec,***where** *x1=s] vpeq-reflexive*
*unrelated locally-respects[THEN spec,THEN spec,THEN spec,***where** *x1=next-state s execs* **and** *x=v* **and** *x2=the (next-action s execs)] prec-s*
have *vpeq v (next-state s execs) (step (next-state s execs) (next-action s execs))*
unfolding *step-def precondition-def B-def*
by *(cases next-action s execs,auto)*
from *unrelated this locally-respects-next-state vpeq-transitive* **have** *vpeq v s (step (next-state s execs) (next-action s execs))* **by** *blast*
from *this* **and** *ifp-v-u* **and** *vpeq-s-t* **and** *vpeq-symmetric* **and** *vpeq-transitive* **have** *vpeq v (step (next-state s execs) (next-action s execs)) t* **by** *metis*
}
thus *?thesis* **by** *auto*
qed

A proof that when both states s and t perform a step, the states remain equivalent for any domain v that may interfere with u. It assumes that the current domain *can* interact with u (the domain for which is purged).

lemma *vpeq-ns-nt-ifp-u*:
assumes *vpeq-s-t*: *∀ v . ifp^∗∗ v u ⟶ vpeq v s t′*
and *current-s-t*: *current s = current t′*
shows *precondition (next-state s execs) a ∧ precondition (next-state t′ execs) a ⟹ (ifp^∗∗ (current s) u ⟹ (∀ v . ifp^∗∗ v u ⟶ vpeq v (step (next-state s execs) a) (step (next-state t′ execs) a)))*
proof–
fix *a*
assume *precs*: *precondition (next-state s execs) a ∧ precondition (next-state t′ execs) a*
assume *ifp-curr*: *ifp^∗∗ (current s) u*
from *vpeq-s-t* **have** *vpeq-curr-s-t*: *ifp^∗∗ (current s) u ⟶ vpeq (current s) s t′* **by** *auto*
from *ifp-curr precs*

next-state-consistent[*THEN spec*,*THEN spec*,**where** *x1=s* **and** *x=t'*] *vpeq-curr-s-t vpeq-s-t*
current-next-state current-s-t weakly-step-consistent[*THEN spec*,*THEN spec*,*THEN spec*,*THEN spec*,**where** *x3=next-state s execs* **and** *x2=next-state t' execs* **and** *x=the a*]
show ∀ *v* . *ifp*^∗∗ *v u* ⟶ *vpeq v* (*step* (*next-state s execs*) *a*) (*step* (*next-state t' execs*) *a*)
unfolding *step-def precondition-def B-def*
by (*cases a*,*auto*)
qed

A proof that when both states s and t perform a step, the states remain equivalent for any domain v that may interfere with u. It assumes that the current domain *cannot* interact with u (the domain for which is purged).

lemma *vpeq-ns-nt-not-ifp-u*:
assumes *purged-a-a2*: *purged-relation u execs execs2*
and *prec-s*: *precondition* (*next-state s execs*) (*next-action s execs*)
and *current-s-t*: *current s* = *current t'*
and *vpeq-s-t*: ∀ *v* . *ifp*^∗∗ *v u* ⟶ *vpeq v s t'*
shows ¬*ifp*^∗∗ (*current s*) *u* ∧ *precondition* (*next-state t' execs2*) (*next-action t' execs2*) ⟶ (∀ *v* . *ifp*^∗∗ *v u* ⟶ *vpeq v* (*step* (*next-state s execs*) (*next-action s execs*)) (*step* (*next-state t' execs2*) (*next-action t' execs2*)))
proof–
{
assume *not-ifp*: ¬*ifp*^∗∗ (*current s*) *u*
assume *prec-t*: *precondition* (*next-state t' execs2*) (*next-action t' execs2*)
fix *a a' v*
assume *ifp-v-u*: *ifp*^∗∗ *v u*
from *not-ifp* **and** *purged-a-a2* **have** ¬*ifp*^∗∗ (*current s*) *u* **unfolding** *purged-relation-def* **by** *auto*
from *this* **and** *ifp-v-u* **have** *not-ifp-curr-v*: ¬*ifp*^∗∗ (*current s*) *v* **using** *rtranclp-trans* **by** *metis*
from *this current-next-state*[*THEN spec*,*THEN spec*,**where** *x1=s* **and** *x=execs*] *prec-s vpeq-reflexive*
locally-respects[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x1=next-state s execs* **and** *x2=the* (*next-action s execs*) **and** *x=v*]
have *vpeq v* (*next-state s execs*) (*step* (*next-state s execs*) (*next-action s execs*))
unfolding *step-def precondition-def B-def*
by (*cases next-action s execs*,*auto*)
from *not-ifp-curr-v this locally-respects-next-state vpeq-transitive*
have *vpeq-s-ns*: *vpeq v s* (*step* (*next-state s execs*) (*next-action s execs*))
by *blast*
from *not-ifp-curr-v current-s-t current-next-state*[*THEN spec*,*THEN spec*,**where** *x1=t'* **and** *x=execs2*] *prec-t*
locally-respects[*THEN spec*,*THEN spec*,**where** *x=next-state t' execs2*] *vpeq-reflexive*
have *0*: *vpeq v* (*next-state t' execs2*) (*step* (*next-state t' execs2*) (*next-action t' execs2*))
unfolding *step-def precondition-def B-def*
by (*cases next-action t' execs2*,*auto*)
from *not-ifp-curr-v current-s-t current-next-state* **have** *1*: ¬*ifp*^∗∗ (*current t'*) *v*
using *rtranclp-trans* **by** *auto*
from *0 1 locally-respects-next-state vpeq-transitive*
have *vpeq-t-nt*: *vpeq v t'* (*step* (*next-state t' execs2*) (*next-action t' execs2*))
by *blast*
from *vpeq-s-ns* **and** *vpeq-t-nt* **and** *vpeq-s-t* **and** *ifp-v-u* **and** *vpeq-symmetric* **and** *vpeq-transitive*
have *vpeq-ns-nt*: *vpeq v* (*step* (*next-state s execs*) (*next-action s execs*)) (*step* (*next-state t' execs2*) (*next-action t' execs2*))
by *blast*
}
thus *?thesis* **by** *auto*
qed

A run with a purged list of actions appears identical to a run without purging, when starting from two states that appear identical.

lemma *unwinding-implies-view-partitioned-ind*:
shows *view-partitioned-ind*

```
proof–
{
 fix execs execs2 s t n u
  have equivalent-states s t u ∧ purged-relation u execs execs2 ⟶ equivalent-states (run n s execs) (run n t
execs2) u
 proof (induct n s execs arbitrary: t u execs2 rule: run.induct)
   case (1 s execs t u execs2)
    show ?case by auto
   next
   case (2 n execs t u execs2)
    show ?case by simp
   next
   case (3 n s execs t u execs2)
   assume interrupt-s: interrupt (Suc n)
   assume IH: (⋀t u execs2.
        equivalent-states (Some (cswitch (Suc n) s)) t u ∧ purged-relation u execs execs2 ⟶
        equivalent-states (run n (Some (cswitch (Suc n) s)) execs) (run n t execs2) u)
   {
    fix t'
    assume t = Some t'
    fix rs
    assume rs: run (Suc n) (Some s) execs = Some rs
    fix rt
    assume rt: run (Suc n) (Some t') execs2 = Some rt

    assume vpeq-s-t: ∀ v . ifp^** v u ⟶ vpeq v s t'
    assume current-s-t: current s = current t'
    assume purged-a-a2: purged-relation u execs execs2
```

— The following terminology is used: states rs and rt (for: run-s and run-t) are the states after a run. States ns and nt (for: next-s and next-t) are the states after one step.

— We prove two properties: the states rs and rt have equal active domains (current-rs-rt) and are vpeq for all domains v that may influence u (vpeq-rs-rt). Both are proven using the IH. To use the IH, we have to prove that the properties hold for the next step (in this case, a context switch). Statement current-ns-nt states that after one step states ns and nt have the same active domain. Statement vpeq-ns-nt states that after one step states ns and nt are vpeq for all domains v that may influence u (vpeq-rs-rt).

```
    from current-s-t cswitch-independent-of-state
     have current-ns-nt: current (cswitch (Suc n) s) = current (cswitch (Suc n) t') by blast
    from cswitch-consistency vpeq-s-t
     have vpeq-ns-nt: ∀ v . ifp^** v u ⟶ vpeq v (cswitch (Suc n) s) (cswitch (Suc n) t') by auto
     from current-ns-nt vpeq-ns-nt interrupt-s vpeq-reflexive purged-a-a2 current-s-t IH[where u=u and t=Some
(cswitch (Suc n) t') and ?execs2.0=execs2]
    have current-rs-rt: current rs = current rt using rs rt by(auto)
    {
     fix v
     assume ia: ifp^** v u
     from current-ns-nt vpeq-ns-nt ia interrupt-s vpeq-reflexive purged-a-a2 IH[where u=u and t=Some (cswitch
(Suc n) t') and ?execs2.0=execs2]
     have vpeq-rs-rt: vpeq v rs rt using rs rt by(auto)
    }
    from current-rs-rt and this have equivalent-states (Some rs) (Some rt) u by auto
   }
   thus ?case by(simp add:option.splits,cases t,simp+)
   next
   case (4 n execs s t u execs2)
   assume not-interrupt: ¬interrupt (Suc n)
   assume thread-empty-s: thread-empty(execs (current s))
```

assume *IH*: $(\bigwedge t\ u\ execs2.\ equivalent\text{-}states\ (Some\ s)\ t\ u \wedge purged\text{-}relation\ u\ execs\ execs2 \longrightarrow equivalent\text{-}states\ (run\ n\ (Some\ s)\ execs)\ (run\ n\ t\ execs2)\ u)$
{
fix t'
assume *t*: $t = Some\ t'$
fix *rs*
assume *rs*: $run\ (Suc\ n)\ (Some\ s)\ execs = Some\ rs$
fix *rt*
assume *rt*: $run\ (Suc\ n)\ (Some\ t')\ execs2 = Some\ rt$

assume *vpeq-s-t*: $\forall\ v\ .\ ifp\hat{}{**}\ v\ u \longrightarrow vpeq\ v\ s\ t'$
assume *current-s-t*: $current\ s = current\ t'$
assume *purged-a-a2*: *purged-relation u execs execs2*

— The following terminology is used: states rs and rt (for: run-s and run-t) are the states after a run. States ns and nt (for: next-s and next-t) are the states after one step.
— We prove two properties: the states rs and rt have equal active domains (current-rs-rt) and are vpeq for all domains v that may influence u (vpeq-rs-rt). Both are proven using the IH. To use the IH, we have to prove that the properties hold for the next step (in this case, nothing happens in s as the thread is empty). Statement current-ns-nt states that after one step states ns and nt have the same active domain. Statement $vpeq\_ns\_nt$ states that after one step states ns and nt are vpeq for all domains v that may influence u (vpeq-rs-rt).

from *ifp-reflexive* **and** *vpeq-s-t* **have** *vpeq-s-t-u*: $vpeq\ u\ s\ t'$ **by** *auto*
from *thread-empty-s* **and** *purged-a-a2* **and** *current-s-t* **have** *purged-a-na2*: $\neg ifp\hat{}{**}\ (current\ t')\ u \longrightarrow purged\text{-}relation\ u\ execs\ (next\text{-}execs\ t'\ execs2)$
by(*unfold next-execs-def ,unfold purged-relation-def ,auto*)
from *step-atomicity current-next-state current-s-t* **have** *current-s-nt*: $current\ s = current\ (step\ (next\text{-}state\ t'\ execs2)\ (next\text{-}action\ t'\ execs2))$
unfolding *step-def*
by (*cases next-action* t' *execs2,auto*)

— The proof is by case distinction. If the current thread is empty in state t as well (case t-empty), then nothing happens and the proof is trivial. Otherwise (case t-not-empty), since the current thread has different executions in states s and t, we now show that it cannot influence u (statement not-ifp-curr-t). If in state t the precondition holds (case t-prec), locally respects shows that the states remain vpeq. Otherwise, (case t-not-prec), everything holds vacuously.
have *current-rs-rt*: $current\ rs = current\ rt$
proof (*cases thread-empty*(*execs2* (*current* t')) *rule* :*case-split*[*case-names t-empty t-not-empty*])
case *t-empty*
from *purged-a-a2* **and** *vpeq-s-t* **and** *current-s-t IH*[**where** $t=Some\ t'$ **and** $u=u$ **and** *?execs2.0=execs2*]
have *equivalent-states* (*run n* (*Some s*) *execs*) (*run n* (*Some* t') *execs2*) *u* **using** *rs rt* **by**(*auto*)
from *this not-interrupt t-empty thread-empty-s*
show *?thesis* **using** *rs rt* **by**(*auto*)
next
case *t-not-empty*
from *t-not-empty current-next-state* **and** *vpeq-s-t-u* **and** *thread-empty-s* **and** *purged-a-a2* **and** *current-s-t*
have *not-ifp-curr-t*: $\neg ifp\hat{}{**}\ (current\ (next\text{-}state\ t'\ execs2))\ u$ **unfolding** *purged-relation-def* **by** *auto*
show *?thesis*
proof (*cases precondition* (*next-state* t' *execs2*) (*next-action* t' *execs2*) *rule* :*case-split*[*case-names t-prec t-not-prec*])
case *t-prec*
from *locally-respects-next-state current-next-state t-prec not-ifp-curr-t vpeq-s-t locally-respects vpeq-s-nt*
have *vpeq-s-nt*: $(\forall\ v\ .\ ifp\hat{}{**}\ v\ u \longrightarrow vpeq\ v\ s\ (step\ (next\text{-}state\ t'\ execs2)\ (next\text{-}action\ t'\ execs2)))$ **by** *auto*
from *vpeq-s-nt purged-a-na2 this current-s-nt not-ifp-curr-t current-next-state*
IH[**where** $t=Some\ (step\ (next\text{-}state\ t'\ execs2)\ (next\text{-}action\ t'\ execs2))$ **and** $u=u$ **and** *?execs2.0=next-execs* t' *execs2*]
have *equivalent-states* (*run n* (*Some s*) *execs*) (*run n* (*Some* (*step* (*next-state* t' *execs2*) (*next-action* t'

```
execs2))) (next-execs t' execs2)) u
      using rs rt by auto
     from t-not-empty t-prec vpeq-s-nt this thread-empty-s not-interrupt
      show ?thesis using rs rt  by auto
    next
    case t-not-prec
     thus ?thesis using rt t-not-empty not-interrupt by(auto)
    qed
   qed
   {
    fix v
    assume ia: ifp^** v u
    have vpeq v rs rt
    proof (cases thread-empty(execs2 (current t')) rule :case-split[case-names t-empty t-not-empty])
     case t-empty
      from purged-a-a2 and vpeq-s-t and current-s-t IH[where t=Some t' and u=u and ?execs2.0=execs2]
       have equivalent-states (run n (Some s) execs) (run n (Some t') execs2) u using rs rt by(auto)
      from ia this not-interrupt t-empty thread-empty-s
       show ?thesis using rs rt by(auto)
     next
     case t-not-empty
      show ?thesis
      proof (cases precondition (next-state t' execs2) (next-action t' execs2) rule :case-split[case-names t-prec
t-not-prec])
      case t-prec
       from t-not-empty current-next-state and vpeq-s-t-u and thread-empty-s and purged-a-a2 and current-s-t
        have not-ifp-curr-t: ¬ifp^** (current (next-state t' execs2)) u unfolding purged-relation-def
        by auto
          from t-prec current-next-state locally-respects-next-state this and vpeq-s-t and locally-respects and
vpeq-s-nt
        have vpeq-s-nt: (∀ v . ifp^** v u ⟶ vpeq v s (step (next-state t' execs2) (next-action t' execs2))) by
auto
       from purged-a-na2 this current-s-nt not-ifp-curr-t current-next-state
        IH[where t=Some (step (next-state t' execs2) (next-action t' execs2)) and u=u and ?execs2.0=next-execs
t' execs2]
        have equivalent-states (run n (Some s) execs) (run n (Some (step (next-state t' execs2) (next-action t'
execs2))) (next-execs t' execs2)) u
        using rs rt by(auto)
       from ia t-not-empty t-prec vpeq-s-nt this thread-empty-s not-interrupt
       show ?thesis using rs rt by auto
      next
      case t-not-prec
       thus ?thesis using rt t-not-empty not-interrupt by(auto)
      qed
    qed
   }
   from current-rs-rt and this have equivalent-states (Some rs) (Some rt) u by auto
  }
  thus ?case by(simp add:option.splits,cases t,simp+)
  next
  case (5 n execs s t u execs2)
  assume not-interrupt: ¬interrupt (Suc n)
  assume thread-not-empty-s: ¬thread-empty(execs (current s))
  assume not-prec-s: ¬ precondition (next-state s execs) (next-action s execs)
  — Whenever the precondition does not hold, the entire theorem flattens to True and everything holds vacuously.

  hence run (Suc n) (Some s) execs = None using not-interrupt thread-not-empty-s by simp
```

thus *?case* **by**(*simp add*:*option.splits*)
next
case (*6 n execs s t u execs2*)
assume *not-interrupt*: $\neg$*interrupt* (*Suc n*)
assume *thread-not-empty-s*: $\neg$*thread-empty*(*execs* (*current s*))
assume *prec-s*: *precondition* (*next-state s execs*) (*next-action s execs*)
assume *IH*: ($\bigwedge$*t u execs2*.
equivalent-states (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) *t u* $\wedge$
purged-relation u (*next-execs s execs*) *execs2* $\longrightarrow$
equivalent-states
(*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*))
(*run n t execs2*) *u*)
{
fix t'
assume *t*: *t* = *Some* t'
fix *rs*
assume *rs*: *run* (*Suc n*) (*Some s*) *execs* = *Some rs*
fix *rt*
assume *rt*: *run* (*Suc n*) (*Some* t') *execs2* = *Some rt*

assume *vpeq-s-t*: $\forall$ *v* . *ifp*^** *v u* $\longrightarrow$ *vpeq v s* t'
assume *current-s-t*: *current s* = *current* t'
assume *purged-a-a2*: *purged-relation u execs execs2*

— The following terminology is used: states rs and rt (for: run-s and run-t) are the states after a run. States ns and nt (for: next-s and next-t) are the states after one step.

— We prove two properties: the states rs and rt have equal active domains (current-rs-rt) and are vpeq for all domains v that may influence u (vpeq-rs-rt). Both are proven using the IH. To use the IH, we have to prove that the properties hold for the next step (in this case, state s executes an action). Statement current-ns-nt states that after one step states ns and nt have the same active domain. Statement vpeq-ns-nt states that after one step states ns and nt are vpeq for all domains v that may influence u (vpeq-rs-rt).

— Some lemma's used in the remainder of this case.
from *ifp-reflexive* **and** *vpeq-s-t* **have** *vpeq-s-t-u*: *vpeq u s* t' **by** *auto*
from *step-atomicity* **and** *current-s-t current-next-state*
have *current-ns-nt*: *current* (*step* (*next-state s execs*) (*next-action s execs*)) = *current* (*step* (*next-state* t' *execs2*) (*next-action* t' *execs2*))
unfolding *step-def*
by (*cases next-action s execs*,*cases next-action* t' *execs2*,*simp*,*simp*,*cases next-action* t' *execs2*,*simp*,*simp*)
from *vpeq-s-t* **have** *vpeq-curr-s-t*: *ifp*^** (*current s*) *u* $\longrightarrow$ *vpeq* (*current s*) *s* t' **by** *auto*
from *prec-s involved-ifp*[*THEN spec*,*THEN spec*,**where** *x1*=*next-state s execs* **and** *x*=*next-action s execs*] *vpeq-s-t* **have** *vpeq-involved*: *ifp*^** (*current s*) *u* $\longrightarrow$ ($\forall$ *d* $\in$ *involved* (*next-action s execs*) . *vpeq d s* t')
using *current-next-state*
unfolding *involved-def precondition-def B-def*
by(*cases next-action s execs*,*simp*,*auto*,*metis converse-rtranclp-into-rtranclp*)
from *current-s-t next-execs-consistent vpeq-curr-s-t vpeq-involved*
have *next-execs-t*: *ifp*^** (*current s*) *u* $\longrightarrow$ *next-execs* t' *execs* = *next-execs s execs*
unfolding *next-execs-def*
by(*auto*)
from *current-s-t purged-a-a2 thread-not-empty-s next-action-consistent*[*THEN spec*,*THEN spec*,**where** *x1*=*s* **and** *x*=t'] *vpeq-curr-s-t vpeq-involved*
have *next-action-s-t*: *ifp*^** (*current s*) *u* $\longrightarrow$ *next-action* t' *execs2* = *next-action s execs*
by(*unfold next-action-def* ,*unfold purged-relation-def* ,*auto*)
from *purged-a-a2 current-s-t next-execs-consistent*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2*=*s* **and** *x1*=t' **and** *x*=*execs*]
vpeq-curr-s-t vpeq-involved
have *purged-na-na2*: *purged-relation u* (*next-execs s execs*) (*next-execs* t' *execs2*)

unfolding *next-execs-def purged-relation-def*
by(*auto*)
from *purged-a-a2* **and** *purged-relation-def* **and** *thread-not-empty-s* **and** *current-s-t* **have** *thread-not-empty-t*: *ifp^** (current s) u ⟶ ¬thread-empty(execs2 (current t′))* **by** *auto*
from *step-atomicity current-s-t current-next-state* **have** *current-ns-t*: *current (step (next-state s execs) (next-action s execs)) = current t′*
unfolding *step-def*
by (*cases next-action s execs,auto*)
from *step-atomicity* **and** *current-s-t* **have** *current-s-nt*: *current s = current (step t′ (next-action t′ execs2))*
unfolding *step-def*
by (*cases next-action t′ execs2,auto*)
from *purged-a-a2* **have** *purged-na-a*: *¬ifp^** (current s) u ⟶ purged-relation u (next-execs s execs) execs2*
by(*unfold next-execs-def ,unfold purged-relation-def ,auto*)

— The proof is by case distinction. If the current domain can interact with u (case curr-ifp-u), then either in state t the precondition holds (case t-prec) or not. If it holds, then lemma vpeq-ns-nt-ifp-u applies. Otherwise, the proof is trivial as the theorem holds vacuously. If the domain cannot interact with u, (case curr-not-ifp-u), then lemma vpeq-ns-nt-not-ifp-u applies.
have *current-rs-rt*: *current rs = current rt*
proof (*cases ifp^** (current s) u rule :case-split[case-names curr-ifp-u curr-not-ifp-u]*)
case *curr-ifp-u*
show *?thesis*
proof (*cases precondition (next-state t′ execs2) (next-action t′ execs2) rule :case-split[case-names prec-t prec-not-t]*)
case *prec-t*
have *thread-not-empty-t*: *¬thread-empty(execs2 (current t′))* **using** *thread-not-empty-t curr-ifp-u* **by** *auto*
from
current-ns-nt next-execs-t next-action-s-t purged-a-a2
curr-ifp-u prec-t prec-s vpeq-ns-nt-ifp-u[**where** *a=(next-action s execs)*] *vpeq-s-t current-s-t*
have *equivalent-states (Some (step (next-state s execs) (next-action s execs))) (Some (step (next-state t′ execs2) (next-action t′ execs2))) u*
unfolding *purged-relation-def next-state-def*
by *auto*
from *this*
IH[**where** *u=u* **and** *?execs2.0=(next-execs t′ execs2)* **and** *t=Some (step (next-state t′ execs2) (next-action t′ execs2))*]
current-ns-nt purged-na-na2
have *equivalent-states (run n (Some (step (next-state s execs) (next-action s execs))) (next-execs s execs)) (run n (Some (step (next-state t′ execs2) (next-action t′ execs2))) (next-execs t′ execs2)) u*
by *auto*
from *prec-t thread-not-empty-t prec-s* **and** *this* **and** *not-interrupt* **and** *thread-not-empty-s* **and** *next-action-s-t*
show *?thesis* **using** *rs rt* **by** *auto*
next
case *prec-not-t*
from *curr-ifp-u prec-not-t thread-not-empty-t not-interrupt* **show** *?thesis* **using** *rt* **by** *simp*
qed
next
case *curr-not-ifp-u*
show *?thesis*
proof (*cases thread-empty(execs2 (current t′)) rule :case-split[case-names t-empty t-not-empty]*)
case *t-not-empty*
show *?thesis*
proof (*cases precondition (next-state t′ execs2) (next-action t′ execs2) rule :case-split[case-names t-prec t-not-prec]*)
case *t-prec*
from *curr-not-ifp-u t-prec IH*[**where** *u=u* **and** *?execs2.0=(next-execs t′ execs2)* **and** *t=Some (step (next-state t′ execs2) (next-action t′ execs2))*]

current-ns-nt next-execs-t purged-na-na2 vpeq-ns-nt-not-ifp-u current-s-t vpeq-s-t prec-s purged-a-a2
have *equivalent-states* (*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*))
(*run n* (*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))) (*next-execs t′ execs2*)) *u* **by** *auto*
from *this t-prec curr-not-ifp-u t-not-empty prec-s not-interrupt thread-not-empty-s* **show** *?thesis* **using** *rs rt* **by** *auto*
next
case *t-not-prec*
from *t-not-prec t-not-empty not-interrupt* **show** *?thesis* **using** *rt* **by** *simp*
qed
next
case *t-empty*
from *curr-not-ifp-u* **and** *prec-s* **and** *vpeq-s-t* **and** *locally-respects* **and** *vpeq-ns-t current-next-state locally-respects-next-state*
have *vpeq-ns-t*: (∀ *v* . *ifp*^∗∗ *v u* ⟶ *vpeq v* (*step* (*next-state s execs*) (*next-action s execs*)) *t′*)
by *blast*
from *curr-not-ifp-u IH*[**where** *t=Some t′* **and** *u=u* **and** *?execs2.0=execs2*] **and** *current-ns-t* **and** *next-execs-t* **and** *purged-na-a* **and** *vpeq-ns-t* **and** *this*
have *equivalent-states* (*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*))

(*run n* (*Some t′*) *execs2*) *u* **by** *auto*
from *this not-interrupt thread-not-empty-s t-empty prec-s* **show** *?thesis* **using** *rs rt* **by** *auto*
qed
qed
{
fix *v*
assume *ia*: *ifp*^∗∗ *v u*

have *vpeq v rs rt*
proof (*cases ifp*^∗∗ (*current s*) *u rule* :*case-split*[*case-names curr-ifp-u curr-not-ifp-u*])
case *curr-ifp-u*
show *?thesis*
proof (*cases precondition* (*next-state t′ execs2*) (*next-action t′ execs2*) *rule* :*case-split*[*case-names t-prec t-not-prec*])
case *t-prec*
have *thread-not-empty-t*: ¬*thread-empty*(*execs2* (*current t′*)) **using** *thread-not-empty-t curr-ifp-u* **by** *auto*
from
current-ns-nt next-execs-t next-action-s-t purged-a-a2
curr-ifp-u t-prec prec-s vpeq-ns-nt-ifp-u[**where** *a=*(*next-action s execs*)] *vpeq-s-t current-s-t*
have *equivalent-states* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))) *u*
unfolding *purged-relation-def next-state-def*
by *auto*
from *this*
IH[**where** *u=u* **and** *?execs2.0=*(*next-execs t′ execs2*) **and** *t=Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))]
current-ns-nt purged-na-na2
have *equivalent-states* (*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*))
(*run n* (*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))) (*next-execs t′ execs2*)) *u*
by *auto*
from *ia curr-ifp-u t-prec thread-not-empty-t prec-s* **and** *this* **and** *not-interrupt* **and** *thread-not-empty-s* **and** *next-action-s-t*
show *?thesis* **using** *rs rt* **by** *auto*
next
case *t-not-prec*

from *curr-ifp-u t-not-prec thread-not-empty-t not-interrupt* **show** *?thesis* **using** *rt* **by** *simp*
qed
next
case *curr-not-ifp-u*
show *?thesis*
proof (*cases thread-empty*(*execs2* (*current t′*)) *rule* :*case-split*[*case-names t-empty t-not-empty*])
case *t-not-empty*
show *?thesis*
proof (*cases precondition* (*next-state t′ execs2*) (*next-action t′ execs2*) *rule* :*case-split*[*case-names t-prec t-not-prec*])
case *t-prec*
from *curr-not-ifp-u t-prec IH*[**where** *u*=*u* **and** *?execs2.0*=(*next-execs t′ execs2*) **and** *t*=*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))]
current-ns-nt next-execs-t purged-na-na2 vpeq-ns-nt-not-ifp-u current-s-t vpeq-s-t prec-s purged-a-a2
have *equivalent-states* (*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*))
(*run n* (*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))) (*next-execs t′ execs2*)) *u* **by** *auto*
from *ia this t-prec curr-not-ifp-u t-not-empty prec-s not-interrupt thread-not-empty-s* **show** *?thesis* **using** *rs rt* **by** *auto*
next
case *t-not-prec*
from *t-not-prec t-not-empty not-interrupt* **show** *?thesis* **using** *rt* **by** *simp*
qed
next
case *t-empty*
from *curr-not-ifp-u prec-s* **and** *vpeq-s-t* **and** *locally-respects* **and** *vpeq-ns-t current-next-state locally-respects-next-state*
have *vpeq-ns-t*: (∀ *v* . *ifp*^** *v u* ⟶ *vpeq v* (*step* (*next-state s execs*) (*next-action s execs*)) *t′*)
by *blast*
from *curr-not-ifp-u IH*[**where** *t*=*Some t′* **and** *u*=*u* **and** *?execs2.0*=*execs2*] **and** *current-ns-t* **and** *next-execs-t* **and** *purged-na-a* **and** *vpeq-ns-t* **and** *this*
have *equivalent-states* (*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*))

(*run n* (*Some t′*) *execs2*) *u* **by** *auto*
from *ia this not-interrupt thread-not-empty-s t-empty prec-s* **show** *?thesis* **using** *rs rt* **by** *auto*
qed
qed
}
from *current-rs-rt* **and** *this* **have** *equivalent-states* (*Some rs*) (*Some rt*) *u* **by** *auto*
}
thus *?case* **by**(*simp add*:*option.splits*,*cases t*,*simp*+)
qed
}
thus *?thesis*
unfolding *view-partitioned-ind-def* **by** *auto*
qed

From the previous lemma, we can prove that the system is view partitioned. The previous lemma was inductive, this lemma just instantiates the previous lemma replacing s and t by the initial state.

lemma *unwinding-implies-view-partitioned*:
shows *view-partitioned*
proof–
from *assms unwinding-implies-view-partitioned-ind* **have** *view-partitioned-inductive*: *view-partitioned-ind*
by *blast*
have *purged-relation*: ∀ *u execs* . *purged-relation u execs* (*purge execs u*)
by(*unfold purged-relation-def*, *unfold purge-def*, *auto*)
{
fix *execs s t n u*

```
  assume 1: equivalent-states s t u
  from this view-partitioned-inductive purged-relation
    have equivalent-states (run n s execs) (run n t (purge execs u)) u
    unfolding view-partitioned-ind-def by auto
  from this ifp-reflexive
    have run n s execs ∥ run n t (purge execs u) ⇀ (λrs rt. vpeq u rs rt ∧ current rs = current rt)
    using r-into-rtranclp unfolding B-def
    by(cases run n s execs,simp,cases run n t (purge execs u),simp,auto)
}
thus ?thesis unfolding view-partitioned-def Let-def by auto
qed
```

Domains that many not interfere with each other, do not interfere with each other.

```
theorem unwinding-implies-NI-unrelated:
shows NI-unrelated
proof–
  {
    fix execs a n
    from assms unwinding-implies-view-partitioned
      have vp: view-partitioned by blast
    from vp and vpeq-reflexive
      have 1: ∀ u . (run n (Some s0) execs
                ∥ run n (Some s0) (purge execs u)
                  ⇀ (λrs rt. vpeq u rs rt ∧ current rs = current rt))
      unfolding view-partitioned-def by auto
     have run n (Some s0) execs ⇀ (λs-f. run n (Some s0) (purge execs (current s-f)) ⇀ (λs-f2. output-f s-f a =
output-f s-f2 a ∧ current s-f = current s-f2))
    proof(cases run n (Some s0) execs)
    case None
      thus ?thesis unfolding B-def by simp
    next
    case (Some rs)
      thus ?thesis
      proof(cases run n (Some s0) (purge execs (current rs)))
      case None
        from Some this show ?thesis unfolding B-def by simp
      next
      case (Some rt)
        from ‹run n (Some s0) execs = Some rs› Some 1[THEN spec,where x=current rs]
          have vpeq: vpeq (current rs) rs rt ∧ current rs = current rt
          unfolding B-def by auto
        from this output-consistent have output-f rs a = output-f rt a
           by auto
        from this vpeq ‹run n (Some s0) execs = Some rs› Some
          show ?thesis unfolding B-def by auto
      qed
    qed
  }
  thus ?thesis unfolding NI-unrelated-def by auto
qed
```

3.2.2 Security for indirectly interfering domains

Consider the following security policy over three domains A, B and C: $A \rightsquigarrow B \rightsquigarrow C$, but $A \not\rightsquigarrow C$. The semantics of this policy is that A may communicate with C, but *only* via B. No direct communication from A to C is allowed. We formalize these semantics as follows: without intermediate domain B, domain A cannot flow information to C. In other words, from the point of view of domain C the run where domain B is inactive must be equivalent to the run where domain B is inactive and domain A is replaced by an attacker. Domain C must be independent of domain A, when domain B is inactive.

