Dual adjacency matrix : exploring link groups in dense networks

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Dual Adjacency Matrix: Exploring Link Groups in Dense Networks

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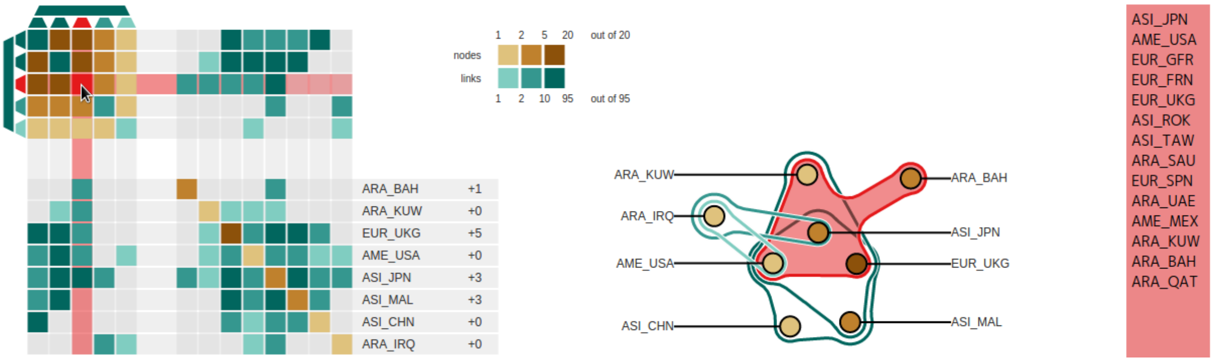


Figure 1: *Dual adjacency matrix (left), node-link-contour diagram (middle), and highlighted group nodes (right) that depict a trade network of countries (nodes) with substantial changes in volume traded (links) over a fifty year period. One group of links is selected in the top-left matrix, which covers countries such as Japan (ASI_JPN) and the USA (AME_USA).*

Abstract

Node grouping is a common way of adding structure and information to networks that aids their interpretation. However, certain networks benefit from the grouping of links instead of nodes. Link communities, for example, are a form of link groups that describe high-quality overlapping node communities. There is a conceptual gap between node groups and link groups that poses an interesting visualization challenge. We introduce the Dual Adjacency Matrix to bridge this gap. This matrix combines node and link group techniques via a generalization that also enables it to be coordinated with a node-link-contour diagram. These methods have been implemented in a prototype that we evaluated with an information scientist and neuroscientist via interviews and prototype walk-throughs. We demonstrate this prototype with the analysis of a trade network and an fMRI correlation network.

Categories and Subject Descriptors (according to ACM CCS):
Computer Graphics [I.3.3]: Picture/Image Generation–Line and curve generation—

1. Introduction

Many networks are derived through experimental observation of real-world systems for analysis purposes. Social networks, for example, describe interactions between people and provide insights about the functioning of society (see Fig. 2(a)). Some of these networks are dense, in which most nodes are so interconnected that their individual roles are of less interest than the concert of their interactions. This phenomenon appears as the notion of a network module, or *community*, which is a dense network section (with a high link-to-node ratio) that reflects part of a system that is likely to have a special role (see Fig. 2(b)). For example, communities in the social network of a company could be correlated to departmentalization, where team members are likely to interact. Likewise, communities emerge via natural selection in organisms and appear in protein interaction and

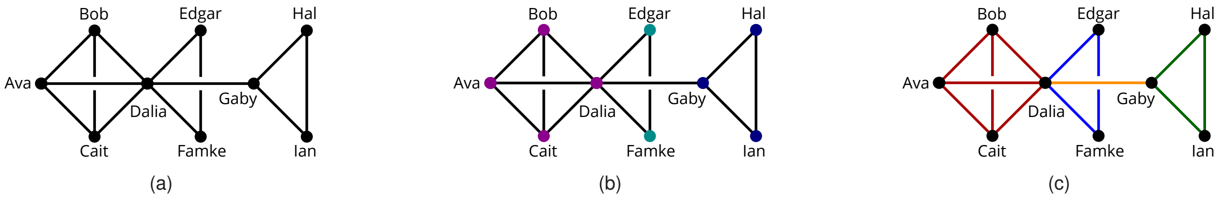


Figure 2: *Example of a network and derived community structures: (a) Plain social network, people are nodes (dots) and their interactions are links (connecting lines); (b) Densely interconnected nodes of (a) have been grouped into communities, in which* Dalia *is part of a single community in spite of her widespread interactions; (c) Densely interconnected links of (a) have been grouped into communities, in which* Dalia *is part of multiple communities.*

metabolic networks [RSM*02, YOB04]. Communities are often defined in terms of node groups.

Node group A group (or cluster) of nodes that together fulfill a role within a network. Node groups are disjoint.

Node groups ease the visual aggregation of networks into several joint nodes for which information is summarized. However, certain networks benefit from grouping (or clustering) links instead of nodes.

Link group A group (or cluster) of links that together fulfill a role within a network. Link groups are disjoint.

Suppose the links of a network are accompanied by a time series. Grouping these links by similar behavior over time will expose links–and the nodes that they connect–that act in concert, indicating a shared role. Likewise, link groups that are clustered by network connectivity can be used to determine high-quality overlapping node communities [ABL10], as shown in Fig. 2(c).

