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An Exact Algorithm for the Vehicle Routing Problem with Time Windows and Shifts

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This paper introduces the Vehicle Routing Problem with Time Windows and Shifts (VRPTWS). At the depot, several shifts with non-overlapping operating periods are available to load the planned trucks. Each shift has a limited loading capacity. We solve the VRPTWS exactly by a branch-and-cut-and-price algorithm. The master problem is a set partitioning with an additional constraint for every shift. Each of these constraints requires the total quantity loaded in a shift to be less than its loading capacity. For every shift, a pricing subproblem is solved by a label setting algorithm. Shift capacity constraints define knapsack inequalities, hence we use valid inequalities inspired from knapsack inequalities to strengthen the LP-relaxation of the master problem when solved by column generation. In particular, we use a family of tailored and new cover inequalities defined both on the flow variables and on the master variables. Numerical results show that cover inequalities defined directly on the master variables significantly improve the algorithm.

Key words: vehicle routing problem; column generation; shift capacity; branch-and-cut-and-price
History:

1. Introduction

In the vehicle routing problem with time windows (VRPTW), a homogeneous fleet of vehicles with limited capacity delivers goods to a set of geographically scattered customers. Each customer requires the delivery of a certain amount of goods within a specified time window. The objective of the problem is to determine a set of routes that minimizes the total operational cost while ensuring that all customers are served, that time windows are respected and that the capacity limit of the vehicles is not violated. It is assumed that all vehicles start and end their routes at a common depot, and that travel cost and travel time between each pair of locations in the problem is known.

Due to its practical relevance, the VRPTW is extensively studied in the literature (see, e.g., Gendreau and Tarantilis (2010) and Baldacci et al. (2012) for some recent surveys). Consequently, many (meta-) heuristics and exact methods are successfully developed to solve this problem. However, most existing models assume that vehicles are simultaneously dispatched at the depot. In many real-life situations, this assumption is not realistic. In fact, depots consist of a number of shifts (e.g., the day, the evening and the night shift), each with a limited loading capacity. A shift loading capacity is, for instance, the number of full-truck loads that can be realized in that shift. Obviously, when the total quantity to be delivered exceeds a shift loading capacity, multiple shifts must be used to load the vehicles. As shifts have non-overlapping operating periods (e.g., the day shift (7:00-15:00), the evening shift (15:00-23:00) and the night shift (23:00-7:00)), some of the vehicles must be dispatched at a later time. Consequently, due to customers delivery time windows, solutions derived from the VRPTW could be unfeasible when implemented in real-life.

We consider the variant of the vehicle routing problem with time windows where multiple shifts with limited loading capacity are considered and denote this variant the VRPTW with shifts (VRPTWS). We divide the depot's operating period (e.g., a day) into several non-overlapping time zones where a different shift is associated with each of these zones. Consequently, the depot's operating period consists of multiple shifts each with a start and end time, and a limited loading capacity. In this paper, we determine the set of routes that minimizes the total distance traveled. Additionally, the assignment of routes to the different shifts must take the shift loading capacity into consideration.

We solve the VRPTWS to optimality using a branch-and-cut-and-price (BCP) algorithm. In a BCP algorithm, the linear relaxation of the master problem in each branch-and-bound node is solved by column generation. In case of the VRPTWS, the master problem of the column generation is a set partitioning with an additional constraint for every shift. Each of these constraints requires that the total quantity loaded in a shift must not exceed its loading capacity. For every shift, a pricing subproblem, which is an elementary shortest path problem with resource constraints (ESPPRC), is solved by means of a label setting algorithm (Desrochers 1986). To tighten the linear relaxation of the master problem, we include several valid inequalities defined both on the compact variables and directly on the master variables. While the former can easily be handled in the BCP algorithm, the later are shown to be stronger, but increase the complexity of the pricing subproblem. The developed valid inequalities could be applied to several combinatorial optimization problems where knapsack inequalities appear in the formulation.

The main contributions of this paper are summarized as follows. First, we introduce a new problem that extends the classical VRPTW by considering shifts limited loading capacity. Secondly, we present an exact solution based on a BCP algorithm. For every shift, a separate pricing subproblem is solved by means of a label setting algorithm. By exploiting the structure of the problem, we develop new valid inequalities to strengthen the LP-relaxation of the master problem when solved by column generation. The added valid inequalities are shown to be useful when solving the VRPTWS and could be used in the solution of related problems where knapsack inequalities are part of the formulation. We include two types of valid inequalities, i.e., valid inequalities defined on the compact variables, and valid inequalities based on the master variables. While the former can easily be handled in the BCP algorithm, the later increases the complexity of the pricing subproblem. In fact, valid inequalities defined on the compact variables does not change the pricing subproblem as their dual variables can simply subtracted from the edge costs and the label setting algorithm remains unchanged. To reflect the additional cost incurred by dual variables stemming from valid inequalities defined on the master variables, it may be necessary to modify the pricing problem by adding more resources to the label setting algorithm. We show how to deal with this complexity due to including valid inequalities based on the master variables.

The paper is organized as follows. Section 2 reviews the literature relevant to our problem. In Section 3, a formal description of the studied problem along with its arc flow formulation is provided. In Sections 4 and 5, the column generation algorithm and the branching decisions are respectively described. Section 6 introduces the valid inequalities used in the BCP framework. In Section 7, we show how valid inequalities are handled in the pricing problem. In Section 8, extensive numerical experiments are conducted. Finally, Section 9 concludes the paper.

2. Literature Review

This non-exhaustive literature review roughly deals with two broad topics. We discuss the relevant literature both from an application point of view and from a related methodology point of view. For both cases, our paper significantly adds to the mentioned literature.

An abundant number of publications is devoted to the vehicle routing problem (see Laporte (1992), Toth and Vigo (2002), and Laporte (2007) for some reviews). For good reviews on the VRPTW, the reader is referred to Bräysy and Gendreau (2005a,b), Kallehauge (2008) and Gendreau and Tarantilis (2010). Column generation was successfully implemented for the VRPTW. For an overview of column generation algorithms, the reader is referred to Lübbecke and Desrosiers (2005). Column generation in the context of the VRPTW was first introduced by Desrochers et al. (1992). Later, Kohl et al. (1999) introduced subtour elimination constraints and 2-path cuts into the column generation approach and Cook and Rich (1999) applied the more general k-path cuts. In the nineties, the pricing problem of choice was the shortest path problem with resource constraints and two cycle elimination, in Irnich and Villeneuve (2006) an algorithm for k-cycle elimination was introduced which led to tighter bounds and, Feillet et al. (2004) and Chabrier (2006) proposed algorithms for the elementary shortest path problem with resource constraints (ESPPRC) which further improved lower bounds. Righini and Salani (2006, 2008) proposed various techniques to speed up the ESPPRC algorithm, including bi-directional search and decremental state space relaxation. Jespen et al. (2008) further improved lower bounds by proposing a column generation algorithm with valid inequalities based on the master problem variables (up to that paper inequalities had been expressed in the variables of the equivalent compact formulation). To accelerate the pricing problem solution, Desaulniers et al. (2008) proposed a tabu search heuristic for the ESPPRC. Furthermore, elementarity was relaxed for a subset of nodes, and both 2-path and subset-row inequalities were used. Baldacci et al. (2011b) introduced a new route relaxation, called *ng*-route, used to solve the pricing problem. Their framework proved to be very effective in solving difficult instances of the VRPTW with wide time windows, they solved all but one of the 56 famous Solomon instances.

In this paper, we apply two types of valid inequalities inspired from cover inequalities for knapsack problems. First, we include *compact cover* inequalities defined on the compact problem variables. These inequalities were first discovered separately by Balas (1975) and Wolsey (1975). We also include a strengthened version of these inequalities, i.e., the lifted compact cover inequalities. Zonghao et al. (1998) developed similar inequalities and investigated their implementation issues when applied in a branch-and-cut algorithm for 0-1 integer programs. Kaparis and Letchford (2008) applied lifted cover inequalities in the context of a 0-1 multidimensional knapsack problem. Secondly, we include *master cover* inequalities. They are new valid inequalities defined on the master problem variables. Including master cover inequalities increases the complexity of the pricing problem as each inequality leads to an additional resource. The introduced master cover inequalities can be applied to several combinatorial optimization problems when solved by column generation and when knapsack constraints are part of the set-partitioning formulation. Some example are the capacitated location routing problem (Baldacci et al. 2011a) and the more general two-echelon capacitated vehicle routing problem (Baldacci et al. 2013) where a depot capacity is modeled as a knapsack constraint. In Muter et al. (2014), a branch-and-price algorithm is used to solve the multidepot VRP with interdepot routes where vehicles are allowed to stop at any depot to replenish and continue with a another route. A set of routes traversed by a vehicle is called a rotation. The rotation duration must not exceed a maximum D, hence the total duration of the routes included in a rotation is bounded by D. This is again modeled by a knapsack constraint in the set partitioning formulation. Another problem where master cover inequalities can be applied is described in Degraeve and Jans (2007). In this paper, a capacitated lot-sizing problem is solved by means of column generation. In each period, a limited time capacity is available to produce products and each product incurs a set up time before starting its production.

Closely related to the VRPTWS, Gromicho et al. (2012) consider a combination of vehicle routing and loading dock scheduling, including synchronized routing. Examples of physical constraints mentioned in their paper include a limited number of loading docks and a limited size of loading crews. Additionally, time windows and obedience of compulsory working time directives are considered as well. This problem is solved using a heuristic based column generation. Cases obtained from two large retailers are used to demonstrate the value of their approach. These cases also dealt with an heterogeneous fleet with different dock capacity constraints, similar to our paper. Ren et al. (2010) consider a VRPTW with multi-shift and overtime. Their problem, inspired by a routing problem in healthcare, where the vehicles continuously operate in shifts, and overtime is allowed. They introduced a shift dependent tabu search based heuristic that takes overtime into account in the routing. The authors developed lower bounds by solving the LP relaxation of an MIP model with a number of specialized cuts. These cuts give improved bounds on minimum number of required routes, but also give insights on the minimum overtime needed and aim at eliminating two-node cycles. There are also similarities between our problem and the multi-depot VRP (Contardo and Martinelli (2014)) and the multi-period VRP (Mourgaya and Venderbeck (2007)) as depots and periods can be seen as shifts. There are also similarities between our problem and the location-routing problem presented in Baldacci et al. (2011a). In fact, a depot is equivalent to a shift and its capacity is equivalent to a shift loading capacity.

3. Problem Description

Consider a graph $G = (V, A)$ where $V = \{0, 1, ..., n, n+1\}$ is the set of nodes and $V_c = V \setminus \{0, n+1\}$ represents the set of customers while nodes 0 and $n+1$ represent the depot, the two nodes are the start and end, respectively, of any route. Let $[a_i, b_i]$ be the time window, d_i be the demand and s_i be the service time of node $i \in V$. We assume, without loss of generality, that $s_0 = s_{n+1} = d_0 = d_{n+1} = a_0 = 0$. Let τ_{ij} and c_{ij} denote the travel time (it includes service time at i) and the travel cost, respectively, from node i to node j. We consider an unlimited fleet of homogeneous vehicles K, each having a finite capacity Q. We can now define the set of feasible arcs as $A = \{(i,j) \in V \times V : i \neq j \text{ and } a_i + \tau_{ij} \leq b_j \text{ and } d_i + d_j \leq Q\}$. Furthermore, we assume that an operating period at the depot consists of a set of shifts S. Each shift $s \in S$ has a start time l_s, end time u_s and a limited loading capacity L_s. We assume that vehicles planned in shift s can be dispatched at time l_s.

We present an MIP arc flow formulation based on the flow variables $x^s_{ijk}, s \in S, k \in K, (i,j) \in A$, that take the value 1 if and only if the arc (i,j) is traversed by the vehicle k that is loaded in shift s, and the time variables $\omega^s_{ik}, s \in S, k \in K, i \in V$, representing the start time of service at node i. Furthermore, for every subset $A' \subseteq A$, vehicle $k \in K$ and shift $s \in S$, we denote $x^s_k(A') = \sum_{(i,j) \in A'} x^s_{ijk}$, and we let $\gamma^+(i)$ and $\gamma^-(i)$ be the set of arcs originating from i and the set of arcs ending in j respectively. The arc flow formulation of the VRPTWS is as follows:

$$\min z = \sum_{s\in S}\sum_{k\in K}\sum_{(i,j)\in A} c_{ij}x^s_{ijk} \tag{1}$$

subject to

$$\sum_{s\in S}\sum_{k\in K} x^s_k(\gamma^+(i)) = 1 \qquad \forall i \in V_c \tag{2}$$

$$\sum_{k\in K}\sum_{i\in V} d_i x^s_k(\gamma^+(i)) \leq L_s \qquad \forall s \in S \tag{3}$$

$$x^s_k(\gamma^+(0)) = x^s_k(\gamma^-(n+1)) = 1 \qquad \forall s \in S, \forall k \in K \tag{4}$$

$$x^s_k(\gamma^+(i)) = x^s_k(\gamma^-(i)) \qquad \forall s \in S, \forall k \in K, \forall i \in V_c \tag{5}$$

$$x^s_{ijk}\left(\omega^s_{ik} + \tau_{ij}\right) \leq \omega^s_{jk} \qquad \forall s \in S, \forall k \in K, \forall (i,j) \in A \tag{6}$$

$$a_i \leq \omega^s_{ik} \leq b_i \qquad \forall s \in S, \forall k \in K, \forall i \in V_c \tag{7}$$

$$l_s \leq \omega^s_{0k} \leq u_s \qquad \forall s \in S, \forall k \in K \tag{8}$$

$$\sum_{i\in V} d_i x^s_k(\gamma^+(i)) \leq Q \qquad \forall s \in S, \forall k \in K \tag{9}$$

$$w^s_{ik} \geq 0 \qquad \forall s \in S, \forall k \in K, \forall i \in V \tag{10}$$

$$x^s_{ijk} \in \{0,1\} \qquad \forall s \in S, \forall k \in K, \forall (i,j) \in A \tag{11}$$

The objective function (1) expresses the total cost to be minimized. Constraints (2) ensure that every customer is assigned to exactly one vehicle, and every vehicle is assigned to exactly one shift. Constraints (3) guarantee that shifts loading capacity is respected. Constraints (4)-(5) are related to the flow of arcs on the path traversed by a vehicle $k \in K$ that is loaded in shift $s \in S$. Furthermore, constraints (6), (7) and (8) guarantee feasibility with respect to time considerations. Constraints (9) make sure that the vehicles' capacity is respected. Finally, constraints (10) ensure that the time variables are non-negative, and constraints (11) impose binary conditions on the flow variables.