The aim of this subsection is to formalize the semantics where A can write to C via B *only*. We define to two ipurge functions. The first purges all domains d that are *intermediary* for some other domain v. An intermediary for u is defined as a domain d for which there exists an information flow from some domain v to u via d, but no direct information flow from v to u is allowed.

definition *intermediary* :: *'dom-t ⇒ 'dom-t ⇒ bool*
where *intermediary d u ≡ ∃ v . ifp^** v d ∧ ifp d u ∧ ¬ifp v u ∧ d ≠ u*
primrec *remove-gateway-communications* :: *'dom-t ⇒ 'action-t execution ⇒ 'action-t execution*
where *remove-gateway-communications u* [] = []
| *remove-gateway-communications u* (*aseq#exec*) = (*if ∃ a ∈ set aseq . ∃ v . intermediary v u ∧ v ∈ involved* (*Some a*) *then* [] *else aseq*)#(*remove-gateway-communications u exec*)

definition *ipurge-l* ::
(*'dom-t ⇒ 'action-t execution*) ⇒ *'dom-t* ⇒ (*'dom-t ⇒ 'action-t execution*) **where**
ipurge-l execs u ≡ λ d . if intermediary d u then
[]
else if d = u then
remove-gateway-communications u (*execs u*)
else execs d

The second ipurge removes both the intermediaries and the *indirect sources*. An indirect source for u is defined as a domain that may indirectly flow information to u, but not directly.

abbreviation *ind-source* :: *'dom-t ⇒ 'dom-t ⇒ bool*
where *ind-source d u ≡ ifp^** d u ∧ ¬ifp d u*
definition *ipurge-r* ::
(*'dom-t ⇒ 'action-t execution*) ⇒ *'dom-t* ⇒ (*'dom-t ⇒ 'action-t execution*) **where**
ipurge-r execs u ≡ λ d . if intermediary d u then
[]
else if ind-source d u then
SOME alpha . realistic-execution alpha
else if d = u then
remove-gateway-communications u (*execs u*)
else
execs d

For a system with an intransitive policy to be called secure for domain u any indirect source may not flow information towards u when the intermediaries are purged out. This definition of security allows the information flow $A \rightsquigarrow B \rightsquigarrow C$, but prohibits $A \rightsquigarrow C$.

definition *NI-indirect-sources* ::*bool*
where *NI-indirect-sources*
≡ ∀ *execs a n. run n* (*Some s0*) *execs* ⇀
(λ *s-f* . (*run n* (*Some s0*) (*ipurge-l execs* (*current s-f*)) ‖
run n (*Some s0*) (*ipurge-r execs* (*current s-f*)) ⇀
(λ *s-l s-r* . *output-f s-l a = output-f s-r a*)))

This definition concerns indirect sources only. It does not enforce that an *unrelated* domain may not flow information to u. This is expressed by "secure".

This allows us to define security over intransitive policies.

definition *isecure*::*bool*
where *isecure ≡ NI-indirect-sources ∧ NI-unrelated*

abbreviation *iequivalent-states* :: *'state-t option ⇒ 'state-t option ⇒ 'dom-t ⇒ bool*
where *iequivalent-states s t u ≡ s* ‖ *t* ⇀ (λ *s t* . (∀ *v . ifp v u ∧ ¬intermediary v u ⟶ vpeq v s t*) ∧ *current s = current t*)

definition *does-not-communicate-with-gateway*
where *does-not-communicate-with-gateway u execs* ≡ ∀ *a* . *a* ∈ *actions-in-execution* (*execs u*) ⟶ (∀ *v* . *intermediary v u* ⟶ *v* ∉ *involved* (*Some a*))

definition *iview-partitioned*::*bool* **where** *iview-partitioned*
  ≡ ∀ *execs ms mt n u* . *iequivalent-states ms mt u* ⟶
    (*run n ms* (*ipurge-l execs u*) ∥
    run n mt (*ipurge-r execs u*) ⇀
    (λ *rs rt* . *vpeq u rs rt* ∧ *current rs* = *current rt*))

definition *ipurged-relation1* :: *'dom-t* ⇒ (*'dom-t* ⇒ *'action-t execution*) ⇒ (*'dom-t* ⇒ *'action-t execution*) ⇒ *bool*
where *ipurged-relation1 u execs1 execs2* ≡ ∀ *d* . (*ifp d u* ⟶ *execs1 d* = *execs2 d*) ∧ (*intermediary d u* ⟶ *execs1 d* = [])

Proof that if the current is not an intermediary for u, then all domains involved in the next action are vpeq.

lemma *vpeq-involved-domains*:
assumes *ifp-curr*: *ifp* (*current s*) *u*
  and *not-intermediary-curr*: ¬*intermediary* (*current s*) *u*
  and *no-gateway-comm*: *does-not-communicate-with-gateway u execs*
  and *vpeq-s-t*: ∀ *v* . *ifp v u* ∧ ¬*intermediary v u* ⟶ *vpeq v s t'*
  and *prec-s*: *precondition* (*next-state s execs*) (*next-action s execs*)
 shows ∀ *d* ∈ *involved* (*next-action s execs*) . *vpeq d s t'*
proof–
{
 fix *v*
 assume *involved*: *v* ∈ *involved* (*next-action s execs*)
 from *this prec-s involved-ifp*[*THEN spec*,*THEN spec*,**where** *x1*=*next-state s execs* **and** *x*=*next-action s execs*]
  have *ifp-v-curr*: *ifp v* (*current s*)
  using *current-next-state*
  unfolding *involved-def precondition-def B-def*
   by(*cases next-action s execs*,*auto*)
 have *vpeq v s t'*
 proof–
 {
  assume *ifp v u* ∧ ¬*intermediary v u*
  from *this vpeq-s-t*
   have *vpeq v s t'* **by** (*auto*)
 }
 moreover
 {
  assume *not-intermediary-v*: *intermediary v u*
  from *ifp-curr not-intermediary-curr ifp-v-curr not-intermediary-v* **have** *curr-is-u*: *current s* = *u*
   using *rtranclp-trans r-into-rtranclp*
   by (*metis intermediary-def*)
   from *curr-is-u next-action-from-execs*[*THEN spec*,*THEN spec*,**where** *x*=*execs* **and** *x1*=*s*] *not-intermediary-v involved*
   no-gateway-comm[*unfolded does-not-communicate-with-gateway-def*,*THEN spec*,**where** *x*=*the* (*next-action s execs*)]
   have *False*
   unfolding *involved-def B-def*
   by (*cases next-action s execs*,*auto*)
  hence *vpeq v s t'* **by** *auto*
 }
 moreover
 {

assume *intermediary-v*: ¬ *ifp v u*
from *ifp-curr not-intermediary-curr ifp-v-curr intermediary-v*
have *False* **unfolding** *intermediary-def* **by** *auto*
hence *vpeq v s t′* **by** *auto*
}
ultimately
show *vpeq v s t′* **unfolding** *intermediary-def* **by** *auto*
qed
}
thus *?thesis* **by** *auto*
qed

Proof that purging removes communications of the gateway to domain u.

lemma *ipurge-l-removes-gateway-communications*:
shows *does-not-communicate-with-gateway u* (*ipurge-l execs u*)
proof–
{
fix *aseq u execs a v*
assume *1*: *aseq* ∈ *set* (*remove-gateway-communications u* (*execs u*))
assume *2*: *a* ∈ *set aseq*
assume *3*: *intermediary v u*
have *4*: *v* ∉ *involved* (*Some a*)
proof–
{
fix *a*::′*action-t*
fix *aseq u exec v*
have *aseq* ∈ *set* (*remove-gateway-communications u exec*) ∧ *a* ∈ *set aseq* ∧ *intermediary v u* ⟶ *v* ∉ *involved* (*Some a*)
by(*induct exec*,*auto*)
}
from *1 2 3 this* **show** *?thesis* **by** *metis*
qed
}
from *this*
show *?thesis*
unfolding *does-not-communicate-with-gateway-def ipurge-l-def actions-in-execution-def*
by *auto*
qed

Proof of view partitioning. The lemma is structured exactly as lemma unwinding_implies_view_partitioned_ind and uses the same convention for naming.

lemma *iunwinding-implies-view-partitioned1*:
shows *iview-partitioned*
proof–
{
fix *u execs execs2 s t n*
have *does-not-communicate-with-gateway u execs* ∧ *iequivalent-states s t u* ∧ *ipurged-relation1 u execs execs2* ⟶ *iequivalent-states* (*run n s execs*) (*run n t execs2*) *u*
proof (*induct n s execs arbitrary*: *t u execs2 rule*: *run.induct*)
case (*1 s execs t u execs2*)
show *?case* **by** *auto*
next
case (*2 n execs t u execs2*)
show *?case* **by** *simp*
next
case (*3 n s execs t u execs2*)
assume *interrupt-s*: *interrupt* (*Suc n*)
assume *IH*: (⋀*t u execs2*. *does-not-communicate-with-gateway u execs* ∧

```
      iequivalent-states (Some (cswitch (Suc n) s)) t u ∧ ipurged-relation1 u execs execs2 ⟶
      iequivalent-states (run n (Some (cswitch (Suc n) s)) execs) (run n t execs2) u)
  {
   fix t' :: 'state-t
   assume t = Some t'
   fix rs
   assume rs: run (Suc n) (Some s) execs = Some rs
   fix rt
   assume rt: run (Suc n) (Some t') execs2 = Some rt

   assume no-gateway-comm: does-not-communicate-with-gateway u execs
   assume vpeq-s-t: ∀ v . ifp v u ∧ ¬intermediary v u ⟶ vpeq v s t'
   assume current-s-t: current s = current t'
   assume purged-a-a2: ipurged-relation1 u execs execs2

   from current-s-t cswitch-independent-of-state
    have current-ns-nt: current (cswitch (Suc n) s) = current (cswitch (Suc n) t')
    by blast
   from cswitch-consistency vpeq-s-t
    have vpeq-ns-nt: ∀ v . ifp v u ∧ ¬intermediary v u ⟶ vpeq v (cswitch (Suc n) s) (cswitch (Suc n) t')
    by auto
   from no-gateway-comm current-ns-nt vpeq-ns-nt interrupt-s vpeq-reflexive current-s-t purged-a-a2 IH[where
u=u and t=Some (cswitch (Suc n) t') and ?execs2.0=execs2]
    have current-rs-rt: current rs = current rt using rs rt by(auto)
   {
    fix v
    assume ia: ifp v u ∧ ¬intermediary v u
       from no-gateway-comm interrupt-s current-ns-nt vpeq-ns-nt vpeq-reflexive ia current-s-t purged-a-a2
IH[where u=u and t=Some (cswitch (Suc n) t') and ?execs2.0=execs2]
     have vpeq v rs rt using rs rt by(auto)
   }
   from current-rs-rt and this have iequivalent-states (Some rs) (Some rt) u by auto
  }
  thus ?case by(simp add:option.splits,cases t,simp+)
 next
 case (4 n execs s t u execs2)
  assume not-interrupt: ¬interrupt (Suc n)
  assume thread-empty-s: thread-empty(execs (current s))

   assume IH: (⋀t u execs2. does-not-communicate-with-gateway u execs ∧ iequivalent-states (Some s) t u ∧
ipurged-relation1 u execs execs2 ⟶ iequivalent-states (run n (Some s) execs) (run n t execs2) u)
  {
   fix t'

   assume t: t = Some t'
   fix rs
   assume rs: run (Suc n) (Some s) execs = Some rs
   fix rt
   assume rt: run (Suc n) (Some t') execs2 = Some rt

   assume no-gateway-comm: does-not-communicate-with-gateway u execs
   assume vpeq-s-t: ∀ v . ifp v u ∧ ¬intermediary v u ⟶ vpeq v s t'
   assume current-s-t: current s = current t'
   assume purged-a-a2: ipurged-relation1 u execs execs2

   from ifp-reflexive vpeq-s-t have vpeq-u-s-t: vpeq u s t' unfolding intermediary-def by auto
    from step-atomicity current-next-state current-s-t have current-s-nt: current s = current (step (next-state t'
```

execs2) (*next-action t′ execs2*))
unfolding *step-def*
by (*cases next-action s execs*,*cases next-action t′ execs2*,*simp*,*simp*,*cases next-action t′ execs2*,*simp*,*simp*)
from *vpeq-s-t* **have** *vpeq-curr-s-t*: *ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u* ⟶ *vpeq* (*current s*) *s t′* **by** *auto*
have *iequivalent-states* (*run* (*Suc n*) (*Some s*) *execs*) (*run* (*Suc n*) (*Some t′*) *execs2*) *u*
proof(*cases thread-empty*(*execs2* (*current t′*)))
case *True*
from *purged-a-a2* **and** *vpeq-s-t* **and** *current-s-t IH*[**where** *t*=*Some t′* **and** *u*=*u* **and** *?execs2.0*=*execs2*] *no-gateway-comm*
have *iequivalent-states* (*run n* (*Some s*) *execs*) (*run n* (*Some t′*) *execs2*) *u* **using** *rs rt* **by**(*auto*)
from *this not-interrupt True thread-empty-s*
show *?thesis* **using** *rs rt* **by**(*auto*)
next
case *False*
have *prec-t*: *precondition* (*next-state t′ execs2*) (*next-action t′ execs2*)
proof–
{
assume *not-prec-t*: ¬*precondition* (*next-state t′ execs2*) (*next-action t′ execs2*)
hence *run* (*Suc n*) (*Some t′*) *execs2* = *None* **using** *not-interrupt False not-prec-t* **by** (*simp*)
from *this* **have** *False* **using** *rt* **by**(*simp add*:*option.splits*)
}
thus *?thesis* **by** *auto*
qed

from *False purged-a-a2 thread-empty-s current-s-t*
have *1*: *ind-source* (*current t′*) *u* ∨ *unrelated* (*current t′*) *u* **unfolding** *ipurged-relation1-def intermediary-def* **by** *auto*
{
fix *v*
assume *ifp-v*: *ifp v u*
assume *v-not-intermediary*: ¬*intermediary v u*

from *1 ifp-v v-not-intermediary* **have** *not-ifp-curr-v*: ¬*ifp* (*current t′*) *v* **unfolding** *intermediary-def* **by** *auto*
from *not-ifp-curr-v prec-t locally-respects*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x1*=*next-state t′ execs2* **and** *x*=*v* **and** *x2*=*the* (*next-action t′ execs2*)]
current-next-state vpeq-reflexive
have *vpeq v* (*next-state t′ execs2*) (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))
unfolding *step-def precondition-def B-def*
by (*cases next-action t′ execs2*,*auto*)
from *this vpeq-transitive not-ifp-curr-v locally-respects-next-state*
have *vpeq-t-nt*: *vpeq v t′* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))
by *blast*
from *vpeq-s-t ifp-v v-not-intermediary vpeq-t-nt vpeq-transitive vpeq-symmetric vpeq-reflexive*
have *vpeq v s* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))
by (*metis*)
}
hence *vpeq-ns-nt*: ∀ *v* . *ifp v u* ∧ ¬*intermediary v u* ⟶ *vpeq v s* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*)) **by** *auto*
from *False purged-a-a2 current-s-t thread-empty-s* **have** *purged-a-na2*: *ipurged-relation1 u execs* (*next-execs t′ execs2*)
unfolding *ipurged-relation1-def next-execs-def* **by**(*auto*)
from *vpeq-ns-nt no-gateway-comm*
and *IH*[**where** *t*=*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*)) **and** *?execs2.0*=(*next-execs t′ execs2*) **and** *u*=*u*]
and *current-s-nt purged-a-na2*
have *eq-ns-nt*: *iequivalent-states* (*run n* (*Some s*) *execs*)

(*run n* (*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))) (*next-execs t′ execs2*)) *u* **by** *auto*
from *prec-t eq-ns-nt not-interrupt False thread-empty-s*
show *?thesis* **using** *t rs rt* **by**(*auto*)
qed
}
thus *?case* **by**(*simp add:option.splits,cases t,simp+*)
next
case (*5 n execs s t u execs2*)
assume *not-interrupt*: ¬*interrupt* (*Suc n*)
assume *thread-not-empty-s*: ¬*thread-empty*(*execs* (*current s*))
assume *not-prec-s*: ¬ *precondition* (*next-state s execs*) (*next-action s execs*)
hence *run* (*Suc n*) (*Some s*) *execs* = *None* **using** *not-interrupt thread-not-empty-s* **by** *simp*
thus *?case* **by**(*simp add:option.splits*)
next
case (*6 n execs s t u execs2*)
assume *not-interrupt*: ¬*interrupt* (*Suc n*)
assume *thread-not-empty-s*: ¬*thread-empty*(*execs* (*current s*))
assume *prec-s*: *precondition* (*next-state s execs*) (*next-action s execs*)
assume *IH*: (⋀*t u execs2*. *does-not-communicate-with-gateway u* (*next-execs s execs*) ∧
iequivalent-states (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) *t u* ∧
ipurged-relation1 u (*next-execs s execs*) *execs2* ⟶
iequivalent-states
(*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*))
(*run n t execs2*) *u*)
{
fix *t′*
assume *t*: *t* = *Some t′*
fix *rs*
assume *rs*: *run* (*Suc n*) (*Some s*) *execs* = *Some rs*
fix *rt*
assume *rt*: *run* (*Suc n*) (*Some t′*) *execs2* = *Some rt*

assume *no-gateway-comm*: *does-not-communicate-with-gateway u execs*
assume *vpeq-s-t*: ∀ *v* . *ifp v u* ∧ ¬*intermediary v u* ⟶ *vpeq v s t′*
assume *current-s-t*: *current s* = *current t′*
assume *purged-a-a2*: *ipurged-relation1 u execs execs2*

from *ifp-reflexive vpeq-s-t* **have** *vpeq-u-s-t*: *vpeq u s t′* **unfolding** *intermediary-def* **by** *auto*
from *step-atomicity* **and** *current-s-t current-next-state*
have *current-ns-nt*: *current* (*step* (*next-state s execs*) (*next-action s execs*)) = *current* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))
unfolding *step-def*
by (*cases next-action s execs,cases next-action t′ execs2,simp,simp,cases next-action t′ execs2,simp,simp*)

from *step-atomicity current-next-state current-s-t* **have** *current-ns-t*: *current* (*step* (*next-state s execs*) (*next-action s execs*)) = *current t′*
unfolding *step-def*
by (*cases next-action s execs,auto*)
from *vpeq-s-t* **have** *vpeq-curr-s-t*: *ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u* ⟶ *vpeq* (*current s*) *s t′* **unfolding** *intermediary-def* **by** *auto*
from *current-s-t purged-a-a2*
have *eq-execs*: *ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u* ⟶ *execs* (*current s*) = *execs2* (*current s*)
by(*auto simp add*: *ipurged-relation1-def*)
from *vpeq-involved-domains no-gateway-comm vpeq-s-t vpeq-involved-domains prec-s*
have *vpeq-involved*: *ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u* ⟶ (∀ *d* ∈ *involved* (*next-action s execs*) . *vpeq d s t′*)

by *blast*
from *current-s-t next-execs-consistent*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2=s* **and** *x1=t′* **and** *x=execs*] *vpeq-curr-s-t vpeq-involved*
have *next-execs-t*: *ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u* ⟶ *next-execs t′ execs* = *next-execs s execs*
by(*auto simp add*: *next-execs-def*)
from *current-s-t* **and** *purged-a-a2* **and** *thread-not-empty-s next-action-consistent*[*THEN spec*,*THEN spec*,**where** *x1=s* **and** *x=t′*] *vpeq-curr-s-t vpeq-involved*
have *next-action-s-t*: *ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u* ⟶ *next-action t′ execs2* = *next-action s execs*
by(*unfold next-action-def*,*unfold ipurged-relation1-def*,*auto*)
from *purged-a-a2* **and** *thread-not-empty-s* **and** *current-s-t*
have *thread-not-empty-t*: *ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u* ⟶ ¬*thread-empty*(*execs2* (*current t′*))
unfolding *ipurged-relation1-def* **by** *auto*
have *vpeq-ns-nt-1*: ⋀ *a* . *precondition* (*next-state s execs*) *a* ∧ *precondition* (*next-state t′ execs*) *a* ⟹ *ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u* ⟹ (∀ *v* . *ifp v u* ∧ ¬*intermediary v u* ⟶ *vpeq v* (*step* (*next-state s execs*) *a*) (*step* (*next-state t′ execs*) *a*))
proof–
fix *a*
assume *precs*: *precondition* (*next-state s execs*) *a* ∧ *precondition* (*next-state t′ execs*) *a*
assume *ifp-curr*: *ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u*
from *ifp-curr precs*
next-state-consistent[*THEN spec*,*THEN spec*,**where** *x1=s* **and** *x=t′*] *vpeq-curr-s-t vpeq-s-t*
current-next-state current-s-t weakly-step-consistent[*THEN spec*,*THEN spec*,*THEN spec*,*THEN spec*,**where** *x3=next-state s execs* **and** *x2=next-state t′ execs* **and** *x=the a*]
show ∀ *v* . *ifp v u* ∧ ¬*intermediary v u* ⟶ *vpeq v* (*step* (*next-state s execs*) *a*) (*step* (*next-state t′ execs*) *a*)
unfolding *step-def precondition-def B-def*
by (*cases a*,*auto*)
qed
have *no-gateway-comm-na*: *does-not-communicate-with-gateway u* (*next-execs s execs*)
proof–
{
fix *a*
assume *a* ∈ *actions-in-execution* (*next-execs s execs u*)
from *this no-gateway-comm*[*unfolded does-not-communicate-with-gateway-def*,*THEN spec*,**where** *x=a*]
next-execs-subset[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2=s* **and** *x1=execs* **and** *x0=u*]
have ∀ *v*. *intermediary v u* ⟶ *v* ∉ *involved* (*Some a*)
unfolding *actions-in-execution-def*
by(*auto*)
}
thus *?thesis* **unfolding** *does-not-communicate-with-gateway-def* **by** *auto*
qed
have *iequivalent-states* (*run* (*Suc n*) (*Some s*) *execs*) (*run* (*Suc n*) (*Some t′*) *execs2*) *u*
proof (*cases ifp* (*current s*) *u* ∧ ¬*intermediary* (*current s*) *u rule* :*case-split*[*case-names T F*])
case *T*
show *?thesis*
proof (*cases thread-empty*(*execs2* (*current t′*)) *rule* :*case-split*[*case-names T2 F2*])
case *F2*
show *?thesis*
proof (*cases precondition* (*next-state t′ execs2*) (*next-action t′ execs2*) *rule* :*case-split*[*case-names T3 F3*])

case *T3*
from *T purged-a-a2 current-s-t*
next-execs-consistent[*THEN spec*,*THEN spec*,**where** *x1=s* **and** *x=t′*] *vpeq-curr-s-t vpeq-involved*
have *purged-na-na2*: *ipurged-relation1 u* (*next-execs s execs*) (*next-execs t′ execs2*)
unfolding *ipurged-relation1-def next-execs-def*
by *auto*

from *IH*[**where** *t=Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*)) **and** *?execs2.0=next-execs t′ execs2* **and** *u=u*]
purged-na-na2 current-ns-nt vpeq-ns-nt-1[**where** *a*=(*next-action s execs*)] *T T3 prec-s*
next-action-s-t eq-execs current-s-t no-gateway-comm-na
have *eq-ns-nt*: *iequivalent-states* (*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*))
(*run n* (*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))) (*next-execs t′ execs2*)) *u*
unfolding *next-state-def*
by (*auto*,*metis*)
from *this not-interrupt thread-not-empty-s prec-s F2 T3*
have *current-rs-rt*: *current rs* = *current rt* **using** *rs rt* **by** *auto*
{
fix *v*
assume *ia*: *ifp v u* ∧ ¬*intermediary v u*
from *this eq-ns-nt not-interrupt thread-not-empty-s prec-s F2 T3*
have *vpeq v rs rt* **using** *rs rt* **by** *auto*
}
from *this* **and** *current-rs-rt* **show** *?thesis* **using** *rs rt* **by** *auto*
next
case *F3*
from *F3 F2 not-interrupt* **show** *?thesis* **using** *rt* **by** *simp*
qed
next
case *T2*
from *T2 T purged-a-a2 thread-not-empty-s current-s-t prec-s next-action-s-t vpeq-u-s-t*
have *ind-source*: *False* **unfolding** *ipurged-relation1-def* **by** *auto*
thus *?thesis* **by** *auto*
qed
next
case *F*
hence *1*: *ind-source* (*current s*) *u* ∨ *unrelated* (*current s*) *u* ∨ *intermediary* (*current s*) *u*
unfolding *intermediary-def*
by *auto*
from *purged-a-a2* **and** *thread-not-empty-s*
have *2*: ¬*intermediary* (*current s*) *u* **unfolding** *ipurged-relation1-def* **by** *auto*

let *?nt* = *if thread-empty*(*execs2* (*current t′*)) *then t′ else step* (*next-state t′ execs2*) (*next-action t′ execs2*)
let *?na2* = *if thread-empty*(*execs2* (*current t′*)) *then execs2 else next-execs t′ execs2*

have *prec-t*: ¬*thread-empty*(*execs2* (*current t′*)) ⟹ *precondition* (*next-state t′ execs2*) (*next-action t′ execs2*)
proof–
assume *thread-not-empty-t*: ¬*thread-empty*(*execs2* (*current t′*))
{
assume *not-prec-t*: ¬*precondition* (*next-state t′ execs2*) (*next-action t′ execs2*)
hence *run* (*Suc n*) (*Some t′*) *execs2* = *None* **using** *not-interrupt thread-not-empty-t not-prec-t* **by** (*simp*)
from *this* **have** *False* **using** *rt* **by**(*simp add:option.splits*)
}
thus *?thesis* **by** *auto*
qed

show *?thesis*
proof–
{
fix *v*
assume *ifp-v*: *ifp v u*

assume *v-not-intermediary*: ¬*intermediary v u*

have *not-ifp-curr-v*: ¬*ifp* (*current s*) *v*
proof
assume *ifp-curr-v*: *ifp* (*current s*) *v*
thus *False*
proof–
{
assume *ind-source* (*current s*) *u*
from *this ifp-curr-v ifp-v* **have** *intermediary v u* **unfolding** *intermediary-def* **by** *auto*
from *this v-not-intermediary* **have** *False* **unfolding** *intermediary-def* **by** *auto*
}
moreover
{
assume *unrelated*: *unrelated* (*current s*) *u*
from *this ifp-v ifp-curr-v* **have** *False* **using** *rtranclp-trans r-into-rtranclp* **by** *metis*
}
ultimately show *?thesis* **using** *1 2* **by** *auto*
qed
qed
from *this current-next-state*[*THEN spec*,*THEN spec*,**where** *x1=s* **and** *x=execs*] *prec-s*
locally-respects[*THEN spec*,*THEN spec*,**where** *x=next-state s execs*] *vpeq-reflexive*
have *vpeq v* (*next-state s execs*) (*step* (*next-state s execs*) (*next-action s execs*))
unfolding *step-def precondition-def B-def*
by (*cases next-action s execs*,*auto*)
from *not-ifp-curr-v this locally-respects-next-state vpeq-transitive*
have *vpeq-s-ns*: *vpeq v s* (*step* (*next-state s execs*) (*next-action s execs*))
by *blast*
from *not-ifp-curr-v current-s-t current-next-state*[*THEN spec*,*THEN spec*,**where** *x1=t′* **and** *x=execs2*] *prec-t*
locally-respects[*THEN spec*,*THEN spec*,**where** *x=next-state t′ execs2*]
F vpeq-reflexive
have *0*: ¬ *thread-empty* (*execs2* (*current t′*)) ⟶ *vpeq v* (*next-state t′ execs2*) (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))
unfolding *step-def precondition-def B-def*
by (*cases next-action t′ execs2*,*auto*)
from *0 not-ifp-curr-v current-s-t locally-respects-next-state*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2=t′* **and** *x1=v* **and** *x=execs2*]
vpeq-transitive
have *vpeq-t-nt*: ¬ *thread-empty* (*execs2* (*current t′*)) ⟶ *vpeq v t′* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*)) **by** *metis*
from *this vpeq-reflexive*
have *vpeq-t-nt*: *vpeq v t′ ?nt*
by *auto*
from *vpeq-s-t ifp-v v-not-intermediary*
have *vpeq v s t′* **by** *auto*
from *this vpeq-s-ns vpeq-t-nt vpeq-transitive vpeq-symmetric vpeq-reflexive*
have *vpeq v* (*step* (*next-state s execs*) (*next-action s execs*)) *?nt*
by (*metis* (*hide-lams*, *no-types*))
}
hence *vpeq-ns-nt*: ∀ *v* . *ifp v u* ∧ ¬*intermediary v u* ⟶ *vpeq v* (*step* (*next-state s execs*) (*next-action s execs*)) *?nt* **by** *auto*
from *vpeq-s-t 2 F purged-a-a2 current-s-t thread-not-empty-s* **have** *purged-na-na2*: *ipurged-relation1 u* (*next-execs s execs*) *?na2*
unfolding *ipurged-relation1-def next-execs-def intermediary-def* **by**(*auto*)
from *current-ns-nt current-ns-t current-next-state* **have** *current-ns-nt*:
current (*step* (*next-state s execs*) (*next-action s execs*)) = *current ?nt*
by *auto*

from *prec-s vpeq-ns-nt no-gateway-comm-na*
and *IH*[**where** *t=Some ?nt* **and** *?execs2.0=?na2* **and** *u=u*]
and *current-ns-nt purged-na-na2*
have *eq-ns-nt*: *iequivalent-states* (*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*))
(*run n* (*Some ?nt*) *?na2*) *u* **by** *auto*

from *this not-interrupt thread-not-empty-s prec-t prec-s*
have *current-rs-rt*: *current rs* = *current rt* **using** *rs rt* **by** (*cases thread-empty* (*execs2* (*current t'*)),*simp*,*simp*)
{
fix *v*
assume *ia*: *ifp v u* ∧ ¬*intermediary v u*
from *this eq-ns-nt not-interrupt thread-not-empty-s prec-s prec-t*
have *vpeq v rs rt*
using *rs rt* **by** (*cases thread-empty*(*execs2* (*current t'*)),*simp*,*simp*)
}
from *current-rs-rt* **and** *this* **show** *?thesis* **using** *rs rt* **by** *auto*
qed
qed
}
thus *?case* **by**(*simp add*:*option.splits*,*cases t*,*simp+*)
qed
}
hence *iview-partitioned-inductive*: ∀ *u s t execs execs2 n*. *does-not-communicate-with-gateway u execs* ∧ *iequivalent-states s t u* ∧ *ipurged-relation1 u execs execs2* ⟶ *iequivalent-states* (*run n s execs*) (*run n t execs2*) *u*
by *blast*
have *ipurged-relation*: ∀ *u execs* . *ipurged-relation1 u* (*ipurge-l execs u*) (*ipurge-r execs u*)
by(*unfold ipurged-relation1-def* ,*unfold ipurge-l-def* ,*unfold ipurge-r-def* ,*auto*)
{
fix *execs s t n u*
assume *1*: *iequivalent-states s t u*
from *ifp-reflexive*
have *dir-source*: ∀ *u* . *ifp u u* ∧ ¬*intermediary u u* **unfolding** *intermediary-def* **by** *auto*
from *ipurge-l-removes-gateway-communications*
have *does-not-communicate-with-gateway u* (*ipurge-l execs u*)
by *auto*
from *1 this iview-partitioned-inductive ipurged-relation*
have *iequivalent-states* (*run n s* (*ipurge-l execs u*)) (*run n t* (*ipurge-r execs u*)) *u* **by** *auto*
from *this dir-source*
have *run n s* (*ipurge-l execs u*) ∥ *run n t* (*ipurge-r execs u*) ⇀ (λ*rs rt*. *vpeq u rs rt* ∧ *current rs* = *current rt*)
using *r-into-rtranclp* **unfolding** *B-def*
by(*cases run n s* (*ipurge-l execs u*),*simp*,*cases run n t* (*ipurge-r execs u*),*simp*,*auto*)
}
thus *?thesis* **unfolding** *iview-partitioned-def Let-def* **by** *auto*
qed