Link groups are more difficult to grasp as a concept than node groups and therefore pose an interesting visualization challenge. We bridge this conceptual gap by contributing:

- Generalization and combination of node and link group techniques into a Dual Adjacency Matrix (DAM) and node-link-contour diagram;
- A prototype implementation and demonstration of our approach on a trade network and fMRI correlation network;
- Informal evaluation by an information scientist and neuroscientist via interviews and prototype walk-throughs.

2. Related Work

Visualization of network topology has been the subject of much research [HMM00, vLKS*11], where node groups often occur to, for example, support node time series or multivariate analysis [HSCW13, vdEvW14]. Shifting focus from nodes to links has already appeared in various forms. Bundling links by the position of their nodes in a predefined hierarchy reveals correlations between links and the properties of their nodes [Hol06]. Visual manipulation of network topology can be avoided via explicit visualization of link to link relations by introducing an extra type of link [VHTW13].

An overview of node community visualization can be found in [VRW13]. Many approaches involve node-link diagrams in which community memberships are visualized by layout [VRW13] and color [APF*06, IMMS09]. A dual, community-centric approach is taken in [APF*06], where communities are depicted as nodes and their overlaps as weighted links. These techniques have all proved effective, either for arbitrary overlapping communities or those that result from specific detection algorithms. However, to the best of our knowledge, no visualization techniques have been explored for link groups and the node groups that they induce.

Visualizing overlapping node groups while abstracting from the underlying network topology is equivalent to the visualization of a set system or undirected hyper graph. Venn and Euler diagrams represent these set systems as overlapping shapes with elements placed in the shapes according to their set memberships. Here the layout of shapes and elements plays an important role [BE01, SAA09, HRD10] and this layout is sometimes constrained as well [CPC09, MRS*13]. Some methods give priority to visualizing the distribution of elements among sets, instead of set system topology [KBH06, AAMH13, SMDS14]. Various matrix-like representations exist as well [KJ13, LGS*14]. An overview of set system visualization can be found in [AMA*14].

3. Link Group Analysis Tasks

Related research [LPP*06, HRD10, DvKSW12, SSK14] has formalized a list of important tasks performed by analysts on networks, set systems, and node groups in networks. Network tasks capture how nodes are related to each other via their *link connectivity*. As a dual to this, we consider relating links to each other via their imposed groups as well as *node connectivity*, and identify a set of analogous tasks, which capture how link groups are related to each other via shared nodes. Additional link attributes, possibly used to derive link groups, are included in these tasks (see Table 1).

Table 1: *Analysis tasks from Node-centric and Link-centric perspectives.*

	Node-centric	*Link-centric*
T1. Membership	Find link groups that cover given nodes	Find the nodes of a given link group
T2. Overlap	Find link groups that share given nodes	Find nodes shared by given link groups
T3. Path	Find paths between nodes via link groups	Find paths between link groups via nodes
T4. Cluster (clique)	Find multiple nodes that share many link groups	Find multiple link groups that share many nodes
T5. Component	Find (dis-)connected components of nodes	Find (dis-)connected components of link groups
T6. Hub	Find a node that is covered by many link groups	Find a link group that covers many nodes
T7. Bridge	Find nodes that are sole link group connectors	Find link groups that are sole node connectors
A. Attribute	Compare attributes of links that cover given nodes	Compare attributes of given link groups

4. Concept

To the best of our knowledge, visual aggregation and navigation techniques have so far only focused on node groups. We transfer established techniques from node groups to link groups and introduce visualizations that couple node and link group perspectives.

Dual Adjacency Matrix. Both node and link perspectives are combined in the dual adjacency matrix, which consists of four quadrants, as shown in Fig. 3.

The bottom-right quadrant (see Fig. 3(b)) is the familiar matrix that shows adjacencies between node groups. Node groups (along the diagonal) are colored brown according to their size, and the number of links that connect two node groups is shown with a green color scale (see Fig. 3(e)). For example, a node group in Fig. 3 is highlighted in blue, which includes its matrix row and column. This node group consists of *Ava* and two additional people (indicated by a *+2*), and it is connected to only the node group of *Dalia*.

The top-left quadrant is the dual of the matrix at the bottom-right; it shows adjacencies between link groups. Again, link groups (along the diagonal) are colored green according to their size, and the number of nodes shared between two link groups is color coded in brown. For example, one link group in Fig. 3(a) is highlighted in red and shares nodes with two other link groups.

The bottom-left and top-right quadrants are symmetric and connect the top-left link groups to the bottom-right node groups, showing which node groups are covered by which link groups. For example, the bottom-left and top-right quadrant tiles that are highlighted in black in Fig. 3(c) show the connection between the blue and red node and link groups respectively. Here, the green color coding shows the number of links in a link group that cover nodes from a node group.

Node-link diagram. Matrix visualizations are suited for the visualization of dense networks, where link group overlaps are common. In case of simple link group topology, we also show how to create node-link diagrams. Coordinating these diagrams with a dual adjacency matrix via interactive highlighting eases the transition from reading a familiar node-link diagram to reading an unfamiliar matrix. For example, the node-link diagram of Fig. 3(d) shows the node group of *Ava* highlighted in blue and covered by a link group highlighted in red. It also shows that the red link group covers the *Dalia* node group, as can be seen in Fig. 3(c).