4. Set Partitioning Formulation and Column Generation

To derive the set partitioning formulation for the VRPTWS, we define Ω^s as the set of feasible paths corresponding to shift $s \in S$. For a given shift, a path is feasible if it is loaded within the shift operating period, satisfies customers delivery time windows and vehicle and shift capacity constraints. For each path $p \in \Omega^s$, c_p denotes its cost (i.e., the total distance traveled) and m_p its respective load. Let σ_{ip} be a constant that counts the number of times node i is visited by the path p. Furthermore, if y_p is a binary variable that takes the value 1 if and only if the path p is included in the solution, the VRPTWS is formulated as the following set partitioning problem:

$$\min \sum_{s\in S}\sum_{p\in\Omega^s} c_p y_p \tag{12}$$

subject to

$$\sum_{s\in S}\sum_{p\in\Omega^s}\sigma_{ip}y_p = 1 \qquad \forall i\in V_c \tag{13}$$

$$\sum_{p\in\Omega^s} m_p y_p \le L_s \qquad \forall s\in S \tag{14}$$

$$y_p \in\{0,1\} \qquad \forall s\in S, \forall p\in\Omega^s. \tag{15}$$

The objective function (12) minimizes the cost of the chosen routes. Constraints (13) guarantee that each node is visited exactly once. Constraints (14) ensure that the shifts loading capacities are respected. We use column generation to solve the LP-relaxation of (12)–(15): starting with a small subset of variables, we generate additional variables for the master problem by solving, for each shift $s\in S$, a pricing subproblem that searches for variables with negative reduced cost. Let $\pi_i > 0, i\in V_c$, be the dual variables associated with constraints (13), and $\mu_s < 0, s\in S$, the dual variables associated with constraints (14). The reduced cost of a variable (path) is defined as:

$$\overline{c}_p^s = c_p - \sum_{i\in V_c}\sigma_{ip}\pi_i - m_p\mu_s \tag{16}$$

The dual variable μ_s is negative and therefore will be acting as a penalty when subtracted from the path's reduced cost. If we let x_{ijp} be a binary variable that takes the value one if and only if arc (i,j) is used in path p, the path's load m_p can be expressed as:

$$m_p = \sum_{(i,j)\in A} d_i x_{ijp} \tag{17}$$

Hence, the reduced cost of path p is expressed as follows:

$$\overline{c}_p^s = \sum_{(i,j)\in A}(c_{ij} - \pi_i - d_i\mu_s)\, x_{ijp} \tag{18}$$

For an overview of column generation algorithms, the reader is refereed to Lübbecke and Desrosiers (2005) and Desaulniers et al. (2005).

5. Branching

The branch and bound tree is explored using a best bound strategy. First, the algorithm branches on the number of vehicles $\sum_{s\in S}\sum_{j\in V} x_{0j}^s$ over all shifts. It creates two branches $\sum_{s\in S}\sum_{j\in V} x_{0j}^s \le \lfloor\sum_{s\in S}\sum_{j\in V} x_{0j}^s\rfloor$ and $\sum_{s\in S}\sum_{j\in V} x_{0j}^s \ge \lceil\sum_{s\in S}\sum_{j\in V} x_{0j}^s\rceil$. If the number of vehicles for all shifts is integer, the algorithm branches on the number of vehicles per shift. It looks for the shift $s\in S$ with the most fractional number of vehicles and creates two branches $\sum_{j\in V} x_{0j}^s \le \lfloor\sum_{j\in V} x_{0j}^s\rfloor$ and $\sum_{j\in V} x_{0j}^s \ge \lceil\sum_{j\in V} x_{0j}^s\rceil$. If for all shifts the number of vehicles is integer, the algorithm branches on the arc variables x_{ij}^s. It looks for pairs $(i,j), i,j\in V_c$ and shifts $s\in S$ such that $x_{ij}^{s*} + x_{ji}^{s*}$ is close to

0.5 (x^* is the current fractional solution expressed in the arc variables) and imposes two branches $x^s_{ij} + x^s_{ji} \leq \lfloor x^{s*}_{ij} + x^{s*}_{ji} \rfloor$ and $x^s_{ij} + x^s_{ji} \geq \lceil x^{s*}_{ij} + x^{s*}_{ji} \rceil$. If $x^{s*}_{ij} + x^{s*}_{ji}$ is integer for all pairs $(i,j), i,j \in V_c$ and shifts $s \in S$, then the algorithm looks for an arc $(i,j) \in A$ and a shift $s \in S$ for which x^{s*}_{ij} is fractional and branches on that instead. Strong branching is used, that is, the impact of branching on several candidates is investigated every time a branching decision has to be made. For each branch candidate, we estimate the lower bound in the two child nodes by solving the associated LP-relaxation using a quick pricing heuristic. The branch that maximizes the lower bound in the weakest of the two child nodes is chosen. We considers 30 branch candidates in the first 20 nodes of the branch and bound tree, and 20 candidates in the rest.

6. Cover Inequalities

Cover inequalities are well-known valid inequalities for the knapsack problem. The polytops defined by the compact formulation (1)–(11) and the master problem (12)–(15) includes 0/1-knapsack inequalities defined by, respectively, the shift capacity constraints (3) and (14). Therefore, it is logical to think in this direction and apply valid inequalities inspired from the knapsack problem to strengthen the LP-relaxation of the master problem when solved by column generation. We include a family of tailored and new valid cover inequalities defined both on the compact variables and directly on the master variables. We call cover inequalities expressed in the compact variable *compact cover inequalities*, cover inequalities expressed in the master variables are called *master cover inequalities*.

6.1. Compact Cover Inequalities

For every shift $s \in S$, the corresponding shift capacity constraint (3), along with the flow variables $\mathbf{x^s} = \{x^s_a : a \in A\}$ of the compact formulation (1)–(11), defines the 0/1-knapsack structure

$$X_s = \{\mathbf{x^s} \in \mathbb{B}^{|A|} : \sum_{a \in A} d_a x^s_a \leq L_s\} \tag{19}$$

in which the items are the arcs in A, the weight d_a of each arc $a = (i,j) \in A$ is the demand d_i of its start node i, and the knapsack capacity is equal to the shift capacity L_s. Therefore, valid inequalities for the convex hall of X_s defined on the compact variables $\mathbf{x^s}$ can be used to strengthen the LP-relaxation of the master problem. A subset $C \subseteq A$ is called a cover if $\sum_{a \in C} d_a > L_s$. Moreover, C is a minimal cover if no proper subset of C is also a cover, that is, for every $a' \in C$, it holds that $\sum_{a \in C \setminus \{a'\}} d_a \leq L_s$. For any minimal cover C, the inequality

$$\sum_{a \in C} x^s_a \leq |C| - 1 \tag{20}$$

is a compact cover inequality and is valid for the convex hall of X_s. It simply says that a subset of customers with a total demand that is larger than the shift loading capacity cannot all be planned on vehicles loaded in the same shift. A compact cover inequality can be extended by the arcs in the set $\overline{C} = \{a \in A \setminus C : d_a \geq \tau\}$, where $\tau = \max\{d_a : a \in C\}$ is called the inequality threshold. Hence, the inequality

$$\sum_{a \in C \cup \overline{C}} x_a^s \leq |C| - 1 \tag{21}$$

is also a valid compact cover inequality for the convex hall of X_s.

6.1.1. Separation of Compact Cover Inequalities For a given shift $s \in S$ and its corresponding fractional solution $\mathbf{x^{*s}}$, the separation of the compact cover inequalities (20) implies finding a subset of arcs C (i.e., a cover) such that the total quantity delivered on these arcs exceeds the shift capacity L_s, and $\sum_{a \in C} x_a^{*s} > |C| - 1$. Introducing the binary variable z_a^s that takes the value 1 if and only if $a \in C$, the separation problem for the compact cover inequalities is equivalent to:

$$\xi = \min \left\{ \sum_{a \in A} (1 - x_a^{*s}) z_a^s : \sum_{a \in A} d_a z_a^s > L_s \right\} \tag{22}$$

A violated compact cover inequality is found if and only if $\xi < 1$. The separation problem (22) is equivalent to a knapsack problem, and can be solved by dynamic programming.

6.2. Lifted Compact Cover Inequalities

Compact cover inequalities (20) can also be strengthened by lifting up the variables corresponding to the arcs in $A \setminus C$ and adding them to the left hand side of the inequalities. The resulting lifted compact cover inequalities are of the form:

$$\sum_{a \in C} x_a^s + \sum_{a \in A \setminus C} \alpha_a x_a^s \leq |C| - 1 \tag{23}$$

where the non-negative integers α_a are as large as possible. In this paper, we use the procedures as described in Zonghao et al. (1998) and Kaparis and Letchford (2008) to generate violated lifted compact cover inequalities. We denote $\mathcal{CC}$ the set of (lifted) compact cover inequalities (21) and (23) dded to the LP-relaxation of the master problem.

6.3. Master Cover Inequalities

In this section, we introduce a family of valid inequalities for the VRPTWS defined directly on the path variables. For every shift $s \in S$, the corresponding shift capacity constraint (14), along with the path variables $\mathbf{y^s} = \{y_p : p \in \Omega^s\}$ of the master problem, defines the 0/1-knapsack structure

$$Y_s = \{\mathbf{y^s} \in \mathbb{B}^{|\Omega^s|} : \sum_{p \in \Omega^s} m_p y_p \leq L_s\} \tag{24}$$

in which the items are the paths in Ω^s, the weight of each path is its load m_p, and the knapsack capacity is equal to the shift capacity L_s. In the sequel, we introduce several valid inequalities for the convex hall of Y_s.

6.3.1. Master k-Cover Inequalities: In this section, we introduce a family of master cover inequalities, we call the *master k-cover inequality*. For shift $s \in S$ and integer $k \geq 1$, we define a k-cover

$$C = \left\{ p \in \Omega^s : m_p > \frac{L_s}{k} \right\} \tag{25}$$

as the subset of paths with a load larger than the threshold $\tau = \frac{L_s}{k}$. Now, we can define the master k-cover inequalities as follows

DEFINITION 1. For shift $s \in S$, consider the knapsack structure Y_s and the k-cover C for some $k \geq 1$. The master k-cover inequality is defined as:

$$\sum_{p \in C} y_p \leq k - 1 \tag{26}$$

Obviously, the master k-cover inequality is valid for the convex hall of Y_s. The inequality cuts off fractional solutions that plan more than k paths each with a load larger than the threshold τ in the same shift. These cuts are easy to separate with a simple and fast enumeration.

EXAMPLE 1. Consider the fractional solution in Tabel 1 obtained after solving the master problem for an instance of 25 customers and 3 shifts each with loading capacity 200, and after adding all the (lifted) compact cover inequalities. The first column shows the paths indices, the second column corresponds to a path's weight in the LP solution, the third column shows the shifts in which a path is planned, the fourth column represents a path's load and the fifth column shows the sequence of a path. For shift $s = 0$ and $k = 2$, the 2-cover $C = \{2, 3, 4\}$ defines the inequality

Table 1

p	y_p	s	m_p	Route
1	0.67	0	70	5, 3, 7, 8, 10
2	0.01	0	190	13, 17, 18, 19, 15, 16, 14, 12
3	1.00	0	100	20, 24, 25, 23, 22, 21
4	0.26	0	160	5, 3, 7, 8, 10, 11, 9, 6, 4, 2, 1
5	0.99	1	190	13, 17, 18, 19, 15, 16, 14, 12
6	0.07	1	160	5, 3, 7, 8, 10, 11, 9, 6, 4, 2, 1
7	0.67	2	90	11, 9, 6, 4, 2, 1

$$y_2 + y_3 + y_4 \leq 1 \tag{27}$$

which is a violated master 2-cover inequality with a threshold $\tau = \frac{200}{2} = 100$. □

We denote $\mathcal{MC}_1$, the set of master k-cover inequalities (26) added to the LP-relaxation of the master problem (12)-(15).

6.3.2. Master p-Cover Inequalities: In this section, we introduce another family of valid inequalities, we call the *master p-cover inequality.* For shift $s \in S$, a subset $C \subseteq \Omega^s$ is a p-cover if $\sum_{p\in C} m_p > L_s$. C is minimal if any proper subset of it is not a p-cover. For any minimal p-cover C, the inequality

$$\sum_{p\in C} y_p \leq |C| - 1 \tag{28}$$

is valid for the convex hall of Y_s. The separation of C is done by solving a knapsack problem using dynamic programming.

Example 2. Consider the fractional solution in Tabel 1. It is easy to see that $y_3 + y_4 \leq 1$ is a violated valid inequality for shift 0, and $y_5 + y_6 \leq 1$ is a violated valid inequality for shift 1. □

Let's call $\tau = \max\{m_p : p \in C\}$ the inequality threshold, and let $V(p)$ be the set of nodes visited along the path $p \in \Omega^s$. Furthermore, let's call path p a super path of path p' if $V(p') \subseteq V(p)$. The inequality (28) can be strengthened by adding all the variables corresponding to super paths of the paths in C to its left hand side. Moreover, all paths with a load at least equal to τ are added to the inequality left hand side. The strengthened inequality has the form:

$$\sum_{p'\in C} \sum_{\substack{p\in\Omega^s \\ V(p')\subseteq V(p)}} y_p + \sum_{p\in\overline{C}} y_p \leq |C| - 1 \tag{29}$$

where $\overline{C} = \{p \in \Omega^s \setminus C : m_p \geq \tau\}$. In fact, if $p \in \Omega^s$ is a super path of some path $p' \in C$, then $m_p \geq m_{p'}$. Additionally, paths p and p' cannot be both in a feasible solution as customers must be visited exactly once. Therefore, the inequality (29) is valid.

We can further strengthen the inequalities (29) by trimming the paths in the p-cover C. For all paths in C, we reduce the sets $V(p)$ by deleting the nodes with the least load. Trimming the set $V(p)$ results in the trimmed set $\widetilde{V}(p)$. Each time a node is deleted we check wether the inequality is still valid by checking wether $\sum_{p\in C} \sum_{i\in\widetilde{V}(p)} d_i > L_s$. We can now introduce the following definition:

Definition 2. For shift $s \in S$, consider the knapsack structure Y_s and let C be a p-cover. The master p-cover inequality is defined as

$$\sum_{p'\in C} \sum_{\substack{p\in\Omega^s \\ \widetilde{V}(p')\subseteq V(p)}} y_p + \sum_{p\in\overline{C}} y_p \leq |C| - 1 \tag{30}$$

The master p-cover inequality (30) is valid for the convex hall of Y_s. Moreover, it is stronger than (29) as we will add more super paths on the left hand side of the inequality. We denote $\mathcal{MC}_2$ the set of master p-cover inequalities added to the LP-relaxation of the master problem (12)-(15).

Example 3. Considering the fractional solution of Tabel 1. For shift 0, paths 3 and 4 define the p-cover $C = \{3, 4\}$ that results in the violated master p-cover inequality

$$y_3 + y_4 \leq 1 \tag{31}$$

The threshold for inequality (31) is τ =160. Moreover, the subsets of visited nodes on path 3 is $V(3) = \{20, 21, 22, 23, 24, 25\}$, and the subset of visited nodes on path 4 is $V(4) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. Every path with a load at least equal to 160, and all super paths of paths 3 and 4, must be added to the left hand side of inequality (31).