Returns True iff and only if the two states have the same active domain, *or* if one of the states is None.

definition *mcurrents* :: *'state-t option* ⇒ *'state-t option* ⇒ *bool*
where *mcurrents m1 m2* ≡ *m1* ∥ *m2* ⇀ (λ *s t* . *current s* = *current t*)

Proof that switching/interrupts are purely time-based and happen independent of the actions done by the domains. As all theorems in this locale, it holds vacuously whenever one of the states is None, i.e., whenver at some point a precondition does not hold.

lemma *current-independent-of-domain-actions*:
assumes *current-s-t*: *mcurrents s t*

```
 shows mcurrents (run n s execs) (run n t execs2)
proof-
{
 fix n s execs t execs2
 have mcurrents s t ⟶ mcurrents (run n s execs) (run n t execs2)
 proof (induct n s execs arbitrary: t execs2 rule: run.induct)
 case (1 s execs t execs2)
  from this show ?case using current-s-t unfolding B-def by auto
 next
 case (2 n execs t execs2)
  show ?case unfolding mcurrents-def by(auto)
 next
 case (3 n s execs t execs2)
  assume interrupt: interrupt (Suc n)
  assume IH: (⋀t execs2. mcurrents (Some (cswitch (Suc n) s)) t ⟶ mcurrents (run n (Some (cswitch (Suc n)
s)) execs) (run n t execs2))
  {
   fix t′
   assume t: t = (Some t′)
   assume curr: mcurrents (Some s) t
   from t curr cswitch-independent-of-state[THEN spec,THEN spec,THEN spec,where x1=s] have current-ns-nt:
current (cswitch (Suc n) s) = current (cswitch (Suc n) t′)
     unfolding mcurrents-def by simp
   from current-ns-nt IH[where t=Some (cswitch (Suc n) t′) and ?execs2.0=execs2]
     have mcurrents-ns-nt: mcurrents (run n (Some (cswitch (Suc n) s)) execs) (run n (Some (cswitch (Suc n)
t′)) execs2)
    unfolding mcurrents-def by(auto)
   from mcurrents-ns-nt interrupt t
    have mcurrents (run (Suc n) (Some s) execs) (run (Suc n) t execs2)
    unfolding mcurrents-def B2-def B-def by(cases run n (Some (cswitch (Suc n) s)) execs, cases run (Suc n) t
execs2,auto)
  }
  thus ?case unfolding mcurrents-def B2-def by(cases t,auto)
 next
 case (4 n execs s t execs2)
  assume not-interrupt: ¬interrupt (Suc n)
  assume thread-empty-s: thread-empty(execs (current s))
  assume IH: (⋀t execs2. mcurrents (Some s) t ⟶ mcurrents (run n (Some s) execs) (run n t execs2))
  {
   fix t′
   assume t: t = (Some t′)
   assume curr: mcurrents (Some s) t
   {
    assume thread-empty-t: thread-empty(execs2 (current t′))
    from t curr not-interrupt thread-empty-s this IH[where ?execs2.0=execs2 and t=Some t′]
     have mcurrents (run (Suc n) (Some s) execs) (run (Suc n) t execs2)
     by auto
   }
   moreover
   {
    assume not-prec-t: ¬thread-empty(execs2 (current t′)) ∧ ¬precondition (next-state t′ execs2) (next-action t′
execs2)
    from t this not-interrupt
     have mcurrents (run (Suc n) (Some s) execs) (run (Suc n) t execs2)
     unfolding mcurrents-def by (simp add: rewrite-B2-cases)
   }
   moreover
```

{
assume *step-t*: ¬*thread-empty*(*execs2* (*current t′*)) ∧ *precondition* (*next-state t′ execs2*) (*next-action t′ execs2*)
have *mcurrents* (*Some s*) (*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*)))
using *step-atomicity curr t current-next-state* **unfolding** *mcurrents-def*
unfolding *step-def*
by (*cases next-action t′ execs2,auto*)
from *t step-t curr not-interrupt thread-empty-s this IH*[**where** *?execs2.0=next-execs t′ execs2* **and** *t=Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))]
have *mcurrents* (*run* (*Suc n*) (*Some s*) *execs*) (*run* (*Suc n*) *t execs2*)
by *auto*
}
ultimately have *mcurrents* (*run* (*Suc n*) (*Some s*) *execs*) (*run* (*Suc n*) *t execs2*) **by** *blast*
}
thus *?case* **unfolding** *mcurrents-def B2-def* **by**(*cases t,auto*)
next
case (*5 n execs s t execs2*)
assume *not-interrupt-s*: ¬*interrupt* (*Suc n*)
assume *thread-not-empty-s*: ¬*thread-empty*(*execs* (*current s*))
assume *not-prec-s*: ¬ *precondition* (*next-state s execs*) (*next-action s execs*)
hence *run* (*Suc n*) (*Some s*) *execs* = *None* **using** *not-interrupt-s thread-not-empty-s* **by** *simp*
thus *?case* **unfolding** *mcurrents-def* **by**(*simp add*:*option.splits*)
next
case (*6 n execs s t execs2*)
assume *not-interrupt*: ¬*interrupt* (*Suc n*)
assume *thread-not-empty-s*: ¬*thread-empty*(*execs* (*current s*))
assume *prec-s*: *precondition* (*next-state s execs*) (*next-action s execs*)
assume *IH*: (⋀*t execs2*.
mcurrents (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) *t* ⟶
mcurrents (*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*)) (*run n t execs2*))
{
fix *t′*
assume *t*: *t* = (*Some t′*)
assume *curr*: *mcurrents* (*Some s*) *t*
{
assume *thread-empty-t*: *thread-empty*(*execs2* (*current t′*))
have *mcurrents* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*Some t′*)
using *step-atomicity curr t current-next-state* **unfolding** *mcurrents-def*
unfolding *step-def*
by (*cases next-action s execs,auto*)
from *t curr not-interrupt thread-not-empty-s prec-s thread-empty-t this IH*[**where** *?execs2.0=execs2* **and** *t=Some t′*]
have *mcurrents* (*run* (*Suc n*) (*Some s*) *execs*) (*run* (*Suc n*) *t execs2*)
by *auto*
}
moreover
{
assume *not-prec-t*: ¬*thread-empty*(*execs2* (*current t′*)) ∧ ¬*precondition* (*next-state t′ execs2*) (*next-action t′ execs2*)
from *t this not-interrupt*
have *mcurrents* (*run* (*Suc n*) (*Some s*) *execs*) (*run* (*Suc n*) *t execs2*)
unfolding *mcurrents-def B2-def* **by** (*auto*)
}
moreover
{
assume *step-t*: ¬*thread-empty*(*execs2* (*current t′*)) ∧ *precondition* (*next-state t′ execs2*) (*next-action t′*

execs2)
have *mcurrents* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*)))
using *step-atomicity curr t current-next-state* **unfolding** *mcurrents-def*
unfolding *step-def*
by (*cases next-action s execs,simp,cases next-action t′ execs2,simp,simp,cases next-action t′ execs2,simp,simp*)
from *current-next-state t step-t curr not-interrupt thread-not-empty-s prec-s this IH*[**where** *?execs2.0=next-execs t′ execs2* **and** *t=Some* (*step* (*next-state t′ execs2*) (*next-action t′ execs2*))]
have *mcurrents* (*run* (*Suc n*) (*Some s*) *execs*) (*run* (*Suc n*) *t execs2*)
by *auto*
}
ultimately have *mcurrents* (*run* (*Suc n*) (*Some s*) *execs*) (*run* (*Suc n*) *t execs2*) **by** *blast*
}
thus *?case* **unfolding** *mcurrents-def B2-def* **by**(*cases t,auto*)
qed
}
thus *?thesis* **using** *current-s-t* **by** *auto*
qed

theorem *unwinding-implies-NI-indirect-sources*:
shows *NI-indirect-sources*
proof–
{
fix *execs a n*
from *assms iunwinding-implies-view-partitioned1*
have *vp*: *iview-partitioned* **by** *blast*
from *vp* **and** *vpeq-reflexive*
have *1*: ∀ *u* . *run n* (*Some s0*) (*ipurge-l execs u*) ∥ *run n* (*Some s0*) (*ipurge-r execs u*) ⇀ (*λrs rt*. *vpeq u rs rt* ∧ *current rs* = *current rt*)
unfolding *iview-partitioned-def* **by** *auto*

have *run n* (*Some s0*) *execs* ⇀ (*λs-f* . *run n* (*Some s0*) (*ipurge-l execs* (*current s-f*)) ∥
run n (*Some s0*) (*ipurge-r execs* (*current s-f*)) ⇀
(*λs-l s-r*. *output-f s-l a* = *output-f s-r a*))
proof(*cases run n* (*Some s0*) *execs*)
case *None*
thus *?thesis* **unfolding** *B-def* **by** *simp*
next
case (*Some s-f*)
thus *?thesis*
proof(*cases run n* (*Some s0*) (*ipurge-l execs* (*current s-f*)))
case *None*
from *Some this* **show** *?thesis* **unfolding** *B-def* **by** *simp*
next
case (*Some s-ipurge-l*)
show *?thesis*
proof(*cases run n* (*Some s0*) (*ipurge-r execs* (*current s-f*)))
case *None*
from ‹*run n* (*Some s0*) *execs* = *Some s-f*› *Some this* **show** *?thesis* **unfolding** *B-def* **by** *simp*
next
case (*Some s-ipurge-r*)
from *cswitch-independent-of-state*
‹*run n* (*Some s0*) *execs* = *Some s-f*› ‹*run n* (*Some s0*) (*ipurge-l execs* (*current s-f*)) = *Some s-ipurge-l*›
current-independent-of-domain-actions[**where** *n=n* **and** *s=Some s0* **and** *t=Some s0* **and** *execs=execs* **and** *?execs2.0=*(*ipurge-l execs* (*current s-f*))]
have *2*: *current s-ipurge-l* = *current s-f*
unfolding *mcurrents-def B-def* **by** *auto*

from ‹*run n* (*Some s0*) *execs* = *Some s-f*› ‹*run n* (*Some s0*) (*ipurge-l execs* (*current s-f*)) = *Some s-ipurge-l*›
Some 1[*THEN spec*,**where** *x*=*current s-f*]
have *vpeq* (*current s-f*) *s-ipurge-l s-ipurge-r* ∧ *current s-ipurge-l* = *current s-ipurge-r*
unfolding *B-def* **by** *auto*
from *this 2* **have** *output-f s-ipurge-l a* = *output-f s-ipurge-r a*
using *output-consistent* **by** *auto*
from ‹*run n* (*Some s0*) *execs* = *Some s-f*› ‹*run n* (*Some s0*) (*ipurge-l execs* (*current s-f*)) = *Some s-ipurge-l*›
this Some
show *?thesis* **unfolding** *B-def* **by** *auto*
qed
qed
qed
}
thus *?thesis* **unfolding** *NI-indirect-sources-def* **by** *auto*
qed

theorem *unwinding-implies-isecure*:
shows *isecure*
using *unwinding-implies-NI-indirect-sources unwinding-implies-NI-unrelated assms* **unfolding** *isecure-def* **by**(*auto*)

end
end

3.3 ISK (Interruptible Separation Kernel)

theory *ISK*
imports *SK*
begin

At this point, the precondition linking action to state is generic and highly unconstrained. We refine the previous locale by given generic functions "precondition" and "realistic_trace" a definiton. This yields a total run function, instead of the partial one of locale Separation_Kernel.

This definition is based on a set of valid action sequences AS_set. Consider for example the following action sequence:

$$\gamma = [COPY\_INIT, COPY\_CHECK, COPY\_COPY]$$

If action sequence γ is a member of AS_set, this means that the attack surface contains an action COPY, which consists of three consecutive atomic kernel actions. Interrupts can occur anywhere between these atomic actions.

Given a set of valid action sequences such as γ, generic function precondition can be defined. It now consists of 1.) a generic invariant and 2.) more refined preconditions for the current action.

These preconditions need to be proven inductive only according to action sequences. Assume, e.g., that $\gamma \in$ AS_set and that d is the currently active domain in state s. The following constraints are assumed and must therefore be proven for the instantiation:

- "AS_precondition s d COPY_INIT"
  since COPY_INIT is the start of an action sequence.
- "AS_precondition (step s COPY_INIT) d COPY_CHECK"
  since (COPY_INIT, COPY_CHECK) is a sub sequence.
- "AS_precondition (step s COPY_CHECK) d COPY_COPY"
  since (COPY_CHECK, COPY_COPY) is a sub sequence.

Additionally, the precondition for domain d must be consistent when a context switch occurs, or when ever some other domain d' performs an action.

Locale Interruptible_Separation_Kernel refines locale Separation_Kernel in two ways. First, there is a definition of realistic executions. A realistic trace consists of action sequences from AS_set.

Secondly, the generic *control* function has been refined by additional assumptions. It is now assumed that control conforms to one of four possibilities:

1. The execution of the currently active domain is empty and the control function returns no action.
2. The currently active domain is executing the action sequence at the head of the execution. It returns the next kernel action of this sequence and updates the execution accordingly.
3. The action sequence is delayed.
4. The action sequence that is at the head of the execution is skipped and the execution is updated accordingly.

As for the state update, this is still completely unconstrained and generic as long as it respects the generic invariant and the precondition.

locale *Interruptible-Separation-Kernel* = *Separation-Kernel kstep output-f s0 current cswitch interrupt kprecondition realistic-execution control kinvolved ifp vpeq*
for *kstep* :: *'state-t* ⇒ *'action-t* ⇒ *'state-t*
and *output-f* :: *'state-t* ⇒ *'action-t* ⇒ *'output-t*
and *s0* :: *'state-t*
and *current* :: *'state-t* => *'dom-t* — Returns the currently active domain
and *cswitch* :: *time-t* ⇒ *'state-t* ⇒ *'state-t* — Switches the current domain
and *interrupt* :: *time-t* ⇒ *bool* — Returns t iff an interrupt occurs in the given state at the given time
and *kprecondition* :: *'state-t* ⇒ *'action-t* ⇒ *bool* — Returns t if an precondition holds that relates the current action to the state
and *realistic-execution* :: *'action-t execution* ⇒ *bool* — In this locale, this function is completely unconstrained.
and *control* :: *'state-t* ⇒ *'dom-t* ⇒ *'action-t execution* ⇒ ((*'action-t option*) × *'action-t execution* × *'state-t*)
and *kinvolved* :: *'action-t* ⇒ *'dom-t set*
and *ifp* :: *'dom-t* ⇒ *'dom-t* ⇒ *bool*
and *vpeq* :: *'dom-t* ⇒ *'state-t* ⇒ *'state-t* ⇒ *bool*
+
fixes *AS-set* :: (*'action-t list*) *set* — Returns a set of valid action sequences, i.e., the attack surface
and *invariant* :: *'state-t* ⇒ *bool*
and *AS-precondition* :: *'state-t* ⇒ *'dom-t* ⇒ *'action-t* ⇒ *bool*
and *aborting* :: *'state-t* ⇒ *'dom-t* ⇒ *'action-t* ⇒ *bool*
and *waiting* :: *'state-t* ⇒ *'dom-t* ⇒ *'action-t* ⇒ *bool*
assumes *empty-in-AS-set*: [] ∈ *AS-set*
and *invariant-s0*: *invariant s0*
and *invariant-after-cswitch*: ∀ *s n* . *invariant s* ⟶ *invariant* (*cswitch n s*)
and *precondition-after-cswitch*: ∀ *s d n a*. *AS-precondition s d a* ⟶ *AS-precondition* (*cswitch n s*) *d a*
and *AS-prec-first-action*: ∀ *s d aseq* . *invariant s* ∧ *aseq* ∈ *AS-set* ∧ *aseq* ≠ [] ⟶ *AS-precondition s d* (*hd aseq*)
and *AS-prec-after-step*: ∀ *s a a'* . (∃ *aseq* ∈ *AS-set* . *is-sub-seq a a' aseq*) ∧ *invariant s* ∧ *AS-precondition s* (*current s*) *a* ∧ ¬*aborting s* (*current s*) *a* ∧ ¬ *waiting s* (*current s*) *a* ⟶ *AS-precondition* (*kstep s a*) (*current s*) *a'*
and *AS-prec-dom-independent*: ∀ *s d a a'* . *current s* ≠ *d* ∧ *AS-precondition s d a* ⟶ *AS-precondition* (*kstep s a'*) *d a*
and *spec-of-invariant*: ∀ *s a* . *invariant s* ⟶ *invariant* (*kstep s a*)

and *kprecondition-def*: *kprecondition s a* ≡ *invariant s* ∧ *AS-precondition s* (*current s*) *a*
and *realistic-execution-def*: *realistic-execution aseq* ≡ *set aseq* ⊆ *AS-set*
and *control-spec*: ∀ *s d aseqs* . *case control s d aseqs of* (*a*,*aseqs'*,*s'*) ⇒
(*thread-empty aseqs* ∧ (*a*,*aseqs'*) = (*None*,[])) ∨ (∗ *Nothing happens* ∗)
(*aseqs* ≠ [] ∧ *hd aseqs* ≠ [] ∧ ¬*aborting s' d* (*the a*) ∧ ¬ *waiting s' d* (*the a*) ∧ (*a*,*aseqs'*) = (*Some* (*hd* (*hd aseqs*)), (*tl* (*hd aseqs*))#(*tl aseqs*))) ∨ (∗ *Execute the first action of the current action sequence* ∗)

$(aseqs \neq [] \wedge hd\ aseqs \neq [] \wedge waiting\ s'\ d\ (the\ a) \wedge (a,aseqs',s') = (Some\ (hd\ (hd\ aseqs)),aseqs,s)) \vee$ (* *Nothing happens, waiting to execute the next action* *)
$(a,aseqs') = (None,tl\ aseqs)$
and *next-action-after-cswitch*: $\forall\ s\ n\ d\ aseqs\ .\ fst\ (control\ (cswitch\ n\ s)\ d\ aseqs) = fst\ (control\ s\ d\ aseqs)$
and *next-action-after-next-state*: $\forall\ s\ execs\ d\ .\ current\ s \neq d \longrightarrow fst\ (control\ (next\text{-}state\ s\ execs)\ d\ (execs\ d)) = None \vee fst\ (control\ (next\text{-}state\ s\ execs)\ d\ (execs\ d)) = fst\ (control\ s\ d\ (execs\ d))$
and *next-action-after-step*: $\forall\ s\ a\ d\ aseqs\ .\ current\ s \neq d \longrightarrow fst\ (control\ (step\ s\ a)\ d\ aseqs) = fst\ (control\ s\ d\ aseqs)$
and *next-state-precondition*: $\forall\ s\ d\ a\ execs.\ AS\text{-}precondition\ s\ d\ a \longrightarrow AS\text{-}precondition\ (next\text{-}state\ s\ execs)\ d\ a$
and *next-state-invariant*: $\forall\ s\ execs\ .\ invariant\ s \longrightarrow invariant\ (next\text{-}state\ s\ execs)$
and *spec-of-waiting*: $\forall\ s\ a\ .\ waiting\ s\ (current\ s)\ a \longrightarrow kstep\ s\ a = s$
begin

We can now formulate a total run function, since based on the new assumptions the case where the precondition does not hold, will never occur.

function *run-total* :: $time\text{-}t \Rightarrow 'state\text{-}t \Rightarrow ('dom\text{-}t \Rightarrow 'action\text{-}t\ execution) \Rightarrow 'state\text{-}t$
where $run\text{-}total\ 0\ s\ execs = s$
$|\ interrupt\ (Suc\ n) \Longrightarrow run\text{-}total\ (Suc\ n)\ s\ execs = run\text{-}total\ n\ (cswitch\ (Suc\ n)\ s)\ execs$
$|\ \neg interrupt\ (Suc\ n) \Longrightarrow thread\text{-}empty(execs\ (current\ s)) \Longrightarrow run\text{-}total\ (Suc\ n)\ s\ execs = run\text{-}total\ n\ s\ execs$
$|\ \neg interrupt\ (Suc\ n) \Longrightarrow \neg thread\text{-}empty(execs\ (current\ s)) \Longrightarrow$
$run\text{-}total\ (Suc\ n)\ s\ execs = run\text{-}total\ n\ (step\ (next\text{-}state\ s\ execs)\ (next\text{-}action\ s\ execs))\ (next\text{-}execs\ s\ execs)$
using *not0-implies-Suc* **by** (*metis prod-cases3,auto*)
termination by *lexicographic-order*

The major part of the proofs in this locale consist of proving that function run_total is equivalent to function run, i.e., that the precondition does always hold. This assumes that the executions are *realistic*. This means that the execution of each domain contains action sequences that are from AS_set. This ensures, e.g, that a COPY_CHECK is always preceded by a COPY_INIT.

definition *realistic-executions* :: $('dom\text{-}t \Rightarrow 'action\text{-}t\ execution) \Rightarrow bool$
where $realistic\text{-}executions\ execs \equiv \forall\ d\ .\ realistic\text{-}execution\ (execs\ d)$

Lemma run_total_equals_run is proven by doing induction. It is however not inductive and can therefore not be proven directly: a realistic execution is not necessarily realistic after performing one action. We generalize to do induction. Predicate realistic_executions_ind is the inductive version of realistic_executions. All action sequences in the tail of the executions must be complete action sequences (i.e., they must be from AS_set). The first action sequence, however, is being executed and is therefore not necessarily an action sequence from AS_set, but it is *the last part* of some action sequence from AS_set.

definition *realistic-AS-partial* :: $'action\text{-}t\ list \Rightarrow bool$
where $realistic\text{-}AS\text{-}partial\ aseq \equiv \exists\ n\ aseq'\ .\ n \leq length\ aseq' \wedge aseq' \in AS\text{-}set \wedge aseq = lastn\ n\ aseq'$
definition *realistic-executions-ind* :: $('dom\text{-}t \Rightarrow 'action\text{-}t\ execution) \Rightarrow bool$
where $realistic\text{-}executions\text{-}ind\ execs \equiv \forall\ d\ .\ (case\ execs\ d\ of\ [] \Rightarrow True\ |\ (aseq\#aseqs) \Rightarrow realistic\text{-}AS\text{-}partial\ aseq \wedge set\ aseqs \subseteq AS\text{-}set)$

We need to know that invariably, the precondition holds. As this precondition consists of 1.) a generic invariant and 2.) more refined preconditions for the current action, we have to know that these two are invariably true.

definition *precondition-ind* :: $'state\text{-}t \Rightarrow ('dom\text{-}t \Rightarrow 'action\text{-}t\ execution) \Rightarrow bool$
where $precondition\text{-}ind\ s\ execs \equiv invariant\ s \wedge (\forall\ d\ .\ fst(control\ s\ d\ (execs\ d)) \rightharpoonup AS\text{-}precondition\ s\ d)$

Proof that "execution is realistic" is inductive, i.e., assuming the current execution is realistic, the execution returned by the control mechanism is realistic.

lemma *next-execution-is-realistic-partial*:
assumes *na-def*: $next\text{-}execs\ s\ execs\ d = aseq\ \#\ aseqs$
and *d-is-curr*: $d = current\ s$
and *realistic*: $realistic\text{-}executions\text{-}ind\ execs$
and *thread-not-empty*: $\neg thread\text{-}empty(execs\ (current\ s))$

shows *realistic-AS-partial aseq* ∧ *set aseqs* ⊆ *AS-set*
proof−
let *?c* = *control s* (*current s*) (*execs* (*current s*))
{
assume *c-empty*: *let* (*a*,*aseqs*′,*s*′) = *?c in*
(*a*,*aseqs*′) = (*None*,[])
from *na-def d-is-curr c-empty*
have *?thesis*
unfolding *realistic-executions-ind-def next-execs-def* **by** (*auto*)
}
moreover
{
let *?ct*= *execs* (*current s*)
let *?execs*′ = (*tl* (*hd ?ct*))#(*tl ?ct*)
let *?a*′ = *Some* (*hd* (*hd ?ct*))
assume *hd-thread-not-empty*: *hd* (*execs* (*current s*)) ≠ []
assume *c-executing*: *let* (*a*,*aseqs*′,*s*′) = *?c in*
(*a*,*aseqs*′) = (*?a*′, *?execs*′)
from *na-def c-executing d-is-curr*
have *as-defs*: *aseq* = *tl* (*hd ?ct*) ∧ *aseqs* = *tl ?ct*
unfolding *next-execs-def* **by** (*auto*)
from *realistic*[*unfolded realistic-executions-ind-def*,*THEN spec*,**where** *x*=*d*] *d-is-curr*
have *subset*: *set* (*tl ?execs*′) ⊆ *AS-set*
unfolding *Let-def realistic-AS-partial-def*
by (*cases execs d*,*auto*)
from *d-is-curr thread-not-empty hd-thread-not-empty realistic*[*unfolded realistic-executions-ind-def*,*THEN spec*,**where** *x*=*d*]
obtain *n aseq*′ **where** *n-aseq*′: *n* ≤ *length aseq*′ ∧ *aseq*′ ∈ *AS-set* ∧ *hd ?ct* = *lastn n aseq*′
unfolding *realistic-AS-partial-def*
by (*cases execs d*,*auto*)
from *this hd-thread-not-empty* **have** *n* > *0* **unfolding** *lastn-def* **by**(*cases n*,*auto*)
from *this n-aseq*′ *lastn-one-less*[**where** *n*=*n* **and** *x*=*aseq*′ **and** *a*=*hd* (*hd ?ct*) **and** *y*=*tl* (*hd ?ct*)] *hd-thread-not-empty*
have *n* − *1* ≤ *length aseq*′ ∧ *aseq*′ ∈ *AS-set* ∧ *tl* (*hd ?ct*) = *lastn* (*n* − *1*) *aseq*′
by *auto*
from *this as-defs subset* **have** *?thesis*
unfolding *realistic-AS-partial-def*
by *auto*
}
moreover
{
let *?ct*= *execs* (*current s*)
let *?execs*′ = *?ct*
let *?a*′ = *Some* (*hd* (*hd ?ct*))
assume *c-waiting*: *let* (*a*,*aseqs*′,*s*′) = *?c in*
(*a*,*aseqs*′) = (*?a*′, *?execs*′)
from *na-def c-waiting d-is-curr*
have *as-defs*: *aseq* = *hd ?execs*′ ∧ *aseqs* = *tl ?execs*′
unfolding *next-execs-def* **by** (*auto*)
from *realistic*[*unfolded realistic-executions-ind-def*,*THEN spec*,**where** *x*=*d*] *d-is-curr set-tl-is-subset*[**where** *x*=*?execs*′]
have *subset*: *set* (*tl ?execs*′) ⊆ *AS-set*
unfolding *Let-def realistic-AS-partial-def*
by (*cases execs d*,*auto*)
from *na-def c-waiting d-is-curr*
have *?execs*′ ≠ [] **unfolding** *next-execs-def* **by** *auto*
from *realistic*[*unfolded realistic-executions-ind-def*,*THEN spec*,**where** *x*=*d*] *d-is-curr thread-not-empty*
obtain *n aseq*′ **where** *witness*: *n* ≤ *length aseq*′ ∧ *aseq*′ ∈ *AS-set* ∧ *hd*(*execs d*) = *lastn n aseq*′

```
    unfolding realistic-AS-partial-def by (cases execs d,auto)
  from d-is-curr this subset as-defs have ?thesis
    unfolding realistic-AS-partial-def
    by auto
}
moreover
{
  let ?ct= execs (current s)
  let ?execs' = tl ?ct
  let ?a' = None
  assume c-aborting: let (a,aseqs',s') = ?c in
                     (a,aseqs') = (?a', ?execs')
  from na-def c-aborting d-is-curr
    have as-defs: aseq = hd ?execs' ∧ aseqs = tl ?execs'
    unfolding next-execs-def by (auto)
  from realistic[unfolded realistic-executions-ind-def,THEN spec,where x=d] d-is-curr set-tl-is-subset[where x=?execs']
    have subset: set (tl ?execs') ⊆ AS-set
    unfolding Let-def realistic-AS-partial-def
    by (cases execs d,auto)
  from na-def c-aborting d-is-curr
    have ?execs' ≠ [] unfolding next-execs-def by auto
  from empty-in-AS-set this
    realistic[unfolded realistic-executions-ind-def,THEN spec,where x=d] d-is-curr
      have length (hd ?execs') ≤ length (hd ?execs') ∧ (hd ?execs') ∈ AS-set ∧ hd ?execs' = lastn (length (hd ?execs')) (hd ?execs')
    unfolding lastn-def
    by (cases execs (current s),auto)
  from this subset as-defs have ?thesis
    unfolding realistic-AS-partial-def
    by auto
}
ultimately
show ?thesis
  using control-spec[THEN spec,THEN spec,THEN spec,where x2=s and x1=current s and x=execs (current s)]
      d-is-curr thread-not-empty
  by (auto simp add: Let-def)
qed
```