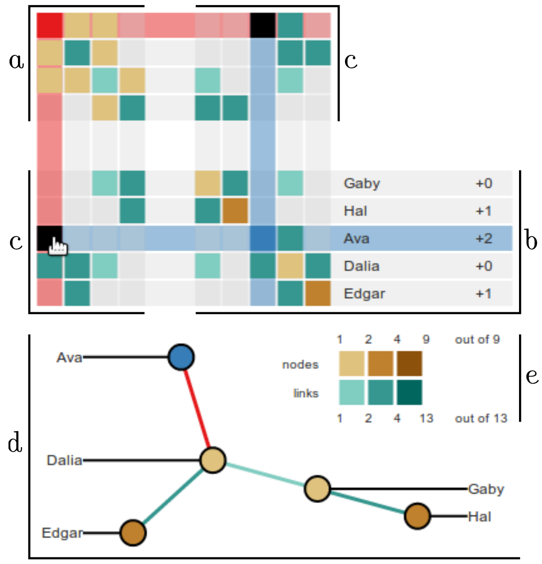


Figure 3: *Concept of the dual adjacency matrix, in which the network of Fig. 2(c) is depicted while aggregated according to its link groups: (a) The rows and columns of the top-left quadrant represent link groups, in which the diagonal shows the number of links in each link group, and the quadrant remainder shows the number of nodes that connect the row and column link groups; (b) The rows and columns of the bottom-right quadrant represent node groups, in which the diagonal shows the number of nodes in each node group, and the quadrant remainder shows the number of links that connect row and column node groups; (c) The rows and columns of (a) and (b) extend into the two remaining quadrants that show which node groups are covered by which link groups; (d) Node-link diagrams that match the dual matrix are used to depict link and node group topology when it is sparse; (e) Numbers of nodes and links are encoded by color scales, and selected groups are shown in red and blue.*

5. Network Aggregation

We regard node and link groups as each other's dual, where nodes and links can be interchanged. The dual of a regular network is also known as a *link-to-node dual* or *line graph*.

Node groups. Node groups are derived from node attributes [AMA08, LNS11] or topology. For example, nodes can be grouped by similar attribute values, short topological distance, or neighborhood similarity. The node groups in Fig. 4(b), indicated by color, induce the aggregated network of Fig. 4(d). Every node in the aggregated network represents a group of people and every link represents the presence of an interaction between one or more members of the two groups. Both networks can also be represented as *node adjacency matrices*, shown in Fig. 4(f) and (h).

Link groups. The grouping of links can also be expressed as a grouping of nodes in the dual of Fig. 4(b), which can be represented by either a node-link diagram or a link adjacency matrix, as shown in Fig. 4(c) and (g) respectively. The aggregated adjacency matrix of Fig. 4(i) depicts every link group as a row and column in the matrix (with accompanying labels to the sides), and shared nodes as the dots at intersections.

The conceptual gap between grouping nodes and grouping links can be bridged via a bipartite graph interpretation (see Fig. 4(a)). Nodes and links are shown as solid and hollow dots that are connected if the associated nodes and links are connected in the original network. This interpretation structures the node-centric versus the link-centric tasks of Section 3, for which an observer traces connections between the two groups of the bipartite graph and the only difference between the task categories is the type of node that the observer starts from.

6. Construction of a Dual Adjacency Matrix

The link adjacency matrices of Fig. 4(g) and (i) display overlaps of only two link groups at a time, while there is a need to oversee the intersection of an arbitrary number of groups (tasks T2 and T4). For example, *D* in Fig. 4(a) is covered by three link groups, but this is difficult to see in Fig. 4(g). We therefore introduce the *Dual Adjacency Matrix (DAM)*, which consists of a link adjacency matrix, a node adjacency matrix, and two additional matrix quadrants representing which link groups cover which node groups.

Link group intersections. The need to oversee intersections of arbitrary numbers of link groups can be met by grouping nodes according to the link groups that cover them, as shown in Fig. 5(a) & (b). This is similar to visualizing overlapping sets by grouping nodes according to set membership combinations [AMA*14, LGS*14]. However, in this case each set consists of the nodes covered by a link group. The resulting link group to node group relationships can also be shown as the matrix in Fig. 5(c), where node groups are not predefined (as in Fig. 4) but derive from link groups. These node