The total load of the p-cover $C = \{3, 4\}$ is 260, and shift's 0 loading capacity is 200. Therefore, there is room for trimming the subsets $V(3)$ and $V(4)$. Trimming the p-cover results in the trimmed subsets $\widetilde{V}(3) = \{21, 22, 25\}$ and $\widetilde{V}(4) = \{2, 5, 6, 7, 8, 9, 10, 11\}$. After trimming, the p-cover has a total load of 210 which is still larger than the the shift capacity 200. Now, for path p to be added to inequality (31), it suffices that its set of visited nodes $V(p)$ includes one of the trimmed sets. □

6.3.3. Master q-Cover Inequalities: In this section, we introduce a family of valid inequalities, we call the *master q-cover inequality*. For shift $s \in S$, customer $i \in V_c$ and integer $q \geq 1$ we define

$$\Omega^s(i, q) = \{p \in \Omega^s : i \in V(p) \wedge m_p \geq q\} \tag{32}$$

as the subset of paths that visit customer i and have a load larger than or equal to q. We can rewrite the master p-cover inequality (31) as:

$$\sum_{p \in \Omega^0(20,110) \cup \Omega^0(5,160)} y_p \leq 1 \tag{33}$$

Obviously both paths 3 and 4 are in the subset $\Omega^0(20, 110) \cup \Omega^0(5, 160)$, so the inequality (33) is stronger than the inequality (31). Moreover, it is easy to see that inequality (33) is valid as choosing two or more paths from $\Omega^0(20, 110) \cup \Omega^0(5, 160)$ would imply that at least 220 units of capacity is needed in shift 0, which exceeds its capacity of 200.

Let's consider another example in which the validity of the inequality is less obvious.

Table 2

p	y_p	s	m_p	Route
1	1.00	0	42	12, 18, 8, 17
2	0.72	0	69	11, 19, 7, 10, 20, 1
3	1.00	0	68	5, 14, 16, 6
4	0.27	1	69	11, 19, 7, 10, 20, 1
5	1.00	1	77	21, 23, 22, 4
6	1.00	1	38	2, 15, 13
7	1.00	2	38	9, 3, 24, 25

Example 4. Consider the fractional solution in Tabel 2 obtained after solving an instance of 25 customers and 3 shifts each with loading capacity of 160, and after adding all violated (lifted) compact cover inequalities.

We notice that the master p-cover inequality

$$y_1 + y_2 + y_3 \leq 2 \tag{34}$$

is violated for shift 0. Using the subsets $\Omega^0(i,q)$, we can define the set $C = \Omega^0(12,42) \cup \Omega^0(11,69) \cup \Omega^0(5,68)$ and rewrite inequality (34) as:

$$\sum_{p \in C} y_p \leq 2 \tag{35}$$

Obviously, it is not possible to select more than three paths from the set C in a feasible solution, because otherwise at least one node from the subset of nodes $\{5,11,12\}$ must be visited at least twice. For the same reason, selecting exactly three paths from C, implies selecting one path from $\Omega^0(12,42)$, one path from $\Omega^0(11,69)$ and one path from $\Omega^0(5,68)$. Consequently, the total load of the selected paths will be at least 179 which exceeds shift's 0 loading capacity. Hence, at most two paths can be selected from C, and the inequality (35) is valid. We potentially can include more paths on the left hand side, and hence strengthen the inequality, by reducing the q in the $\Omega^0(i,q)$ sets used to construct the set C. For example, replacing the set C in equation (35) by $\Omega^0(12,41) \cup \Omega^0(11,60) \cup \Omega^0(5,60)$ or $\Omega^0(12,24) \cup \Omega^0(11,69) \cup \Omega^0(5,68)$, leads to stronger valid inequalities. Furthermore, we can again extend, and hence strengthen, the inequality by adding paths, with a load exceeding 69, to its left hand side. If we let $\Omega^s(\tau)$ be the set of paths for shift s with a load at least equal to τ then

$$\sum_{p \in C \cup \Omega^0(69)} y_p \leq 2 \tag{36}$$

is a valid inequality. Since $\Omega^0(11,69) \subseteq \Omega(69)$, the set C can be simplified to $C = \Omega^0(12,24) \cup \Omega^0(5,68) \cup \Omega^0(69)$. $\square$

In general, we introduce the following definition:

Definition 3. For shift $s \in S$ and integer $k \geq 1$, let $\mathcal{F} = \{f_1, ..., f_k\}$ be a set of k distinct customers, and $\mathcal{Q} = \{q_1, ..., q_k\}$ a set of k integers representing minimum path loads. Let η be the maximum number of distinct items from $\mathcal{Q}$ that can be packed in a knapsack with capacity L_s, and $\tau = \max_{i=1,...,k}\{q_i\}$. A q-cover C is defined as

$$C = \left(\bigcup_{i=1}^{k} \Omega^s(f_i, q_i)\right) \cup \Omega^s(\tau) \tag{37}$$

and the master q-cover inequality is defined as

$$\sum_{p \in C} y_p \leq \eta \tag{38}$$

Furthermore, we can prove the following proposition

Proposition 1. *For shift $s \in S$, the master q-cover inequalities (38) are valid for the convex hall of Y_s.*

Proof of Proposition 1: **TO DO** □

In the sequel, we present an example where no master k-cover and p-cover inequality is violated, but a violated master q-cover inequality is found.

Example 5. Consider the fractional solution in Tabel 3 obtained after solving an instance of 25 customers and 3 shifts each with loading capacity of 190, and after adding all violated (lifted) compact cover inequalities.

Table 3

p	y_p	s	m_p	Route
1	0.33	0	50	7, 11, 19, 10
2	0.33	0	25	8, 10
3	0.33	0	38	18, 6, 13
4	0.02	0	65	14, 16, 6, 13
5	0.33	0	54	7, 11, 8, 17, 5
6	0.67	0	12	18
7	0.33	0	45	11, 19, 10
8	1.00	0	48	3, 9, 20, 1
9	1.00	0	62	21, 23, 24, 12
10	0.65	1	65	14, 16, 6, 13
11	1.00	1	58	15, 22, 4, 25
12	0.33	1	67	16, 17, 5
13	0.33	1	59	7, 19, 8, 17, 5

For this example it is not possible to find a violated master k-cover or p-cover inequality. For shift 0, consider the master q-cover inequality defined by the sets $\mathcal{F} = \{3, 6, 11, 21\}$ and

$\mathcal{Q} = \{48, 38, 45, 62\}$. Demands sum to 193, hence η (as defined in Proposition 1) is equal to 3. Paths $\{1, 3, 4, 5, 7, 8, 9\}$ contribute to the master q-cover inequality left hand side which takes the value 3.35. □

We denote $\mathcal{MC}_3$, the set of master q-cover inequalities added to the LP-relaxation of the master problem (12)-(15).

6.3.4. Separation of Master q-Cover Inequalities: For a shift $s \in S$, the separation of the master q-cover inequalities can be defined as follows: given the current LP solution $\mathbf{y^s} = \{y_p : p \in \Omega^s\}$, find the set of nodes $\mathcal{F}$ and the set of minimum loads $\mathcal{Q}$ that define the most violated master q-cover inequality.

Let Ω_i be the set of paths in shift s visiting customer i in the current LP solution, and $D_i = \left\{q_i^1, q_i^2, ..., q_i^{|D_i|}\right\}$ the set of possible loads to associate with customer i. D_i is found by taking the union of the demands of the paths in Ω_i. Furthermore, we define V_s as the of customers assigned to shift s in the current LP solution, and α_{pk} is 1 if path's $p \in \Omega_i$ load m_p is larger than q_i^k, 0 otherwise. We let z_i be a binary variable that takes value 1 if and only if i is included in the set $\mathcal{F}$, and ξ_{ik} be a binary variable that takes value 1 if and only if load $q_i^k \in D_i$ is associated with i. Finally, we let x_{ip} be a binary variable that takes value 1 if and only if path p is in the set $\Omega^s(i, q)$, and δ_p a binary variable that takes the value 1 if and only if path p is included in the q-cover we are trying to separate. The separation problem is formulated as an integer program as follows:

$$\max \sum_{i \in V_s} \sum_{p \in \Omega_i} y_p \delta_p - \sum_{i \in V_s} z_i \tag{39}$$

subject to

$$\sum_{i \in V_s} \sum_{k=1}^{|D_i|} q_i^k \xi_{ik} \geq L_s + 1 \tag{40}$$

$$\sum_{k=1}^{|D_i|} \xi_{ik} \leq z_i \qquad \forall i \in V_s \tag{41}$$

$$x_{ip} \leq \sum_{k=1}^{|D_i|} \alpha_{pk} \xi_{ik} \qquad \forall i \in V_s, \forall p \in \Omega_i \tag{42}$$

$$\delta_p \leq \sum_{i \in V_s} x_{ip} \qquad \forall p \in \Omega_i \tag{43}$$

$$z_i, \xi_{ik}, x_{ip}, \delta_p \in \{0, 1\} \qquad \forall i \in V_s, \forall p \in \Omega_i, \forall k \in \{1, 2, ..., |D_i|\} \tag{44}$$

The objective function (39) maximizes the violation of the found inequality. The terms $\sum_{i \in V_s} \sum_{p \in \Omega_i} y_p \delta_p$ and $\sum_{i \in V_s} z_i$ correspond to the right hand and the left hand side, respectively, of the inequality (38). A violated inequality is detected if the objective value is greater than -1. Constraint (40) ensures that the sum of the selected loads is larger than the shift capacity. Constraints (41) ensure that at most one load is selected per customer. Constraints (42) guarantee that a path can be included in the set $\Omega^s(i,q)$ if its load is larger than q. Furthermore, constraints (43) ensure that we can only add a path to the q-cover we try to separate if it is in at least one of the $\Omega(i,q)$ sets.

7. The Label Setting Algorithm

Each shift defines a pricing subproblem which corresponds to an ESPPRC, where the constrained resources are time and vehicle capacity. Our ESPPRC algorithm is based on standard label setting techniques presented by **(cite the relevant papers)**. Let $p(L)$ be the partial path associated with a label L. The label L is coded using the following attributes:

$v(L)$ Last node visited on the partial path $p(L)$.
$c(L)$ Reduced cost of the partial path $p(L)$.
$d(L)$ Total quantity delivered along the partial path $p(L)$.
$t(L)$ Ready time at node $v(L)$ when reached through the partial path $p(L)$.
$V(L)$ Set of nodes visited along the partial path $p(L)$.

Furthermore, we denote $\overline{V}(L)$ as the set $V(L)$ extended by the nodes that cannot be visited by label L because of time windows and vehicle capacity.

In the labeling algorithm, for every label, all possible extensions are derived and stored. It ends when all labels are processed. However, the number of labels that can be processed is typically very large. To reduce the number of labels, a dominance test is introduced. Let $E(L)$ denote the set of feasible extensions of the label L to node $n+1$. More formally, $E(L)$ is the set of all partial paths that can depart at node $v(L)$ and reach node $n+1$ without violating time windows, which has total demand less than $Q-d(L)$ and which do not use nodes from $V(L)$. If $L' \in E(L)$, we denote $L \oplus L'$ as the label resulting from extending L by L'. Dominance is defined as follows:

Definition 4. Label L_2 is dominated by label L_1 if:

1. $v(L_1) = v(L_2)$
2. $E(L_2) \subseteq E(L_1)$
3. $c(L_1 \oplus L) \leq c(L_2 \oplus L), \forall L \in E(L_2)$

Definition 4 states that any feasible extension of label L_2 is also feasible for label L_1. Furthermore, extending L_1 should always result in a better route. However, it is not straightforward to verify the conditions of Definition 4 as it requires the computation and the evaluation of all feasible extensions of both labels L_1 and L_2. Consequently, sufficient dominance criteria that are computationally less expensive are desirable. Therefore, in Proposition 2 below, the sufficient conditions 1 to 5 are introduced.

PROPOSITION 2. *(Feillet et al. (2004)) Label L_2 is dominated by label L_1 if:*

1. $v(L_1) = v(L_2)$
2. $c(L_1) \leq c(L_2)$
3. $t(L_1) \leq t(L_2)$
4. $d(L_1) \leq d(L_2)$
5. $\overline{V}(L_1) \subseteq \overline{V}(L_2)$

7.1. Solving the Modified Pricing Problem

The compact cover inequalities are so-called robust cuts. They can easily be added to the LP-relaxation of the master problem without increasing the complexity of the pricing problem. For shift $s \in S$, if we let $\lambda_s < 0$ be the dual variable corresponding to a compact cover inequality, the reduced cost of path $p \in \Omega^s$ is expressed as follows:

$$\overline{c}_p^s = \sum_{(i,j)\in A} (c_{ij} - \pi_i - d_i\mu_s - \lambda_s)\, x_{ijp} \tag{45}$$

Including master cover inequalities is not straightforward as the pricing becomes more expensive. For shift $s \in S$, consider some valid master cover inequality $C \in \mathcal{MC}_1 \cup \mathcal{MC}_2 \cup \mathcal{MC}_3$ (for convenience, we denote C the master cover inequality defined by the cover C). Let $\xi_C < 0$ be its corresponding dual variable. The dual variable ξ_C is negative and hence will be acting as a penalty when subtracted from a path's reduced cost. When generating paths for shift s, we must take ξ_C into account. If a path in C is regenerated, its reduced cost must be penalized by ξ_C. Hence,

$$\overline{c}_p^s = \sum_{(i,j)\in A} (c_{ij} - \pi_i - d_i\mu_s - \lambda_s)\, x_{ijp} - \begin{cases} \xi_C & \text{if } p \in C \\ 0 & \text{otherwise} \end{cases} \tag{46}$$

However, we only know a path in C is regenerated when the path is complete (i.e., when the path reaches the end node). Therefore, the standard dominance test of Proposition 2 cannot be directly used, because partial paths, that will be hit by ξ_C when they reach the end node, might erroneously dominate other partial paths that will not lead to a path in C. Considering the fractional solution of Tabel 1, by applying standard dominance criteria as described in

Proposition 2, partial path (0, 20, 24) might dominate partial path (0, 19, 24). However, when extended all the way to the end node, we might have that (0, 19, 24, 25, 23, 22, 21, 0) is a better path than (0, 20, 24, 25, 23, 22, 21, 0), because the later gets penalized by the master p-cover inequality (31) dual variable, while the former does not. Next, we will focus on how we handle the complications stemming from adding master cover inequalities in the pricing problem.

7.1.1. Handeling Master k-Cover Inequalities: Master k-cover inequalities $\mathcal{MC}_1$ are easily handled in the pricing subproblem. For every generated path, we just need to subtract the dual variables corresponding to the master k-cover inequalities in $\mathcal{MC}_1$ for which the inequality threshold is surpassed by the path's load when the end node is reached. Furthermore, we can use standard dominance test as described in Proposition 2. Condition 4 ensures that if any extension of label L_1 by some label L into a path that must be added to a master k-cover inequality in $\mathcal{MC}_1$, extending L_2 by L must be added to the same inequality. In fact, if the load of path $p(L_1 \oplus L)$ surpasses the inequality threshold, the load of path $p(L_2 \oplus L)$ must surpass the inequality threshold as well. So, dominance does not have to know about all the paths in the master k-cover inequalities $\mathcal{MC}_1$.