The lemma that proves that the total run function is equivalent to the partial run function, i.e., that in this refinement the case of the run function where the precondition is False will never occur.

```
lemma run-total-equals-run:
  assumes realistic-exec: realistic-executions execs
      and invariant: invariant s
    shows strict-equal (run n (Some s) execs) (run-total n s execs)
proof–
{
  fix n ms s execs
   have strict-equal ms s ∧ realistic-executions-ind execs ∧ precondition-ind s execs ⟶ strict-equal (run n ms execs) (run-total n s execs)
  proof (induct n ms execs arbitrary: s rule: run.induct)
  case (1 s execs sa)
    show ?case by auto
  next
  case (2 n execs s)
    show ?case unfolding strict-equal-def by auto
  next
  case (3 n s execs sa)
```

assume *interrupt*: *interrupt* (*Suc n*)
assume *IH*: (⋀*sa*. *strict-equal* (*Some* (*cswitch* (*Suc n*) *s*)) *sa* ∧ *realistic-executions-ind execs* ∧ *precondition-ind sa execs* ⟶
strict-equal (*run n* (*Some* (*cswitch* (*Suc n*) *s*)) *execs*) (*run-total n sa execs*))
{
assume *equal-s-sa*: *strict-equal* (*Some s*) *sa*
assume *realistic*: *realistic-executions-ind execs*
assume *inv-sa*: *precondition-ind sa execs*
have *inv-nsa*: *precondition-ind* (*cswitch* (*Suc n*) *sa*) *execs*
proof–
{
fix *d*
have *fst* (*control* (*cswitch* (*Suc n*) *sa*) *d* (*execs d*)) ⇀ *AS-precondition* (*cswitch* (*Suc n*) *sa*) *d*
using *next-action-after-cswitch inv-sa*[*unfolded precondition-ind-def*,*THEN conjunct2*,*THEN spec*,**where** *x*=*d*]
precondition-after-cswitch
unfolding *Let-def B-def precondition-ind-def*
by(*cases fst* (*control* (*cswitch* (*Suc n*) *sa*) *d* (*execs d*)),*auto*)
}
thus *?thesis* **using** *inv-sa invariant-after-cswitch* **unfolding** *precondition-ind-def* **by** *auto*
qed
from *equal-s-sa realistic inv-nsa inv-sa IH*[**where** *sa*=*cswitch* (*Suc n*) *sa*]
have *equal-ns-nt*: *strict-equal* (*run n* (*Some* (*cswitch* (*Suc n*) *s*)) *execs*) (*run-total n* (*cswitch* (*Suc n*) *sa*) *execs*)
unfolding *strict-equal-def* **by**(*auto*)
}
from *this interrupt* **show** *?case* **by** *auto*
next
case (*4 n execs s sa*)
assume *not-interrupt*: ¬*interrupt* (*Suc n*)
assume *thread-empty*: *thread-empty*(*execs* (*current s*))
assume *IH*: (⋀*sa*. *strict-equal* (*Some s*) *sa* ∧ *realistic-executions-ind execs* ∧ *precondition-ind sa execs* ⟶ *strict-equal* (*run n* (*Some s*) *execs*) (*run-total n sa execs*))
have *current-s-sa*: *strict-equal* (*Some s*) *sa* ⟶ *current s* = *current sa* **unfolding** *strict-equal-def* **by** *auto*
{
assume *equal-s-sa*: *strict-equal* (*Some s*) *sa*
assume *realistic*: *realistic-executions-ind execs*
assume *inv-sa*: *precondition-ind sa execs*
from *equal-s-sa realistic inv-sa IH*[**where** *sa*=*sa*]
have *equal-ns-nt*: *strict-equal* (*run n* (*Some s*) *execs*) (*run-total n sa execs*)
unfolding *strict-equal-def* **by**(*auto*)
}
from *this current-s-sa thread-empty not-interrupt* **show** *?case* **by** *auto*
next
case (*5 n execs s sa*)
assume *not-interrupt*: ¬*interrupt* (*Suc n*)
assume *thread-not-empty*: ¬*thread-empty*(*execs* (*current s*))
assume *not-prec*: ¬ *precondition* (*next-state s execs*) (*next-action s execs*)
— In locale ISK, the precondition can be proven to hold at all times. This case cannot happen, and we can prove False.
{
assume *equal-s-sa*: *strict-equal* (*Some s*) *sa*
assume *realistic*: *realistic-executions-ind execs*
assume *inv-sa*: *precondition-ind sa execs*
from *equal-s-sa* **have** *s-sa*: *s* = *sa* **unfolding** *strict-equal-def* **by** *auto*
from *inv-sa* **have**
next-action sa execs ⇀ *AS-precondition sa* (*current sa*)

unfolding *precondition-ind-def B-def next-action-def*
by (*cases next-action sa execs,auto*)
from *this next-state-precondition*
have *next-action sa execs* ⇀ *AS-precondition* (*next-state sa execs*) (*current sa*)
unfolding *precondition-ind-def B-def*
by (*cases next-action sa execs,auto*)
from *inv-sa this s-sa next-state-invariant current-next-state*
have *prec-s*: *precondition* (*next-state s execs*) (*next-action s execs*)
unfolding *precondition-ind-def kprecondition-def precondition-def B-def*
by (*cases next-action sa execs,auto*)
from *this not-prec* **have** *False* **by** *auto*
}
thus *?case* **by** *auto*
next
case (*6 n execs s sa*)
assume *not-interrupt*: ¬*interrupt* (*Suc n*)
assume *thread-not-empty*: ¬*thread-empty*(*execs* (*current s*))
assume *prec*: *precondition* (*next-state s execs*) (*next-action s execs*)
assume *IH*: (⋀*sa. strict-equal* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) *sa* ∧ *realistic-executions-ind* (*next-execs s execs*) ∧ *precondition-ind sa* (*next-execs s execs*) ⟶ *strict-equal* (*run n* (*Some* (*step* (*next-state s execs*) (*next-action s execs*))) (*next-execs s execs*)) (*run-total n sa* (*next-execs s execs*)))
have *current-s-sa*: *strict-equal* (*Some s*) *sa* ⟶ *current s* = *current sa* **unfolding** *strict-equal-def* **by** *auto*
{
assume *equal-s-sa*: *strict-equal* (*Some s*) *sa*
assume *realistic*: *realistic-executions-ind execs*
assume *inv-sa*: *precondition-ind sa execs*

from *equal-s-sa* **have** *s-sa*: *s* = *sa* **unfolding** *strict-equal-def* **by** *auto*

let *?a* = *next-action s execs*
let *?ns* = *step* (*next-state s execs*) *?a*
let *?na* = *next-execs s execs*
let *?c* = *control s* (*current s*) (*execs* (*current s*))

have *equal-ns-nsa*: *strict-equal* (*Some ?ns*) *?ns* **unfolding** *strict-equal-def* **by** *auto*
from *inv-sa equal-s-sa* **have** *inv-s*: *invariant s* **unfolding** *strict-equal-def precondition-ind-def* **by** *auto*

— Two things are proven inductive. First, the assumptions that the execution is realistic (statement realistic-na). This proof uses lemma next-execution-is-realistic-partial. Secondly, the precondition: if the precondition holds for the current action, then it holds for the next action (statement invariant-na).
have *realistic-na*: *realistic-executions-ind ?na*
proof–
{
fix *d*
have *case ?na d of* [] ⇒ *True* | *aseq* # *aseqs* ⇒ *realistic-AS-partial aseq* ∧ *set aseqs* ⊆ *AS-set*
proof(*cases ?na d,simp,rename-tac aseq aseqs,simp,cases d* = *current s*)
case *False*
fix *aseq aseqs*
assume *next-execs s execs d* = *aseq* # *aseqs*
from *False this realistic*[*unfolded realistic-executions-ind-def,THEN spec,***where** *x=d*]
show *realistic-AS-partial aseq* ∧ *set aseqs* ⊆ *AS-set*
unfolding *next-execs-def* **by** *simp*
next
case *True*
fix *aseq aseqs*
assume *na-def*: *next-execs s execs d* = *aseq* # *aseqs*

from *next-execution-is-realistic-partial na-def True realistic thread-not-empty*
show *realistic-AS-partial aseq* ∧ *set aseqs* ⊆ *AS-set* **by** *blast*
qed
}
thus *?thesis* **unfolding** *realistic-executions-ind-def* **by** *auto*
qed
have *invariant-na*: *precondition-ind ?ns ?na*
proof−
from *spec-of-invariant inv-sa next-state-invariant s-sa* **have** *inv-ns*: *invariant ?ns*
unfolding *precondition-ind-def step-def*
by (*cases next-action sa execs*,*auto*)
have ∀ *d*. *fst* (*control ?ns d* (*?na d*)) ⇀ *AS-precondition ?ns d*
proof−
{
fix *d*
{
let *?a′* = *fst* (*control ?ns d* (*?na d*))
assume *snd-action-not-none*: *?a′* ≠ *None*
have *AS-precondition ?ns d* (*the ?a′*)
proof (*cases d* = *current s*)
case *True*
{
have *?thesis*
proof (*cases ?a*)
case (*Some a*)
— Assuming that the current domain executes some action a, and assuming that the action a' after that is not None (statement snd-action-not-none), we prove that the precondition is inductive, i.e., it will hold for a'. Two cases arise: either action a is delayed (case waiting) or not (case executing).
show *?thesis*
proof(*cases ?na d* = *execs* (*current s*) *rule* :*case-split*[*case-names waiting executing*])
case *executing* — The kernel is executing two consecutive actions a and a'. We show that [a,a'] is a subsequence in some action in AS-set. The PO's ensure that the precondition is inductive.
from *executing True Some control-spec*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2*=*s* **and** *x1*=*d* **and** *x*=*execs d*]
have *a-def*: *a* = *hd* (*hd* (*execs* (*current s*))) ∧ *?na d* = (*tl* (*hd* (*execs* (*current s*))))#(*tl* (*execs* (*current s*)))
unfolding *next-action-def next-execs-def Let-def*
by(*auto*)
from *a-def True snd-action-not-none control-spec*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2*=*?ns* **and** *x1*=*d* **and** *x*=*?na d*]
second-elt-is-hd-tl[**where** *x*= *hd* (*execs* (*current s*)) **and** *a*=*hd*(*tl*(*hd* (*execs* (*current s*)))) **and** *x′*=*tl* (*tl*(*hd* (*execs* (*current s*))))]
have *na-def*: *the ?a′* = (*hd* (*execs* (*current s*)))!*1*
unfolding *next-execs-def*
by(*auto*)
from *Some realistic*[*unfolded realistic-executions-ind-def* ,*THEN spec*,**where** *x*=*d*] *True thread-not-empty*
obtain *n aseq′* **where** *witness*: *n* ≤ *length aseq′* ∧ *aseq′* ∈ *AS-set* ∧ *hd*(*execs d*) = *lastn n aseq′*
unfolding *realistic-AS-partial-def* **by** (*cases execs d*,*auto*)
from *True executing length-lt-2-implies-tl-empty*[**where** *x*=*hd* (*execs* (*current s*))]
Some control-spec[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2*=*s* **and** *x1*=*d* **and** *x*=*execs d*]
snd-action-not-none control-spec[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2*=*?ns* **and** *x1*=*d* **and** *x*=*?na d*]
have *in-action-sequence*: *length* (*hd* (*execs* (*current s*))) ≥ *2*
unfolding *next-action-def next-execs-def*
by *auto*
from *this witness consecutive-is-sub-seq*[**where** *a*=*a* **and** *b*=*the ?a′* **and** *n*=*n* **and** *y*=*aseq′* **and** *x*=*tl* (*tl* (*hd* (*execs* (*current s*))))]

a-def na-def True in-action-sequence
x-is-hd-snd-tl[**where** *x=hd (execs (current s))*]
have *1*: *∃ aseq′ ∈ AS-set . is-sub-seq a (the ?a′) aseq′*
by(*auto*)
from *True Some inv-sa*[*unfolded precondition-ind-def*,*THEN conjunct2*,*THEN spec*,**where** *x=current s*] *s-sa*
have *2*: *AS-precondition s (current s) a*
unfolding *strict-equal-def next-action-def B-def* **by** *auto*
from *executing True Some control-spec*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2=s* **and** *x1=d* **and** *x=execs d*]
have *not-aborting*: *¬aborting (next-state s execs) (current s) (the ?a)*
unfolding *next-action-def next-state-def next-execs-def*
by *auto*
from *executing True Some control-spec*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2=s* **and** *x1=d* **and** *x=execs d*]
have *not-waiting*: *¬waiting (next-state s execs) (current s) (the ?a)*
unfolding *next-action-def next-state-def next-execs-def*
by *auto*
from *True this*
1 2 inv-s
sub-seq-in-prefixes[**where** *X=AS-set*] *Some next-state-invariant*
current-next-state[*THEN spec*,*THEN spec*,**where** *x1=s* **and** *x=execs*]
AS-prec-after-step[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2=next-state s execs* **and** *x1=a* **and** *x=the ?a′*]
next-state-precondition not-aborting not-waiting
show *?thesis*
unfolding *step-def*
by *auto*
next
case *waiting* — The kernel is delaying action a. Thus the action after a, which is a', is equal to a.
from *tl-hd-x-not-tl-x*[**where** *x=execs d*] *True waiting control-spec*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2=s* **and** *x1=d* **and** *x=execs d*] *Some*
have *a-def*: *?na d = execs (current s) ∧ next-state s execs = s ∧ waiting s d (the ?a)*
unfolding *next-action-def next-execs-def next-state-def*
by(*auto*)
from *Some waiting a-def True snd-action-not-none control-spec*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x2=?ns* **and** *x1=d* **and** *x=?na d*]
have *na-def*: *the ?a′ = hd (hd (execs (current s)))*
unfolding *next-action-def next-execs-def*
by(*auto*)
from *spec-of-waiting a-def True*
have *no-step*: *step s ?a = s* **unfolding** *step-def* **by** (*cases next-action s execs*,*auto*)
from *no-step Some True a-def*
inv-sa[*unfolded precondition-ind-def*,*THEN conjunct2*,*THEN spec*,**where** *x=current s*] *s-sa*
have *2*: *AS-precondition s (current s) (the ?a′)*
unfolding *next-action-def B-def*
by(*auto*)
from *a-def na-def this True Some no-step*
show *?thesis*
unfolding *step-def*
by(*auto*)
qed
next
case *None*
— Assuming that the current domain does not execute an action, and assuming that the action a' after that is not None (statement snd-action-not-none), we prove that the precondition is inductive, i.e., it will hold for a'. This holds, since the control mechanism will ensure that action a' is the start of a new action sequence in AS-set.

from None True snd-action-not-none control-spec[THEN spec,THEN spec,THEN spec,**where** x2=?ns **and** x1=d **and** x=?na d]
control-spec[THEN spec,THEN spec,THEN spec,**where** x2=s **and** x1=d **and** x=execs d]
have na-def: the ?a′ = hd (hd (tl (execs (current s)))) ∧ ?na d = tl (execs (current s))
unfolding next-action-def next-execs-def
by(auto)
from True None snd-action-not-none control-spec[THEN spec,THEN spec,THEN spec,**where** x2=?ns **and** x1=d **and** x=?na d]
this
have 1: tl (execs (current s)) ≠ [] ∧ hd (tl (execs (current s))) ≠ []
by auto
from this realistic[unfolded realistic-executions-ind-def,THEN spec,**where** x=d] True thread-not-empty
have hd (tl (execs (current s))) ∈ AS-set
by (cases execs d,auto)
from True snd-action-not-none this
inv-ns this na-def 1
AS-prec-first-action[THEN spec,THEN spec,THEN spec,**where** x2=?ns **and** x=hd (tl (execs (current s))) **and** x1=d]
show ?thesis **by** auto
qed
}
thus ?thesis
using control-spec[THEN spec,THEN spec,THEN spec,**where** x2=?ns **and** x1=current s **and** x=?na (current s)]
thread-not-empty True snd-action-not-none
by (auto simp add: Let-def)
next
case False
from False **have** equal-na-a: ?na d = execs d
unfolding next-execs-def **by** auto
from this False current-next-state next-action-after-step
have ?a′ = fst (control (next-state s execs) d (next-execs s execs d))
unfolding next-action-def **by** auto
from inv-sa[unfolded precondition-ind-def,THEN conjunct2,THEN spec,**where** x=d] s-sa equal-na-a this
next-action-after-next-state[THEN spec,THEN spec,THEN spec,**where** x=d **and** x2=s **and** x1=execs]
snd-action-not-none False
have AS-precondition s d (the ?a′)
unfolding precondition-ind-def next-action-def B-def **by** (cases fst (control sa d (execs d)),auto)
from equal-na-a False this next-state-precondition current-next-state
AS-prec-dom-independent[THEN spec,THEN spec,THEN spec,THEN spec,**where** x3=next-state s execs **and** x2=d **and** x=the ?a **and** x1=the ?a′]
show ?thesis
unfolding step-def
by (cases next-action s execs,auto)
qed
}
hence fst (control ?ns d (?na d)) ⇀ AS-precondition ?ns d **unfolding** B-def
by (cases fst (control ?ns d (?na d)),auto)
}
thus ?thesis **by** auto
qed
from this inv-ns **show** ?thesis
unfolding precondition-ind-def B-def Let-def
by (auto)
qed
from equal-ns-nsa realistic-na invariant-na s-sa IH[**where** sa=?ns]

have *equal-ns-nt*: *strict-equal* (*run n* (*Some ?ns*) *?na*) (*run-total n* (*step* (*next-state sa execs*) (*next-action sa execs*)) (*next-execs sa execs*))
by(*auto*)
}
from *this current-s-sa thread-not-empty not-interrupt prec* **show** *?case* **by** *auto*
qed
}
hence *thm-inductive*: ∀ *m s execs n* . *strict-equal m s* ∧ *realistic-executions-ind execs* ∧ *precondition-ind s execs* ⟶ *strict-equal* (*run n m execs*) (*run-total n s execs*) **by** *blast*
have *1*: *strict-equal* (*Some s*) *s* **unfolding** *strict-equal-def* **by** *simp*
have *2*: *realistic-executions-ind execs*
proof–
{
fix *d*
have *case execs d of* [] ⇒ *True* | *aseq* # *aseqs* ⇒ *realistic-AS-partial aseq* ∧ *set aseqs* ⊆ *AS-set*
proof(*cases execs d*,*simp*)
case (*Cons aseq aseqs*)
from *Cons realistic-exec*[*unfolded realistic-executions-def*,*THEN spec*,**where** *x*=*d*]
have *0*: *length aseq* ≤ *length aseq* ∧ *aseq* ∈ *AS-set* ∧ *aseq* = *lastn* (*length aseq*) *aseq*
unfolding *lastn-def realistic-execution-def* **by** *auto*
hence *1*: *realistic-AS-partial aseq* **unfolding** *realistic-AS-partial-def* **by** *auto*
from *Cons realistic-exec*[*unfolded realistic-executions-def*,*THEN spec*,**where** *x*=*d*]
have *2*: *set aseqs* ⊆ *AS-set*
unfolding *realistic-execution-def* **by** *auto*
from *Cons 1 2* **show** *?thesis* **by** *auto*
qed
}
thus *?thesis* **unfolding** *realistic-executions-ind-def* **by** *auto*
qed
have *3*: *precondition-ind s execs*
proof–
{
fix *d*
{
assume *not-empty*: *fst* (*control s d* (*execs d*)) ≠ *None*
from *not-empty realistic-exec*[*unfolded realistic-executions-def*,*THEN spec*,**where** *x*=*d*]
have *current-aseq-is-realistic*: *hd* (*execs d*) ∈ *AS-set*
using *control-spec*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x*=*execs d* **and** *x1*=*d* **and** *x2*=*s*]
unfolding *realistic-execution-def* **by**(*cases execs d*,*auto*)
from *not-empty current-aseq-is-realistic invariant AS-prec-first-action*[*THEN spec*,*THEN spec*,*THEN spec*, **where** *x2*=*s* **and** *x1*=*d* **and** *x*=*hd* (*execs d*)]
have *AS-precondition s d* (*the* (*fst* (*control s d* (*execs d*))))
using *control-spec*[*THEN spec*,*THEN spec*,*THEN spec*,**where** *x*=*execs d* **and** *x1*=*d* **and** *x2*=*s*]
by *auto*
}
hence *fst* (*control s d* (*execs d*)) ⇀ *AS-precondition s d*
unfolding *B-def*
by (*cases fst* (*control s d* (*execs d*)),*auto*)
}
from *this invariant* **show** *?thesis* **unfolding** *precondition-ind-def* **by** *auto*
qed
from *thm-inductive 1 2 3* **show** *?thesis* **by** *auto*
qed

Theorem unwinding_implies_isecure gives security for all realistic executions. For unrealistic executions, it holds vacuously and therefore does not tell us anything. In order to prove security for this refinement (i.e., for function run_total), we have to prove that purging yields realistic runs.

lemma *realistic-purge*:
shows ∀ *execs d . realistic-executions execs* ⟶ *realistic-executions* (*purge execs d*)
proof–
{
fix *execs d*
assume *realistic-executions execs*
hence *realistic-executions* (*purge execs d*)
using *someI*[**where** *P=realistic-execution* **and** *x=execs d*]
unfolding *realistic-executions-def purge-def* **by**(*simp*)
}
thus *?thesis* **by** *auto*
qed

lemma *remove-gateway-comm-subset*:
shows *set* (*remove-gateway-communications d exec*) ⊆ *set exec* ∪ {[]}
by(*induct exec,auto*)

lemma *realistic-ipurge-l*:
shows ∀ *execs d . realistic-executions execs* ⟶ *realistic-executions* (*ipurge-l execs d*)
proof–
{
fix *execs d*
assume *1*: *realistic-executions execs*
from *empty-in-AS-set remove-gateway-comm-subset*[**where** *d=d* **and** *exec=execs d*] *1* **have** *realistic-executions* (*ipurge-l execs d*)
unfolding *realistic-execution-def realistic-executions-def ipurge-l-def* **by**(*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *realistic-ipurge-r*:
shows ∀ *execs d . realistic-executions execs* ⟶ *realistic-executions* (*ipurge-r execs d*)
proof–
{
fix *execs d*
assume *1*: *realistic-executions execs*
from *empty-in-AS-set remove-gateway-comm-subset*[**where** *d=d* **and** *exec=execs d*] *1* **have** *realistic-executions* (*ipurge-r execs d*)
using *someI*[**where** *P=λ x . realistic-execution x* **and** *x=execs d*]
unfolding *realistic-execution-def realistic-executions-def ipurge-r-def* **by**(*auto*)
}
thus *?thesis* **by** *auto*
qed

We now have sufficient lemma's to prove security for run_total. The definition of security is similar to that in Section 3.2. It now assumes that the executions are realistic and concerns function run_total instead of function run.

definition *NI-unrelated-total*::*bool*
where *NI-unrelated-total*
≡ ∀ *execs a n . realistic-executions execs* ⟶
(*let s-f = run-total n s0 execs in*
output-f s-f a = output-f (*run-total n s0* (*purge execs* (*current s-f*))) *a*
∧ *current s-f = current* (*run-total n s0* (*purge execs* (*current s-f*))))

definition *NI-indirect-sources-total*::*bool*
where *NI-indirect-sources-total*
≡ ∀ *execs a n. realistic-executions execs* ⟶

(*let s-f = run-total n s0 execs in*
output-f (*run-total n s0* (*ipurge-l execs* (*current s-f*))) *a* =
output-f (*run-total n s0* (*ipurge-r execs* (*current s-f*))) *a*)

definition *isecure-total*::*bool*
where *isecure-total* ≡ *NI-unrelated-total* ∧ *NI-indirect-sources-total*

theorem *unwinding-implies-isecure-total*:
shows *isecure-total*
proof–
from *assms unwinding-implies-isecure* **have** *secure-partial*: *NI-unrelated* **unfolding** *isecure-def* **by** *blast*
from *assms unwinding-implies-isecure* **have** *isecure1-partial*: *NI-indirect-sources* **unfolding** *isecure-def* **by** *blast*

have *NI-unrelated-total*: *NI-unrelated-total*
proof–
{
fix *execs a n*
assume *realistic*: *realistic-executions execs*
from *assms invariant-s0 realistic run-total-equals-run*[**where** *n*=*n* **and** *s*=*s0* **and** *execs*=*execs*]
have *1*: *strict-equal* (*run n* (*Some s0*) *execs*) (*run-total n s0 execs*) **by** *auto*

have *let s-f* = *run-total n s0 execs in output-f s-f a* = *output-f* (*run-total n s0* (*purge execs* (*current s-f*))) *a* ∧ *current s-f* = *current* (*run-total n s0* (*purge execs* (*current s-f*)))
proof (*cases run n* (*Some s0*) *execs*)
case *None*
thus *?thesis* **using** *1* **unfolding** *NI-unrelated-total-def strict-equal-def* **by** *auto*
next
case (*Some s-f*)
from *realistic-purge assms invariant-s0 realistic run-total-equals-run*[**where** *n*=*n* **and** *s*=*s0* **and** *execs*=*purge execs* (*current s-f*)]
have 2: *strict-equal* (*run n* (*Some s0*) (*purge execs* (*current s-f*))) (*run-total n s0* (*purge execs* (*current s-f*)))
by *auto*
show *?thesis* **proof**(*cases run n* (*Some s0*) (*purge execs* (*current s-f*)))
case *None*
from 2 *None* **show** *?thesis* **using** 2 **unfolding** *NI-unrelated-total-def strict-equal-def* **by** *auto*
next
case (*Some s-f2*)
from ‹*run n* (*Some s0*) *execs* = *Some s-f*› *Some 1 2 secure-partial*[*unfolded NI-unrelated-def*,*THEN spec*,*THEN spec*,*THEN spec*,**where** *x*=*n* **and** *x2*=*execs*]
show *?thesis*
unfolding *strict-equal-def NI-unrelated-def*
by(*simp add*: *Let-def B-def B2-def*)
qed
qed
}
thus *?thesis* **unfolding** *NI-unrelated-total-def* **by** *auto*
qed
have *NI-indirect-sources-total*: *NI-indirect-sources-total*
proof–
{
fix *execs a n*
assume *realistic*: *realistic-executions execs*
from *assms invariant-s0 realistic run-total-equals-run*[**where** *n*=*n* **and** *s*=*s0* **and** *execs*=*execs*]
have *1*: *strict-equal* (*run n* (*Some s0*) *execs*) (*run-total n s0 execs*) **by** *auto*

have *let s-f* = *run-total n s0 execs in output-f* (*run-total n s0* (*ipurge-l execs* (*current s-f*))) *a* = *output-f*

(*run-total n s0* (*ipurge-r execs* (*current s-f*))) *a*
  proof (*cases run n* (*Some s0*) *execs*)
  case *None*
   thus *?thesis* **using** *1* **unfolding** *NI-unrelated-total-def strict-equal-def* **by** *auto*
  next
  case (*Some s-f*)
   from *realistic-ipurge-l assms invariant-s0 realistic run-total-equals-run*[**where** *n=n* **and** *s=s0* **and** *execs=ipurge-l execs* (*current s-f*)]
     have *2*: *strict-equal* (*run n* (*Some s0*) (*ipurge-l execs* (*current s-f*))) (*run-total n s0* (*ipurge-l execs* (*current s-f*)))
     by *auto*
   from *realistic-ipurge-r assms invariant-s0 realistic run-total-equals-run*[**where** *n=n* **and** *s=s0* **and** *execs=ipurge-r execs* (*current s-f*)]
     have *3*: *strict-equal* (*run n* (*Some s0*) (*ipurge-r execs* (*current s-f*))) (*run-total n s0* (*ipurge-r execs* (*current s-f*)))
     by *auto*

   show *?thesis* **proof**(*cases run n* (*Some s0*) (*ipurge-l execs* (*current s-f*)))
   case *None*
    from *2 None* **show** *?thesis* **using** *2* **unfolding** *NI-unrelated-total-def strict-equal-def* **by** *auto*
   next
   case (*Some s-ipurge-l*)
    show *?thesis*
    proof(*cases run n* (*Some s0*) (*ipurge-r execs* (*current s-f*)))
    case *None*
     from *3 None* **show** *?thesis* **using** *2* **unfolding** *NI-unrelated-total-def strict-equal-def* **by** *auto*
     next
     case (*Some s-ipurge-r*)
     from ‹*run n* (*Some s0*) *execs* = *Some s-f*› ‹*run n* (*Some s0*) (*ipurge-l execs* (*current s-f*)) = *Some s-ipurge-l*›
       Some 1 2 3 isecure1-partial[*unfolded NI-indirect-sources-def* ,*THEN spec*,*THEN spec*,*THEN spec*,**where** *x=n* **and** *x2=execs*]
      show *?thesis*
       unfolding *strict-equal-def NI-unrelated-def*
       by(*simp add*: *Let-def B-def B2-def*)
    qed
   qed
  qed
  }
  thus *?thesis* **unfolding** *NI-indirect-sources-total-def* **by** *auto*
 qed
 from *NI-unrelated-total NI-indirect-sources-total* **show** *?thesis* **unfolding** *isecure-total-def* **by** *auto*
qed

end
end

3.4 CISK (Controlled Interruptible Separation Kernel)

theory *CISK*
 imports *ISK*
begin

This section presents a generic model of a Controlled Interruptible Separation Kernel (CISK). It formulates security, i.e., intransitive noninterference. For a presentation of this model, see Section 2 of [31].