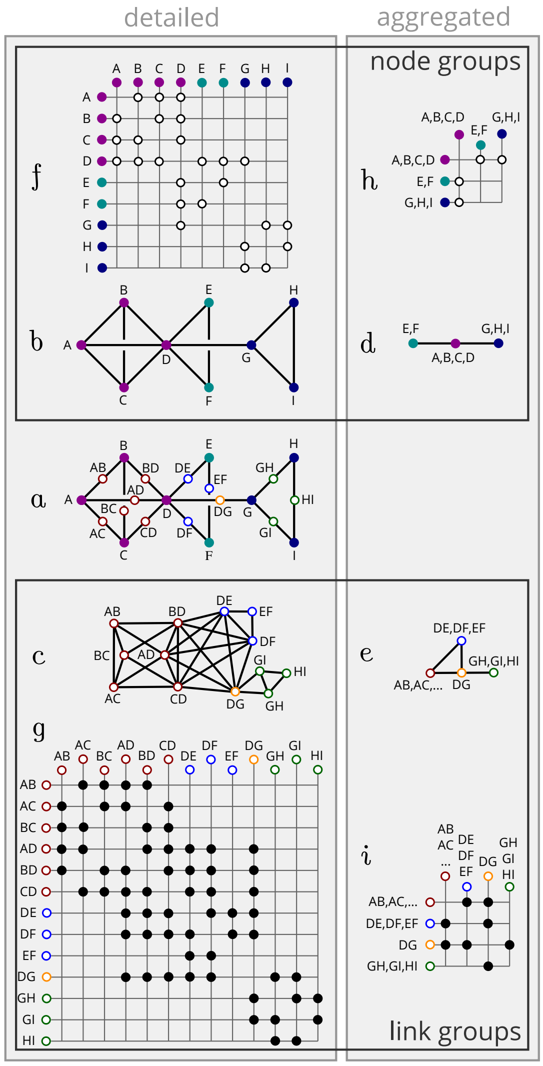


Figure 4: *Overview of network aggregations, applied to the network of Fig. 2: (a) Bipartite network that bridges the node and link duality of (b) and (c), in which the node-to-node (solid dots) and link-to-link (hollow dots) connections correspond to node-link-node and link-node-link paths respectively; (b) Node-link diagram in which nodes (solid dots) are colored by group; (c) Node-link diagram of the dual of (b) in which links (hollow dots) are colored by group; (d) and (e) Node-link diagrams in which the respective groups of (b) and (c) are aggregated into single nodes; (f),(g),(h), and (i) Adjacency matrices of the (b),(c),(d), and (e) networks respectively.*

groups also derive the node adjacency matrix of Fig. 5(d), in which row and column intersections depict links that are shared between node groups. This type of adjacency matrix

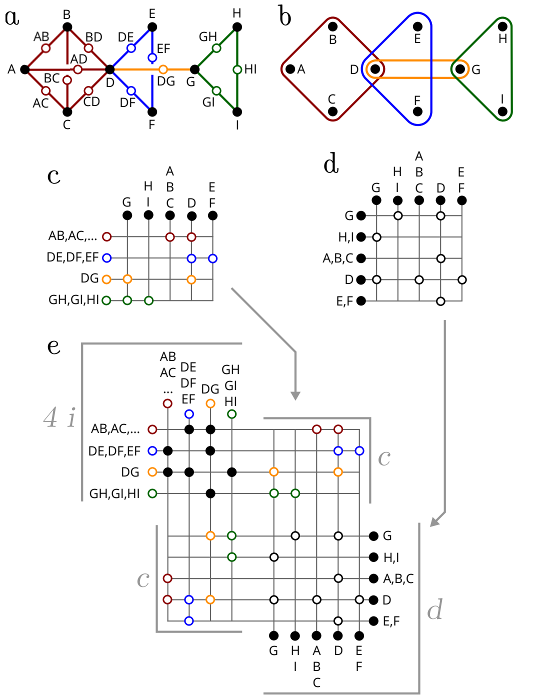


Figure 5: *Providing a node group interpretation of link groups with a dual adjacency matrix: (a) Network of Fig. 4(a), but with links colored to emphasize their groups; (b) Euler diagram of the set system that is induced by the link groups of (a), in which every set contains those nodes covered by its corresponding link group; (c) Set membership table (or matrix) of the set system of (b) that depicts all link group overlaps as the composition of node groups; (d) Node adjacency matrix of the node groups of (c); (e) Combination of (c), (d), and Fig. 4(i) that forms a dual adjacency matrix.*

enables node-centric hub (T6) and bridge (T7) identification, which is familiar to analysts.

Combination and extension. The matrix of Fig. 5(c) forms the bridge between node and link groups that enables combination of the node and link adjacency matrices into the dual adjacency matrix shown in Fig. 5(e). This combination supports both node- and link-centric tasks, in particular membership (T1) and overlap (T2).

The top-left quadrant of a dual adjacency matrix shows the nodes that connect link groups and the bottom-right quadrant shows the links that connect node groups. This leaves the quadrants of Fig. 5(c) open to what their cells (row and column intersections) represent; either connecting nodes or connecting links. Showing connecting nodes is superfluous, because our link groups induce node groups such that any cover of a node group by a link group is complete (all nodes are covered). Representing links does provide additional information however, as shown in Fig. 3(c). It conveys which links of a link group cover a node group; a column shows how a link group decomposes over node groups, and a row shows how the neighborhood links of a node group distribute over link groups.

7. Construction of a Node-link Diagram

The dual adjacency matrix focuses on the visualization of link groups and their overlaps (tasks T2, T4, and T6) as an extension of common adjacency matrices. Adjacency matrices are difficult to read and they perform poorly on global topology tasks (T3, T5, and T7) in comparison to node-link diagrams [GFC04]. We have therefore also explored link group visualization as forms of node-link diagram.

Detailed node-link diagram. Early prototypes featured a node-link diagram of the entire network, in which nodes have a pre-computed position (see Fig. 6(a)). Interactions with the link and node groups of the duality matrix are coordinated with this node-link diagram, where nodes and links are colored according to hovered groups. This approach was valued by users for small and sparse link groups but did not scale due to a lack of aggregation.