Example 6. Let's consider again the fractional solution in Tabel 1. For shift $s = 0$ and integer $k = 2$, the 2-cover $C = \{2, 3, 4\}$ defines the master 2-cover inequality with threshold $\tau = 100$, depicted by equation (27). Furthermore, consider two labels L_1 and L_2 such that $p(L_1) = (20, 21, 16)$ and $d(L_1) = m_{p(L_1)} = 70$, and $p(L_2) = (14, 15, 16)$ and $d(L_2) = m_{p(L_2)} = 90$. Moreover, we have that $\overline{V}(L_1) = V(L_1)$ and $\overline{V}(L_2) = V(L_2) \cup \{10, 20, 21\}$. Let L be an extension of label L_1 such that $d(L) = 40$. The total demand of the extended label $L_1 \oplus L$ is $d(L_1 \oplus L) = 110 > \tau$, hence path $p(L_1 \oplus L)$ must be added to the 2-cover C. Obviously, $p(L_2 \oplus L)$ must be added to C as well, since $d(L_2 \oplus L) = 130 > \tau$. In other words, for any label L, it will never happen that $p(L_1 \oplus L)$ will be penalized by 2-cover's C dual variable and $p(L_2 \oplus L)$ will not. Therefore, condition 2 of the standard dominance test of Proposition 2 is still handeling labels cost correctly. □

7.1.2. Handeling Master p-Cover and q-Cover Inequalities: Handeling master cover inequalities $\mathcal{MC}_2$ and $\mathcal{MC}_3$ in the pricing problem is more complicated. For every valid master inequality $C \in \mathcal{MC}_2 \cup \mathcal{MC}_3$, we need to ensure that its dual variable is subtracted from the reduced cost of a path p that contributes to its violation. This is easily done by checking wether the path's load m_p surpasses the inequality threshold τ. Moreover, In case C defines a master p-cover inequality, p must be added to C if $\widetilde{V}(p') \subseteq V(p)$ for some $p' \in C$. In case C defines a master q-cover inequality, p is added to C if it is in one of the subsets $\Omega^s(i,q)$ used to construct C. The complexity comes in the dominance test where we have to account for the possibility that one of the labels that needs to be compared might contribute to C and the other might not. Next, we will discuss the impact of including master cover inequalities in $\mathcal{MC}_2 \cup \mathcal{MC}_3$ on the dominance criterion.

In case, for any label L, the elementarity constraint in the pricing problem is handled through the set of visited nodes $V(L)$, the standard dominance test will require that $V(L_1) \subseteq V(L_2)$ if label L_1 should dominate label L_2. This condition, together with condition 4 of the dominance test of Proposition 2, is sufficient for handeling master cover inequalities in $\mathcal{MC}_2 \cup \mathcal{MC}_3$. In fact, if L is a feasible extension of L_1 such that $p(L_1 \oplus L)$ must be added to a master cover inequality $C \in \mathcal{MC}_2 \cup \mathcal{MC}_3$, extending L_2 by the same extension L will imply that path $p(L_2 \oplus L)$ must be added to C as well. In fact, if $p(L_1 \oplus L)$ is added to C because its load surpasses the threshold τ, condition 4 of Proposition 2 will force $p(L_2 \oplus L)$ to be added to C. If $C \in \mathcal{MC}_2$ and $p(L_1 \oplus L)$ is added to C because $\widetilde{V}(p') \subseteq V(L_1 \oplus L)$ for some $p' \in C$, then condition 5 of Proposition 2 ensures that $\widetilde{V}(p') \subseteq V(L_2 \oplus L)$, and hence $p(L_2 \oplus L)$ must be added to C. Furthermore, If $C \in \mathcal{MC}_3$ and $p(L_1 \oplus L)$ is added to C because $p(L_1 \oplus L) \in \Omega^s(i,q)$ for some $i \in V_c$ and integer q, then conditions 4 and 5 of Proposition 2 imply that $p(L_2 \oplus L) \in \Omega^s(i,q)$, and hence $p(L_2 \oplus L)$ must be added to C. Therefore, the dominance criterion will be similar to the one in Proposition 2 with the only difference that condition $\overline{V}(L_1) \subseteq \overline{V}(L_2)$ must be relaxed to $V(L_1) \subseteq V(L_2)$.

If elementarity is handled by keeping track of the nodes that cannot be visited by a label L (i.e., using the set $\overline{V}(L)$), then we need more information to do the dominance test correctly. In fact, we need to keep the set of nodes that are really visited by the partial path $p(L)$, to be able to judge whether an extension of label L might lead to a path that must be included in a master cover inequality in $\mathcal{MC}_2 \cup \mathcal{MC}_3$, and subtract the corresponding dual variable from the reduced cost of the partial path $p(L)$. If we consider label L_2 as described in Example 6, it is not possible, knowing only $\overline{V}(L_2)$, to judge wether an extension of L_2 might, in the worst case, lead to a path that contributes to some master cover inequality in $\mathcal{MC}_2 \cup \mathcal{MC}_3$. For labels L_1 and L_2 of Example 6, and the fraction solution in Tabel 1, it is clear that, in the worst case, label L_1 might be extended to a path that must be added to the p-cover $C = \{3, 4\}$. In fact, the partial path $p(L_1)$ has already visited customers 20 and 21 that are also visited by path 3.

7.2. A Modified Dominance Criterion

In general, if, for a subset of nodes $\mathcal{N} \subseteq V_c$, we let

$$\alpha(\mathcal{N}) = \left\{ C \in \mathcal{MC}_2 : \left(\bigcup_{p \in C} \widetilde{V}(p) \right) \cap \mathcal{N} \neq \emptyset \right\}$$
$$\cup \{ C \in \mathcal{MC}_3 : \mathcal{F} \cap \mathcal{N} \neq \emptyset \}$$

be the subset of master cover inequalities in $\mathcal{MC}_2 \cup \mathcal{MC}_3$ that "use a node from $\mathcal{N}$". The dominance test can now be written as:

Proposition 3. *Label L_2 is dominated by label L_1 if:*

1. $v(L_1) = v(L_2)$
2. $c(L_1) - \sum_{C \in \alpha(V(L_1) \setminus V(L_2))} \xi_C \leq c(L_2)$
3. $t(L_1) \leq t(L_2)$
4. $d(L_1) \leq d(L_2)$
5. $\overline{V}(L_1) \subseteq \overline{V}(L_2)$

Proof of Proposition 3: **TO DO** □

The idea of condition 2 in the dominance test of Proposition 3 is that we, in the worst case, need to subtract all the dual variables corresponding to the master cover inequalities in $\mathcal{MC}_2 \cup \mathcal{MC}_3$ that are active in the extension of label L_1, but not in the extension of label L_2.

The dominance test can be further improved as we can determine that some of the master cover inequalities in $\mathcal{MC}_2 \cup \mathcal{MC}_3$ will never be active for a given path. Furthermore, we can also determine that some inequalities will for sure be active for any extension of label L_2. Let

$$\beta(L_1) = \left\{ C \in \mathcal{MC}_2 : \forall p \in C, \;\; \widetilde{V}(p) \cap \left(\overline{V}(L_1) \setminus V(L_1) \right) \neq \emptyset \right\}$$
$$\cup \left\{ C \in \mathcal{MC}_3 : \mathcal{F} \subseteq \overline{V}(L_1) \setminus V(L_1) \right\}$$

be the subset of master cover inequalities that will never be active for a path extended from label L_1. $\overline{V}(L_1) \setminus V(L_1)$ is the set of nodes that have not been visited in path $p(L_1)$ and cannot be visited in any extension of L_1. If this set intersects with all the paths defining a master p-cover inequality in $\mathcal{MC}_2$, or includes the set of nodes $\mathcal{F}$ in case of a master q-cover inequality in $\mathcal{MC}_3$, then any extension of L_1 will never contribute to the inequality. Considering Example 2, the master p-cover inequality defined by the p-cover $C = \{3, 4\}$ will never be active in a path that is extended from label L_2.

Furthermore, let

$$\begin{aligned}\varphi(L_2) = &\{C \in \mathcal{MC}_2 \cup \mathcal{MC}_3 : d(L_2) \geq \tau\} \\ &\cup \left\{C \in \mathcal{MC}_2 : \exists p \in C, \widetilde{V}(p) \subseteq V(L_2)\right\} \\ &\cup \{C \in \mathcal{MC}_3 : \exists (f_i, q_i) \in \mathcal{F} \times \mathcal{Q}, p(L_2) \in \Omega^s(f_i, q_i)\}\end{aligned}$$

be the subset of master cover inequalities in $\mathcal{MC}_2 \cup \mathcal{MC}_3$ for which we know for sure that label L_2 will be extended into a path that will contribute to one of its master cover inequalities.

If we now define $\theta(L_1, L_2) = \alpha(V(L_1) \setminus V(L_2)) \setminus (\beta(L_1) \cup \varphi(L_2))$, we get the improved dominance criterion:

Proposition 4. *Label L_2 is dominated by label L_1 if:*

1. $v(L_1) = v(L_2)$
2. $c(L_1) - \sum_{C \in \theta(L_1, L_2)} \xi_C \leq c(L_2)$
3. $t(L_1) \leq t(L_2)$
4. $d(L_1) \leq d(L_2)$
5. $\overline{V}(L_1) \subseteq \overline{V}(L_2)$

Proof of Proposition 4: **TO DO** □

8. Computational Results

The branch-and-cut-and-price algorithm is implemented in C++ on a Intel Core i5 CPU, 2.6 GHz with 4 GB of memory. For all experiments, we use a time limit of 2 hours. The LP solver CLP from the open source framework COIN (COIN CLP (2011)) is used to solve the LP relaxation of the master problem. Furthermore, Cplex is used to solve the master q-cover inequalities separation problem (39)-(44). For our numerical study, we use the well known Solomon's data sets (Solomon 1987) that follow a naming convention of $DTm.n$. D is the geographic distribution of the customers which can be R (Random), C (Clustered) or RC (Randomly Clustered). T is the instance type which can be either 1 (instances with tight time windows) or 2 (instances with wide time windows). m denotes the number of the instance, and n the number of customers that need to be served. For all instances, we consider three shifts with equal loading capacity, which is calculated as $\rho \frac{\sum_{i \in V_c} d_i}{3}$, where $\rho \in \{1.05, 1.2, 1.5\}$. Furthermore, the depot's operating period is divided into three equally long periods with length $\frac{b_{n+1}}{3}$ such that each period is assigned to a different shift. We consider the situation where shifts 2 and 3 of day $X-1$ and shift 1 of day X are used to load vehicles delivering demand of day X.

Table 4 Algorithms Overview

	$\mathcal{CC}$	$\mathcal{MC}_1$	$\mathcal{MC}_2$	$\mathcal{MC}_3$
$\mathcal{A}_1$				
$\mathcal{A}_2$	X			
$\mathcal{A}_3$	X	X		
$\mathcal{A}_4$	X	X	X	
$\mathcal{A}_5$	X	X		X

8.1. General Findings

As expected adding shifts loading capacities to the vehicle routing problem with time windows adds to its complexity. However, it is remarkable how complicated the resulting problem (i.e., the VRPTWS) becomes. This complexity is reflected by the solution running times and the large size of the branching trees, especially when shifts loading capacities are binding (e.g., instances rc101.25, rc105.25 and rc108.25 for $\rho = 1.05$). Furthermore, the shift loading capacities have a significant impact on the costs. If we call $1-\rho$, the excess of the total shifts loading capacity, decreasing the loading capacity excess from 0.5 to 0.2 results in an increase of 7.63% in cost in average, with a maximum increase of 24.27% and a minimum increase of 0%. Moreover, if we further decrease the loading capacity excess to 0.05, the costs increase by 14.37% on average, with a maximum increase of 34.58% and a minimum increase of 1.47%.

8.2. Impact of the Valid Inequalities

We run all the instances using 5 different algorithms (see Table 4). $\mathcal{A}_1$ is the basic algorithm where we don't include any of the valid inequalities. Algorithm $\mathcal{A}_2$ implements (lifted) compact cover inequalities ($\mathcal{CC}$) but none of the master cover inequalities. Algorithm $\mathcal{A}_3$ implements, in addition to $\mathcal{CC}$, master k-cover inequalities ($\mathcal{MC}_1$). Furthermore, algorithm $\mathcal{A}_4$ supports master p-cover inequalities ($\mathcal{MC}_2$), and algorithm $\mathcal{A}_5$ supports master q-cover inequalities ($\mathcal{MC}_3$).