First, a locale is defined that defines all generic functions and all proof obligations (see Section 2.3 of [31]).

locale *Controllable-Interruptible-Separation-Kernel* = — CISK
fixes *kstep* :: *'state-t* ⇒ *'action-t* ⇒ *'state-t* — Executes one atomic kernel action
and *output-f* :: *'state-t* ⇒ *'action-t* ⇒ *'output-t* — Returns the observable behavior
and *s0* :: *'state-t* — The initial state
and *current* :: *'state-t* => *'dom-t* — Returns the currently active domain
and *cswitch* :: *time-t* ⇒ *'state-t* ⇒ *'state-t* — Performs a context switch
and *interrupt* :: *time-t* ⇒ *bool* — Returns t iff an interrupt occurs in the given state at the given time
and *kinvolved* :: *'action-t* ⇒ *'dom-t set* — Returns the set of domains that are involved in the given action
and *ifp* :: *'dom-t* ⇒ *'dom-t* ⇒ *bool* — The security policy.
and *vpeq* :: *'dom-t* ⇒ *'state-t* ⇒ *'state-t* ⇒ *bool* — View partitioning equivalence
and *AS-set* :: (*'action-t list*) *set* — Returns a set of valid action sequences, i.e., the attack surface
and *invariant* :: *'state-t* ⇒ *bool* — Returns an inductive state-invariant
and *AS-precondition* :: *'state-t* ⇒ *'dom-t* ⇒ *'action-t* ⇒ *bool* — Returns the preconditions under which the given action can be executed.
and *aborting* :: *'state-t* ⇒ *'dom-t* ⇒ *'action-t* ⇒ *bool* — Returns true iff the action is aborted.
and *waiting* :: *'state-t* ⇒ *'dom-t* ⇒ *'action-t* ⇒ *bool* — Returns true iff execution of the given action is delayed.
and *set-error-code* :: *'state-t* ⇒ *'action-t* ⇒ *'state-t* — Sets an error code when actions are aborted.
assumes *vpeq-transitive*: ∀ *a b c u*. (*vpeq u a b* ∧ *vpeq u b c*) ⟶ *vpeq u a c*
and *vpeq-symmetric*: ∀ *a b u*. *vpeq u a b* ⟶ *vpeq u b a*
and *vpeq-reflexive*: ∀ *a u*. *vpeq u a a*
and *ifp-reflexive*: ∀ *u* . *ifp u u*
and *weakly-step-consistent*: ∀ *s t u a*. *vpeq u s t* ∧ *vpeq* (*current s*) *s t* ∧ *invariant s* ∧ *AS-precondition s* (*current s*) *a* ∧ *invariant t* ∧ *AS-precondition t* (*current t*) *a* ∧ *current s* = *current t* ⟶ *vpeq u* (*kstep s a*) (*kstep t a*)
and *locally-respects*: ∀ *a s u*. ¬*ifp* (*current s*) *u* ∧ *invariant s* ∧ *AS-precondition s* (*current s*) *a* ⟶ *vpeq u s* (*kstep s a*)
and *output-consistent*: ∀ *a s t*. *vpeq* (*current s*) *s t* ∧ *current s* = *current t* ⟶ (*output-f s a*) = (*output-f t a*)
and *step-atomicity*: ∀ *s a* . *current* (*kstep s a*) = *current s*
and *cswitch-independent-of-state*: ∀ *n s t* . *current s* = *current t* ⟶ *current* (*cswitch n s*) = *current* (*cswitch n t*)
and *cswitch-consistency*: ∀ *u s t n* . *vpeq u s t* ⟶ *vpeq u* (*cswitch n s*) (*cswitch n t*)
and *empty-in-AS-set*: [] ∈ *AS-set*
and *invariant-s0*: *invariant s0*
and *invariant-after-cswitch*: ∀ *s n* . *invariant s* ⟶ *invariant* (*cswitch n s*)
and *precondition-after-cswitch*: ∀ *s d n a*. *AS-precondition s d a* ⟶ *AS-precondition* (*cswitch n s*) *d a*
and *AS-prec-first-action*: ∀ *s d aseq* . *invariant s* ∧ *aseq* ∈ *AS-set* ∧ *aseq* ≠ [] ⟶ *AS-precondition s d* (*hd aseq*)
and *AS-prec-after-step*: ∀ *s a a'* . (∃ *aseq* ∈ *AS-set* . *is-sub-seq a a' aseq*) ∧ *invariant s* ∧ *AS-precondition s* (*current s*) *a* ∧ ¬*aborting s* (*current s*) *a* ∧ ¬ *waiting s* (*current s*) *a* ⟶ *AS-precondition* (*kstep s a*) (*current s*) *a'*
and *AS-prec-dom-independent*: ∀ *s d a a'* . *current s* ≠ *d* ∧ *AS-precondition s d a* ⟶ *AS-precondition* (*kstep s a'*) *d a*
and *spec-of-invariant*: ∀ *s a* . *invariant s* ⟶ *invariant* (*kstep s a*)
and *aborting-switch-independent*: ∀ *n s* . *aborting* (*cswitch n s*) = *aborting s*
and *aborting-error-update*: ∀ *s d a' a* . *current s* ≠ *d* ∧ *aborting s d a* ⟶ *aborting* (*set-error-code s a'*) *d a*
and *aborting-after-step*: ∀ *s a d* . *current s* ≠ *d* ⟶ *aborting* (*kstep s a*) *d* = *aborting s d*
and *aborting-consistent*: ∀ *s t u* . *vpeq u s t* ⟶ *aborting s u* = *aborting t u*
and *waiting-switch-independent*: ∀ *n s* . *waiting* (*cswitch n s*) = *waiting s*
and *waiting-error-update*: ∀ *s d a' a* . *current s* ≠ *d* ∧ *waiting s d a* ⟶ *waiting* (*set-error-code s a'*) *d a*
and *waiting-consistent*: ∀ *s t u a* . *vpeq* (*current s*) *s t* ∧ (∀ *d* ∈ *kinvolved a* . *vpeq d s t*) ∧ *vpeq u s t* ⟶ *waiting s u a* = *waiting t u a*
and *spec-of-waiting*: ∀ *s a* . *waiting s* (*current s*) *a* ⟶ *kstep s a* = *s*
and *set-error-consistent*: ∀ *s t u a* . *vpeq u s t* ⟶ *vpeq u* (*set-error-code s a*) (*set-error-code t a*)
and *set-error-locally-respects*: ∀ *s u a* . ¬*ifp* (*current s*) *u* ⟶ *vpeq u s* (*set-error-code s a*)
and *current-set-error-code*: ∀ *s a* . *current* (*set-error-code s a*) = *current s*
and *precondition-after-set-error-code*: ∀ *s d a a'*. *AS-precondition s d a* ∧ *aborting s* (*current s*) *a'* ⟶ *AS-precondition* (*set-error-code s a'*) *d a*
and *invariant-after-set-error-code*: ∀ *s a* . *invariant s* ⟶ *invariant* (*set-error-code s a*)
and *involved-ifp*: ∀ *s a* . ∀ *d* ∈ (*kinvolved a*) . *AS-precondition s* (*current s*) *a* ⟶ *ifp d* (*current s*)

begin

3.4.1 Execution semantics

Control is based on generic functions *aborting*, *waiting* and *set_error_code*. Function *aborting* decides whether a certain action is aborting, given its domain and the state. If so, then function set_error_code will be used to update the state, possibly communicating to other domains that an action has been aborted. Function *waiting* can delay the execution of an action. This behavior is implemented in function CISK_control.

function *CISK-control* :: *'state-t ⇒ 'dom-t ⇒ 'action-t execution ⇒ ('action-t option × 'action-t execution × 'state-t)*
where *CISK-control s d* [] = (*None*,[],*s*) — The thread is empty
| *CISK-control s d* ([]#[]) = (*None*,[],*s*) — The current action sequence has been finished and the thread has no next action sequences to execute
| *CISK-control s d* ([]#(*as'*#*execs'*)) = (*None*,*as'*#*execs'*,*s*) — The current action sequence has been finished. Skip to the next sequence
| *CISK-control s d* ((*a*#*as*)#*execs'*) = (*if aborting s d a then*
(*None*, *execs'*,*set-error-code s a*)
else if waiting s d a then
(*Some a*, (*a*#*as*)#*execs'*,*s*)
else
(*Some a*, *as*#*execs'*,*s*)) — Executing an action sequence
by *pat-completeness auto*
termination by *lexicographic-order*

Function *run* defines the execution semantics. This function is presented in [31] by pseudo code (see Algorithm 1). Before defining the run function, we define accessor functions for the control mechanism. Functions next_action, next_execs and next_state correspond to "control.a", "control.x" and "control.s" in [31].

abbreviation *next-action::'state-t ⇒ ('dom-t ⇒ 'action-t execution) ⇒ 'action-t option*
where *next-action ≡ Kernel.next-action current CISK-control*
abbreviation *next-execs::'state-t ⇒ ('dom-t ⇒ 'action-t execution) ⇒ ('dom-t ⇒ 'action-t execution)*
where *next-execs ≡ Kernel.next-execs current CISK-control*
abbreviation *next-state::'state-t ⇒ ('dom-t ⇒ 'action-t execution) ⇒ 'state-t*
where *next-state ≡ Kernel.next-state current CISK-control*

A thread is empty iff either it has no further action sequences to execute, or when the current action sequence is finished and there are no further action sequences to execute.

abbreviation *thread-empty::'action-t execution ⇒ bool*
where *thread-empty exec ≡ exec* = [] ∨ *exec* = [[]]

The following function defines the execution semantics of CISK, using function CISK_control.

function *run* :: *time-t ⇒ 'state-t ⇒ ('dom-t ⇒ 'action-t execution) ⇒ 'state-t*
where *run 0 s execs = s*
| *interrupt* (*Suc n*) ⟹ *run* (*Suc n*) *s execs* = *run n* (*cswitch* (*Suc n*) *s*) *execs*
| ¬*interrupt* (*Suc n*) ⟹ *thread-empty*(*execs* (*current s*)) ⟹ *run* (*Suc n*) *s execs* = *run n s execs*
| ¬*interrupt* (*Suc n*) ⟹ ¬*thread-empty*(*execs* (*current s*)) ⟹
run (*Suc n*) *s execs* = (*let control-a = next-action s execs*;
control-s = next-state s execs;
control-x = next-execs s execs in
case control-a of None ⇒ run n control-s control-x
| (*Some a*) ⇒ *run n* (*kstep control-s a*) *control-x*)
using *not0-implies-Suc* **by** (*metis prod-cases3*,*auto*)
termination by *lexicographic-order*

3.4.2 Formulations of security

The definitions of security as presented in Section 2.2 of [31].

abbreviation *kprecondition*
 where *kprecondition s a* ≡ *invariant s* ∧ *AS-precondition s* (*current s*) *a*
definition *realistic-execution*
where *realistic-execution aseq* ≡ *set aseq* ⊆ *AS-set*
definition *realistic-executions* :: (*'dom-t* ⇒ *'action-t execution*) ⇒ *bool*
where *realistic-executions execs* ≡ ∀ *d*. *realistic-execution* (*execs d*)
abbreviation *involved* **where** *involved* ≡ *Kernel.involved kinvolved*
abbreviation *step* **where** *step* ≡ *Kernel.step kstep*
abbreviation *purge* **where** *purge* ≡ *Separation-Kernel.purge realistic-execution ifp*
abbreviation *ipurge-l* **where** *ipurge-l* ≡ *Separation-Kernel.ipurge-l kinvolved ifp*
abbreviation *ipurge-r* **where** *ipurge-r* ≡ *Separation-Kernel.ipurge-r realistic-execution kinvolved ifp*

definition *NI-unrelated*::*bool*
where *NI-unrelated*
 ≡ ∀ *execs a n* . *realistic-executions execs* ⟶
  (*let s-f* = *run n s0 execs in*
   output-f s-f a = *output-f* (*run n s0* (*purge execs* (*current s-f*))) *a*)
definition *NI-indirect-sources*::*bool*
where *NI-indirect-sources*
 ≡ ∀ *execs a n*. *realistic-executions execs* ⟶
  (*let s-f* = *run n s0 execs in*
   output-f (*run n s0* (*ipurge-l execs* (*current s-f*))) *a* =
   output-f (*run n s0* (*ipurge-r execs* (*current s-f*))) *a*)
definition *isecure*::*bool*
where *isecure* ≡ *NI-unrelated* ∧ *NI-indirect-sources*

3.4.3 Proofs

The final theorem is unwinding_implies_isecure_CISK. This theorem shows that any interpretation of locale CISK is secure.

To prove this theorem, the refinement framework is used. CISK is a refinement of ISK, as the only idfference is the control function. In ISK, this function is a generic function called *control*, in CISK it is interpreted in function *CISK_control*. It is proven that function *CISK_control* satisfies all the proof obligations concerning generic function *control*. In other words, *CISK_control* is proven to be an interpretation of control. Therefore, all theorems on run_total apply to the run function of CISK as well.

lemma *next-action-consistent*:
shows ∀ *s t execs* . *vpeq* (*current s*) *s t* ∧ (∀ *d* ∈ *involved* (*next-action s execs*) . *vpeq d s t*) ∧ *current s* = *current t* ⟶ *next-action s execs* = *next-action t execs*
proof–
{
 fix *s t execs*
 assume *vpeq*: *vpeq* (*current s*) *s t*
 assume *vpeq-involved*: ∀ *d* ∈ *involved* (*next-action s execs*) . *vpeq d s t*
 assume *current-s-t*: *current s* = *current t*
 from *aborting-consistent current-s-t vpeq*
  have *aborting t* (*current s*) = *aborting s* (*current s*) **by** *auto*
 from *current-s-t this waiting-consistent vpeq-involved*
  have *next-action s execs* = *next-action t execs*
  unfolding *Kernel.next-action-def*
  by(*cases* (*s*,(*current s*),*execs* (*current s*)) *rule*: *CISK-control.cases*,*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *next-execs-consistent*:
shows ∀ *s t execs* . *vpeq* (*current s*) *s t* ∧ (∀ *d* ∈ *involved* (*next-action s execs*) . *vpeq d s t*) ∧ *current s* = *current t* ⟶ *fst* (*snd* (*CISK-control s* (*current s*) (*execs* (*current s*)))) = *fst* (*snd* (*CISK-control t* (*current s*) (*execs* (*current s*))))
proof–
{
fix *s t execs*
assume *vpeq*: *vpeq* (*current s*) *s t*
assume *vpeq-involved*: ∀ *d* ∈ *involved* (*next-action s execs*) . *vpeq d s t*
assume *current-s-t*: *current s* = *current t*
from *aborting-consistent current-s-t vpeq*
have *1*: *aborting t* (*current s*) = *aborting s* (*current s*) **by** *auto*
from *1 vpeq current-s-t vpeq-involved waiting-consistent*[*THEN spec*,*THEN spec*,*THEN spec*,*THEN spec*,**where** *x3=s* **and** *x2=t* **and** *x1=current s* **and** *x=the* (*next-action s execs*)]
have *fst* (*snd* (*CISK-control s* (*current s*) (*execs* (*current s*)))) = *fst* (*snd* (*CISK-control t* (*current s*) (*execs* (*current s*))))
unfolding *Kernel.next-action-def Kernel.involved-def*
by(*cases* (*s*,(*current s*),*execs* (*current s*)) *rule*: *CISK-control.cases*,*auto split add*: *split-if-asm*)
}
thus *?thesis* **by** *auto*
qed

lemma *next-state-consistent*:
shows ∀ *s t u execs* . *vpeq* (*current s*) *s t* ∧ *vpeq u s t* ∧ *current s* = *current t* ⟶ *vpeq u* (*next-state s execs*) (*next-state t execs*)
proof–
{
fix *s t u execs*
assume *vpeq-s-t*: *vpeq* (*current s*) *s t* ∧ *vpeq u s t*
assume *current-s-t*: *current s* = *current t*
from *vpeq-s-t current-s-t*
have *vpeq u* (*next-state s execs*) (*next-state t execs*)
unfolding *Kernel.next-state-def*
using *aborting-consistent set-error-consistent*
by(*cases* (*s*,(*current s*),*execs* (*current s*)) *rule*: *CISK-control.cases*,*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *current-next-state*:
shows ∀ *s execs* . *current* (*next-state s execs*) = *current s*
proof–
{
fix *s execs*
have *current* (*next-state s execs*) = *current s*
unfolding *Kernel.next-state-def*
using *current-set-error-code*
by(*cases* (*s*,(*current s*),*execs* (*current s*)) *rule*: *CISK-control.cases*,*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *locally-respects-next-state*:
shows ∀ *s u execs*. ¬*ifp* (*current s*) *u* ⟶ *vpeq u s* (*next-state s execs*)
proof–
{

fix *s u execs*
assume ¬*ifp* (*current s*) *u*
hence *vpeq u s* (*next-state s execs*)
unfolding *Kernel.next-state-def*
using *vpeq-reflexive set-error-locally-respects*
by(*cases* (*s*,(*current s*),*execs* (*current s*)) *rule*: *CISK-control.cases*,*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *CISK-control-spec*:
shows ∀ *s d aseqs*.
case CISK-control s d aseqs of
(*a*, *aseqs′*, *s′*) ⇒
thread-empty aseqs ∧ (*a*, *aseqs′*) = (*None*, []) ∨
aseqs ≠ [] ∧ *hd aseqs* ≠ [] ∧ ¬ *aborting s′ d* (*the a*) ∧ ¬ *waiting s′ d* (*the a*) ∧ (*a*, *aseqs′*) = (*Some* (*hd* (*hd aseqs*)), *tl* (*hd aseqs*) # *tl aseqs*) ∨
aseqs ≠ [] ∧ *hd aseqs* ≠ [] ∧ *waiting s′ d* (*the a*) ∧ (*a*, *aseqs′*, *s′*) = (*Some* (*hd* (*hd aseqs*)), *aseqs*, *s*) ∨ (*a*, *aseqs′*) = (*None*, *tl aseqs*)
proof–
{
fix *s d aseqs*
have *case CISK-control s d aseqs of*
(*a*, *aseqs′*, *s′*) ⇒
thread-empty aseqs ∧ (*a*, *aseqs′*) = (*None*, []) ∨
aseqs ≠ [] ∧ *hd aseqs* ≠ [] ∧ ¬ *aborting s′ d* (*the a*) ∧ ¬ *waiting s′ d* (*the a*) ∧ (*a*, *aseqs′*) = (*Some* (*hd* (*hd aseqs*)), *tl* (*hd aseqs*) # *tl aseqs*) ∨
aseqs ≠ [] ∧ *hd aseqs* ≠ [] ∧ *waiting s′ d* (*the a*) ∧ (*a*, *aseqs′*, *s′*) = (*Some* (*hd* (*hd aseqs*)), *aseqs*, *s*) ∨ (*a*, *aseqs′*) = (*None*, *tl aseqs*)
by(*cases* (*s*,*d*,*aseqs*) *rule*: *CISK-control.cases*,*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *next-action-after-cswitch*:
shows ∀ *s n d aseqs* . *fst* (*CISK-control* (*cswitch n s*) *d aseqs*) = *fst* (*CISK-control s d aseqs*)
proof–
{
fix *s n d aseqs*
have *fst* (*CISK-control* (*cswitch n s*) *d aseqs*) = *fst* (*CISK-control s d aseqs*)
using *aborting-switch-independent waiting-switch-independent*
by(*cases* (*s*,*d*,*aseqs*) *rule*: *CISK-control.cases*,*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *next-action-after-next-state*:
shows ∀ *s execs d* . *current s* ≠ *d* ⟶ *fst* (*CISK-control* (*next-state s execs*) *d* (*execs d*)) = *None* ∨ *fst* (*CISK-control* (*next-state s execs*) *d* (*execs d*)) = *fst* (*CISK-control s d* (*execs d*))
proof–
{
fix *s execs d aseqs*
assume *1*: *current s* ≠ *d*
have *fst* (*CISK-control* (*next-state s execs*) *d aseqs*) = *None* ∨ *fst* (*CISK-control* (*next-state s execs*) *d aseqs*) = *fst* (*CISK-control s d aseqs*)
proof(*cases* (*s*,*d*,*aseqs*) *rule*: *CISK-control.cases*,*simp*,*simp*,*simp*)

case (*4 sa da a as execs'*)
thus *?thesis*
unfolding *Kernel.next-state-def*
using *aborting-error-update waiting-error-update 1*
by(*cases* (*sa*,*current sa*,*execs* (*current sa*)) *rule*: *CISK-control.cases*,*auto split*: *split-if-asm*)
qed
}
thus *?thesis* **by** *auto*
qed

lemma *next-action-after-step*:
shows $\forall$ *s a d aseqs* . *current s* $\neq$ *d* $\longrightarrow$ *fst* (*CISK-control* (*step s a*) *d aseqs*) = *fst* (*CISK-control s d aseqs*)
proof–
{
fix *s a d aseqs*
assume *1*: *current s* $\neq$ *d*
from *this aborting-after-step*
have *fst* (*CISK-control* (*step s a*) *d aseqs*) = *fst* (*CISK-control s d aseqs*)
unfolding *Kernel.step-def*
by(*cases* (*s*,*d*,*aseqs*) *rule*: *CISK-control.cases*,*simp*,*simp*,*simp*,*cases a*,*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *next-state-precondition*:
shows $\forall$ *s d a execs*. *AS-precondition s d a* $\longrightarrow$ *AS-precondition* (*next-state s execs*) *d a*
proof–
{
fix *s a d execs*
assume *AS-precondition s d a*
hence *AS-precondition* (*next-state s execs*) *d a*
unfolding *Kernel.next-state-def*
using *precondition-after-set-error-code*
by(*cases* (*s*,(*current s*),*execs* (*current s*)) *rule*: *CISK-control.cases*,*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *next-state-invariant*:
shows $\forall$ *s execs*. *invariant s* $\longrightarrow$ *invariant* (*next-state s execs*)
proof–
{
fix *s execs*
assume *invariant s*
hence *invariant* (*next-state s execs*)
unfolding *Kernel.next-state-def*
using *invariant-after-set-error-code*
by(*cases* (*s*,(*current s*),*execs* (*current s*)) *rule*: *CISK-control.cases*,*auto*)
}
thus *?thesis* **by** *auto*
qed

lemma *next-action-from-execs*:
shows $\forall$ *s execs* . *next-action s execs* $\rightharpoonup$ (λ *a* . *a* $\in$ *actions-in-execution* (*execs* (*current s*)))
proof–
{
fix *s execs*

```
 {
  fix a
  assume 1: next-action s execs = Some a
  from 1 have a ∈ actions-in-execution (execs (current s))
   unfolding Kernel.next-action-def actions-in-execution-def
   by (cases (s,(current s),execs (current s)) rule: CISK-control.cases,auto split add: split-if-asm)
 }
 hence next-action s execs ⇀ (λ a . a ∈ actions-in-execution (execs (current s)))
  unfolding B-def
  by (cases next-action s execs,auto)
}
thus ?thesis unfolding B-def by (auto)
qed

lemma next-execs-subset:
shows ∀ s execs u . actions-in-execution (next-execs s execs u) ⊆ actions-in-execution (execs u)
proof–
{
 fix s execs u
 have actions-in-execution (next-execs s execs u) ⊆ actions-in-execution (execs u)
  unfolding Kernel.next-execs-def actions-in-execution-def
  by (cases (s,(current s),execs (current s)) rule: CISK-control.cases,auto split add: split-if-asm)
}
thus ?thesis by auto
qed


theorem unwinding-implies-isecure-CISK:
shows isecure
proof–
 interpret int: Interruptible-Separation-Kernel kstep output-f s0 current cswitch interrupt kprecondition realistic-execution
CISK-control kinvolved ifp vpeq AS-set invariant AS-precondition aborting waiting
  proof (unfold-locales)
   show ∀ a b c u. vpeq u a b ∧ vpeq u b c ⟶ vpeq u a c
    using vpeq-transitive by blast
   show ∀ a b u. vpeq u a b ⟶ vpeq u b a
    using vpeq-symmetric by blast
   show ∀ a u. vpeq u a a
    using vpeq-reflexive by blast
   show ∀ u. ifp u u
    using ifp-reflexive by blast
   show ∀  s t u a. vpeq u s t ∧ vpeq (current s) s t ∧ kprecondition s a ∧ kprecondition t a ∧ current s = current t
⟶ vpeq u (kstep s a) (kstep t a)
    using weakly-step-consistent by blast
   show ∀  a s u. ¬ifp (current s) u  ∧ kprecondition s a ⟶ vpeq u s (kstep s a)
    using locally-respects by blast
   show ∀  a s t. vpeq (current s) s t ∧ current s = current t ⟶ (output-f s a) = (output-f t a)
    using output-consistent by blast
   show ∀  s a . current (kstep s a) = current s
    using step-atomicity by blast
   show ∀  n s t . current s = current t ⟶ current (cswitch n s) = current (cswitch n t)
    using cswitch-independent-of-state by blast
   show ∀  u s t n . vpeq u s t ⟶ vpeq u (cswitch n s) (cswitch n t)
    using cswitch-consistency by blast
   show ∀ s t execs. vpeq (current s) s t ∧ (∀  d ∈ involved (next-action s execs) . vpeq d s t) ∧ current s = current
t ⟶ next-action s execs = next-action t execs
    using next-action-consistent by blast
```

show $\forall$ *s t execs*.
vpeq (*current s*) *s t* $\wedge$ ($\forall$ *d* $\in$ *involved* (*next-action s execs*) . *vpeq d s t*) $\wedge$ *current s* = *current t* $\longrightarrow$
fst (*snd* (*CISK-control s* (*current s*) (*execs* (*current s*)))) = *fst* (*snd* (*CISK-control t* (*current s*) (*execs* (*current s*))))
using *next-execs-consistent* **by** *blast*
show $\forall$ *s t u execs*. *vpeq* (*current s*) *s t* $\wedge$ *vpeq u s t* $\wedge$ *current s* = *current t* $\longrightarrow$ *vpeq u* (*next-state s execs*) (*next-state t execs*)
using *next-state-consistent* **by** *auto*
show $\forall$ *s execs*. *current* (*next-state s execs*) = *current s*
using *current-next-state* **by** *auto*
show $\forall$ *s u execs*. $\neg$ *ifp* (*current s*) *u* $\longrightarrow$ *vpeq u s* (*next-state s execs*)
using *locally-respects-next-state* **by** *auto*
show [] $\in$ *AS-set*
using *empty-in-AS-set* **by** *blast*
show $\forall$ *s n* . *invariant s* $\longrightarrow$ *invariant* (*cswitch n s*)
using *invariant-after-cswitch* **by** *blast*
show $\forall$ *s d n a*. *AS-precondition s d a* $\longrightarrow$ *AS-precondition* (*cswitch n s*) *d a*
using *precondition-after-cswitch* **by** *blast*
show *invariant s0*
using *invariant-s0* **by** *blast*
show $\forall$ *s d aseq* . *invariant s* $\wedge$ *aseq* $\in$ *AS-set* $\wedge$ *aseq* $\neq$ [] $\longrightarrow$ *AS-precondition s d* (*hd aseq*)
using *AS-prec-first-action* **by** *blast*
show $\forall$ *s a a'*. ($\exists$ *aseq*$\in$*AS-set*. *is-sub-seq a a' aseq*) $\wedge$ *invariant s* $\wedge$ *AS-precondition s* (*current s*) *a* $\wedge$ $\neg$ *aborting s* (*current s*) *a* $\wedge$ $\neg$ *waiting s* (*current s*) *a* $\longrightarrow$
AS-precondition (*kstep s a*) (*current s*) *a'*
using *AS-prec-after-step* **by** *blast*
show $\forall$ *s d a a'* . *current s* $\neq$ *d* $\wedge$ *AS-precondition s d a* $\longrightarrow$ *AS-precondition* (*kstep s a'*) *d a*
using *AS-prec-dom-independent* **by** *blast*
show $\forall$ *s a* . *invariant s* $\longrightarrow$ *invariant* (*kstep s a*)
using *spec-of-invariant* **by** *blast*
show $\bigwedge$*s a*. *kprecondition s a* $\equiv$ *kprecondition s a*
by *auto*
show $\bigwedge$*aseq*. *realistic-execution aseq* $\equiv$ *set aseq* $\subseteq$ *AS-set*
unfolding *realistic-execution-def*
by *auto*
show $\forall$ *s a*. $\forall$ *d* $\in$ *involved a*. *kprecondition s* (*the a*) $\longrightarrow$ *ifp d* (*current s*)
using *involved-ifp* **unfolding** *Kernel.involved-def* **by** (*auto split*: *option.splits*)
show $\forall$ *s execs*. *next-action s execs* $\rightharpoonup$ (λ*a*. *a* $\in$ *actions-in-execution* (*execs* (*current s*)))
using *next-action-from-execs* **by** *blast*
show $\forall$ *s execs u*. *actions-in-execution* (*next-execs s execs u*) $\subseteq$ *actions-in-execution* (*execs u*)
using *next-execs-subset* **by** *blast*
show $\forall$ *s d aseqs*.
case CISK-control s d aseqs of
(*a*, *aseqs'*, *s'*) $\Rightarrow$
thread-empty aseqs $\wedge$ (*a*, *aseqs'*) = (*None*, []) $\vee$
aseqs $\neq$ [] $\wedge$ *hd aseqs* $\neq$ [] $\wedge$ $\neg$ *aborting s' d* (*the a*) $\wedge$ $\neg$ *waiting s' d* (*the a*) $\wedge$ (*a*, *aseqs'*) = (*Some* (*hd* (*hd aseqs*)), *tl* (*hd aseqs*) # *tl aseqs*) $\vee$
aseqs $\neq$ [] $\wedge$ *hd aseqs* $\neq$ [] $\wedge$ *waiting s' d* (*the a*) $\wedge$ (*a*, *aseqs'*, *s'*) = (*Some* (*hd* (*hd aseqs*)), *aseqs*, *s*) $\vee$ (*a*, *aseqs'*) = (*None*, *tl aseqs*)
using *CISK-control-spec* **by** *blast*
show $\forall$ *s n d aseqs*. *fst* (*CISK-control* (*cswitch n s*) *d aseqs*) = *fst* (*CISK-control s d aseqs*)
using *next-action-after-cswitch* **by** *auto*
show $\forall$ *s execs d*.
current s $\neq$ *d* $\longrightarrow$
fst (*CISK-control* (*next-state s execs*) *d* (*execs d*)) = *None* $\vee$ *fst* (*CISK-control* (*next-state s execs*) *d* (*execs d*)) = *fst* (*CISK-control s d* (*execs d*))
using *next-action-after-next-state* **by** *auto*

show ∀ *s a d aseqs*. *current s* ≠ *d* ⟶ *fst* (*CISK-control* (*step s a*) *d aseqs*) = *fst* (*CISK-control s d aseqs*)
  using *next-action-after-step* **by** *auto*
 show ∀ *s d a execs*. *AS-precondition s d a* ⟶ *AS-precondition* (*next-state s execs*) *d a*
  using *next-state-precondition* **by** *auto*
 show ∀ *s execs*. *invariant s* ⟶ *invariant* (*next-state s execs*)
  using *next-state-invariant* **by** *auto*
 show ∀ *s a*. *waiting s* (*current s*) *a* ⟶ *kstep s a* = *s*
  using *spec-of-waiting* **by** *blast*
qed

note *interpreted=‹Interruptible-Separation-Kernel kstep output-f s0 current cswitch kprecondition realistic-execution CISK-control kinvolved ifp vpeq AS-set invariant AS-precondition aborting waiting›*
note *run-total-induct = Interruptible-Separation-Kernel.run-total.induct*[*of kstep output-f s0 current cswitch kprecondition realistic-execution*
CISK-control kinvolved ifp vpeq AS-set invariant AS-precondition aborting waiting - interrupt]
have *run-equals-run-total*:
 ⋀ *n s execs* . *run n s execs* ≡ *Interruptible-Separation-Kernel.run-total kstep current cswitch interrupt CISK-control n s execs*
 proof–
  fix *n s execs*
  show *run n s execs* ≡ *Interruptible-Separation-Kernel.run-total kstep current cswitch interrupt CISK-control n s execs*
   using *interpreted int.step-def*
   by(*induct n s execs rule*: *run-total-induct*,*auto split*: *option.splits*)
 qed
from *interpreted*
have *0*: *Interruptible-Separation-Kernel.isecure-total kstep output-f s0 current cswitch interrupt realistic-execution CISK-control kinvolved ifp*
 by (*metis int.unwinding-implies-isecure-total*)
from *0 run-equals-run-total*
 have *1*: *NI-unrelated*
  by (*metis realistic-executions-def int.isecure-total-def int.realistic-executions-def int.NI-unrelated-total-def NI-unrelated-def*)
from *0 run-equals-run-total*
 have *2*: *NI-indirect-sources*
 by (*metis realistic-executions-def int.NI-indirect-sources-total-def int.isecure-total-def int.realistic-executions-def NI-indirect-sources-def*)
from *1 2* **show** *?thesis* **unfolding** *isecure-def* **by** *auto*
qed

end
end

4 Instantiation by a separation kernel with concrete actions

In the previous section, no concrete actions for the step function were given. The foremost point we want to make by this instantiation is to show that we can instantiate the CISK model of the previous section with an implementation that, for the step function, as actions, provides events and interprocess communication (IPC). System call invocations that can be interrupted at certain interrupt points are split into several atomic steps. A communication interface of events and IPC is less "trivial" than it may seem it at a first glance, for example the L4 microkernel API *only* provided IPC as communication primitive [16]. In particular, the concrete actions illustrate how an application of the CISK framework can be used to separate policy enforcement from other computations unrelated to policy enforcement.

Our separation kernel instantiation also has a notion of *partitions*. A *partition* is a logical unit that serves to encapsulate a group of CISK threads by, amongst others, enforcing a static per-partition access control policy to system resources. In the following instantiation, while the subjects of the step function are individual threads, the

information flow policy ifp is defined at the granularity of partitions, which is realistic for many separation kernel implementations.
Lastly, as a limited manipulation of an access control policy is often needed, we also provide an invariant for having a dynamic access control policy whose maximal closure is bounded by the static per-partition access control policy. That the dynamic access control policy is a subset of a static access control policy is expressed by the invariant sp_subset. A use case for this is when you have statically configured access to files by subjects, but whether a file can be read/written also depends on whether the file has been dynamically opened or not. The instantiation provides infrastructure for such an invariant on the relation of a dynamic policy to a static policy, and shows how the invariant is maintained, without modeling any API for an open/close operation.

4.1 Model of a separation kernel configuration

theory *Step-configuration*
 imports *Main*
begin

4.1.1 Type definitions

The separation kernel partitions are considered to be the "subjects" of the information flow policy *ifp*. A file provider is a partition that, via a file API (read/write), provides files to other partitions. The configuration statically defines which partitions can act as a file provider and also the access rights (right/write) of other partitions to the files provided by the file provider. Some separation kernels include a management for address spaces (tasks), that may be hierachically structured. Such a task hierarchy is not part of this model.

typedecl *partition-id-t*
typedecl *thread-id-t*

typedecl *page-t* — physical address of a memory page
typedecl *filep-t* — name of file provider

datatype *obj-id-t* =
  PAGE page-t
 | *FILEP filep-t*

datatype *mode-t* =
  READ — The subject has right to read from the memory page, from the files served by a file provider.
 | *WRITE* — The subject has right to write to the memory page, from the files served by a file provider.
 | *PROVIDE* — The subject has right serve as the file provider. This mode is not used for memory pages or ports.