Aggregated node-link diagram. Aggregating the detailed node-link diagram down to the groups of the dual adjacency matrix gives a less cluttered but more abstract visualization: Every node group appears as a node and every link group as multiple lines; one line runs between two nodes for every link group that covers them. This approach is common for the visualization of multiple (overlapping) link types, but in this case it shows link groups.

Node-link-contour diagram. Link groups facilitate the creation of the overlapping shapes of an Euler diagram because their individual links can be inflated and combined (see Fig. 6(c)). The links that underlie a community therefore act as a skeleton, which can be inflated to form hulls [LQB12, MRS*13]: a contour is derived per link group by dilating its links (the application of a Minkowski sum [DBVKOS00] with a circle), dilating covered node groups with a greater radius for emphasis, then eroding the contour to smoothen it, and finally subtracting the areas around non-member nodes to avoid invalid overlaps. Contours are separated by dilating with different radii at nodes that are shared by multiple link groups (see the nested contours in Fig. 6(c)). Contours are also cohesive per link group, provided that the link groups themselves are cohesive, which is often the case [ABL10]. It is also possible to visualize aggregated links (see Fig. 6(d)). This provides more insight into network topology at the expense of some additional clutter.

8. Prototype Design

We implemented the DAM in a prototype that was refined in an iterative fashion with feedback from link group experts. The prototype includes additional node and link information.

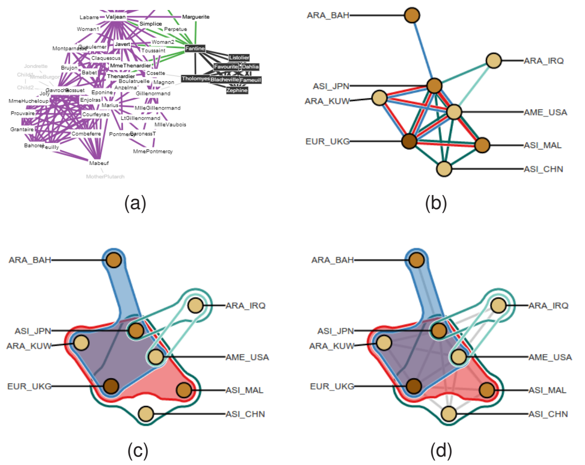


Figure 6: *Node-link diagrams that incorporate link groups: (a) Detailed, full node-link diagram with color coding of nodes and links via interaction; (b) Aggregated node-link diagram with links between node group pairs, color coded like (a); (c) Link group contours derived (and color coded) from the links of (b); (d) Addition of links between group pairs to (c) for improved depiction of topology.*

Node labels. Node groups are labeled to provide an indication of their contents. This label consists of the name of the most important node in the group (by input score) and is shown at an extension of its matrix row (see Fig. 3(b)). It also shows the remaining number of nodes in the group to emphasize it being a group.

Multi-level link groups. We expect input link groups to be the result of clustering, based on link topology and/or attributes like a time series. Such clusterings are often hierarchic, which is why the prototype supports a link hierarchy. This hierarchy is mirrored at the top and side of the top-left matrix quadrant as icicle plots that extend to the rows and columns of the matrix (see Fig. 1). The sides of this icicle plot are tapered to get a tree-like representation with pronounced hierarchy branches. Interactive hierarchy navigation is enabled for better scalability, where changes to the visualizations due to splitting high-level groups are animated.

Aesthetics. The colors for highlighted groups are derived from color brewer [HB03]. Strong contrast and hard outlines are avoided, though black is used for highlights and node legibility. Matrix rows and columns are of a translucent gray such that their intersections are more pronounced. A large space is placed between the dual matrix quadrants to make them appear cohesive and emphasize their difference. The color scales for node and link abundance (see Fig. 3(e)) are divided into four levels: the first level encodes zero abundance, the second level encodes a single node or link, and the remaining levels follow a log scale.

Highlighting. Highlighting is enabled via mouse-over of up to two matrix rows, columns, or diagram contours. If two of such elements belong to the same group, then this group is highlighted in red (and translucent red in the background), as shown in Fig. 1. If two different groups are hovered, then one group is shown in red, the other group in blue, and their overlap in black (see Fig. 3). This simultaneous highlighting enables the comparison of two groups in coordinated views, which include additional information (task A).