Tabels 5-7, we report the instances for which we could at least solve the root node of the branch-and-bound tree using algorithm $\mathcal{A}_1$. Each table reports the results for different values of ρ. The first column indicates the name of the instance. The columns denoted as "Time" shows the time (in seconds) spent to solve an instance. The columns denoted as "Root LB" show the the lower bounds in the root node. The columns "Best LB" and "UB" show, respectively, the lower and upper bound

Table 5 Results for Algorithm $\mathcal{A}_1$ and Instances with 25 Customers

	ρ=1.05					ρ=1.2					ρ=1.5				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	19.2	6,978.5	7,109.0	7,109	320	12.8	6,560.6	6,719.0	6,719	178	0.2	6,171.0	6,171.0	6,171	2
r102	42.2	5,726.2	5,807.0	5,807	290	13.7	5,523.0	5,574.0	5,574	82	2.4	5,463.3	5,471.0	5,471	10
r103	96.8	4,850.2	5,047.0	5,047	446	32.0	4,688.7	4,782.0	4,782	124	8.0	4,546.0	4,546.0	4,546	12
r104	454.4	4,657.7	4,836.0	4,836	1,384	135.3	4,456.4	4,518.0	4,518	314	389.9	4,142.0	4,208.0	4,208	732
r105	30.4	5,805.1	5,968.0	5,968	320	15.4	5,561.3	5,664.0	5,664	140	0.3	5,519.0	5,519.0	5,519	2
r106	41.8	4,680.0	4,904.0	4,904	132	41.9	4,634.7	4,832.0	4,832	92	7.1	4,573.0	4,654.0	4,654	16
r107	168.0	4,350.5	4,572.0	4,572	252	39.7	4,284.2	4,433.0	4,433	108	23.0	4,221.0	4,258.0	4,258	36
r108	229.1	4,170.1	4,377.0	4,377	60	167.1	4,064.6	4,269.0	4,269	52	162.6	3,930.7	4,043.0	4,043	168
r109	88.9	4,676.3	4,973.0	4,973	332	111.5	4,585.4	4,817.0	4,817	510	49.9	4,413.0	4,478.0	4,478	144
r110	438.0	4,417.8	4,671.0	4,671	690	24.7	4,398.2	4,519.0	4,519	52	15.7	4,383.5	4,441.0	4,441	26
r111	143.4	4,473.1	4,717.0	4,717	176	141.7	4,372.8	4,613.0	4,613	310	7.0	4,272.8	4,288.0	4,288	8
r112	620.4	3,962.6	4,279.0	4,279	868	152.0	3,912.2	4,059.0	4,059	44	16.8	3,870.5	3,930.0	3,930	14
c101	21.5	2,632.9	2,872.0	2,872	70	39.4	2,372.9	2,652.0	2,652	86	10.8	1,913.0	2,134.0	2,134	48
c102	70.5	2,446.3	2,638.0	2,638	34	143.7	2,252.8	2,516.0	2,516	74	147.8	1,903.0	2,124.0	2,124	82
c103	880.7	2,368.8	2,518.0	2,518	92	–	2,208.0	2,467.8	2,474	962	987.7	1,903.0	2,075.0	2,075	6
c104	–	2,311.9	2,463.0	2,598	336	–	2,161.1	2,353.8	2,460	612	–	1,869.0	1,907.6	–	4
c105	26.7	2,453.0	2,650.0	2,650	50	74.6	2,274.9	2,588.0	2,588	148	13.6	1,913.0	2,134.0	2,134	58
c106	28.4	2,632.9	2,872.0	2,872	66	47.4	2,372.9	2,652.0	2,652	116	11.9	1,913.0	2,134.0	2,134	52
c107	66.9	2,441.7	2,650.0	2,650	74	95.2	2,270.0	2,566.0	2,566	152	14.9	1,913.0	2,134.0	2,134	48
c108	406.1	2,439.6	2,632.0	2,632	284	2,243.3	2,256.1	2,566.0	2,566	2,316	38.1	1,913.0	2,134.0	2,134	64
c109	1,341.2	2,345.3	2,565.0	2,565	616	–	2,186.9	2,480.7	2,610	4,230	103.9	1,913.0	2,134.0	2,134	68
rc101	–	5,116.0	5,644.5	–	72,164	3.9	4,717.5	5,349.0	5,349	22	8.7	4,066.3	4,627.0	4,627	30
rc102	274.1	4,026.6	4,704.0	4,704	758	21.3	3,803.1	4,177.0	4,177	34	3.9	3,518.0	4,008.0	4,008	8
rc103	184.8	3,812.6	4,278.0	4,278	50	94.8	3,613.1	3,987.0	3,987	64	77.7	3,328.0	3,886.0	3,886	80
rc104	–	3,398.4	3,944.9	3,977	1,260	26.0	3,201.4	3,683.0	3,683	10	109.0	2,997.0	3,610.0	3,610	24
rc105	–	4,955.6	5,346.1	5,474	42,818	84.7	4,623.3	4,837.0	4,837	336	0.8	4,113.0	4,113.0	4,113	2
rc106	301.0	4,499.9	4,729.0	4,729	1,604	24.6	4,118.3	4,607.0	4,607	48	20.9	3,455.0	3,969.0	3,969	28
rc107	75.3	4,011.7	4,348.0	4,348	94	36.6	3,652.8	4,300.0	4,300	34	71.6	2,983.0	3,638.0	3,638	140
rc108	–	3,404.2	3,676.1	–	14,896	30.7	3,194.3	3,634.0	3,634	12	455.2	2,945.0	3,600.0	3,600	488
r201	18.0	4,902.1	5,021.0	5,021	22	238.2	4,701.0	4,964.0	4,964	564	102.8	4,601.0	4,677.0	4,677	446
r202	–	4,195.2	4,375.9	4,378	92	6,391.1	4,110.2	4,294.0	4,294	268	5,668.1	4,105.0	4,105.0	4,105	4
r203	268.2	3,972.8	4,040.0	4,040	2	–	3,929.3	3,952.0	–	4	1.4	3,914.0	3,914.0	3,914	0
r204	–	3,703.4	3,708.7	–	4	–	3,628.0	3,628.0	–	2	–	3,559.4	3,559.4	–	2
r205	1,721.8	4,049.0	4,212.0	4,212	72	19.3	4,002.1	4,026.0	4,026	4	2,098.9	3,948.0	4,026.0	4,026	46
r206	34.9	3,802.2	3,842.0	3,842	6	–	3,765.4	3,769.1	–	4	–	3,736.0	3,742.7	–	6
r207	–	3,689.2	3,719.0	–	4	–	3,623.5	3,623.5	–	2	–	3,600.5	3,600.5	–	4
r208	–	3,612.4	3,644.6	–	4	–	3,533.9	3,533.9	–	2	16.2	3,404.0	3,404.0	3,404	0
r209	–	3,810.6	3,810.6	–	2	–	3,745.8	3,745.8	–	2	–	3,666.0	3,666.0	–	2
r210	–	4,115.7	4,187.0	–	8	–	4,090.8	4,104.3	–	4	–	4,042.6	4,042.6	–	2
r211	–	3,646.0	3,649.0	–	4	–	3,552.7	3,552.7	–	2	–	3,470.9	3,470.9	–	2
c201	9.9	2,865.1	2,889.0	2,889	12	58.5	2,725.3	2,796.0	2,796	64	1.1	2,488.8	2,521.0	2,521	2
c202	634.0	2,802.4	2,817.0	2,817	20	1,930.3	2,666.9	2,729.0	2,729	52	2,807.4	2,428.5	2,471.0	2,471	18
c203	–	2,774.3	2,774.3	–	2	–	2,641.5	2,641.5	–	2	–	2,403.2	2,433.7	–	4
c204	–	2,741.4	2,741.4	–	2	–	2,600.8	2,600.8	–	2	–	2,383.2	2,383.2	–	2
c205	87.4	2,859.6	2,889.0	2,889	20	226.9	2,720.7	2,796.0	2,796	74	58.1	2,484.6	2,513.0	2,513	6
c206	246.9	2,845.7	2,867.0	2,867	36	424.9	2,701.6	2,774.0	2,774	70	5.5	2,476.1	2,487.0	2,487	2
c207	926.3	2,822.9	2,840.0	2,840	20	–	2,671.1	2,745.0	2,745	134	518.2	2,426.3	2,455.0	2,455	4
c208	495.6	2,831.5	2,859.0	2,859	24	1,529.5	2,697.3	2,766.0	2,766	162	273.8	2,458.2	2,472.0	2,472	6
rc201	954.8	4,542.9	4,964.0	4,964	456	43.6	4,306.6	4,466.0	4,466	62	2.8	4,259.0	4,260.0	4,260	4
rc202	397.1	3,474.0	3,810.0	3,810	4	2,392.4	3,380.0	3,380.0	3,380	2	2.3	3,380.0	3,380.0	3,380	0
rc203	–	3,364.5	3,364.5	–	2	8.6	3,269.0	3,269.0	3,269	0	6.0	3,269.0	3,269.0	3,269	0
rc204	–	3,091.8	3,091.8	–	2	–	2,997.0	2,997.0	–	2	8.7	2,997.0	2,997.0	2,997	0
rc205	46.8	3,477.0	3,763.0	3,763	2	4.2	3,380.0	3,380.0	3,380	0	5.4	3,380.0	3,380.0	3,380	0
rc206	55.8	3,429.4	3,695.0	3,695	4	47.6	3,302.4	3,344.0	3,344	4	4.0	3,240.0	3,344.0	3,344	4
rc207	–	3,083.1	3,083.1	–	2	–	2,983.0	2,983.0	–	2	10.0	2,983.0	2,983.0	2,983	0
rc208	–	3,045.1	3,045.1	–	2	–	2,945.0	2,945.0	–	2	–	2,929.8	2,929.8	–	2

found all over a branching tree. In the column "Tree", we report the size of the branching trees. In Tables 8-16, we report all the instances for which the root node is solved by at least one of the algorithms $\mathcal{A}_2$-$\mathcal{A}_5$.

Table 17 provides a comparison of all the implemented algorithms. In general, algorithm $\mathcal{A}_5$ is able to solve more instances to optimality than the other algorithms. The columns "Avg. root LB"" and "Avg. best LB" indicate, respectively, the average of the root lower bound and the average of the best lower bound of the instances for wich all algorithms are able to produce a lower bound. Moreover, the average computation time (in seconds) and the average trees over all the instances for which all algorithms are able to find an upper bound, are reported in the columns "Avg. time" and "Avg. tree", respectively. Clearly, algorithm $\mathcal{A}_5$ supported by the master q-Cover

Table 6 Results for Algorithm $\mathcal{A}_1$ and Instances with 50 Customers

	ρ=1.05					ρ=1.2					ρ=1.5				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	166.9	12,031.6	12,173.0	12,173	658	72.2	11,277.3	11,364.0	11,364	352	17.8	10,435.0	10,490.0	10,490	24
r102	186.0	9,746.8	9,806.0	9,806	280	160.2	9,286.6	9,337.0	9,337	216	15.8	9,028.0	9,028.0	9,028	8
r103	2,987.9	8,057.2	8,203.0	8,203	1,694	2,689.7	7,814.3	7,929.0	7,929	1,550	1,681.4	7,557.7	7,606.0	7,606	614
r104	6,570.4	6,547.5	6,646.0	6,646	214	4,703.0	6,305.4	6,500.0	6,500	738	–	6,115.4	6,229.0	–	892
r105	165.8	10,290.4	10,379.0	10,379	282	157.1	9,668.4	9,820.0	9,820	256	77.3	9,161.7	9,261.0	9,261	70
r106	631.9	8,606.4	8,756.0	8,756	530	793.1	8,264.7	8,410.0	8,410	590	183.4	7,835.6	7,916.0	7,916	88
r107	4,425.9	7,571.2	7,738.0	7,738	1,432	934.2	7,270.1	7,417.0	7,417	260	562.5	6,934.5	7,083.0	7,083	80
r108	–	6,217.9	6,217.9	–	4	7,055.1	6,028.7	6,282.0	6,282	418	–	5,868.2	5,979.1	–	36
r109	3,385.7	8,254.8	8,523.0	8,523	2,740	534.8	7,982.1	8,179.0	8,179	348	315.8	7,753.4	7,919.0	7,919	276
r110	2,609.1	7,269.5	7,490.0	7,490	1,374	1,874.5	7,125.1	7,339.0	7,339	792	246.9	6,951.0	7,052.0	7,052	58
r111	542.2	7,389.7	7,524.0	7,524	198	478.5	7,181.7	7,331.0	7,331	106	–	6,962.9	7,128.3	7,192	3,212
r112	–	6,416.8	6,648.3	6,651	1,890	2,390.1	6,310.5	6,494.0	6,494	446	4,837.6	6,149.3	6,371.0	6,371	916
c101	414.2	4,696.7	4,995.0	4,995	730	348.3	4,356.5	4,551.0	4,551	430	10.4	3,976.0	4,003.0	4,003	6
c102	260.8	4,590.0	4,675.0	4,675	78	5,736.4	4,281.9	4,421.0	4,421	3,016	71.7	3,966.0	3,993.0	3,993	6
c103	–	4,465.7	4,500.1	–	8	–	4,147.7	4,230.3	–	1,626	–	3,614.0	3,661.1	–	4
c104	–	3,962.9	3,962.9	–	2	–	3,794.2	3,794.2	–	2	–	–	–	–	–
c105	46.1	4,587.4	4,666.0	4,666	64	126.8	4,217.2	4,289.0	4,289	106	98.4	3,624.0	3,845.0	3,845	40
c106	321.5	4,694.9	4,995.0	4,995	562	32.7	4,279.8	4,313.0	4,313	22	90.4	3,624.0	3,845.0	3,845	46
c107	79.0	4,523.0	4,648.0	4,648	78	62.1	4,171.3	4,257.0	4,257	46	145.3	3,624.0	3,845.0	3,845	46
c108	360.8	4,503.2	4,624.0	4,624	174	149.1	4,162.7	4,245.0	4,245	64	204.7	3,624.0	3,845.0	3,845	40
c109	6,846.0	4,300.9	4,547.0	4,547	2,242	–	4,043.6	4,216.5	–	2,406	551.9	3,624.0	3,845.0	3,845	60
rc101	–	10,409.2	11,718.0	11,741	19,194	1,218.9	9,722.9	10,986.0	10,986	2,536	–	9,341.8	10,462.2	10,670	8,806
rc102	–	8,604.9	9,720.4	9,796	9,508	–	7,999.1	9,086.0	9,148	7,436	–	7,099.0	8,176.1	8,418	4,092
rc103	–	7,676.3	8,316.3	–	2,636	–	7,172.1	7,830.1	–	2,000	–	6,298.2	6,668.1	7,152	1,320
rc104	–	6,306.3	6,711.5	–	92	–	5,794.2	6,049.9	–	208	–	5,295.0	5,545.8	–	4
rc105	–	9,308.8	10,051.4	–	7,696	–	8,567.6	9,555.2	–	6,640	–	7,624.4	8,560.0	–	5,266
rc106	–	8,516.8	9,517.4	–	7,126	–	7,776.8	8,439.1	–	4,218	–	6,644.3	7,312.0	–	2,964
rc107	–	6,937.6	7,254.8	–	2,014	–	6,542.7	7,078.9	–	2,158	–	6,011.8	6,476.4	–	1,840
rc108	–	6,262.9	6,807.7	–	1,700	–	5,969.0	6,391.6	–	350	–	5,411.7	6,052.9	–	648
r201	1,833.2	8,292.5	8,441.0	8,441	920	202.1	8,108.3	8,197.0	8,197	62	–	7,919.9	8,006.0	8,006	1,608
r202	2,623.4	7,203.0	7,327.0	7,327	192	–	7,079.7	7,081.3	–	6	–	6,985.6	7,002.7	–	346
r203	–	6,156.2	6,170.0	–	4	–	–	–	–	–	–	–	–	–	–
r205	–	6,989.2	7,144.0	–	138	–	6,879.4	6,919.6	–	14	–	–	–	–	–
r206	–	6,356.3	6,408.3	–	10	–	6,306.6	6,325.2	–	6	–	–	–	–	–
r209	–	6,032.6	6,081.6	–	10	–	6,004.7	6,004.7	–	2	–	5,998.3	5,998.3	–	4
c201	2,252.5	4,190.8	4,298.0	4,298	104	271.4	4,068.3	4,123.0	4,123	8	2,895.8	3,887.7	3,959.0	3,959	32
c202	–	4,091.1	4,122.4	–	8	–	3,929.5	3,929.5	–	2	159.5	3,832.0	3,832.0	3,832	0
c203	–	4,014.8	4,014.8	–	2	–	–	–	–	–	393.3	3,769.0	3,769.0	3,769	0
c205	–	4,135.4	4,255.2	–	394	5,830.3	3,999.1	4,119.0	4,119	146	–	3,869.4	3,889.6	3,894	40
c206	–	4,134.7	4,211.6	–	108	–	3,997.9	4,098.4	–	70	–	3,848.0	3,868.8	–	20
c207	–	4,082.3	4,138.7	–	12	–	3,900.2	3,914.6	–	4	1,584.6	3,771.0	3,771.0	3,771	0
c208	–	4,087.7	4,148.1	–	18	–	3,963.4	3,963.4	–	2	–	3,777.5	3,777.5	–	2
rc201	–	7,874.8	8,161.9	–	1,934	–	7,509.1	7,575.8	8,220	680	65.2	6,848.0	7,166.0	7,166	16
rc202	–	6,864.8	7,058.3	7,612	666	–	6,513.2	6,649.2	–	440	531.4	6,136.0	6,363.0	6,363	42
rc203	–	6,251.8	6,284.9	–	4	–	5,926.3	5,939.1	–	6	–	5,553.0	5,553.0	–	2
rc205	–	7,022.9	7,138.6	–	168	4,573.9	6,656.4	6,803.0	6,803	310	–	6,302.0	6,527.0	6,575	10
rc206	–	6,284.0	6,596.0	–	370	–	6,100.0	6,112.6	6,680	2,030	10.6	6,100.0	6,100.0	6,100	0
rc207	–	5,611.2	5,687.7	–	6	–	5,601.5	5,601.5	–	4	1,815.3	5,586.0	5,602.0	5,602	14

inequalities again outperforms the other algorithms. Compared to algorithm $\mathcal{A}_1$, we could solve 26 more instances, reduce computation times and the size of the branching trees by about 20% and 38%, respectively. Furthermore, The root and the best lower bounds are improved.