4.1.2 Configuration

The information flow policy is implicitly specified by the configuration. The configuration does not contain the communication rights between partitions (subjects). However, the rights can be derived from the configuration. For example, if two partitions p and p' can access a file *f*, then p and p' can communicate. See below.

consts
 configured-subj-obj :: *partition-id-t* ⇒ *obj-id-t* ⇒ *mode-t* ⇒ *bool*

Each user thread belongs to a partition. The relation is fixed at system startup. The configuration specifies how many threads a partition can create, but this limit is not part of the model.

consts
 partition :: *thread-id-t* ⇒ *partition-id-t*

end

4.2 Formulation of a subject-subject communication policy and an information flow policy, and showing both can be derived from subject-object configuration data

theory *Step-policies*
 imports *Step-configuration*
begin

4.2.1 Specification

In order to use CISK, we need an information flow policy *ifp* relation. We also express a static subject-subject *sp-spec-subj-obj* and subject-object *sp-spec-subj-subj* access control policy for the implementation of the model. The following locale summarizes all properties we need.

locale *policy-axioms* =
 fixes *sp-spec-subj-obj* :: *'a ⇒ obj-id-t ⇒ mode-t ⇒ bool*
  and *sp-spec-subj-subj* :: *'a ⇒ 'a ⇒ bool*
  and *ifp* :: *'a ⇒ 'a ⇒ bool*

 assumes *sp-spec-file-provider*: ∀ *p1 p2 f m1 m2* .
  sp-spec-subj-obj p1 (*FILEP f*) *m1* ∧
  sp-spec-subj-obj p2 (*FILEP f*) *m2* ⟶ *sp-spec-subj-subj p1 p2*

 and *sp-spec-no-wronly-pages*:
  ∀ *p x* . *sp-spec-subj-obj p* (*PAGE x*) *WRITE* ⟶ *sp-spec-subj-obj p* (*PAGE x*) *READ*

 and *ifp-reflexive*:
  ∀ *p* . *ifp p p*

 and *ifp-compatible-with-sp-spec*:
  ∀ *a b* . *sp-spec-subj-subj a b* ⟶ *ifp a b* ∧ *ifp b a*

 and *ifp-compatible-with-ipc*:
  ∀ *a b c x* . (*sp-spec-subj-subj a b*
   ∧ *sp-spec-subj-obj b* (*PAGE x*) *WRITE* ∧ *sp-spec-subj-obj c* (*PAGE x*) *READ*)
   ⟶ *ifp a c*
begin end

4.2.2 Derivation

The configuration data only consists of a subject-object policy. We derive the subject-subject policy and the information flow policy from the configuration data and prove that properties we specified in Section 4.2.1 are satisfied.

locale *abstract-policy-derivation* =
 fixes *configuration-subj-obj* :: *'a ⇒ obj-id-t ⇒ mode-t ⇒ bool*
begin

 definition *sp-spec-subj-obj a x m* ≡
  configuration-subj-obj a x m ∨ (∃ *y* . *x* = *PAGE y* ∧ *m* = *READ* ∧ *configuration-subj-obj a x WRITE*)

 definition *sp-spec-subj-subj a b* ≡
  ∃ *f m1 m2* . *sp-spec-subj-obj a* (*FILEP f*) *m1* ∧ *sp-spec-subj-obj b* (*FILEP f*) *m2*

 definition *ifp a b* ≡
  sp-spec-subj-subj a b
  ∨ *sp-spec-subj-subj b a*
  ∨ (∃ *c y* . *sp-spec-subj-subj a c*
   ∧ *sp-spec-subj-obj c* (*PAGE y*) *WRITE*

$\quad\quad \wedge$ *sp-spec-subj-obj b* (*PAGE y*) *READ*)
$\vee$ (*a* = *b*)

Show that the policies specified in Section 4.2.1 can be derived from the configuration and their definitions.

lemma *correct*:
 shows *policy-axioms sp-spec-subj-obj sp-spec-subj-subj ifp*
proof (*unfold-locales*)
 show *sp-spec-file-provider*:
  ∀ *p1 p2 f m1 m2* .
   sp-spec-subj-obj p1 (*FILEP f*) *m1* ∧
   sp-spec-subj-obj p2 (*FILEP f*) *m2* ⟶ *sp-spec-subj-subj p1 p2*
  unfolding *sp-spec-subj-subj-def* **by** *auto*
 show *sp-spec-no-wronly-pages*:
  ∀ *p x* . *sp-spec-subj-obj p* (*PAGE x*) *WRITE* ⟶ *sp-spec-subj-obj p* (*PAGE x*) *READ*
  unfolding *sp-spec-subj-obj-def* **by** *auto*
 show *ifp-reflexive*:
  ∀ *p* . *ifp p p*
  unfolding *ifp-def* **by** *auto*
 show *ifp-compatible-with-sp-spec*:
  ∀ *a b* . *sp-spec-subj-subj a b* ⟶ *ifp a b* ∧ *ifp b a*
  unfolding *ifp-def* **by** *auto*
 show *ifp-compatible-with-ipc*:
  ∀ *a b c x* . (*sp-spec-subj-subj a b*
     ∧ *sp-spec-subj-obj b* (*PAGE x*) *WRITE* ∧ *sp-spec-subj-obj c* (*PAGE x*) *READ*)
     ⟶ *ifp a c*
  unfolding *ifp-def* **by** *auto*
qed
end

type-synonym *sp-subj-subj-t* = *partition-id-t* ⇒ *partition-id-t* ⇒ *bool*
type-synonym *sp-subj-obj-t* = *partition-id-t* ⇒ *obj-id-t* ⇒ *mode-t* ⇒ *bool*

interpretation *Policy*: *abstract-policy-derivation configured-subj-obj*.
interpretation *Policy-properties*: *policy-axioms Policy.sp-spec-subj-obj Policy.sp-spec-subj-subj Policy.ifp*
 using *Policy.correct* **by** *auto*

lemma *example-how-to-use-properties-in-proofs*:
 shows ∀ *p* . *Policy.ifp p p*
 using *Policy-properties.ifp-reflexive* **by** *auto*

end

4.3 Separation kernel state and atomic step function

theory *Step*
 imports *Step-policies*
begin

4.3.1 Interrupt points

To model concurrency, each system call is split into several atomic steps, while allowing interrupts between the steps. The state of a thread is represented by an "interrupt point" (which corresponds to the value of the program counter saved by the system when a thread is interrupted).

datatype *ipc-direction-t* = *SEND* | *RECV*
datatype *ipc-stage-t* = *PREP* | *WAIT* | *BUF page-t*

datatype *ev-consume-t* = *EV-CONSUME-ALL* | *EV-CONSUME-ONE*
datatype *ev-wait-stage-t* = *EV-PREP* | *EV-WAIT* | *EV-FINISH*
datatype *ev-signal-stage-t* = *EV-SIGNAL-PREP* | *EV-SIGNAL-FINISH*

datatype *int-point-t* =
SK-IPC ipc-direction-t ipc-stage-t thread-id-t page-t — The thread is executing a sending / receiving IPC.
| *SK-EV-WAIT ev-wait-stage-t ev-consume-t* — The thread is waiting for an event.
| *SK-EV-SIGNAL ev-signal-stage-t thread-id-t* — The thread is sending an event.
| *NONE* — The thread is not executing any system call.

4.3.2 System state

typedecl *obj-t* — value of an object

Each thread belongs to a partition. The relation is fixed (in this instantiation of a separation kernel).

consts
partition :: *thread-id-t* ⇒ *partition-id-t*

The state contains the dynamic policy (the communication rights in the current state of the system, for example).

record *thread-t* =

ev-counter :: *nat* — event counter

record *state-t* =
sp-impl-subj-subj :: *sp-subj-subj-t* — current subject-subject policy
sp-impl-subj-obj :: *sp-subj-obj-t* — current subject-object policy
current :: *thread-id-t* — current thread
obj :: *obj-id-t* ⇒ *obj-t* — values of all objects
thread :: *thread-id-t* ⇒ *thread-t* — internal state of threads

Later (Section 4.4), the system invariant *sp-subset* will be used to ensure that the dynamic policies (sp_impl_...) are a subset of the corresponding static policies (sp_spec_...).

4.3.3 Atomic step

Helper functions Set new value for an object.

definition *set-object-value* :: *obj-id-t* ⇒ *obj-t* ⇒ *state-t* ⇒ *state-t* **where**
set-object-value obj-id val s =
s (| *obj* := *fun-upd* (*obj s*) *obj-id val* |)

Return a representation of the opposite direction of IPC communication.

definition *opposite-ipc-direction* :: *ipc-direction-t* ⇒ *ipc-direction-t* **where**
opposite-ipc-direction dir ≡ *case dir of SEND* ⇒ *RECV* | *RECV* ⇒ *SEND*

Add an access right from one partition to an object. In this model, not available from the API, but shows how dynamic changes of access rights could be implemented.

definition *add-access-right* :: *partition-id-t* => *obj-id-t* => *mode-t* => *state-t* => *state-t* **where**
add-access-right part-id obj-id m s =
s (| *sp-impl-subj-obj* := $\lambda\, q\, q'\, q''.\ (part\text{-}id = q \wedge obj\text{-}id = q' \wedge m = q'')$
$\vee$ *sp-impl-subj-obj* $s\, q\, q'\, q''$ |)

Add a communication right from one partition to another. In this model, not available from the API.

definition *add-comm-right* :: *partition-id-t* ⇒ *partition-id-t* ⇒ *state-t* ⇒ *state-t* **where**
add-comm-right $p\ p'\ s$ ≡
s (| *sp-impl-subj-subj* := $\lambda\, q\, q'.\ (p = q \wedge p' = q') \vee$ *sp-impl-subj-subj* $s\, q\, q'$ |)

Model of IPC system call We model IPC with the following simplifications:

1. The model contains the system calls for sending an IPC (SEND) and receiving an IPC (RECV), often implementations have a richer API (e.g. combining SEND and RECV in one invocation).
2. We model only a copying ("BUF") mode, not a memory-mapping mode.
3. The model always copies one page per syscall.

definition *ipc-precondition* :: *thread-id-t* ⇒ *ipc-direction-t* ⇒ *thread-id-t* ⇒ *page-t* ⇒ *state-t* ⇒ *bool* **where**
ipc-precondition tid dir partner page s ≡
let sender = (*case dir of SEND* ⇒ *tid* | *RECV* ⇒ *partner*) *in*
let receiver = (*case dir of SEND* ⇒ *partner* | *RECV* ⇒ *tid*) *in*
let local-access-mode = (*case dir of SEND* ⇒ *READ* | *RECV* ⇒ *WRITE*) *in*
(*sp-impl-subj-subj s* (*partition sender*) (*partition receiver*)
∧ *sp-impl-subj-obj s* (*partition tid*) (*PAGE page*) *local-access-mode*)

definition *atomic-step-ipc* :: *thread-id-t* ⇒ *ipc-direction-t* ⇒ *ipc-stage-t* ⇒ *thread-id-t* ⇒ *page-t* ⇒ *state-t* ⇒ *state-t* **where**
atomic-step-ipc tid dir stage partner page s ≡
case stage of
PREP ⇒
s
| *WAIT* ⇒
s
| *BUF page'* ⇒
(*case dir of*
SEND ⇒
(*set-object-value* (*PAGE page'*) (*obj s* (*PAGE page*)) *s*)
| *RECV* ⇒ *s*)

Model of event syscalls **definition** *ev-signal-precondition* :: *thread-id-t* ⇒ *thread-id-t* ⇒ *state-t* ⇒ *bool* **where**
ev-signal-precondition tid partner s ≡
(*sp-impl-subj-subj s* (*partition tid*) (*partition partner*))

definition *atomic-step-ev-signal* :: *thread-id-t* ⇒ *thread-id-t* ⇒ *state-t* ⇒ *state-t* **where**
atomic-step-ev-signal tid partner s =
s (| *thread* := *fun-upd* (*thread s*) *partner* (*thread s partner* (| *ev-counter* := *Suc* (*ev-counter* (*thread s partner*)) |)) |)

definition *atomic-step-ev-wait-one* :: *thread-id-t* ⇒ *state-t* ⇒ *state-t* **where**
atomic-step-ev-wait-one tid s =
s (| *thread* := *fun-upd* (*thread s*) *tid* (*thread s tid* (| *ev-counter* := (*ev-counter* (*thread s tid*) − *1*) |)) |)

definition *atomic-step-ev-wait-all* :: *thread-id-t* ⇒ *state-t* ⇒ *state-t* **where**
atomic-step-ev-wait-all tid s =
s (| *thread* := *fun-upd* (*thread s*) *tid* (*thread s tid* (| *ev-counter* := *0* |)) |)

Instantiation of CISK aborting and waiting In this instantiation of CISK, the *aborting* function is used to indicate security policy enforcement. An IPC call aborts in its *PREP* stage if the precondition for the calling thread does not hold. An event signal call aborts in its *EV-SIGNAL-PREP* stage if the precondition for the calling thread does not hold.

definition *aborting* :: *state-t* ⇒ *thread-id-t* ⇒ *int-point-t* ⇒ *bool*
where *aborting s tid a* ≡ *case a of SK-IPC dir PREP partner page* ⇒

¬ipc-precondition tid dir partner page s
| SK-EV-SIGNAL EV-SIGNAL-PREP partner ⇒
¬ev-signal-precondition tid partner s
| - => False

The *waiting* function is used to indicate synchronization. An IPC call waits in its *WAIT* stage while the precondition for the partner thread does not hold. An EV_WAIT call waits until the event counter is not zero.

definition *waiting* :: *state-t* ⇒ *thread-id-t* ⇒ *int-point-t* ⇒ *bool*
where *waiting s tid a* ≡
case a of SK-IPC dir WAIT partner page ⇒
¬ipc-precondition partner (opposite-ipc-direction dir) tid (SOME page′ . True) s
| SK-EV-WAIT EV-PREP - ⇒ False
| SK-EV-WAIT EV-WAIT - ⇒ ev-counter (thread s tid) = 0
| SK-EV-WAIT EV-FINISH - ⇒ False
| - ⇒ False

The atomic step function. In the definition of *atomic-step* the arguments to an interrupt point are not taken from the thread state – the argument given to *atomic-step* could have an arbitrary value. So, seen in isolation, *atomic-step* allows more transitions than actually occur in the separation kernel. However, the CISK framework (1) restricts the atomic step function by the *waiting* and *aborting* functions as well (2) the set of realistic traces as attack sequences *rAS-set* (Section 4.8). An additional condition is that (3) the dynamic policy used in *aborting* is a subset of the static policy. This is ensured by the invariant *sp-subset*.

definition *atomic-step* :: *state-t* ⇒ *int-point-t* ⇒ *state-t* **where**
atomic-step s ipt ≡
case ipt of
SK-IPC dir stage partner page ⇒
atomic-step-ipc (current s) dir stage partner page s
| SK-EV-WAIT EV-PREP consume ⇒ s
| SK-EV-WAIT EV-WAIT consume ⇒ s
| SK-EV-WAIT EV-FINISH consume ⇒
case consume of
EV-CONSUME-ONE ⇒ atomic-step-ev-wait-one (current s) s
| EV-CONSUME-ALL ⇒ atomic-step-ev-wait-all (current s) s
| SK-EV-SIGNAL EV-SIGNAL-PREP partner ⇒ s
| SK-EV-SIGNAL EV-SIGNAL-FINISH partner ⇒
atomic-step-ev-signal (current s) partner s
| NONE ⇒ s

end

4.4 Preconditions and invariants for the atomic step

theory *Step-invariants*
imports *Step*
begin

The dynamic/implementation policies have to be compatible with the static configuration.

definition *sp-subset s* ≡
$(\forall\ p1\ p2\ .\ sp\text{-}impl\text{-}subj\text{-}subj\ s\ p1\ p2 \longrightarrow Policy.sp\text{-}spec\text{-}subj\text{-}subj\ p1\ p2)$
$\wedge (\forall\ p1\ p2\ m.\ sp\text{-}impl\text{-}subj\text{-}obj\ s\ p1\ p2\ m \longrightarrow Policy.sp\text{-}spec\text{-}subj\text{-}obj\ p1\ p2\ m)$

The following predicate expresses the precondition for the atomic step. The precondition depends on the type of the atomic action.

definition *atomic-step-precondition* :: *state-t* ⇒ *thread-id-t* ⇒ *int-point-t* ⇒ *bool* **where**
atomic-step-precondition s tid ipt ≡
case ipt of
SK-IPC dir WAIT partner page ⇒
(∗ *the thread managed it past PREP stage* ∗)
ipc-precondition tid dir partner page s
| *SK-IPC dir* (*BUF page*′) *partner page* ⇒
(∗ *both the calling thread and its communication partner*
managed it past PREP and WAIT stages ∗)
ipc-precondition tid dir partner page s
∧ *ipc-precondition partner* (*opposite-ipc-direction dir*) *tid page*′ *s*
| *SK-EV-SIGNAL EV-SIGNAL-FINISH partner* ⇒
ev-signal-precondition tid partner s
| - ⇒
(∗ *No precondition for other interrupt points.* ∗)
True

The invariant to be preserved by the atomic step function. The invariant is independent from the type of the atomic action.

definition *atomic-step-invariant* :: *state-t* ⇒ *bool* **where**
atomic-step-invariant s ≡
sp-subset s

4.4.1 Atomic steps of SK_IPC preserve invariants

lemma *set-object-value-invariant*:
shows *atomic-step-invariant s* = *atomic-step-invariant* (*set-object-value ob va s*)
proof –
show *?thesis* **using** *assms*
unfolding *atomic-step-invariant-def atomic-step-precondition-def ipc-precondition-def*
sp-subset-def set-object-value-def Let-def
by (*simp split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*)
qed

lemma *set-thread-value-invariant*:
shows *atomic-step-invariant s* = *atomic-step-invariant* (*s* (| *thread* := *thrst* |))
proof –
show *?thesis* **using** *assms*
unfolding *atomic-step-invariant-def atomic-step-precondition-def ipc-precondition-def*
sp-subset-def set-object-value-def Let-def
by (*simp split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*)
qed

lemma *atomic-ipc-preserves-invariants*:
fixes *s* :: *state-t*
and *tid* :: *thread-id-t*
assumes *atomic-step-invariant s*
shows *atomic-step-invariant* (*atomic-step-ipc tid dir stage partner page s*)
proof –
show *?thesis*
proof (*cases stage*)
case *PREP*
from *this assms* **show** *?thesis*
unfolding *atomic-step-ipc-def atomic-step-invariant-def* **by** *auto*
next
case *WAIT*
from *this assms* **show** *?thesis*
unfolding *atomic-step-ipc-def atomic-step-invariant-def* **by** *auto*

```
  next
  case BUF
    show ?thesis
      using assms BUF set-object-value-invariant
      unfolding atomic-step-ipc-def
      by (simp split add: ipc-direction-t.splits)
  qed
qed

lemma atomic-ev-wait-one-preserves-invariants:
 fixes s :: state-t
   and tid :: thread-id-t
 assumes atomic-step-invariant s
 shows atomic-step-invariant (atomic-step-ev-wait-one tid s)
 proof –
  from assms show ?thesis
  unfolding atomic-step-ev-wait-one-def atomic-step-invariant-def sp-subset-def
  by auto
 qed

lemma atomic-ev-wait-all-preserves-invariants:
 fixes s :: state-t
   and tid :: thread-id-t
 assumes atomic-step-invariant s
 shows atomic-step-invariant (atomic-step-ev-wait-all tid s)
 proof –
  from assms show ?thesis
  unfolding atomic-step-ev-wait-all-def atomic-step-invariant-def sp-subset-def
  by auto
 qed

lemma atomic-ev-signal-preserves-invariants:
 fixes s :: state-t
   and tid :: thread-id-t
 assumes atomic-step-invariant s
 shows atomic-step-invariant (atomic-step-ev-signal tid  partner s)
 proof –
  from assms show ?thesis
  unfolding atomic-step-ev-signal-def atomic-step-invariant-def sp-subset-def
  by auto
 qed
```

4.4.2 Summary theorems on atomic step invariants

Now we are ready to show that an atomic step from the current interrupt point in any thread preserves invariants.

```
theorem atomic-step-preserves-invariants:
 fixes s :: state-t
   and tid :: thread-id-t
 assumes atomic-step-invariant s
 shows atomic-step-invariant (atomic-step s a)
proof (cases a)
 case SK-IPC
   then show ?thesis unfolding atomic-step-def
   using assms atomic-ipc-preserves-invariants
   by simp
 next  case (SK-EV-WAIT ev-wait-stage consume)
```

then show *?thesis*
proof (*cases consume*)
case *EV-CONSUME-ALL*
then show *?thesis* **unfolding** *atomic-step-def*
using *SK-EV-WAIT assms atomic-ev-wait-all-preserves-invariants*
by (*simp split*: *ev-wait-stage-t.splits*)
next case *EV-CONSUME-ONE*
then show *?thesis* **unfolding** *atomic-step-def*
using *SK-EV-WAIT assms atomic-ev-wait-one-preserves-invariants*
by (*simp split*: *ev-wait-stage-t.splits*)
qed
next case *SK-EV-SIGNAL*
then show *?thesis* **unfolding** *atomic-step-def*
using *assms atomic-ev-signal-preserves-invariants*
by (*simp add*: *ev-signal-stage-t.splits*)
next case *NONE*
then show *?thesis* **unfolding** *atomic-step-def*
using *assms*
by *auto*
qed

Finally, the invariants do not depend on the current thread. That is, the context switch preserves the invariants, and an atomic step that is not a context switch does not change the current thread.

theorem *cswitch-preserves-invariants*:
fixes *s* :: *state-t*
and *new-current* :: *thread-id-t*
assumes *atomic-step-invariant s*
shows *atomic-step-invariant* (*s* (| *current* := *new-current* |))
proof –
let *?s1* = *s* (| *current* := *new-current* |)
have *sp-subset s* = *sp-subset ?s1*
unfolding *sp-subset-def* **by** *auto*
from *assms this* **show** *?thesis*
unfolding *atomic-step-invariant-def* **by** *metis*
qed

theorem *atomic-step-does-not-change-current-thread*:
shows *current* (*atomic-step s ipt*) = *current s*
proof –
show *?thesis*
unfolding *atomic-step-def*
and *atomic-step-ipc-def*
and *set-object-value-def Let-def*
and *atomic-step-ev-wait-one-def atomic-step-ev-wait-all-def*
and *atomic-step-ev-signal-def*
by (*simp split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*
ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)
qed

end

4.5 The view-partitioning equivalence relation

theory *Step-vpeq*
imports *Step Step-invariants*
begin

The view consists of

1. View of object values.
2. View of subject-subject dynamic policy. The threads can discover the policy at runtime, e.g. by calling ipc() and observing success or failure.
3. View of subject-object dynamic policy. The threads can discover the policy at runtime, e.g. by calling open() and observing success or failure.

definition *vpeq-obj* :: *partition-id-t* ⇒ *state-t* ⇒ *state-t* ⇒ *bool* **where**
vpeq-obj u s t ≡ ∀ *obj-id* . *Policy.sp-spec-subj-obj u obj-id READ* ⟶ (*obj s*) *obj-id* = (*obj t*) *obj-id*

definition *vpeq-subj-subj* :: *partition-id-t* ⇒ *state-t* ⇒ *state-t* ⇒ *bool* **where**
vpeq-subj-subj u s t ≡
∀ *v* . ((*Policy.sp-spec-subj-subj u v* ⟶ *sp-impl-subj-subj s u v* = *sp-impl-subj-subj t u v*)
∧ (*Policy.sp-spec-subj-subj v u* ⟶ *sp-impl-subj-subj s v u* = *sp-impl-subj-subj t v u*))

definition *vpeq-subj-obj* :: *partition-id-t* ⇒ *state-t* ⇒ *state-t* ⇒ *bool* **where**
vpeq-subj-obj u s t ≡
∀ *ob m p1* .
(*Policy.sp-spec-subj-obj u ob m* ⟶ *sp-impl-subj-obj s u ob m* = *sp-impl-subj-obj t u ob m*)
∧ (*Policy.sp-spec-subj-obj p1 ob PROVIDE* ∧ (*Policy.sp-spec-subj-obj u ob READ* ∨ *Policy.sp-spec-subj-obj u ob WRITE*) ⟶
sp-impl-subj-obj s p1 ob PROVIDE = *sp-impl-subj-obj t p1 ob PROVIDE*)

definition *vpeq-local* :: *partition-id-t* ⇒ *state-t* ⇒ *state-t* ⇒ *bool* **where**
vpeq-local u s t ≡
∀ *tid* . (*partition tid*) = *u* ⟶ (*thread s tid*) = (*thread t tid*)

definition *vpeq u s t* ≡
vpeq-obj u s t ∧ *vpeq-subj-subj u s t* ∧ *vpeq-subj-obj u s t* ∧ *vpeq-local u s t*

4.5.1 Elementary properties

lemma *vpeq-rel*:
shows *vpeq-refl*: *vpeq u s s*
and *vpeq-sym* [*sym*]: *vpeq u s t* ⟹ *vpeq u t s*
and *vpeq-trans* [*trans*]: ⟦ *vpeq u s1 s2* ; *vpeq u s2 s3* ⟧ ⟹ *vpeq u s1 s3*
unfolding *vpeq-def vpeq-obj-def vpeq-subj-subj-def vpeq-subj-obj-def vpeq-local-def*
by *auto*

Auxiliary equivalence relation.

lemma *set-object-value-ign*:
assumes *eq-obs*: ~ *Policy.sp-spec-subj-obj u x READ*
shows *vpeq u s* (*set-object-value x y s*)
proof –
from *assms* **show** *?thesis*
unfolding *vpeq-def vpeq-obj-def vpeq-subj-subj-def vpeq-subj-obj-def set-object-value-def vpeq-local-def*
by *auto*
qed

Context-switch and fetch operations are also consistent with vpeq and locally respect everything.

theorem *cswitch-consistency-and-respect*:
fixes *u* :: *partition-id-t*
and *s* :: *state-t*
and *new-current* :: *thread-id-t*

assumes *atomic-step-invariant s*
shows *vpeq u s* (*s* (| *current* := *new-current* |))
proof –
show *?thesis*
unfolding *vpeq-def vpeq-obj-def vpeq-subj-subj-def vpeq-subj-obj-def vpeq-local-def*
by *auto*
qed

end

4.6 Atomic step locally respects the information flow policy

theory *Step-vpeq-locally-respects*
imports *Step Step-invariants Step-vpeq*
begin

The notion of locally respects is common usage. We augment it by assuming that the *atomic-step-invariant* holds (see [31]).

4.6.1 Locally respects of atomic step functions

lemma *ipc-respects-policy*:
assumes *no*: ¬ *Policy.ifp* (*partition tid*) *u*
and *inv*: *atomic-step-invariant s*
and *prec*: *atomic-step-precondition s tid* (*SK-IPC dir stage partner pag*)
and *ipt-case*: *ipt* = *SK-IPC dir stage partner page*
shows *vpeq u s* (*atomic-step-ipc tid dir stage partner page s*)
proof(*cases stage*)
case *PREP*
thus *?thesis*
unfolding *atomic-step-ipc-def*
using *vpeq-refl* **by** *simp*
next
case *WAIT*
thus *?thesis*
unfolding *atomic-step-ipc-def*
using *vpeq-refl* **by** *simp*
next case (*BUF mypage*)
show *?thesis*
proof(*cases dir*)
case *RECV*
thus *?thesis*
unfolding *atomic-step-ipc-def*
using *vpeq-refl BUF* **by** *simp*
next
case *SEND*
have *Policy.sp-spec-subj-subj* (*partition tid*) (*partition partner*)
and *Policy.sp-spec-subj-obj* (*partition partner*) (*PAGE mypage*) *WRITE*
using *BUF SEND inv prec ipt-case*
unfolding *atomic-step-invariant-def sp-subset-def*
unfolding *atomic-step-precondition-def ipc-precondition-def opposite-ipc-direction-def*
by *auto*
hence ¬ *Policy.sp-spec-subj-obj u* (*PAGE mypage*) *READ*
using *no Policy-properties.ifp-compatible-with-ipc*
by *auto*

```
      thus ?thesis
        using BUF SEND assms
        unfolding atomic-step-ipc-def set-object-value-def
        unfolding vpeq-def vpeq-obj-def vpeq-subj-obj-def vpeq-subj-subj-def vpeq-local-def
        by auto
    qed
  qed

lemma ev-signal-respects-policy:
  assumes no: ¬ Policy.ifp (partition tid) u
    and inv: atomic-step-invariant s
    and prec: atomic-step-precondition s tid (SK-EV-SIGNAL EV-SIGNAL-FINISH partner)
    and ipt-case: ipt = SK-EV-SIGNAL EV-SIGNAL-FINISH partner
  shows vpeq u s (atomic-step-ev-signal tid partner s)
 proof –
  from inv no have ¬ sp-impl-subj-subj s (partition tid) u
   unfolding Policy.ifp-def atomic-step-invariant-def sp-subset-def
   by auto
  with prec have 1:(partition partner) ≠ u
  unfolding atomic-step-precondition-def ev-signal-precondition-def
    by (auto simp add: ev-signal-stage-t.splits)
  then have 2:vpeq-local u s (atomic-step-ev-signal tid partner s)
  unfolding vpeq-local-def atomic-step-ev-signal-def
   by simp
  have 3:vpeq-obj u s (atomic-step-ev-signal tid partner s)
  unfolding vpeq-obj-def atomic-step-ev-signal-def
   by simp
  have 4:vpeq-subj-subj u s (atomic-step-ev-signal tid partner s)
  unfolding vpeq-subj-subj-def atomic-step-ev-signal-def
   by simp
  have 5:vpeq-subj-obj u s (atomic-step-ev-signal tid partner s)
  unfolding vpeq-subj-obj-def atomic-step-ev-signal-def
   by simp
  with 2 3 4 5 show ?thesis
  unfolding vpeq-def
   by simp
qed

lemma ev-wait-all-respects-policy:
  assumes no: ¬ Policy.ifp (partition tid) u
    and inv: atomic-step-invariant s
    and prec: atomic-step-precondition s tid ipt
    and ipt-case: ipt = SK-EV-WAIT ev-wait-stage EV-CONSUME-ALL
  shows vpeq u s (atomic-step-ev-wait-all tid s)
  proof –
  from assms have 1:(partition tid) ≠ u
  unfolding Policy.ifp-def
   by simp
  then have 2:vpeq-local u s (atomic-step-ev-wait-all tid s)
  unfolding vpeq-local-def atomic-step-ev-wait-all-def
   by simp
  have 3:vpeq-obj u s (atomic-step-ev-wait-all tid s)
  unfolding vpeq-obj-def atomic-step-ev-wait-all-def
   by simp
  have 4:vpeq-subj-subj u s (atomic-step-ev-wait-all tid s)
  unfolding vpeq-subj-subj-def atomic-step-ev-wait-all-def
   by simp
```

have *5*:*vpeq-subj-obj u s* (*atomic-step-ev-wait-all tid s*)
unfolding *vpeq-subj-obj-def atomic-step-ev-wait-all-def*
 by *simp*
with *2 3 4 5* **show** *?thesis*
unfolding *vpeq-def*
 by *simp*
qed

lemma *ev-wait-one-respects-policy*:
 assumes *no*: ¬ *Policy.ifp* (*partition tid*) *u*
  and *inv*: *atomic-step-invariant s*
  and *prec*: *atomic-step-precondition s tid ipt*
  and *ipt-case*: *ipt* = *SK-EV-WAIT ev-wait-stage EV-CONSUME-ONE*
 shows *vpeq u s* (*atomic-step-ev-wait-one tid s*)
proof –
 from *assms* **have** *1*:(*partition tid*) ≠ *u*
 unfolding *Policy.ifp-def*
  by *simp*
 then have *2*:*vpeq-local u s* (*atomic-step-ev-wait-one tid s*)
 unfolding *vpeq-local-def atomic-step-ev-wait-one-def*
  by *simp*
 have *3*:*vpeq-obj u s* (*atomic-step-ev-wait-one tid s*)
 unfolding *vpeq-obj-def atomic-step-ev-wait-one-def*
  by *simp*
 have *4*:*vpeq-subj-subj u s* (*atomic-step-ev-wait-one tid s*)
 unfolding *vpeq-subj-subj-def atomic-step-ev-wait-one-def*
  by *simp*
 have *5*:*vpeq-subj-obj u s* (*atomic-step-ev-wait-one tid s*)
 unfolding *vpeq-subj-obj-def atomic-step-ev-wait-one-def*
  by *simp*
 with *2 3 4 5* **show** *?thesis*
 unfolding *vpeq-def*
  by *simp*
qed

4.6.2 Summary theorems on view-partitioning locally respects

Atomic step locally respects the information flow policy (ifp). The policy ifp is not necessarily the same as sp_spec_subj_subj.

theorem *atomic-step-respects-policy*:
 assumes *no*: ¬ *Policy.ifp* (*partition* (*current s*)) *u*
  and *inv*: *atomic-step-invariant s*
  and *prec*: *atomic-step-precondition s* (*current s*) *ipt*
 shows *vpeq u s* (*atomic-step s ipt*)
proof –
 show *?thesis*
  using *assms ipc-respects-policy vpeq-refl*
   ev-signal-respects-policy ev-wait-one-respects-policy
   ev-wait-all-respects-policy
  unfolding *atomic-step-def*
  by (*auto split add*: *int-point-t.splits ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits*)
qed

end

4.7 Weak step consistency

theory *Step-vpeq-weakly-step-consistent*
 imports *Step Step-invariants Step-vpeq*
begin

The notion of weak step consistency is common usage. We augment it by assuming that the *atomic-step-invariant* holds (see [31]).