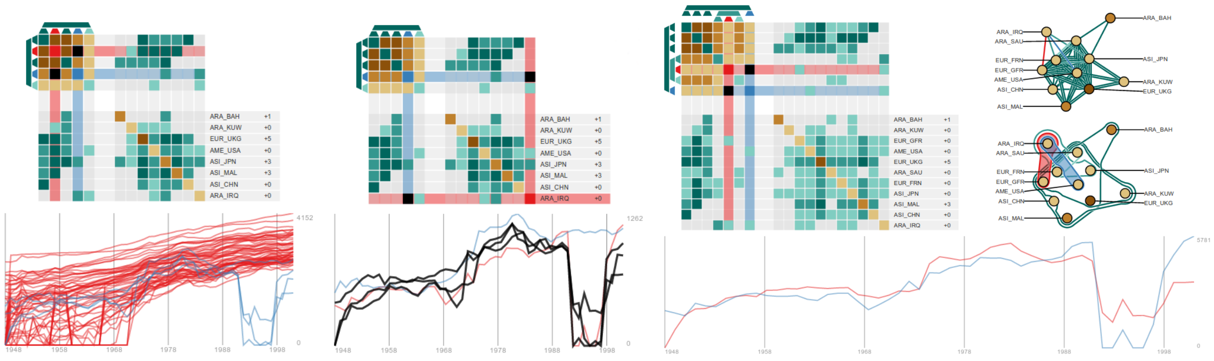
One such coordinated view is shown at the right of Fig. 1, which lists the nodes of a highlighted node group, or those nodes covered by a highlighted link group. Three lists are shown when two groups are highlighted: two lists that show all nodes per group, and one list in the middle that shows overlapping nodes. Links are coupled to time series data in another view (see Fig. 7). It visualizes the time series of up to two highlighted groups as semi-transparent colored trend plots. For a highlighted link group, these are the time series that belong to its links, and for a highlighted node group, these are the time series of the links that cover any of its nodes. Time series of links that are shared by two groups are emphasized as black plots.

9. Exploration Demonstration

We demonstrate the described concepts and the use of the tool by exploring a dense network of countries (20 nodes) and their trade relations (95 links). These trade relations are the combined import and export (in millions of dollars) between countries, measured on a yearly basis between 1948 and 2000 (53 time points).

Preprocessing. The analyzed data set was derived from a larger trade network by selecting the countries from four major regional trading groups (North-America as *AME*, Europe as *EUR*, Arabia as *ARA*, and Asia as *ASI*). In addition, trade relations were filtered for high variance, which leaves the most dynamic relations for analysis. These trade relations were attached to corresponding links as time series, and the links were then grouped according to these time series with a k-means algorithm. This clustering uses angular (cosine) distance for the time series vectors such that links with similar trade dynamics are grouped together. Links from the same group could therefore be trade relations with common external influences, explaining their similar dynamics, or they could have been grouped as a clustering artifact. Clustering was applied twice to get a multi-level grouping, with $k = 5$ for top groups and $k = 3$ for subgroups. The resulting five top link groups can be typified as follows: a large group with consistent trade growth but a lot of missing data, a large group with consistent growth and little missing data, a smaller group with a minor trade slump in the 1980s, a group of four links with major slumps in the '90s, and a group of a single link with the same slump in the '90s.

Matrix perspective. Dual adjacency matrix and node-link-contour visualizations of the initial, top-level link groups are



(a) (b) (c)

Figure 7: *Elucidation of link groups in a trade network, in which time series of highlighted groups are shown as trend plots that show trade volume on a log scale: (a) Comparison of the time series of a large link group (red) that has a mostly steady growth of trade volume, and a smaller link group (blue) with a significant trade slump in the 90's; (b) Overlap of all available trade relations of Iraq (red) with the relations of the smaller link group of (a) (blue), showing that many 90's volume slumps involve trade with Iraq (overlap colored black); (c) The blue link group of (a) is split into three subgroups of which one subgroup is a single link (red, covering* ARA_IRQ *and* EUR_GFR*) that is compared to the single link of a smaller top group (blue, covering* ARA_IRQ *and* AME_USA*).*

shown in Fig. 1. These visualizations provide insights into the trade network topology. For example, the matrix at the top-left shows that all link groups overlap, which indicates that different trade behaviors intermingle. Three link groups at the top-left have a strong overlap (they share many nodes), which is shown by dark brown matrix intersections (T4).

The bottom-left and top-right matrix quadrant confirm a strong overlap of these link groups. These quadrants also reveal a large node group, led by the United Kingdom (*EUR_UKG*), that is exclusively covered by these link groups (T1). Only two other groups of countries, led by the United States (*AME_USA*) and Japan (*ASI_JPN*), are covered by more link groups, making them link group hubs (T6). However, China (*ASI_CHN*) is isolated to one link group, making this link group a bridge (T7), while the bottom right matrix shows that China has links to many countries, making China a conventional hub that is constrained to one link group.

Node-link-contour perspective. The node-link-contour diagram in Fig. 1 enables the same observations as the dual adjacency matrix because the link group topology is plain. The overlap between all link groups is not as apparent in the diagram as in the top-left matrix. On the other hand, sparsely connected node groups such as Iraq (*ARA_IRQ*) and Bahrain (*ARA_BAH*) are easier to spot in the diagram than in the matrices. Moreover, small link groups stand out, such as the two groups that bridge to Iraq.

Inspecting link groups. The three large link groups and two smaller link groups can be explained by inspecting their associated time series in an additional view (task A). In Fig. 7(a) we highlight one of the larger link groups (colored red) and one of the smaller groups (blue) by hovering their intersection in the top-left matrix quadrant. The large group contains a large number of links that appear as numerous red trend lines. These series are noisy until 1970, likely caused by a lack of data (defaulting trade volume to nil), but show consistent trade growth afterwards. Comparing the large link groups to each other confirms that all large link groups share this growth pattern. The smaller group shows a significant trade slump in the 90's and the bottom-left matrix quadrant shows that this link group covers Iraq.