9. Conclusions

In real life loading vehicles is constrained by the shifts loading capacities at the warehouses. In this paper, we explicitly consider shifts loading capacity, which, in our context, leads the vehicle routing problem with time windows ans shifts. Limited shifts loading capacities is modeled by knapsack inequalities, where the knapsacks are the shifts, and the items to pack in are either the customers or the paths. Inspired from valid inequalities for the knapsack problem, we developed tailored and new cover inequalities defines both on the flow variables and on the master variables. The developed valid inequalities can be applicable in to a wide class of problems where knapsack inequalities are part of the formulation. However, further research and implementations are needed to investigate their value when applied in another context that the VRPTWS. Valid inequalities defined on the master variables are clearly stronger, but significantly complicates the pricing problem. We succeed to handle the included inequalities in efficient way, and showed their value in extended computational experiments. The algorithm can handle some instances with up to 100 customers and 3 shifts.

Table 7 Results for Algorithm $\mathcal{A}_1$ and Instances with 100 Customers

	ρ=1.05					ρ=1.2					ρ=1.5				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	1,824.6	18,299.0	18,406.0	18,406	1,478	700.7	17,087.6	17,216.0	17,216	396	51.3	16,739.5	16,794.0	16,794	24
r102	1,048.3	15,322.2	15,354.0	15,354	202	1,870.2	14,984.2	14,995.0	14,995	356	121.4	14,699.5	14,700.0	14,700	6
r103	–	12,214.8	12,235.9	–	328	3,365.4	11,970.6	12,015.0	12,015	180	1,958.4	11,839.4	11,857.0	11,857	50
r104	–	9,925.8	9,987.6	–	30	–	9,676.4	9,685.3	–	6	–	9,426.3	9,459.4	–	22
r105	2,053.4	14,531.5	14,652.0	14,652	610	736.2	13,866.3	13,913.0	13,913	184	965.3	13,514.0	13,614.0	13,614	226
r106	–	12,784.0	12,908.8	–	672	–	12,508.9	12,641.5	–	652	3,601.3	12,207.0	12,280.0	12,280	172
r107	–	–	–	–	–	–	10,602.2	10,730.2	–	234	–	10,339.2	10,432.4	–	84
r108	–	–	–	–	–	–	9,227.1	9,283.4	–	22	–	8,984.5	9,033.4	–	26
r109	–	12,103.9	12,274.5	–	1,170	–	11,716.3	11,848.6	–	730	–	11,340.1	11,436.8	–	632
r110	–	11,019.1	11,150.7	–	276	–	10,740.5	10,879.3	–	354	–	10,554.9	10,625.2	–	256
r111	–	10,812.6	10,928.2	–	268	–	10,567.1	10,650.5	–	266	–	10,345.6	10,428.2	–	160
r112	–	–	–	–	–	–	9,461.6	9,511.3	–	22	–	9,264.7	9,264.7	–	2
c101	1,406.7	11,019.5	11,145.0	11,145	410	4,433.5	9,728.7	9,918.0	9,918	1,336	509.8	8,625.0	8,625.0	8,625	28
c102	4,478.7	10,190.7	10,363.0	10,363	464	–	9,447.0	9,557.1	–	546	495.9	8,625.0	8,625.0	8,625	2
c103	–	9,964.4	10,080.7	–	248	–	9,284.8	9,426.3	–	238	–	8,263.0	8,263.0	–	24
c104	–	9,153.7	9,153.7	–	2	–	8,755.5	8,755.5	–	2	–	–	–	–	–
c105	3,910.3	10,506.4	10,716.0	10,716	988	5,304.4	9,509.1	9,646.0	9,646	1,358	–	8,273.0	8,303.9	8,475	572
c106	–	10,516.0	10,768.5	–	1,448	–	9,571.3	9,700.9	–	1,504	–	8,273.0	8,288.1	8,475	608
c107	1,184.0	10,273.7	10,306.0	10,306	140	1,939.3	9,389.3	9,546.0	9,546	310	–	8,273.0	8,295.8	8,475	536
c108	4,569.0	10,180.1	10,293.0	10,293	502	–	9,377.3	9,538.0	–	892	–	8,273.0	8,276.0	–	208
c109	–	9,565.6	9,828.5	–	448	–	8,975.6	9,093.8	–	386	–	8,273.0	8,273.0	–	112
rc101	–	18,186.2	18,717.8	–	3,368	–	17,116.0	17,566.1	–	3,040	–	16,213.2	16,620.0	–	3,074
rc102	–	15,288.3	15,661.2	–	1,234	–	14,630.8	14,944.5	–	1,166	–	14,090.6	14,306.9	–	854
rc103	–	13,196.8	13,403.6	–	190	–	12,637.5	12,835.8	–	160	–	12,038.3	12,213.4	–	166
rc104	–	11,719.6	11,774.4	–	10	–	11,256.4	11,316.7	–	14	–	10,756.0	10,756.0	–	2
rc105	–	16,605.3	17,001.0	–	1,736	–	15,858.7	16,230.8	–	1,642	–	15,331.7	15,606.2	–	1,656
rc106	–	14,725.9	14,920.6	–	1,216	–	14,005.3	14,187.6	–	1,086	–	13,181.3	13,373.0	–	1,102
rc107	–	12,917.0	13,080.6	–	384	–	12,425.8	12,568.2	–	368	–	11,833.7	11,921.3	–	96
rc108	–	11,593.1	11,706.6	–	74	–	11,189.5	11,290.8	–	86	–	10,733.4	10,785.3	–	72
r201	–	11,782.2	11,881.7	11,885	352	4,252.1	11,599.5	11,677.0	11,677	206	–	11,403.0	11,432.5	–	144
r202	–	–	–	–	–	–	10,253.5	10,276.5	–	22	–	10,222.3	10,232.4	–	26
r203	–	8,754.0	8,754.0	–	2	–	8,697.4	8,697.4	–	2	–	–	–	–	–
r205	–	–	–	–	–	–	9,448.3	9,483.3	–	26	–	9,389.3	9,412.9	–	22
r206	–	–	–	–	–	–	8,694.8	8,694.8	–	2	–	–	–	–	–
r209	–	8,477.1	8,491.0	–	6	–	8,438.3	8,466.1	–	12	–	8,414.0	8,426.1	–	6
r210	–	8,952.6	8,963.9	–	4	–	–	–	–	–	–	8,893.7	8,893.7	–	2
c201	–	6,616.6	6,782.4	–	22	–	6,332.6	6,569.0	–	52	–	5,891.0	5,963.3	–	4
c202	–	6,584.1	6,584.1	–	2	–	6,322.4	6,335.2	–	4	–	5,891.0	5,963.3	–	4
c205	–	6,546.6	6,610.9	–	8	–	6,299.9	6,482.1	–	22	–	5,864.0	5,936.3	–	4
c206	–	6,475.1	6,549.5	–	8	–	6,261.3	6,390.6	–	8	–	5,860.0	5,932.3	–	4
c207	–	6,427.2	6,503.4	–	4	–	6,228.8	6,228.8	–	2	–	5,858.0	5,858.0	–	2
c208	–	6,339.9	6,421.9	–	4	–	6,147.6	6,147.6	–	2	–	5,858.0	5,858.0	–	2
rc201	–	12,830.1	12,928.6	–	84	–	12,680.8	12,793.8	12,798	350	–	12,559.4	12,618.0	12,624	178
rc202	–	11,059.6	11,076.9	–	18	–	10,956.6	10,997.4	–	34	–	10,880.8	10,892.5	–	10
rc205	–	11,582.8	11,670.9	–	62	–	11,494.4	11,537.5	–	46	–	11,476.1	11,479.2	–	14
rc206	–	10,540.7	10,569.1	–	18	–	10,445.1	10,478.5	–	24	–	10,386.0	10,402.5	–	26
rc207	–	9,484.8	9,486.6	–	4	–	9,474.6	9,474.9	–	6	–	9,473.1	9,474.6	–	14