4.7.1 Weak step consistency of auxiliary functions

lemma *ipc-precondition-weakly-step-consistent*:
 assumes *eq-tid*: *vpeq* (*partition tid*) *s1 s2*
  and *inv1*: *atomic-step-invariant s1*
  and *inv2*: *atomic-step-invariant s2*
 shows *ipc-precondition tid dir partner page s1* = *ipc-precondition tid dir partner page s2*
proof –
 let *?sender* = *case dir of SEND* ⇒ *tid* | *RECV* ⇒ *partner*
 let *?receiver* = *case dir of SEND* ⇒ *partner* | *RECV* ⇒ *tid*
 let *?local-access-mode* = *case dir of SEND* ⇒ *READ* | *RECV* ⇒ *WRITE*
 let *?A* = *sp-impl-subj-subj s1* (*partition ?sender*) (*partition ?receiver*)
  = *sp-impl-subj-subj s2* (*partition ?sender*) (*partition ?receiver*)
 let *?B* = *sp-impl-subj-obj s1* (*partition tid*) (*PAGE page*) *?local-access-mode*
  = *sp-impl-subj-obj s2* (*partition tid*) (*PAGE page*) *?local-access-mode*

 have *A*: *?A*
  proof (*cases Policy.sp-spec-subj-subj* (*partition ?sender*) (*partition ?receiver*))
   case *True*
    thus *?A*
     using *eq-tid* **unfolding** *vpeq-def vpeq-subj-subj-def*
     by (*simp split add*: *ipc-direction-t.splits*)
   next case *False*
    have *sp-subset s1* **and** *sp-subset s2*
     using *inv1 inv2* **unfolding** *atomic-step-invariant-def sp-subset-def* **by** *auto*
    hence ¬ *sp-impl-subj-subj s1* (*partition ?sender*) (*partition ?receiver*)
     and ¬ *sp-impl-subj-subj s2* (*partition ?sender*) (*partition ?receiver*)
     using *False* **unfolding** *sp-subset-def* **by** *auto*
    thus *?A* **by** *auto*
  qed
 have *B*: *?B*
  proof (*cases Policy.sp-spec-subj-obj* (*partition tid*) (*PAGE page*) *?local-access-mode*)
   case *True*
    thus *?B*
     using *eq-tid* **unfolding** *vpeq-def vpeq-subj-obj-def*
     by (*simp split add*: *ipc-direction-t.splits*)
   next case *False*
    have *sp-subset s1* **and** *sp-subset s2*
     using *inv1 inv2* **unfolding** *atomic-step-invariant-def sp-subset-def* **by** *auto*
    hence ¬ *sp-impl-subj-obj s1* (*partition tid*) (*PAGE page*) *?local-access-mode*
     and ¬ *sp-impl-subj-obj s2* (*partition tid*) (*PAGE page*) *?local-access-mode*
     using *False* **unfolding** *sp-subset-def* **by** *auto*
    thus *?B* **by** *auto*
  qed
 show *?thesis* **using** *A B* **unfolding** *ipc-precondition-def* **by** *auto*
qed

lemma *ev-signal-precondition-weakly-step-consistent*:
 assumes *eq-tid*: *vpeq* (*partition tid*) *s1 s2*

```
    and inv1: atomic-step-invariant s1
    and inv2: atomic-step-invariant s2
  shows ev-signal-precondition tid partner s1 = ev-signal-precondition tid partner s2
proof –
 let ?A = sp-impl-subj-subj s1 (partition tid) (partition partner)
          = sp-impl-subj-subj s2 (partition tid) (partition partner)
 have A: ?A
  proof (cases Policy.sp-spec-subj-subj (partition tid) (partition partner))
    case True
     thus ?A
      using eq-tid unfolding vpeq-def vpeq-subj-subj-def
      by (simp split add: ipc-direction-t.splits)
    next case False
     have sp-subset s1 and sp-subset s2
      using inv1 inv2 unfolding atomic-step-invariant-def sp-subset-def by auto
     hence ¬ sp-impl-subj-subj s1 (partition tid) (partition partner)
      and ¬ sp-impl-subj-subj s2 (partition tid) (partition partner)
      using False unfolding sp-subset-def by auto
     thus ?A by auto
  qed
 show ?thesis using A unfolding ev-signal-precondition-def by auto
qed


lemma set-object-value-consistent:
 assumes eq-obs: vpeq u s1 s2
  shows vpeq u (set-object-value x y s1) (set-object-value x y s2)
proof –
 let ?s1′ = set-object-value x y s1 and ?s2′ = set-object-value x y s2
 have E1: vpeq-obj u ?s1′ ?s2′
  proof –
   { fix x′
    assume 1: Policy.sp-spec-subj-obj u x′ READ
    have obj ?s1′ x′ = obj ?s2′ x′ proof (cases x = x′)
     case True
      thus obj ?s1′ x′ = obj ?s2′ x′ unfolding set-object-value-def by auto
     next case False
      hence 2: obj ?s1′ x′ = obj s1 x′
       and 3: obj ?s2′ x′ = obj s2 x′
       unfolding set-object-value-def by auto
      have 4: obj s1 x′ = obj s2 x′
       using 1 eq-obs unfolding vpeq-def vpeq-obj-def by auto
      from 2 3 4 show obj ?s1′ x′ = obj ?s2′ x′
       by simp
     qed }
   thus vpeq-obj u ?s1′ ?s2′ unfolding vpeq-obj-def by auto
  qed
 have E4: vpeq-subj-subj u ?s1′ ?s2′
  proof –
   have sp-impl-subj-subj ?s1′ = sp-impl-subj-subj s1
    and sp-impl-subj-subj ?s2′ = sp-impl-subj-subj s2
    unfolding set-object-value-def by auto
   thus vpeq-subj-subj u ?s1′ ?s2′
    using eq-obs unfolding vpeq-def vpeq-subj-subj-def by auto
  qed
 have E5: vpeq-subj-obj u ?s1′ ?s2′
  proof –
```

have *sp-impl-subj-obj ?s1′ = sp-impl-subj-obj s1*
and *sp-impl-subj-obj ?s2′ = sp-impl-subj-obj s2*
unfolding *set-object-value-def* **by** *auto*
thus *vpeq-subj-obj u ?s1′ ?s2′*
using *eq-obs* **unfolding** *vpeq-def vpeq-subj-obj-def* **by** *auto*
qed
from *eq-obs* **have** *E6*: *vpeq-local u ?s1′ ?s2′*
unfolding *vpeq-def vpeq-local-def set-object-value-def*
by *simp*
from *E1 E4 E5 E6*
show *?thesis* **unfolding** *vpeq-def*
by *auto*
qed

4.7.2 Weak step consistency of atomic step functions

lemma *ipc-weakly-step-consistent*:
assumes *eq-obs*: *vpeq u s1 s2*
and *eq-act*: *vpeq (partition tid) s1 s2*
and *inv1*: *atomic-step-invariant s1*
and *inv2*: *atomic-step-invariant s2*
and *prec1*: *atomic-step-precondition s1 tid ipt*
and *prec2*: *atomic-step-precondition s1 tid ipt*
and *ipt-case*: *ipt = SK-IPC dir stage partner page*
shows *vpeq u*
(atomic-step-ipc tid dir stage partner page s1)
(atomic-step-ipc tid dir stage partner page s2)
proof –
have $\bigwedge$ *mypage* . [[*dir = SEND*; *stage = BUF mypage*]] $\Longrightarrow$ *?thesis*
proof –
fix *mypage*
assume *dir-send*: *dir = SEND*
assume *stage-buf*: *stage = BUF mypage*
have *Policy.sp-spec-subj-obj (partition tid) (PAGE page) READ*
using *inv1 prec1 dir-send stage-buf ipt-case*
unfolding *atomic-step-invariant-def sp-subset-def*
unfolding *atomic-step-precondition-def ipc-precondition-def opposite-ipc-direction-def*
by *auto*
hence *obj s1 (PAGE page) = obj s2 (PAGE page)*
using *eq-act* **unfolding** *vpeq-def vpeq-obj-def vpeq-local-def*
by *auto*
thus *vpeq u*
(atomic-step-ipc tid dir stage partner page s1)
(atomic-step-ipc tid dir stage partner page s2)
using *dir-send stage-buf eq-obs set-object-value-consistent*
unfolding *atomic-step-ipc-def*
by *auto*
qed
thus *?thesis*
using *eq-obs* **unfolding** *atomic-step-ipc-def*
by *(cases stage, auto, cases dir, auto)*
qed

lemma *ev-wait-one-weakly-step-consistent*:
assumes *eq-obs*: *vpeq u s1 s2*
and *eq-act*: *vpeq (partition tid) s1 s2*
and *inv1*: *atomic-step-invariant s1*
and *inv2*: *atomic-step-invariant s2*

and *prec1*: *atomic-step-precondition s1* (*current s1*) *ipt*
and *prec2*: *atomic-step-precondition s1* (*current s1*) *ipt*
shows *vpeq u*
(*atomic-step-ev-wait-one tid s1*)
(*atomic-step-ev-wait-one tid s2*)
using *assms*
unfolding *vpeq-def vpeq-subj-subj-def vpeq-obj-def vpeq-subj-obj-def vpeq-local-def atomic-step-ev-wait-one-def*
by *simp*

lemma *ev-wait-all-weakly-step-consistent*:
assumes *eq-obs*: *vpeq u s1 s2*
and *eq-act*: *vpeq* (*partition tid*) *s1 s2*
and *inv1*: *atomic-step-invariant s1*
and *inv2*: *atomic-step-invariant s2*
and *prec1*: *atomic-step-precondition s1* (*current s1*) *ipt*
and *prec2*: *atomic-step-precondition s1* (*current s1*) *ipt*
shows *vpeq u*
(*atomic-step-ev-wait-all tid s1*)
(*atomic-step-ev-wait-all tid s2*)
using *assms*
unfolding *vpeq-def vpeq-subj-subj-def vpeq-obj-def vpeq-subj-obj-def vpeq-local-def atomic-step-ev-wait-all-def*
by *simp*

lemma *ev-signal-weakly-step-consistent*:
assumes *eq-obs*: *vpeq u s1 s2*
and *eq-act*: *vpeq* (*partition tid*) *s1 s2*
and *inv1*: *atomic-step-invariant s1*
and *inv2*: *atomic-step-invariant s2*
and *prec1*: *atomic-step-precondition s1* (*current s1*) *ipt*
and *prec2*: *atomic-step-precondition s1* (*current s1*) *ipt*
shows *vpeq u*
(*atomic-step-ev-signal tid partner s1*)
(*atomic-step-ev-signal tid partner s2*)
using *assms*
unfolding *vpeq-def vpeq-subj-subj-def vpeq-obj-def vpeq-subj-obj-def vpeq-local-def atomic-step-ev-signal-def*
by *simp*

The use of *extend-f* is to provide infrastructure to support use in dynamic policies, currently not used.

definition *extend-f* :: (*partition-id-t* ⇒ *partition-id-t* ⇒ *bool*) ⇒ (*partition-id-t* ⇒ *partition-id-t* ⇒ *bool*) ⇒ (*partition-id-t* ⇒ *partition-id-t* ⇒ *bool*) **where**
extend-f f g ≡ λ *p1 p2* . *f p1 p2* ∨ *g p1 p2*

definition *extend-subj-subj* :: (*partition-id-t* ⇒ *partition-id-t* ⇒ *bool*) ⇒ *state-t* ⇒ *state-t* **where**
extend-subj-subj f s ≡ *s* (| *sp-impl-subj-subj* := *extend-f f* (*sp-impl-subj-subj s*) |)

lemma *extend-subj-subj-consistent*:
fixes *f* :: *partition-id-t* ⇒ *partition-id-t* ⇒ *bool*
assumes *vpeq u s1 s2*
shows *vpeq u* (*extend-subj-subj f s1*) (*extend-subj-subj f s2*)
proof –
let *?g1* = *sp-impl-subj-subj s1* **and** *?g2* = *sp-impl-subj-subj s2*
have ∀ *v* . *Policy.sp-spec-subj-subj u v* ⟶ *?g1 u v* = *?g2 u v*
and ∀ *v* . *Policy.sp-spec-subj-subj v u* ⟶ *?g1 v u* = *?g2 v u*
using *assms* **unfolding** *vpeq-def vpeq-subj-subj-def* **by** *auto*

```
  hence ∀ v . Policy.sp-spec-subj-subj u v ⟶ extend-f f ?g1 u v = extend-f f ?g2 u v
    and ∀ v . Policy.sp-spec-subj-subj v u ⟶ extend-f f ?g1 v u = extend-f f ?g2 v u
    unfolding extend-f-def by auto
  hence 1: vpeq-subj-subj u (extend-subj-subj f s1) (extend-subj-subj f s2)
    unfolding vpeq-subj-subj-def extend-subj-subj-def
    by auto
  have 2: vpeq-obj u (extend-subj-subj f s1) (extend-subj-subj f s2)
    using assms unfolding vpeq-def vpeq-obj-def extend-subj-subj-def by fastforce
  have 3: vpeq-subj-obj u (extend-subj-subj f s1) (extend-subj-subj f s2)
    using assms unfolding vpeq-def vpeq-subj-obj-def extend-subj-subj-def by fastforce
 have 4: vpeq-local u (extend-subj-subj f s1) (extend-subj-subj f s2)
    using assms unfolding vpeq-def vpeq-local-def extend-subj-subj-def by fastforce
  from 1 2 3 4 show ?thesis
    using assms unfolding vpeq-def by fast
qed
```

4.7.3 Summary theorems on view-partitioning weak step consistency

The atomic step is weakly step consistent with view partitioning. Here, the "weakness" is that we assume that the two states are vp-equivalent not only w.r.t. the observer domain *u*, but also w.r.t. the caller domain *Step.partition tid*).

```
theorem atomic-step-weakly-step-consistent:
  assumes eq-obs: vpeq u s1 s2
    and eq-act: vpeq (partition (current s1)) s1 s2
    and inv1:  atomic-step-invariant s1
    and inv2:  atomic-step-invariant s2
    and prec1: atomic-step-precondition s1 (current s1) ipt
    and prec2: atomic-step-precondition s2 (current s2) ipt
    and eq-curr: current s1 = current s2
  shows vpeq u (atomic-step s1 ipt) (atomic-step s2 ipt)
proof –
  show ?thesis
    using assms
        ipc-weakly-step-consistent
        ev-wait-all-weakly-step-consistent
        ev-wait-one-weakly-step-consistent
        ev-signal-weakly-step-consistent
        vpeq-refl ev-signal-stage-t.exhaust
    unfolding atomic-step-def
    apply (cases ipt, auto)
    apply  (simp split add: ev-consume-t.splits ev-wait-stage-t.splits )
    by  (simp split add: ev-signal-stage-t.splits)
   qed
end
```

4.8 Separation kernel model

```
theory Separation-kernel-model
  imports ../../step/Step
        ../../step/Step-invariants
        ../../step/Step-vpeq
        ../../step/Step-vpeq-locally-respects
        ../../step/Step-vpeq-weakly-step-consistent
        CISK
begin
```

First (Section 4.8.1) we instantiate the CISK generic model. Functions that instantiate a generic function of the CISK model are prefixed with an ‘r’, ‘r’ standing for “Rushby’;, as CISK is derived originally from a model by Rushby [31]. For example, ‘rifp’ is the instantiation of the generic ‘ifp’.

Later (Section 4.8.5) all CISK proof obligations are discharged, e.g., weak step consistency, output consistency, etc. These will be used in Section 4.9.

4.8.1 Initial state of separation kernel model

We assume that the initial state of threads and memory is given. The initial state of threads is arbitrary, but the threads are not executing the system call. The purpose of the following definitions is to obtain the initial state without potentially dangerous axioms. The only axioms we admit without proof are formulated using the “consts” syntax and thus safe.

```
consts
  initial-current :: thread-id-t
  initial-obj :: obj-id-t ⇒ obj-t

definition s0 :: state-t where
  s0 ≡ (| sp-impl-subj-subj = Policy.sp-spec-subj-subj,
        sp-impl-subj-obj = Policy.sp-spec-subj-obj,
        current = initial-current,
        obj = initial-obj,
        thread = λ - . (| ev-counter = 0 |)
        |)

lemma initial-invariant:
  shows atomic-step-invariant s0
proof –
  have sp-subset s0
    unfolding sp-subset-def s0-def by auto
  thus ?thesis
    unfolding atomic-step-invariant-def by auto
qed
```

4.8.2 Types for instantiation of the generic model

To simplify formulations, we include the state invariant *atomic-step-invariant* in the state data type. The initial state *s0* serves at witness that *rstate-t* is non-empty.

```
typedef rstate-t = { s . atomic-step-invariant s }
  using initial-invariant by auto

definition abs :: state-t ⇒ rstate-t (↑ -) where abs = Abs-rstate-t
definition rep :: rstate-t ⇒ state-t (↓ -) where rep = Rep-rstate-t

lemma rstate-invariant:
  shows atomic-step-invariant (↓s)
  unfolding rep-def by (metis Rep-rstate-t mem-Collect-eq)

lemma rstate-down-up[simp]:
  shows (↑↓s) = s
  unfolding rep-def abs-def using Rep-rstate-t-inverse by auto

lemma rstate-up-down[simp]:
  assumes atomic-step-invariant s
  shows (↓↑s) = s
  using assms Abs-rstate-t-inverse unfolding rep-def abs-def by auto
```

A CISK action is identified with an interrupt point.

type-synonym *raction-t = int-point-t*

definition *rcurrent* :: *rstate-t* ⇒ *thread-id-t* **where**
rcurrent s = current ↓*s*

definition *rstep* :: *rstate-t* ⇒ *raction-t* ⇒ *rstate-t* **where**
rstep s a ≡ ↑(*atomic-step* (↓*s*) *a*)

Each CISK domain is identified with a thread id.

type-synonym *rdom-t = thread-id-t*

The output function returns the contents of all memory accessible to the subject. The action argument of the output function is ignored.

datatype *visible-obj-t = VALUE obj-t* | *EXCEPTION*
type-synonym *routput-t = page-t* ⇒ *visible-obj-t*

definition *routput-f* :: *rstate-t* ⇒ *raction-t* ⇒ *routput-t* **where**
routput-f s a p ≡
if sp-impl-subj-obj (↓*s*) (*partition* (*rcurrent s*)) (*PAGE p*) *READ then*
VALUE (*obj* (↓*s*) (*PAGE p*))
else
EXCEPTION

The precondition for the generic model. Note that *atomic-step-invariant* is already part of the state.

definition *rprecondition* :: *rstate-t* ⇒ *rdom-t* ⇒ *raction-t* ⇒ *bool* **where**
rprecondition s d a ≡ *atomic-step-precondition* (↓*s*) *d a*
abbreviation *rinvariant*
where *rinvariant s* ≡ *True* — The invariant is already in the state type.

Translate view-partitioning and interaction-allowed relations.

definition *rvpeq* :: *rdom-t* ⇒ *rstate-t* ⇒ *rstate-t* ⇒ *bool* **where**
rvpeq u s1 s2 ≡ *vpeq* (*partition u*) (↓*s1*) (↓*s2*)

definition *rifp* :: *rdom-t* ⇒ *rdom-t* ⇒ *bool* **where**
rifp u v = Policy.ifp (*partition u*) (*partition v*)

Context Switches

definition *rcswitch* :: *nat* ⇒ *rstate-t* ⇒ *rstate-t* **where**
rcswitch n s ≡ ↑((↓*s*) (| *current* := (*SOME t* . *True*) |))

4.8.3 Possible action sequences

An *SK-IPC* consists of three atomic actions *PREP*, *WAIT* and *BUF* with the same parameters.

definition *is-SK-IPC* :: *raction-t list* ⇒ *bool*
where *is-SK-IPC aseq* ≡ ∃ *dir partner page* .
aseq = [*SK-IPC dir PREP partner page*,*SK-IPC dir WAIT partner page*,*SK-IPC dir* (*BUF* (*SOME page*′ . *True*)) *partner page*]

An *SK-EV-WAIT* consists of three atomic actions, one for each of the stages *EV-PREP*, *EV-WAIT* and *EV-FINISH* with the same parameters.

definition *is-SK-EV-WAIT* :: *raction-t list* ⇒ *bool*
where *is-SK-EV-WAIT aseq* ≡ ∃ *consume* .
aseq = [*SK-EV-WAIT EV-PREP consume* ,
SK-EV-WAIT EV-WAIT consume ,
SK-EV-WAIT EV-FINISH consume]

An *SK-EV-SIGNAL* consists of two atomic actions, one for each of the stages *EV-SIGNAL-PREP* and *EV-SIGNAL-FINISH* with the same parameters.

definition *is-SK-EV-SIGNAL* :: *raction-t list* ⇒ *bool*
where *is-SK-EV-SIGNAL aseq* ≡ ∃ *partner* .
aseq = [*SK-EV-SIGNAL EV-SIGNAL-PREP partner*,
SK-EV-SIGNAL EV-SIGNAL-FINISH partner]

The complete attack surface consists of IPC calls, events, and noops.

definition *rAS-set* :: *raction-t list set*
where *rAS-set* ≡ { *aseq* . *is-SK-IPC aseq* ∨ *is-SK-EV-WAIT aseq* ∨ *is-SK-EV-SIGNAL aseq* } ∪ {[]}

4.8.4 Control

When are actions aborting, and when are actions waiting. We do not currently use the *set-error-code* function yet.

abbreviation *raborting*
where *raborting s* ≡ *aborting* (↓*s*)
abbreviation *rwaiting*
where *rwaiting s* ≡ *waiting* (↓*s*)
definition *rset-error-code* :: *rstate-t* ⇒ *raction-t* ⇒ *rstate-t*
where *rset-error-code s a* ≡ *s*

Returns the set of threads that are involved in a certain action. For example, for an IPC call, the *WAIT* stage synchronizes with the partner. This partner is involved in that action.

definition *rkinvolved* :: *int-point-t* ⇒ *rdom-t set*
where *rkinvolved a* ≡
case a of SK-IPC dir WAIT partner page ⇒ {*partner*}
| *SK-EV-SIGNAL EV-SIGNAL-FINISH partner* => {*partner*}
| - ⇒ {}
abbreviation *rinvolved* :: *int-point-t option* ⇒ *rdom-t set*
where *rinvolved* ≡ *Kernel.involved rkinvolved*

4.8.5 Discharging the proof obligations

lemma *inst-vpeq-rel*:
shows *rvpeq-refl*: *rvpeq u s s*
and *rvpeq-sym*: *rvpeq u s1 s2* ⟹ *rvpeq u s2 s1*
and *rvpeq-trans*: ⟦ *rvpeq u s1 s2*; *rvpeq u s2 s3* ⟧ ⟹ *rvpeq u s1 s3*
unfolding *rvpeq-def* **using** *vpeq-rel* **by** *metis+*

lemma *inst-ifp-refl*:
shows ∀ *u* . *rifp u u*
unfolding *rifp-def* **using** *Policy-properties.ifp-reflexive* **by** *fast*

lemma *inst-step-atomicity* [*simp*]:
shows ∀ *s a* . *rcurrent* (*rstep s a*) = *rcurrent s*
unfolding *rstep-def rcurrent-def*
using *atomic-step-does-not-change-current-thread rstate-up-down rstate-invariant atomic-step-preserves-invariants*
by *auto*

lemma *inst-weakly-step-consistent*:
assumes *rvpeq u s t*

```
    and rvpeq (rcurrent s) s t
    and rcurrent s = rcurrent t
    and rprecondition s (rcurrent s) a
    and rprecondition t (rcurrent t) a
  shows rvpeq u (rstep s a) (rstep t a)
using assms atomic-step-weakly-step-consistent rstate-invariant atomic-step-preserves-invariants
unfolding rcurrent-def rstep-def rvpeq-def rprecondition-def
by auto
```

```
lemma inst-local-respect:
 assumes not-ifp: ¬rifp (rcurrent s) u
    and prec: rprecondition s (rcurrent s) a
  shows rvpeq u s (rstep s a)
using assms atomic-step-respects-policy rstate-invariant atomic-step-preserves-invariants
unfolding rifp-def rprecondition-def rvpeq-def rstep-def rcurrent-def
by auto
```

```
lemma inst-output-consistency:
 assumes rvpeq: rvpeq (rcurrent s) s t
 and     current-eq: rcurrent s = rcurrent t
 shows   routput-f s a = routput-f t a
proof–
 have ∀ a s t. rvpeq (rcurrent s) s t ∧ rcurrent s = rcurrent t ⟶ routput-f s a = routput-f t a
  proof–
   { fix a :: raction-t
    fix s t :: rstate-t
    fix p :: page-t
    assume 1: rvpeq (rcurrent s) s t
      and 2: rcurrent s = rcurrent t
    let ?part = partition (rcurrent s)
    have routput-f s a p = routput-f t a p
     proof (cases Policy.sp-spec-subj-obj ?part (PAGE p) READ
         rule: case-split [case-names Allowed Denied])
      case Allowed
       have 5: obj (↓s) (PAGE p) = obj (↓t) (PAGE p)
        using 1 Allowed unfolding rvpeq-def vpeq-def vpeq-obj-def  by auto
       have 6: sp-impl-subj-obj (↓s) ?part (PAGE p) READ = sp-impl-subj-obj (↓t) ?part (PAGE p) READ
        using 1 2 Allowed unfolding rvpeq-def vpeq-def vpeq-subj-obj-def by auto
       show routput-f s a p = routput-f t a p
        unfolding routput-f-def using 2 5 6 by auto
      next case Denied
       hence sp-impl-subj-obj (↓s) ?part (PAGE p) READ = False
        and sp-impl-subj-obj (↓t) ?part (PAGE p) READ = False
        using rstate-invariant unfolding atomic-step-invariant-def sp-subset-def
        by auto
       thus routput-f s a p = routput-f t a p
        using 2 unfolding routput-f-def by simp
     qed }
    thus ∀ a s t. rvpeq (rcurrent s) s t ∧ rcurrent s = rcurrent t ⟶ routput-f s a = routput-f t a
     by auto
  qed
 thus ?thesis using assms by auto
```

qed

lemma *inst-cswitch-independent-of-state*:
assumes *rcurrent s = rcurrent t*
shows *rcurrent* (*rcswitch n s*) = *rcurrent* (*rcswitch n t*)
using *rstate-invariant cswitch-preserves-invariants* **unfolding** *rcurrent-def rcswitch-def* **by** *simp*

lemma *inst-cswitch-consistency*:
assumes *rvpeq u s t*
shows *rvpeq u* (*rcswitch n s*) (*rcswitch n t*)
proof–
have *1*: *vpeq* (*partition u*) (↓*s*) ↓(*rcswitch n s*)
using *rstate-invariant cswitch-consistency-and-respect cswitch-preserves-invariants*
unfolding *rcswitch-def*
by *auto*
have *2*: *vpeq* (*partition u*) (↓*t*) ↓(*rcswitch n t*)
using *rstate-invariant cswitch-consistency-and-respect cswitch-preserves-invariants*
unfolding *rcswitch-def*
by *auto*
from *1 2 assms* **show** *?thesis* **unfolding** *rvpeq-def* **using** *vpeq-rel* **by** *metis*
qed

For the *PREP* stage (the first stage of the IPC action sequence) the precondition is True.

lemma *prec-first-IPC-action*:
assumes *is-SK-IPC aseq*
shows *rprecondition s d* (*hd aseq*)
using *assms*
unfolding *is-SK-IPC-def rprecondition-def atomic-step-precondition-def*
by *auto*

For the the first stage of the *EV-WAIT* action sequence the precondition is True.

lemma *prec-first-EV-WAIT-action*:
assumes *is-SK-EV-WAIT aseq*
shows *rprecondition s d* (*hd aseq*)
using *assms*
unfolding *is-SK-EV-WAIT-def rprecondition-def atomic-step-precondition-def*
by *auto*

For the first stage of the *EV-SIGNAL* action sequence the precondition is True.

lemma *prec-first-EV-SIGNAL-action*:
assumes *is-SK-EV-SIGNAL aseq*
shows *rprecondition s d* (*hd aseq*)
using *assms*
unfolding *is-SK-EV-SIGNAL-def rprecondition-def atomic-step-precondition-def*
ev-signal-precondition-def
by *auto*

When not waiting or aborting, the precondition is "1-step inductive", that is at all times the precondition holds initially (for the first step of an action sequence) and after doing one step.

lemma *prec-after-IPC-step*:
assumes *prec*: *rprecondition s* (*rcurrent s*) (*aseq* ! *n*)
and *n-bound*: *Suc n* < *length aseq*
and *IPC*: *is-SK-IPC aseq*
and *not-aborting*: ¬*raborting s* (*rcurrent s*) (*aseq* ! *n*)
and *not-waiting*: ¬*rwaiting s* (*rcurrent s*) (*aseq* ! *n*)

shows *rprecondition* (*rstep s* (*aseq* ! *n*)) (*rcurrent s*) (*aseq* ! *Suc n*)
proof–
{
 fix *dir partner page*
 let *?page′* = (*SOME page′ . True*)
 assume *IPC*: *aseq* = [*SK-IPC dir PREP partner page*,*SK-IPC dir WAIT partner page*,*SK-IPC dir* (*BUF ?page′*) *partner page*]
  {
   assume *0*: *n=0*
   from *0 IPC prec not-aborting*
    have *?thesis*
   unfolding *rprecondition-def atomic-step-precondition-def rstep-def rcurrent-def atomic-step-def atomic-step-ipc-def aborting-def*
    by(*auto*)
  }
  moreover
  {
   assume *1*: *n=1*
   from *1 IPC prec not-waiting*
    have *?thesis*
   unfolding *rprecondition-def atomic-step-precondition-def rstep-def rcurrent-def atomic-step-def atomic-step-ipc-def waiting-def*
    by(*auto*)
  }
  moreover
  from *IPC*
   have *length aseq* = *3*
   by *auto*
  ultimately
   have *?thesis*
   using *n-bound*
   by *arith*
}
thus *?thesis*
 using *IPC*
 unfolding *is-SK-IPC-def*
 by(*auto*)
qed