We request more detail about Iraq in Fig. 7(b) (colored red) by hovering its overlap with a link group (blue). Hovering the row or column of a group of countries shows all of their trade relations in the time series view. Trade volume plots of the blue link group that involve Iraq are colored black like the matrix overlap cell. We see that all of Iraq's relations are covered by the blue link group except for one (the red plot). Nonetheless, all of Iraq's trade relations feature the 90's volume slump, which is likely caused by the regional conflicts during this period.

Cluster assessment. In Fig. 7(a)&(b) the blue link group contains one trade relation that does not slump and which excludes Iraq. This mismatch appears to be a fault in the clustering, so we split the blue link cluster by clicking on its link hierarchy branch at the top-left to get the new matrix configuration in Fig. 7(c). This unveils that the link without the slump, positioned between the blue and red subgroups, does not involve Iraq but Saudi Arabia (*ARA_SAU*) and France (*EUR_FRN*), which explains the trade development that is similar to Iraq up to the war and why their trade links were clustered together. Comparing one of the link subgroups that involve Iraq with the only other link group that involves Iraq (colored red and blue respectively in Fig. 7(c)) shows similar trade developments, but the isolation of the cluster (being the trade link with the United States) could be explained by its very strong increase in trade volume before the turn of the millennium.

10. Preliminary Expert Feedback

We performed interviews and walk-throughs of our prototype with two experts from different fields to gather feedback on our approach. We first performed a series of interviews with each expert to collect several of their datasets and associated analysis questions. Then, during an one-hour session, one of the authors walked the experts through the prototype while they used their own data.

Information science expert. Our first expert, Jevin West, is an academic researcher at the information science school of the University of Washington, with a solid expertise in analyzing complex networks and a focus on understanding communities. The datasets he was primarily interested in were scientific journal citation networks, composed of several hundreds of nodes and several thousands of links that are hierarchically partitioned into link communities [ABL10]. After our presentation of the tool, it took about 15 minutes for Jevin to interact with the system and make effective use of the dual adjacency matrix to explore his data. Jevin commented that the tool had a steep learning curve, but is powerful once understood. This enables him to answer questions on link and node communities that were not easy to answer before. In particular, it helped him make the leap from link groups to shared node groups. For example, he identified nodes acting as hubs by glancing at the link to node group matrix and reviewing node groups. He also pointed out link communities sharing many nodes, commenting that this was potentially helpful to identify noise in the data or regions with a low clustering quality.

Jevin was quickly familiar with the link group intersection matrix and used the hierarchical navigation several times to adjust the level of link groups to the desired granularity. Jevin concentrated most of his analysis on the link to node group matrix and the conventional node group matrix. He explained that "the most useful feature here [pointing at the node group adjacency matrix] is [seeing] what is not connected because we usually know about the [presence of] clusters [themselves]". He commented that it was compelling to see holes in this matrix and argued that this could be useful to identify missing data or, if not missing, serve to make predictions on future connections, which he mentioned is of interest in many scenarios.

Neuroscience expert. Our second expert, Tara Madhyastha, is an academic researcher at the radiology department of the University of Washington, with expertise in analyzing functional brain connectivity networks extracted from magnetic resonance imaging. She was primarily interested in dynamic weighted networks, composed of two dozens of regions of interest in the brain and their weighted connections that evolve over time. We augmented the prototype with a spatial brain view to provide a familiar context to the nodes and weighted connections. In our initial interview, Tara explained that she had experimented with several link grouping algorithms before. The output usually contained one large group of links and many small ones, from which she concluded it was not worth pursuing this type of analysis. Aware of these past attempts, we experimented with several other algorithms to get to a more balanced distribution of the number of links in groups. We presented the output of k-means clustering on vectors of link weight to Tara in our walk-through session. Tara had never before used visual tools to inspect link and node groups at the same time.

Tara had a very different exploration process than Jevin. She spent most of the session inspecting the content of the link group matrix coupled with the time series view. As we presented the tool, she immediately identified a link group of interest, pointing at the time series view, and asked a series of questions about the clustering algorithm. From this point, Tara used the prototype to visually assess the quality of the link groups. In a later session we combined Tara's own link clusters of three large-scale intrinsic brain zones [MG14] for her to explore a subset of individuals with Parkinson's Disease (PD) and age-matched controls. Research, including Tara's, suggests that the coupling of these zones may be dysregulated in PD. The DAM enabled her to examine the time series of multiple link communities in multiple zones at the same time, expecting that some are coupled and others are not (see Fig. 8). For example, she discovered partial coupling of the Posterior Default Mode zone and parts of the Fronto-Parietal Task Control zone in a PD subject at rest.

11. Discussion and Limitations

We have demonstrated DAMs for small but dense networks where links are subject to multi-level grouping. Even for networks with few nodes this poses a visualization challenge, because standard node-link diagrams are hard to read (see Fig. 7(c)). However, such a diagram is likely to enable more effective analyses for a sparse network, provided its link groups are coherent (see Fig. 6).

DAMs scale to larger networks by adding interaction techniques to manipulate link groups and their hierarchy: filtering link groups on criteria such as size and density, interactive branch pruning, and automated branch expansion. This involves common hierarchy interaction that is peripheral to the concept of DAMs but was vital to support Jevin's large link community analyses with our prototype. Nonetheless, analyses with DAMs can be impeded by flat or imbalanced link hierarchies.