Table 8 Instances with 25 Customers and $\rho = 1.05$

	$\mathcal{A}_2$					$\mathcal{A}_3$					$\mathcal{A}_4$					$\mathcal{A}_5$				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	28.4	6,978.5	7,109.0	7,109	306	22.6	6,978.5	7,109.0	7,109	290	24.3	6,978.5	7,109.0	7,109	332	7.1	6,989.0	7,109.0	7,109	184
r102	45.2	5,726.2	5,807.0	5,807	302	58.2	5,728.3	5,807.0	5,807	370	64.7	5,728.3	5,807.0	5,807	268	5.8	5,732.1	5,807.0	5,807	148
r103	115.6	4,850.2	5,047.0	5,047	446	116.3	4,850.2	5,047.0	5,047	410	86.8	4,850.2	5,047.0	5,047	236	86.8	4,856.1	5,047.0	5,047	276
r104	410.6	4,657.7	4,836.0	4,836	1,276	378.9	4,657.7	4,836.0	4,836	962	159.9	4,672.4	4,836.0	4,836	326	109.5	4,672.4	4,836.0	4,836	268
r105	33.3	5,805.1	5,968.0	5,968	320	48.3	5,823.7	5,968.0	5,968	370	30.8	5,823.7	5,968.0	5,968	202	29.5	5,845.3	5,968.0	5,968	132
r106	37.0	4,680.0	4,904.0	4,904	118	36.0	4,773.0	4,904.0	4,904	94	47.4	4,774.4	4,904.0	4,904	108	46.6	4,773.0	4,904.0	4,904	124
r107	95.7	4,350.6	4,572.0	4,572	114	86.5	4,389.3	4,572.0	4,572	96	79.0	4,389.3	4,572.0	4,572	80	48.2	4,405.1	4,572.0	4,572	60
r108	213.6	4,170.1	4,377.0	4,377	60	637.3	4,211.1	4,377.0	4,377	56	551.7	4,211.1	4,377.0	4,377	46	416.0	4,211.1	4,377.0	4,377	38
r109	91.3	4,676.3	4,973.0	4,973	332	118.8	4,678.1	4,973.0	4,973	364	71.2	4,699.5	4,973.0	4,973	196	48.7	4,714.6	4,973.0	4,973	138
r110	395.7	4,417.8	4,671.0	4,671	690	356.0	4,443.4	4,671.0	4,671	626	177.7	4,443.4	4,671.0	4,671	278	117.4	4,470.6	4,671.0	4,671	180
r111	130.9	4,473.1	4,717.0	4,717	176	168.9	4,487.9	4,717.0	4,717	174	131.3	4,514.1	4,717.0	4,717	126	86.2	4,546.2	4,717.0	4,717	78
r112	602.1	3,962.6	4,279.0	4,279	868	622.3	4,035.8	4,279.0	4,279	464	428.1	4,035.8	4,279.0	4,279	268	244.2	4,035.8	4,279.0	4,279	124
c101	25.9	2,632.9	2,872.0	2,872	70	21.0	2,638.6	2,872.0	2,872	62	26.9	2,639.8	2,872.0	2,872	74	19.7	2,638.6	2,872.0	2,872	48
c102	82.9	2,446.3	2,638.0	2,638	34	242.6	2,460.1	2,638.0	2,638	28	243.7	2,460.1	2,638.0	2,638	28	209.3	2,460.1	2,638.0	2,638	28
c103	–	2,368.8	2,518.0	2,518	92	729.5	2,368.8	2,518.0	2,518	82	1,075.6	2,368.8	2,518.0	2,518	98	617.4	2,368.8	2,518.0	2,518	86
c104	–	2,311.9	2,460.9	2,598	310	–	2,311.9	2,450.0	2,596	226	–	2,311.9	2,459.8	2,595	302	–	2,311.9	2,442.4	–	190
c105	30.7	2,453.0	2,650.0	2,650	50	32.2	2,465.8	2,650.0	2,650	46	37.0	2,465.8	2,650.0	2,650	46	29.2	2,472.5	2,650.0	2,650	42
c106	25.5	2,632.9	2,872.0	2,872	66	24.0	2,638.6	2,872.0	2,872	66	25.9	2,639.8	2,872.0	2,872	66	20.4	2,643.6	2,872.0	2,872	44
c107	63.2	2,441.7	2,650.0	2,650	74	88.4	2,441.7	2,650.0	2,650	76	65.5	2,441.7	2,650.0	2,650	74	66.1	2,448.9	2,650.0	2,650	70
c108	388.1	2,439.6	2,632.0	2,632	284	524.0	2,439.6	2,632.0	2,632	336	484.2	2,439.6	2,632.0	2,632	306	447.6	2,439.6	2,632.0	2,632	248
c109	1,305.7	2,345.3	2,565.0	2,565	616	2,457.4	2,352.6	2,565.0	2,565	650	2,874.1	2,352.6	2,565.0	2,565	666	1,912.7	2,352.6	2,565.0	2,565	534
rc101	–	5,116.0	5,644.5	–	68,902	–	5,121.4	5,650.5	–	58,746	5,722.1	5,121.4	6,040.0	6,040	43,010	1,900.1	5,179.5	6,040.0	6,040	11,218
rc102	251.7	4,026.6	4,704.0	4,704	758	349.9	4,059.7	4,704.0	4,704	718	161.1	4,059.7	4,704.0	4,704	176	114.0	4,059.9	4,704.0	4,704	140
rc103	184.0	3,812.6	4,278.0	4,278	50	428.0	3,841.2	4,278.0	4,278	48	445.3	3,841.2	4,278.0	4,278	48	407.0	3,841.2	4,278.0	4,278	48
rc104	–	3,399.1	3,940.8	3,977	1,280	–	3,465.5	3,859.0	4,028	338	–	3,465.5	3,858.0	4,028	338	–	3,465.5	3,887.0	4,006	254
rc105	–	4,955.6	5,345.1	5,474	41,602	6,719.7	4,955.6	5,356.0	5,356	32,544	287.0	4,969.7	5,356.0	5,356	908	126.6	5,009.7	5,356.0	5,356	494
rc106	329.2	4,499.9	4,729.0	4,729	1,604	133.9	4,509.9	4,729.0	4,729	300	51.8	4,513.6	4,729.0	4,729	46	34.6	4,509.9	4,729.0	4,729	26
rc107	72.7	4,011.7	4,348.0	4,348	94	78.4	4,037.8	4,348.0	4,348	74	88.5	4,048.1	4,348.0	4,348	36	41.4	4,095.0	4,348.0	4,348	12
rc108	–	3,404.2	3,725.7	4,033	8,668	–	3,465.5	3,702.7	4,015	1,500	–	3,465.5	3,821.6	3,976	1,524	–	3,474.8	3,838.1	3,993	1,084
r201	20.0	4,902.1	5,021.0	5,021	22	8.8	4,902.1	5,021.0	5,021	18	13.1	4,902.1	5,021.0	5,021	18	9.7	4,902.1	5,021.0	5,021	18
r202	–	4,195.2	4,375.9	4,378	92	–	4,203.0	4,375.4	4,403	82	–	4,203.0	4,375.4	4,403	62	–	4,223.5	4,377.6	4,403	48
r203	278.2	3,972.8	4,040.0	4,040	2	504.7	3,989.8	4,040.0	4,040	2	460.4	3,989.8	4,040.0	4,040	2	338.4	3,989.8	4,040.0	4,040	2
r204	–	3,703.4	3,708.7	–	4	355.5	3,714.0	3,754.0	3,754	6	349.7	3,714.0	3,754.0	3,754	6	270.2	3,714.0	3,754.0	3,754	6
r205	1,973.3	4,049.0	4,212.0	4,212	72	929.8	4,049.0	4,212.0	4,212	80	871.9	4,049.0	4,212.0	4,212	78	663.0	4,049.0	4,212.0	4,212	82
r206	32.8	3,802.2	3,842.0	3,842	6	46.2	3,808.9	3,842.0	3,842	2	41.3	3,808.9	3,842.0	3,842	2	33.4	3,808.9	3,842.0	3,842	2
r207	–	3,689.2	3,719.0	–	4	–	3,689.2	3,719.0	–	4	–	3,689.2	3,719.0	–	4	–	3,689.2	3,719.0	–	4
r208	–	3,612.4	3,644.6	–	4	–	3,612.4	3,644.6	–	4	–	3,612.4	3,644.6	–	4	–	3,612.4	3,644.6	–	4
r209	–	3,810.6	3,810.6	–	2	–	3,810.6	3,810.6	–	2	–	3,810.6	3,810.6	–	2	–	3,810.6	3,810.6	–	2
r210	–	4,115.7	4,187.0	–	8	607.6	4,156.3	4,215.0	4,215	12	639.3	4,156.3	4,215.0	4,215	12	492.3	4,165.6	4,215.0	4,215	14
r211	–	3,646.0	3,649.0	–	4	–	3,646.0	3,649.0	–	4	–	3,646.0	3,649.0	–	4	–	3,646.0	3,649.0	–	4
c201	10.4	2,865.1	2,889.0	2,889	12	17.7	2,872.4	2,889.0	2,889	8	11.7	2,872.4	2,889.0	2,889	8	9.7	2,872.4	2,889.0	2,889	8
c202	644.7	2,802.4	2,817.0	2,817	20	808.5	2,802.4	2,817.0	2,817	24	768.8	2,802.4	2,817.0	2,817	24	566.9	2,802.4	2,817.0	2,817	24
c203	–	2,774.3	2,774.3	–	2	–	2,774.3	2,774.3	–	2	–	2,774.3	2,774.3	–	2	–	2,774.3	2,774.3	–	2
c204	–	2,741.4	2,741.4	–	2	–	2,741.4	2,741.4	–	2	–	2,741.4	2,741.4	–	2	–	2,741.4	2,741.4	–	2
c205	94.9	2,859.6	2,889.0	2,889	20	102.0	2,866.0	2,889.0	2,889	24	149.0	2,866.0	2,889.0	2,889	36	90.3	2,866.0	2,889.0	2,889	24
c206	272.4	2,845.7	2,867.0	2,867	36	306.5	2,845.7	2,867.0	2,867	38	324.5	2,845.7	2,867.0	2,867	36	241.8	2,845.7	2,867.0	2,867	38
c207	996.5	2,822.9	2,840.0	2,840	20	816.4	2,829.0	2,840.0	2,840	12	824.8	2,829.0	2,840.0	2,840	12	628.7	2,829.0	2,840.0	2,840	12
c208	477.5	2,831.5	2,859.0	2,859	24	930.7	2,831.5	2,859.0	2,859	44	905.1	2,831.5	2,859.0	2,859	44	731.7	2,831.5	2,859.0	2,859	44
rc201	1,067.5	4,542.9	4,964.0	4,964	456	1,069.1	4,558.6	4,964.0	4,964	654	406.8	4,558.6	4,964.0	4,964	210	243.0	4,593.1	4,964.0	4,964	146
rc202	788.2	3,474.0	3,810.0	3,810	4	795.6	3,474.0	3,810.0	3,810	4	755.6	3,474.0	3,810.0	3,810	4	587.4	3,474.0	3,810.0	3,810	4
rc203	–	3,364.5	3,364.5	–	2	–	3,364.5	3,364.5	–	2	–	3,364.5	3,364.5	–	2	–	3,364.5	3,364.5	–	2
rc204	–	3,091.8	3,091.8	–	2	–	3,091.8	3,091.8	–	2	–	3,091.8	3,091.8	–	2	–	3,091.8	3,091.8	–	2
rc205	79.4	3,477.0	3,763.0	3,763	2	67.4	3,477.0	3,763.0	3,763	2	67.3	3,477.0	3,763.0	3,763	2	55.4	3,477.0	3,763.0	3,763	2
rc206	58.5	3,429.4	3,695.0	3,695	4	62.9	3,466.7	3,695.0	3,695	2	63.0	3,466.7	3,695.0	3,695	2	50.9	3,466.7	3,695.0	3,695	2
rc207	–	3,083.1	3,372.0	–	4	–	3,134.2	3,134.2	–	2	–	3,134.2	3,134.2	–	2	6,714.8	3,134.2	3,372.2	3,372	2
rc208	–	3,045.1	3,045.1	–	2	–	3,045.1	3,045.1	–	2	–	3,045.1	3,045.1	–	2	–	3,045.1	3,045.1	–	2

Table 9 Instances with 25 Customers and $\rho = 1.2$

	$\mathcal{A}_2$					$\mathcal{A}_3$					$\mathcal{A}_4$					$\mathcal{A}_5$				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	15.0	6,560.6	6,719.0	6,719	178	13.7	6,560.6	6,719.0	6,719	180	18.6	6,560.6	6,719.0	6,719	176	16.0	6,560.6	6,719.0	6,719	178
r102	13.6	5,523.0	5,574.0	5,574	78	15.4	5,527.7	5,574.0	5,574	84	11.0	5,527.7	5,574.0	5,574	64	6.8	5,555.6	5,574.0	5,574	22
r103	33.6	4,688.7	4,782.0	4,782	104	62.7	4,692.4	4,782.0	4,782	168	31.1	4,692.4	4,782.0	4,782	78	24.6	4,692.4	4,782.0	4,782	58
r104	228.6	4,456.4	4,518.0	4,518	390	219.5	4,456.4	4,518.0	4,518	344	64.4	4,456.4	4,518.0	4,518	124	43.3	4,456.4	4,518.0	4,518	56
r105	18.3	5,561.3	5,664.0	5,664	150	7.8	5,591.6	5,664.0	5,664	56	7.8	5,591.6	5,664.0	5,664	48	22.1	5,591.6	5,664.0	5,664	94
r106	45.4	4,634.7	4,832.0	4,832	92	47.7	4,641.0	4,832.0	4,832	82	24.8	4,641.0	4,832.0	4,832	46	21.1	4,683.1	4,832.0	4,832	42
r107	50.1	4,284.2	4,433.0	4,433	108	39.0	4,299.2	4,433.0	4,433	78	31.7	4,299.2	4,433.0	4,433	48	39.1	4,312.3	4,433.0	4,433	46
r108	191.4	4,064.6	4,269.0	4,269	52	264.1	4,113.6	4,269.0	4,269	46	246.6	4,113.6	4,269.0	4,269	46	225.4	4,120.2	4,269.0	4,269	42
r109	119.0	4,585.4	4,817.0	4,817	476	117.9	4,597.3	4,817.0	4,817	482	81.0	4,597.3	4,817.0	4,817	306	117.2	4,597.3	4,817.0	4,817	362
r110	27.3	4,398.2	4,519.0	4,519	52	26.6	4,402.7	4,519.0	4,519	54	26.9	4,402.7	4,519.0	4,519	50	32.1	4,413.5	4,519.0	4,519	50
r111	138.3	4,372.8	4,613.0	4,613	310	134.7	4,398.4	4,613.0	4,613	254	93.2	4,398.4	4,613.0	4,613	174	83.0	4,440.8	4,613.0	4,613	130
r112	162.0	3,912.2	4,059.0	4,059	44	152.0	3,970.8	4,059.0	4,059	26	136.5	3,970.8	4,059.0	4,059	26	96.3	3,970.8	4,059.0	4,059	22
c101	40.0	2,372.9	2,652.0	2,652	86	73.7	2,374.9	2,652.0	2,652	126	61.1	2,374.9	2,652.0	2,652	120	48.0	2,374.9	2,652.0	2,652	84
c102	158.9	2,252.8	2,516.0	2,516	74	214.3	2,266.8	2,516.0	2,516	76	226.1	2,266.8	2,516.0	2,516	76	209.5	2,266.8	2,516.0	2,516	76
c103	–	2,208.0	2,463.1	2,474	866	–	2,211.2	2,466.6	2,474	894	–	2,211.2	2,445.1	2,474	586	–	2,211.2	2,462.7	2,475	940
c104	–	2,161.1	2,340.4	2,460	418	–	2,165.0	2,280.3	2,419	104	–	2,165.0	2,280.3	2,419	104	–	2,165.0	2,287.3	2,419	110
c105	76.6	2,274.9	2,588.0	2,588	148	75.6	2,282.2	2,588.0	2,588	118	77.1	2,282.2	2,588.0	2,588	118	104.8	2,282.2	2,588.0	2,588	166
c106	51.7	2,372.9	2,652.0	2,652	116	77.3	2,374.9	2,652.0	2,652	118	75.3	2,374.9	2,652.0	2,652	116	81.2	2,374.9	2,652.0	2,652	114
c107	97.0	2,270.0	2,566.0	2,566	152	93.8	2,277.2	2,566.0	2,566	148	86.2	2,277.2	2,566.0	2,566	142	84.6	2,277.2	2,566.0	2,566	132
c108	2,218.7	2,256.1	2,566.0	2,566	2,316	1,854.0	2,259.5	2,566.0	2,566	1,972	2,690.0	2,259.5	2,566.0	2,566	2,422	1,839.3	2,259.5	2,566.0	2,566	1,734
c109	–	2,186.9	2,481.2	2,610	4,272	–	2,200.7	2,485.1	2,565	4,400	–	2,200.7	2,480.1	2,565	3,798	–	2,207.5	2,484.5	2,572	3,722
rc101	5.1	4,717.5	5,349.0	5,349	22	5.6	4,756.7	5,349.0	5,349	20	4.9	4,756.7	5,349.0	5,349	18	6.7	4,787.8	5,349.0	5,349	22
rc102	29.1	3,803.1	4,177.0	4,177	34	69.9	3,820.7	4,177.0	4,177	36	63.5	3,849.6	4,177.0	4,177	36	57.1	3,920.7	4,177.0	4,177	24
rc103	96.8	3,613.1	3,987.0	3,987	64	167.7	3,630.7	3,987.0	3,987	62	144.8	3,657.6	3,987.0	3,987	62	176.1	3,714.2	3,987.0	3,987	52
rc104	40.6	3,202.9	3,683.0	3,683	10	210.7	3,297.7	3,683.0	3,683	8	227.5	3,297.7	3,683.0	3,683	8	195.9	3,297.7	3,683.0	3,683	8
rc105	79.0	4,623.3	4,837.0	4,837	336	61.1	4,623.3	4,837.0	4,837	258	75.9	4,623.3	4,837.0	4,837	248	106.0	4,623.3	4,837.0	4,837	336
rc106	21.5	4,118.3	4,607.0	4,607	48	49.4	4,124.6	4,607.0	4,607	50	47.3	4,124.6	4,607.0	4,607	46	18.9	4,141.2	4,607.0	4,607	10
rc107	37.0	3,652.8	4,300.0	4,300	34	95.4	3,658.9	4,300.0	4,300	30	110.3	3,658.9	4,300.0	4,300	28	117.4	3,681.8	4,300.0	4,300	24
rc108	58.3	3,194.3	3,634.0	3,634	12	137.6	3,289.5	3,634.0	3,634	10	176.6	3,289.5	3,634.0	3,634	10	186.2	3,303.8	3,634.0	3,634	10
r201	245.5	4,701.0	4,964.0	4,964	564	271.1	4,708.8	4,964.0	4,964	470	202.7	4,734.7	4,964.0	4,964	280	146.0	4,754.6	4,964.0	4,964	222
r202	–	4,110.2	4,286.6	4,329	246	–	4,110.2	4,190.4	4,360	8	2,145.1	4,110.2	4,294.0	4,294	156	1,056.9	4,206.1	4,294.0	4,294	90
r203	–	3,929.3	3,952.0	–	4	23.7	3,933.6	3,959.0	3,959	4	23.8	3,933.6	3,959.0	3,959	4	24.4	3,933.6	3,959.0	3,959	4
r204	–	3,628.0	3,628.0	–	2	–	3,636.9	3,636.9	–	2	–	3,636.9	3,636.9	–	2	–	3,636.9	3,636.9	–	2
r205	2.5	4,002.1	4,026.0	4,026	4	1.5	4,026.0	4,026.0	4,026	0	1.4	4,026.0	4,026.0	4,026	0	2.2	4,026.0	4,026.0	4,026	0
r206	–	3,765.4	3,769.1	–	4	22.2	3,786.3	3,820.0	3,820	6	22.7	3,786.3	3,820.0	3,820	6	20.6	3,786.3	3,820.0	3,820	6
r207	–	3,623.5	3,623.5	–	2	17.9	3,623.5	3,631.0	3,631	2	21.0	3,623.5	3,631.0	3,631	2	18.5	3,623.5	3,631.0	3,631	2
r208	–	3,533.9	3,533.9	–	2	–	3,546.4	3,546.4	–	2	–	3,546.4	3,546.4	–	2	–	3,546.4	3,546.4	–	2
r209	–	3,745.8	3,745.8	–	2	–	3,745.8	3,745.8	–	2	–	3,745.8	3,745.8	–	2	–	3,745.8	3,745.8	–	2
r210	–	4,090.8	4,104.3	–	4	83.1	4,094.8	4,131.0	4,131	6	100.1	4,094.8	4,131.0	4,131	6	89.0	4,094.8	4,131.0	4,131	6
r211	–	3,552.7	3,552.7	–	2	–	3,552.7	3,552.7	–	2	–	3,552.7	3,552.7	–	2	–	3,552.7	3,552.7	–	2
c201	79.5	2,725.3	2,796.0	2,796	64	56.9	2,725.3	2,796.0	2,796	64	61.8	2,725.3	2,796.0	2,796	66	60.9	2,725.3	2,796.0	2,796	66
c202	2,765.1	2,666.9	2,729.0	2,729	52	1,336.2	2,666.9	2,729.0	2,729	52	1,429.3	2,666.9	2,729.0	2,729	52	1,082.5	2,666.9	2,729.0	2,729	52
c203	–	2,641.5	2,641.5	–	2	–	2,641.5	2,641.5	–	2	–	2,641.5	2,641.5	–	2	–	2,641.5	2,641.5	–	2
c204	–	2,600.8	2,600.8	–	2	–	2,600.8	2,600.8	–	2	–	2,600.8	2,600.8	–	2	–	2,600.8	2,600.8	–	2
c205	240.5	2,720.7	2,796.0	2,796	74	254.2	2,720.7	2,796.0	2,796	74	246.2	2,720.7	2,796.0	2,796	74	196.2	2,720.7	2,796.0	2,796	76
c206	413.5	2,701.6	2,774.0	2,774	70	415.3	2,701.6	2,774.0	2,774	68	430.5	2,701.6	2,774.0	2,774	72	355.9	2,701.6	2,774.0	2,774	70
c207	7,044.7	2,671.1	2,745.0	2,745	134	–	2,671.1	2,741.1	2,745	118	–	2,671.1	2,736.3	2,802	154	4715.7	2,671.1	2,745.0	2,745	126
c208	1,519.3	2,697.3	2,766.0	2,766	162	1,543.0	2,697.3	2,766.0	2,766	168	1,760.0	2,697.3	2,766.0	2,766	162	1,099.5	2,697.7	2,766.0	2,766	82
rc201	41.9	4,306.6	4,466.0	4,466	62	38.5	4,338.2	4,466.0	4,466	58	40.5	4,338.2	4,466.0	4,466	52	6.3	4,338.2	4,466.0	4,466	6
rc202	1.6	3,380.0	3,380.0	3,380	2	2.1	3,380.0	3,380.0	3,380	0	2.7	3,380.0	3,380.0	3,380	0	2.6	3,380.0	3,380.0	3,380	0
rc203	11.2	3,269.0	3,269.0	3,269	0	7.4	3,269.0	3,269.0	3,269	0	9.9	3,269.0	3,269.0	3,269	0	7.4	3,269.0	3,269.0	3,269	0
rc204	223.6	2,997.0	2,997.0	2,997	0	180.8	2,997.0	2,997.0	2,997	0	212.7	2,997.0	2,997.0	2,997	0	156.9	2,997.0	2,997.0	2,997	0
rc205	4.9	3,380.0	3,380.0	3,380	0	3.7	3,380.0	3,380.0	3,380	0	4.3	3,380.0	3,380.0	3,380	0	4.1	3,380.0	3,380.0	3,380	0
rc206	50.6	3,302.4	3,344.0	3,344	4	3.1	3,344.0	3,344.0	3,344	0	3.2	3,344.0	3,344.0	3,344	0	3.0	3,344.0	3,344.0	3,344	0
rc207	27.4	2,983.0	2,983.0	2,983	0	27.4	2,983.0	2,983.0	2,983	0	31.1	2,983.0	2,983.0	2,983	0	23.4	2,983.0	2,983.0	2,983	0
rc208	88.4	2,945.0	2,945.0	2,945	0	79.2	2,945.0	2,945.0	2,945	0	91.4	2,945.0	2,945.0	2,945	0	70.4	2,945.0	2,945.0	2,945	0