When not waiting or aborting, the precondition is 1-step inductive.

lemma *prec-after-EV-WAIT-step*:
assumes *prec*: *rprecondition s* (*rcurrent s*) (*aseq* ! *n*)
  and *n-bound*: *Suc n* < *length aseq*
  and *IPC*: *is-SK-EV-WAIT aseq*
  and *not-aborting*: ¬*raborting s* (*rcurrent s*) (*aseq* ! *n*)
  and *not-waiting*: ¬*rwaiting s* (*rcurrent s*) (*aseq* ! *n*)
shows *rprecondition* (*rstep s* (*aseq* ! *n*)) (*rcurrent s*) (*aseq* ! *Suc n*)
proof–
{
 fix *consume*

 assume *WAIT*: *aseq* = [*SK-EV-WAIT EV-PREP consume*,
            SK-EV-WAIT EV-WAIT consume,
            SK-EV-WAIT EV-FINISH consume]
 {
  assume *0*: *n=0*
  from *0 WAIT prec not-aborting*
   have *?thesis*

```
    unfolding rprecondition-def atomic-step-precondition-def
    by(auto)
  }
  moreover
  {
   assume 1: n=1
   from 1 WAIT prec not-waiting
    have ?thesis
    unfolding rprecondition-def atomic-step-precondition-def
    by(auto)
  }
  moreover
  from WAIT
   have length aseq = 3
   by auto
  ultimately
   have ?thesis
   using n-bound
   by arith
}
thus ?thesis
  using assms
  unfolding is-SK-EV-WAIT-def
  by auto
qed
```

When not waiting or aborting, the precondition is 1-step inductive.

```
lemma prec-after-EV-SIGNAL-step:
assumes prec: rprecondition s (rcurrent s) (aseq ! n)
   and n-bound: Suc n < length aseq
   and SIGNAL: is-SK-EV-SIGNAL aseq
   and not-aborting: ¬raborting s (rcurrent s) (aseq ! n)
   and not-waiting: ¬rwaiting s (rcurrent s) (aseq ! n)
shows rprecondition (rstep s (aseq ! n)) (rcurrent s) (aseq ! Suc n)
proof–
{  fix partner
   assume SIGNAL1: aseq = [SK-EV-SIGNAL EV-SIGNAL-PREP partner,
                        SK-EV-SIGNAL EV-SIGNAL-FINISH partner]
  {
   assume 0: n=0
   from 0 SIGNAL1 prec not-aborting
    have ?thesis
    unfolding rprecondition-def atomic-step-precondition-def ev-signal-precondition-def
          aborting-def rstep-def atomic-step-def
    by auto
  }
  moreover
  from SIGNAL1
   have length aseq = 2
   by auto
  ultimately
   have ?thesis
   using n-bound
   by arith
}
thus ?thesis
  using assms
  unfolding is-SK-EV-SIGNAL-def
```

by *auto*
qed

lemma *on-set-object-value*:
shows *sp-impl-subj-subj* (*set-object-value ob val s*) = *sp-impl-subj-subj s*
and *sp-impl-subj-obj* (*set-object-value ob val s*) = *sp-impl-subj-obj s*
unfolding *set-object-value-def* **apply** *simp+* **done**

lemma *prec-IPC-dom-independent*:
assumes *current s* ≠ *d*
and *atomic-step-invariant s*
and *atomic-step-precondition s d a*
shows *atomic-step-precondition* (*atomic-step-ipc* (*current s*) *dir stage partner page s*) *d a*
using *assms on-set-object-value*
unfolding *atomic-step-precondition-def atomic-step-ipc-def ipc-precondition-def*
ev-signal-precondition-def set-object-value-def
by (*auto split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*
ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)

lemma *prec-ev-signal-dom-independent*:
assumes *current s* ≠ *d*
and *atomic-step-invariant s*
and *atomic-step-precondition s d a*
shows *atomic-step-precondition* (*atomic-step-ev-signal* (*current s*) *partner s*) *d a*
using *assms on-set-object-value*
unfolding *atomic-step-precondition-def atomic-step-ev-signal-def ipc-precondition-def*
ev-signal-precondition-def set-object-value-def
by (*auto split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*
ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)

lemma *prec-ev-wait-one-dom-independent*:
assumes *current s* ≠ *d*
and *atomic-step-invariant s*
and *atomic-step-precondition s d a*
shows *atomic-step-precondition* (*atomic-step-ev-wait-one* (*current s*) *s*) *d a*
using *assms on-set-object-value*
unfolding *atomic-step-precondition-def atomic-step-ev-wait-one-def ipc-precondition-def*
ev-signal-precondition-def set-object-value-def
by (*auto split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*
ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)

lemma *prec-ev-wait-all-dom-independent*:
assumes *current s* ≠ *d*
and *atomic-step-invariant s*
and *atomic-step-precondition s d a*
shows *atomic-step-precondition* (*atomic-step-ev-wait-all* (*current s*) *s*) *d a*
using *assms on-set-object-value*
unfolding *atomic-step-precondition-def atomic-step-ev-wait-all-def ipc-precondition-def*
ev-signal-precondition-def set-object-value-def
by (*auto split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*
ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits)

lemma *prec-dom-independent*:
shows ∀ *s d a a′* . *rcurrent s* ≠ *d* ∧ *rprecondition s d a* ⟶ *rprecondition* (*rstep s a′*) *d a*
using *atomic-step-preserves-invariants*
rstate-invariant prec-IPC-dom-independent prec-ev-signal-dom-independent
prec-ev-wait-all-dom-independent prec-ev-wait-one-dom-independent

unfolding *rcurrent-def rprecondition-def rstep-def atomic-step-def*
by(*auto split add*: *int-point-t.splits ev-consume-t.splits ev-wait-stage-t.splits ev-signal-stage-t.splits*)

lemma *ipc-precondition-after-cswitch*[*simp*]:
shows *ipc-precondition d dir partner page* ((↓ *s*)(|*current* := *new-current*|))
    = *ipc-precondition d dir partner page* (↓ *s*)
using *assms*
unfolding *ipc-precondition-def*
by(*auto split add*: *ipc-direction-t.splits*)

lemma *precondition-after-cswitch*:
shows ∀ *s d n a*. *rprecondition s d a* ⟶ *rprecondition* (*rcswitch n s*) *d a*
using *cswitch-preserves-invariants rstate-invariant*
unfolding *rprecondition-def rcswitch-def atomic-step-precondition-def*
    ev-signal-precondition-def
by (*auto split add*: *int-point-t.splits ipc-stage-t.splits ev-signal-stage-t.splits*)

lemma *aborting-switch-independent*:
shows ∀ *n s*. *raborting* (*rcswitch n s*) = *raborting s*
proof–
{
 fix *n s*
 {
  fix *tid a*
  have *raborting* (*rcswitch n s*) *tid a* = *raborting s tid a*
   using *rstate-invariant cswitch-preserves-invariants ev-signal-precondition-weakly-step-consistent*
     cswitch-consistency-and-respect
   unfolding *aborting-def rcswitch-def*
   apply (*auto split add*: *int-point-t.splits ipc-stage-t.splits*
       ev-wait-stage-t.splits ev-signal-stage-t.splits)
   apply (*metis* (*full-types*))
   by *blast*
 }
 hence *raborting* (*rcswitch n s*) = *raborting s* **by** *auto*
}
thus *?thesis* **by** *auto*
qed
lemma *waiting-switch-independent*:
shows ∀ *n s*. *rwaiting* (*rcswitch n s*) = *rwaiting s*
proof–
{
 fix *n s*
 {
  fix *tid a*
  have *rwaiting* (*rcswitch n s*) *tid a* = *rwaiting s tid a*
   using *rstate-invariant cswitch-preserves-invariants*
   unfolding *waiting-def rcswitch-def*
   by(*auto split add*: *int-point-t.splits ipc-stage-t.splits ev-wait-stage-t.splits*)
 }
 hence *rwaiting* (*rcswitch n s*) = *rwaiting s* **by** *auto*
}
thus *?thesis* **by** *auto*
qed

lemma *aborting-after-IPC-step*:
assumes *d1* ≠ *d2*
shows *aborting* (*atomic-step-ipc d1 dir stage partner page s*) *d2 a* = *aborting s d2 a*

unfolding *atomic-step-ipc-def aborting-def set-object-value-def ipc-precondition-def*
ev-signal-precondition-def
by(*auto split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*
ev-signal-stage-t.splits)

lemma *waiting-after-IPC-step*:
assumes *d1* ≠ *d2*
shows *waiting* (*atomic-step-ipc d1 dir stage partner page s*) *d2 a* = *waiting s d2 a*
unfolding *atomic-step-ipc-def waiting-def set-object-value-def ipc-precondition-def*
by(*auto split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*
ev-wait-stage-t.splits)

lemma *raborting-consistent*:
shows ∀ *s t u*. *rvpeq u s t* ⟶ *raborting s u* = *raborting t u*
proof–
{
fix *s t u*
assume *vpeq*: *rvpeq u s t*
{
fix *a*
from *vpeq ipc-precondition-weakly-step-consistent rstate-invariant*
have ⋀ *tid dir partner page* . *ipc-precondition u dir partner page* (⇃*s*)
= *ipc-precondition u dir partner page* (⇃*t*)
unfolding *rvpeq-def*
by *auto*
with *vpeq rstate-invariant* **have** *raborting s u a* = *raborting t u a*
unfolding *aborting-def rvpeq-def vpeq-def vpeq-local-def ev-signal-precondition-def*
vpeq-subj-subj-def atomic-step-invariant-def sp-subset-def rep-def
apply (*auto split add*: *int-point-t.splits ipc-stage-t.splits ev-signal-stage-t.splits*)
by *blast*
}
hence *raborting s u* = *raborting t u* **by** *auto*
}
thus *?thesis* **by** *auto*
qed

lemma *aborting-dom-independent*:
assumes *rcurrent s* ≠ *d*
shows *raborting* (*rstep s a*) *d a′* = *raborting s d a′*
proof –
have ⋀ *tid dir partner page s* . *ipc-precondition tid dir partner page s* = *ipc-precondition tid dir partner page* (*atomic-step s a*)
∧ *ev-signal-precondition tid partner s* = *ev-signal-precondition tid partner* (*atomic-step s a*)

proof –
fix *tid dir partner page s*
let *?s* = *atomic-step s a*
have (∀ *p q* . *sp-impl-subj-subj s p q* = *sp-impl-subj-subj ?s p q*)
∧ (∀ *p x m* . *sp-impl-subj-obj s p x m* = *sp-impl-subj-obj ?s p x m*)
unfolding *atomic-step-def atomic-step-ipc-def*
atomic-step-ev-wait-all-def atomic-step-ev-wait-one-def
atomic-step-ev-signal-def set-object-value-def
by (*auto split add*: *int-point-t.splits ipc-stage-t.splits ipc-direction-t.splits*
ev-wait-stage-t.splits ev-consume-t.splits ev-signal-stage-t.splits)
thus *ipc-precondition tid dir partner page s* = *ipc-precondition tid dir partner page* (*atomic-step s a*)

$\wedge$ *ev-signal-precondition tid partner s = ev-signal-precondition tid partner (atomic-step s a)*
unfolding *ipc-precondition-def ev-signal-precondition-def* **by** *simp*
qed
moreover have $\bigwedge b$. *(↓(↑(atomic-step (↓s) b))) = atomic-step (↓s) b*
using *rstate-invariant atomic-step-preserves-invariants rstate-up-down* **by** *auto*
ultimately show *?thesis*
unfolding *aborting-def rstep-def ev-signal-precondition-def*

by *(simp split add: int-point-t.splits ipc-stage-t.splits ev-wait-stage-t.splits*
ev-signal-stage-t.splits)
qed

lemma *ipc-precondition-of-partner-consistent*:
assumes *vpeq*: ∀ *d* ∈ *rkinvolved (SK-IPC dir WAIT partner page) . rvpeq d s t*
shows *ipc-precondition partner dir′ u page′ (↓ s) = ipc-precondition partner dir′ u page′ ↓ t*
proof–
from *assms ipc-precondition-weakly-step-consistent rstate-invariant*
show *?thesis*
unfolding *rvpeq-def rkinvolved-def*
by *auto*
qed

lemma *ev-signal-precondition-of-partner-consistent*:
assumes *vpeq*: ∀ *d* ∈ *rkinvolved (SK-EV-SIGNAL EV-SIGNAL-FINISH partner) . rvpeq d s t*
shows *ev-signal-precondition partner u (↓ s) = ev-signal-precondition partner u (↓ t)*
proof–
from *assms ev-signal-precondition-weakly-step-consistent rstate-invariant*
show *?thesis*
unfolding *rvpeq-def rkinvolved-def*
by *auto*
qed

lemma *waiting-consistent*:
shows ∀ *s t u a . rvpeq (rcurrent s) s t* ∧ (∀ *d* ∈ *rkinvolved a . rvpeq d s t)*
∧ *rvpeq u s t*
⟶ *rwaiting s u a = rwaiting t u a*
proof–
{
fix *s t u a*
assume *vpeq*: *rvpeq (rcurrent s) s t*
assume *vpeq-involved*: ∀ *d* ∈ *rkinvolved a . rvpeq d s t*
assume *vpeq-u*: *rvpeq u s t*
have *rwaiting s u a = rwaiting t u a* **proof** *(cases a)*
case *SK-IPC*
thus *rwaiting s u a = rwaiting t u a*
using *ipc-precondition-of-partner-consistent vpeq-involved*
unfolding *waiting-def* **by** *(auto split add: ipc-stage-t.splits)*
next case *SK-EV-WAIT*
thus *rwaiting s u a = rwaiting t u a*
using *ev-signal-precondition-of-partner-consistent*
vpeq-involved vpeq vpeq-u
unfolding *waiting-def rkinvolved-def ev-signal-precondition-def*
rvpeq-def vpeq-def vpeq-local-def
by *(auto split add: ipc-stage-t.splits ev-wait-stage-t.splits ev-consume-t.splits)*
qed *(simp add: waiting-def , simp add: waiting-def)*
}
thus *?thesis* **by** *auto*

qed

lemma *ipc-precondition-ensures-ifp*:
assumes *ipc-precondition* (*current s*) *dir partner page s*
  and *atomic-step-invariant s*
shows *rifp partner* (*current s*)
proof –
 let *?sp* = λ *t1 t2* . *Policy.sp-spec-subj-subj* (*partition t1*) (*partition t2*)
 have *?sp* (*current s*) *partner* ∨ *?sp partner* (*current s*)
  using *assms* **unfolding** *ipc-precondition-def atomic-step-invariant-def sp-subset-def*
  by (*cases dir*, *auto*)
 thus *?thesis*
  unfolding *rifp-def* **using** *Policy-properties.ifp-compatible-with-sp-spec* **by** *auto*
qed

lemma *ev-signal-precondition-ensures-ifp*:
assumes *ev-signal-precondition* (*current s*) *partner s*
  and *atomic-step-invariant s*
shows *rifp partner* (*current s*)
proof –
 let *?sp* = λ *t1 t2* . *Policy.sp-spec-subj-subj* (*partition t1*) (*partition t2*)
 have *?sp* (*current s*) *partner* ∨ *?sp partner* (*current s*)
  using *assms* **unfolding** *ev-signal-precondition-def atomic-step-invariant-def sp-subset-def*
  by (*auto*)
 thus *?thesis*
  unfolding *rifp-def* **using** *Policy-properties.ifp-compatible-with-sp-spec* **by** *auto*
qed

lemma *involved-ifp*:
shows ∀ *s a* . ∀ *d* ∈ *rkinvolved a* . *rprecondition s* (*rcurrent s*) *a* ⟶ *rifp d* (*rcurrent s*)
proof–
{
 fix *s a d*
 assume *d-involved*: *d* ∈ *rkinvolved a*
 assume *prec*: *rprecondition s* (*rcurrent s*) *a*
 from *d-involved prec* **have** *rifp d* (*rcurrent s*)
  using *ipc-precondition-ensures-ifp ev-signal-precondition-ensures-ifp rstate-invariant*
  unfolding *rkinvolved-def rprecondition-def atomic-step-precondition-def rcurrent-def Kernel.involved-def*
  by(*cases a*,*simp*,*auto split add*: *int-point-t.splits ipc-stage-t.splits ev-signal-stage-t.splits*)
}
thus *?thesis* **by** *auto*
qed

lemma *spec-of-waiting-ev*:
shows ∀ *s a*. *rwaiting s* (*rcurrent s*) (*SK-EV-WAIT EV-FINISH EV-CONSUME-ALL*)
  ⟶ *rstep s a* = *s*
 unfolding *waiting-def*
 by *auto*

lemma *spec-of-waiting-ev-w*:
shows ∀ *s a*. *rwaiting s* (*rcurrent s*) (*SK-EV-WAIT EV-WAIT EV-CONSUME-ALL*)
  ⟶ *rstep s* (*SK-EV-WAIT EV-WAIT EV-CONSUME-ALL*) = *s*
 unfolding *rstep-def atomic-step-def*
 by (*auto split add*: *int-point-t.splits ipc-stage-t.splits ev-wait-stage-t.splits*)

lemma *spec-of-waiting*:
shows ∀ *s a*. *rwaiting s* (*rcurrent s*) *a* ⟶ *rstep s a* = *s*

unfolding *waiting-def rstep-def atomic-step-def atomic-step-ipc-def*
atomic-step-ev-signal-def atomic-step-ev-wait-all-def
atomic-step-ev-wait-one-def
by(*auto split add*: *int-point-t.splits ipc-stage-t.splits ev-wait-stage-t.splits*)
end

4.9 Link implementation to CISK: the specific separation kernel is an interpretation of the generic model.

theory *Link-separation-kernel-model-to-CISK*
imports *Separation-kernel-model*
begin

We show that the separation kernel instantiation satisfies the specification of CISK.

theorem *CISK-proof-obligations-satisfied*:
shows
Controllable-Interruptible-Separation-Kernel
rstep
routput-f
(↑*s0*)
rcurrent
rcswitch
rkinvolved
rifp
rvpeq
rAS-set
rinvariant
rprecondition
raborting
rwaiting
rset-error-code
proof (*unfold-locales*)
— show that rvpeq is equivalence relation
show ∀ *a b c u*. (*rvpeq u a b* ∧ *rvpeq u b c*) ⟶ *rvpeq u a c*
and ∀ *a b u*. *rvpeq u a b* ⟶ *rvpeq u b a*
and ∀ *a u*. *rvpeq u a a*
using *inst-vpeq-rel* **by** *metis+*
— show output consistency
show ∀ *a s t*. *rvpeq* (*rcurrent s*) *s t* ∧ *rcurrent s* = *rcurrent t* ⟶ *routput-f s a* = *routput-f t a*
using *inst-output-consistency* **by** *metis*
— show reflexivity of ifp
show ∀ *u* . *rifp u u*
using *inst-ifp-refl* **by** *metis*
— show step consistency
show ∀ *s t u a*. *rvpeq u s t* ∧ *rvpeq* (*rcurrent s*) *s t* ∧ *True* ∧ *rprecondition s* (*rcurrent s*) *a* ∧ *True* ∧ *rprecondition t* (*rcurrent t*) *a* ∧ *rcurrent s* = *rcurrent t* ⟶
rvpeq u (*rstep s a*) (*rstep t a*)
using *inst-weakly-step-consistent* **by** *blast*
— show step atomicity
show ∀ *s a* . *rcurrent* (*rstep s a*) = *rcurrent s*
using *inst-step-atomicity* **by** *metis*
show ∀ *a s u*. ¬ *rifp* (*rcurrent s*) *u* ∧ *True* ∧ *rprecondition s* (*rcurrent s*) *a* ⟶ *rvpeq u s* (*rstep s a*)
using *inst-local-respect* **by** *blast*
— show cswitch is independent of state
show ∀ *n s t*. *rcurrent s* = *rcurrent t* ⟶ *rcurrent* (*rcswitch n s*) = *rcurrent* (*rcswitch n t*)
using *inst-cswitch-independent-of-state* **by** *metis*
— show cswitch consistency

show $\forall\, u\ s\ t\ n.\ rvpeq\ u\ s\ t \longrightarrow rvpeq\ u\ (rcswitch\ n\ s)\ (rcswitch\ n\ t)$
using *inst-cswitch-consistency* **by** *metis*
— Show the empt action sequence is in *AS-set*
show $[] \in rAS\text{-}set$
unfolding *rAS-set-def*
by *auto*
— The invariant for the initial state, already encoded in *rstate-t*
show *True*
by *auto*
— Step function of the invariant, already encoded in *rstate-t*
show $\forall\, s\ n.\ True \longrightarrow True$
by *auto*
— The precondition does not change with a context switch
show $\forall\, s\ d\ n\ a.\ rprecondition\ s\ d\ a \longrightarrow rprecondition\ (rcswitch\ n\ s)\ d\ a$
using *precondition-after-cswitch* **by** *blast*
— The precondition holds for the first action of each action sequence
show $\forall\, s\ d\ aseq.\ True \wedge aseq \in rAS\text{-}set \wedge aseq \neq [] \longrightarrow rprecondition\ s\ d\ (hd\ aseq)$
using *prec-first-IPC-action prec-first-EV-WAIT-action prec-first-EV-SIGNAL-action*
unfolding *rAS-set-def is-sub-seq-def*
by *auto*
— The precondition holds for the next action in an action sequence, assuming the sequence is not aborted or delayed
show $\forall\, s\ a\ a'.\ (\exists\, aseq{\in}rAS\text{-}set.\ is\text{-}sub\text{-}seq\ a\ a'\ aseq) \wedge True \wedge rprecondition\ s\ (rcurrent\ s)\ a \wedge \neg\ raborting\ s\ (rcurrent\ s)\ a \wedge \neg\ rwaiting\ s\ (rcurrent\ s)\ a \longrightarrow$
$rprecondition\ (rstep\ s\ a)\ (rcurrent\ s)\ a'$
using *prec-after-IPC-step prec-after-EV-SIGNAL-step prec-after-EV-WAIT-step*
unfolding *rAS-set-def is-sub-seq-def*
by *auto*
— Steps of other domains do not influence the precondition
show $\forall\, s\ d\ a\ a'.\ rcurrent\ s \neq d \wedge rprecondition\ s\ d\ a \longrightarrow rprecondition\ (rstep\ s\ a')\ d\ a$
using *prec-dom-independent* **by** *blast*
— The invariant
show $\forall\, s\ a.\ True \longrightarrow True$
by *auto*
— Aborting does not depend on a context switch
show $\forall\, n\ s.\ raborting\ (rcswitch\ n\ s) = raborting\ s$
using *aborting-switch-independent* **by** *auto*
— Aborting does not depend on actions of other domains
show $\forall\, s\ a\ d.\ rcurrent\ s \neq d \longrightarrow raborting\ (rstep\ s\ a)\ d = raborting\ s\ d$
using *aborting-dom-independent* **by** *auto*
— Aborting is consistent
show $\forall\, s\ t\ u.\ rvpeq\ u\ s\ t \longrightarrow raborting\ s\ u = raborting\ t\ u$
using *raborting-consistent* **by** *auto*
— Waiting does not depend on a context switch
show $\forall\, n\ s.\ rwaiting\ (rcswitch\ n\ s) = rwaiting\ s$
using *waiting-switch-independent* **by** *auto*
— Waiting is consistent
show $\forall\, s\ t\ u\ a.\ rvpeq\ (rcurrent\ s)\ s\ t \wedge (\forall\ \ d \in rkinvolved\ a\ .\ rvpeq\ d\ s\ t)$
$\wedge\ rvpeq\ u\ s\ t$
$\longrightarrow rwaiting\ s\ u\ a = rwaiting\ t\ u\ a$
unfolding *Kernel.involved-def*
using *waiting-consistent* **by** *auto*
— Domains that are involved in an action may influence the domain of the action
show $\forall\, s\ a.\ \forall\ \ d \in rkinvolved\ a.\ rprecondition\ s\ (rcurrent\ s)\ a \longrightarrow rifp\ d\ (rcurrent\ s)$
using *involved-ifp* **by** *blast*
— An action that is waiting does not change the state
show $\forall\, s\ a.\ rwaiting\ s\ (rcurrent\ s)\ a \longrightarrow rstep\ s\ a = s$

using *spec-of-waiting* **by** *blast*
— Proof obligations for *set-error-code*. Right now, they are all trivial
show $\forall$ *s d a′ a. rcurrent s* $\neq$ *d* $\wedge$ *raborting s d a* $\longrightarrow$ *raborting* (*rset-error-code s a′*) *d a*
unfolding *rset-error-code-def*
by *auto*
show $\forall$ *s t u a. rvpeq u s t* $\longrightarrow$ *rvpeq u* (*rset-error-code s a*) (*rset-error-code t a*)
unfolding *rset-error-code-def*
by *auto*
show $\forall$ *s u a.* $\neg$ *rifp* (*rcurrent s*) *u* $\longrightarrow$ *rvpeq u s* (*rset-error-code s a*)
unfolding *rset-error-code-def*
by (*metis* ‹$\forall$ *a u. rvpeq u a a*›)
show $\forall$ *s a. rcurrent* (*rset-error-code s a*) = *rcurrent s*
unfolding *rset-error-code-def*
by *auto*
show $\forall$ *s d a a′. rprecondition s d a* $\wedge$ *raborting s* (*rcurrent s*) *a′* $\longrightarrow$ *rprecondition* (*rset-error-code s a′*) *d a*
unfolding *rset-error-code-def*
by *auto*
show $\forall$ *s d a′ a. rcurrent s* $\neq$ *d* $\wedge$ *rwaiting s d a* $\longrightarrow$ *rwaiting* (*rset-error-code s a′*) *d a*
unfolding *rset-error-code-def*
by *auto*
qed

Now we can instantiate CISK with some initial state, interrupt function, etc.

interpretation *Inst*:
Controllable-Interruptible-Separation-Kernel
rstep — step function, without program stack
routput-f — output function
↑*s0* — initial state
rcurrent — returns the currently active domain
rcswitch — switches the currently active domain
(*op* =) *42* — interrupt function (yet unspecified)
rkinvolved — returns a set of threads involved in the give action
rifp — information flow policy
rvpeq — view partitioning
rAS-set — the set of valid action sequences
rinvariant — the state invariant
rprecondition — the precondition for doing an action
raborting — condition under which an action is aborted
rwaiting — condition under which an action is delayed
rset-error-code — updates the state. Has no meaning in the current model.
using *CISK-proof-obligations-satisfied* **by** *auto*

The main theorem: the instantiation implements the information flow policy *ifp*.

theorem *risecure*:
Inst.isecure
using *Inst.unwinding-implies-isecure-CISK*
by *blast*

end

5 Related Work

We consider various definitions of intransitive (I) nonin- terference (NI). This overview is by no means intended to be complete. We first prune the field by focusing on INI with as granularity the domains: if the security policy states the act "$v \rightsquigarrow u$", this means domain v is permitted to flow any information it has at its disposal to u. We do not consider language-based approaches to noninterference [26], which allow finer granularity mechanisms (i.e., flowing just a subset of the available information). Secondly, several formal verification efforts have been conducted concerning properties similar and related to INI such as no-exfiltration and no-infiltration [9]. Heitmeyer et al. prove these properties for a separation kernel in a Common Criteria certification process [11] (which kernel and which EAL is not clear). Martin et al. proved separation properties over the MASK kernel [18] and Shapiro and Weber verified correctness of the EROS confinement mechanism [28]. Klein provides an excellent overview of OSs for which such properties have been verified [13]. Thirdly, INI definitions can be built upon either state-based automata, trace-based models, or process algebraic models [30]. We do not focus on the latter, as our approach is not based on process algebra.

Transitive NI was first introduced by Goguen and Meseguer in 1982 [7] and has been the topic of heavy research since. Goguen and Meseguer tried to extend their definition with an unless construct to allow such policies [8]. This construct, however, did not capture the notion of INI [17]. The first commonly accepted definition of INI is Rushbys purging-based definition IP-secure [24]. IP- security has been applied to, e.g., smartcards [27] and OS kernel extensions [?]. To the best of our knowledge, Rushbys definition has not been applied in a certification context. Rushbys definition has been subject to heavy scrutiny [22], [29] and a vast array of modifications have been proposed.

Roscoe and Goldsmith provide CSP-based definitions of NI for the transitive and the intransitive case, here dubbed as lazy and mixed independence. The latter one is more restrictive than Rushbys IP-security. Their critique on IP-secure, however, is not universally accepted [?]. Greve at al. provided the GWV framework developed in ACL2 [9]. Their definition is a non-inductive version of noninterference similar to Rushbys step consistency. GWV has been used on various industrial systems. The exact relation between GWV and (I)P-secure, i.e., whether they are of equal strength, is still open. The second property, Declassification, means whether the definition allows assignments in the form of $l := \texttt{declassify}(h)$ (where we use Sabelfelds [26] notation for high and low variables). Information flows from h to l, but only after it has been declassified. In general, NI is coarser than declassification. It allows where downgrading can occur, but now what may be downgraded [17]. Mantel provides a definition of transitive NI where exceptions can be added to allow de-classification by adding intransitive exceptions to the security policy [17].

To deal with concurrency, definitions of NI have been proposed for Non-Deterministic automata. Von Oheimb defined noninfluence for such systems. His definition can be regarded as a "non-deterministic version" of IP-secure. Engelhardt et al. defined nTA-secure, the non-deterministic version of TA-security. Finally, some notions of INI consider models that are in a sense richer than similar counterparts. Leslie extends Rushbys notion of IP-security for a model in which the security policy is Dynamic. Eggert et al. defined i-secure, an extension of IP-secure. Their model extends Rushbys model (Mealy machines) with Local security policies. Murray et al. extends Von Oheimb definition of noninfluence to apply to a model that does not assume a static mapping of actions to domains. This makes it applicable to OSs, as in such a setting such a mapping does not exist [20]. NI-OS has been applied to the seL4 separation kernel [20], [14].

Most definitions have an associated methodology. Various methodologies are based on unwinding [8]. This breaks down the proof of NI into smaller proof obligations (POs). These POs can be checked by some manual proof [24], [10], model checking [32] or dedicated algorithms [5]. The methodology of Murray et al. is a combination of unwinding, automated deduction and manual proofs. Some definitions are undecidable and have no suitable unwinding.

We are aiming to provide a methodology for INI based on a model that is richer in detail than Mealy machines. This places our contribution next to other works that aim to extend IP-security [15], [4] in Figure 2. Similar to those approaches, we take IP-security as a starting point. We add kernel control mechanisms, interrupts and context switches. Ideally, we would simply prove IP-security over CISK. We argue that this is impossible and that a rephrasing is necessary.

Our ultimate goal — certification of PikeOS — is very similar to the work done on seL4 [20][19]. There are two reasons why their approach is not directly applicable to PikeOS. First, seL4 has been developed from scratch. A Haskell specification serves as the medium for the implementation as well as the system model for the kernel [6]. C code is derived from a high level specification. PikeOS, in contrast, is an established industrial OS. Secondly, interrupts are mostly disabled in seL4. Klein et al. side-step dealing with the verification complexity of interrupts by using a mostly atomic API [14]. In contrast, we aim to fully address interrupts.

With respect to attempts to formal operating system verifications, notable works are also the Verisoft I project [1] where also a weak form of a separation property, namely fairness of execution was addressed [3].

6 Conclusion

We have introduced a generic theory of intransitive non-interference for separation kernels with control as a series of locales and extensible record definitions in order to a achieve a modular organization. Moreover, we have shown that it can be instantiated for a simplistic API consisting of IPC and events.

In the ongoing EURO-MILS project, we will extend this generic theory in order make it sufficiently rich to be instantiated with a realistic functional model of PikeOS.

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