An alternative to DAMs of dense networks are regular ad-

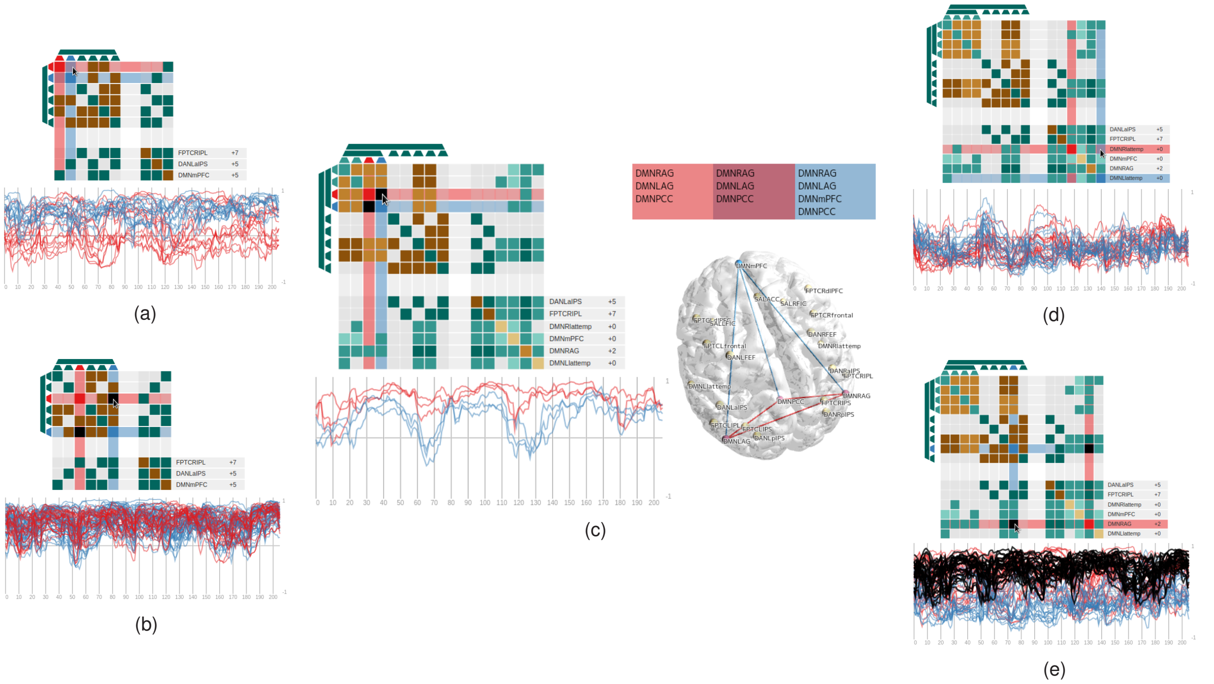


Figure 8: *Exploration of 20 brain regions connected by 183 links that encode (sliding window) fMRI signal correlations along 200 time points. The brain is divided into three zones (DAN, FPTC, and DMN) and links are grouped accordingly: three intra-zone link groups that are subgrouped [MG14], and three inter-zone link groups: (a) The top-left matrix shows no overlap between the three intra-zone link groups, but that they do overlap with the inter-zone link groups. The bottom-right matrix has three node groups that match the zones. The intra-zone link group DMN (red) has mixed signals, while DAN (blue) and FPTC have positive signals. (b) Both the FPTC link group (red) and the link group between FPTC and DAN (blue) have positive signals. (c) Splitting DMN because of its mixed signals reveals its overlapping link subgroups. Two link subgroups have strong overlap, where one group (red) is strongly correlated and tightly positioned in the brain, and the other group (blue) is less correlated and more spread across the brain. (d) The bottom-right adjacency matrix shows a missing link between spatial opposites Rlattemp and Llattemp. Hovering the empty spot compares all neighboring links of Rlattemp and Llattemp, where their signals show a consensus. (e) One node group (red) acts as a hub to DMN link groups. Hovering this node group and the DMN link groups shows that the node group has many anti-correlations within the DMN zone. However, hovering the inter-zone link groups (blue) shows that this node group has mostly positive correlations with the 'remainder' of the network.*

jacency matrices with links color coded by group, and rows and columns arranged by link group similarity. While this regular adjacency matrix might be easier to read, color coding limits the number of link groups shown and complicates interactive link group navigation. One benefit of a DAM is its dual representation of link groups, which enables the attachment of explicit hierarchy representations, and the mouse-over highlighting and comparison of two groups.

12. Conclusion

We have introduced a generalization of the adjacency matrix for exploring link and node groups. This generalization enables analysis of (hierarchical) link groups while providing both node and link group perspectives. Iterative implementation of this concept, while relying on feedback from link group experts, has resulted in an interactive system with coordinated matrix and node-link diagram views. Walk-through sessions with two experts revealed that DAMs help link and node group analysis, bridging the concepts.

The experts had different exploration processes, in which they understood and relied on all quadrants of the adjacency matrix in spite of a steep learning curve. This feedback suggests that our approach enables analysts to bridge the gap between link and node groups. We believe this is an encouraging first step towards visual exploration of link groups.

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