Table 10 Instances with 25 Customers and $\rho = 1.5$

	$\mathcal{A}_2$					$\mathcal{A}_3$					$\mathcal{A}_4$					$\mathcal{A}_5$				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	0.2	6,171.0	6,171.0	6,171	2	0.3	6,171.0	6,171.0	6,171	2	0.4	6,171.0	6,171.0	6,171	2	0.3	6,171.0	6,171.0	6,171	0
r102	2.4	5,463.3	5,471.0	5,471	10	0.6	5,463.3	5,471.0	5,471	4	0.6	5,463.3	5,471.0	5,471	4	0.7	5,463.3	5,471.0	5,471	2
r103	6.0	4,546.0	4,546.0	4,546	12	6.5	4,546.0	4,546.0	4,546	12	7.3	4,546.0	4,546.0	4,546	12	2.1	4,546.0	4,546.0	4,546	4
r104	242.1	4,142.0	4,208.0	4,208	664	–	4,142.0	4,174.5	4,406	33,288	75.8	4,142.0	4,208.0	4,208	118	48.6	4,155.5	4,208.0	4,208	46
r105	0.3	5,519.0	5,519.0	5,519	2	0.3	5,519.0	5,519.0	5,519	0	0.3	5,519.0	5,519.0	5,519	0	0.2	5,519.0	5,519.0	5,519	0
r106	6.2	4,573.0	4,654.0	4,654	16	7.2	4,573.0	4,654.0	4,654	14	12.2	4,573.0	4,654.0	4,654	20	7.5	4,573.0	4,654.0	4,654	14
r107	19.9	4,221.0	4,258.0	4,258	36	23.8	4,221.0	4,258.0	4,258	30	17.8	4,221.0	4,258.0	4,258	20	33.7	4,229.8	4,258.0	4,258	16
r108	141.8	3,930.7	4,043.0	4,043	168	235.9	3,943.2	4,043.0	4,043	104	301.5	3,943.2	4,043.0	4,043	78	166.1	3,943.2	4,043.0	4,043	96
r109	36.0	4,413.0	4,478.0	4,478	144	33.9	4,413.0	4,478.0	4,478	142	26.9	4,413.0	4,478.0	4,478	122	11.7	4,441.3	4,478.0	4,478	20
r110	18.1	4,383.5	4,441.0	4,441	26	11.7	4,383.5	4,441.0	4,441	26	11.6	4,383.5	4,441.0	4,441	18	14.2	4,383.5	4,441.0	4,441	24
r111	6.9	4,272.8	4,288.0	4,288	8	6.7	4,272.8	4,288.0	4,288	8	8.7	4,272.8	4,288.0	4,288	8	0.9	4,288.0	4,288.0	4,288	0
r112	14.3	3,870.5	3,930.0	3,930	14	18.1	3,883.6	3,930.0	3,930	10	22.4	3,883.6	3,930.0	3,930	10	39.6	3,885.9	3,930.0	3,930	12
c101	9.0	1,913.0	2,134.0	2,134	48	8.1	1,913.0	2,134.0	2,134	36	9.9	1,913.0	2,134.0	2,134	26	5.0	2,023.1	2,134.0	2,134	8
c102	151.1	1,903.0	2,124.0	2,124	82	198.9	1,903.0	2,124.0	2,124	66	139.2	1,903.0	2,124.0	2,124	32	145.9	1,998.3	2,124.0	2,124	12
c103	923.3	1,903.0	2,075.0	2,075	6	4,727.4	1,903.0	2,075.0	2,075	8	4,610.4	1,903.0	2,075.0	2,075	8	1,318.4	2,013.0	2,075.0	2,075	6
c104	–	1,869.0	1,907.6	–	4	–	1,869.0	1,924.4	–	4	–	1,919.7	1,946.3	–	6	–	1,959.9	1,960.8	2,076	4
c105	13.4	1,913.0	2,134.0	2,134	58	13.7	1,913.0	2,134.0	2,134	44	11.8	1,913.0	2,134.0	2,134	32	6.4	2,031.2	2,134.0	2,134	10
c106	9.4	1,913.0	2,134.0	2,134	52	8.5	1,913.0	2,134.0	2,134	36	8.3	1,913.0	2,134.0	2,134	28	4.4	1,970.2	2,134.0	2,134	10
c107	14.9	1,913.0	2,134.0	2,134	48	15.3	1,913.0	2,134.0	2,134	46	23.1	1,913.0	2,134.0	2,134	32	12.3	2,015.0	2,134.0	2,134	10
c108	35.8	1,913.0	2,134.0	2,134	64	39.8	1,913.0	2,134.0	2,134	60	43.4	1,913.0	2,134.0	2,134	54	51.4	2,014.6	2,134.0	2,134	12
c109	93.0	1,913.0	2,134.0	2,134	68	150.0	1,913.0	2,134.0	2,134	60	120.2	1,913.0	2,134.0	2,134	54	61.2	2,004.1	2,134.0	2,134	10
rc101	8.6	4,066.3	4,627.0	4,627	30	8.3	4,105.3	4,627.0	4,627	28	8.4	4,105.3	4,627.0	4,627	28	6.9	4,111.8	4,627.0	4,627	24
rc102	3.8	3,518.0	4,008.0	4,008	8	10.7	3,714.6	4,008.0	4,008	2	10.3	3,714.6	4,008.0	4,008	2	17.0	3,770.7	4,008.0	4,008	2
rc103	79.8	3,328.0	3,886.0	3,886	80	185.3	3,524.6	3,886.0	3,886	46	165.5	3,524.6	3,886.0	3,886	38	184.7	3,602.2	3,886.0	3,886	36
rc104	103.2	2,997.0	3,610.0	3,610	24	247.8	3,219.7	3,610.0	3,610	2	225.4	3,219.7	3,610.0	3,610	2	369.7	3,316.6	3,610.0	3,610	2
rc105	1.1	4,113.0	4,113.0	4,113	2	1.0	4,113.0	4,113.0	4,113	0	0.5	4,113.0	4,113.0	4,113	0	0.7	4,113.0	4,113.0	4,113	0
rc106	21.4	3,455.0	3,969.0	3,969	28	22.4	3,658.4	3,969.0	3,969	28	20.6	3,658.4	3,969.0	3,969	28	28.8	3,738.9	3,969.0	3,969	28
rc107	80.0	2,983.0	3,638.0	3,638	140	141.7	3,197.0	3,638.0	3,638	138	91.0	3,197.0	3,638.0	3,638	68	200.8	3,381.3	3,638.0	3,638	46
rc108	384.8	2,945.0	3,600.0	3,600	488	376.2	3,172.9	3,600.0	3,600	132	364.0	3,172.9	3,600.0	3,600	86	449.1	3,282.9	3,600.0	3,600	48
r201	76.6	4,601.0	4,677.0	4,677	278	163.6	4,601.0	4,677.0	4,677	280	27.6	4,601.0	4,677.0	4,677	46	9.3	4,601.0	4,677.0	4,677	14
r202	1.9	4,105.0	4,105.0	4,105	4	1.8	4,105.0	4,105.0	4,105	4	1.8	4,105.0	4,105.0	4,105	4	2.4	4,105.0	4,105.0	4,105	2
r203	1.3	3,914.0	3,914.0	3,914	0	1.2	3,914.0	3,914.0	3,914	0	1.4	3,914.0	3,914.0	3,914	0	1.4	3,914.0	3,914.0	3,914	0
r204	–	3,559.4	3,559.4	–	2	–	3,559.4	3,559.4	–	2	–	3,559.4	3,559.4	–	2	–	3,559.4	3,559.4	–	2
r205	2,108.0	3,948.0	4,026.0	4,026	46	–	3,966.7	4,020.0	4,173	38	–	3,966.7	4,020.0	4,054	22	5,769.54	3,972.4	4,026.0	4,026	20
r206	–	3,736.0	3,742.7	–	6	–	3,739.5	3,744.1	–	6	–	3,739.5	3,744.1	–	6	–	3,744.3	3,750.8	3,786	4
r207	–	3,600.5	3,600.5	–	4	27.3	3,600.5	3,616.0	3,616	4	20.3	3,600.5	3,616.0	3,616	4	17.7	3,600.5	3,616.0	3,616	4
r208	21.5	3,404.0	3,404.0	3,404	0	18.0	3,404.0	3,404.0	3,404	0	28.1	3,404.0	3,404.0	3,404	0	17.0	3,404.0	3,404.0	3,404	0
r209	–	3,666.0	3,666.0	–	2	–	3,666.0	3,666.0	–	2	–	3,666.0	3,666.0	–	2	–	3,666.0	3,666.0	–	2
r210	–	4,042.6	4,042.6	–	2	–	4,042.6	4,042.6	–	2	–	4,042.6	4,042.6	–	2	–	4,042.6	4,042.6	–	2
r211	–	3,470.9	3,470.9	–	2	–	3,470.9	3,470.9	–	2	–	3,470.9	3,470.9	–	2	–	3,470.9	3,470.9	–	2
c201	1.0	2,488.8	2,521.0	2,521	2	1.5	2,488.8	2,521.0	2,521	2	1.0	2,488.8	2,521.0	2,521	2	1.1	2,488.8	2,521.0	2,521	2
c202	2,822.8	2,428.5	2,471.0	2,471	18	3,319.7	2,428.5	2,471.0	2,471	18	2,837.0	2,428.5	2,471.0	2,471	18	2,096.9	2,428.5	2,471.0	2,471	18
c203	–	2,403.2	2,433.7	–	4	–	2,403.2	2,433.7	–	4	–	2,403.2	2,433.7	–	4	–	2,403.2	2,433.7	–	4
c204	–	2,383.2	2,383.2	–	2	–	2,383.2	2,383.2	–	2	–	2,383.2	2,383.2	–	2	–	2,383.2	2,383.2	–	2
c205	59.9	2,484.6	2,513.0	2,513	6	54.0	2,484.6	2,513.0	2,513	6	51.3	2,484.6	2,513.0	2,513	6	48.0	2,484.6	2,513.0	2,513	6
c206	4.8	2,476.1	2,487.0	2,487	2	4.5	2,476.1	2,487.0	2,487	2	4.3	2,476.1	2,487.0	2,487	2	4.5	2,476.1	2,487.0	2,487	2
c207	575.2	2,426.3	2,455.0	2,455	4	514.3	2,426.3	2,455.0	2,455	4	528.0	2,426.3	2,455.0	2,455	4	500.9	2,426.3	2,455.0	2,455	4
c208	278.2	2,458.2	2,472.0	2,472	6	271.2	2,458.2	2,472.0	2,472	6	266.9	2,458.2	2,472.0	2,472	6	127.2	2,458.2	2,472.0	2,472	4
rc201	2.5	4,259.0	4,260.0	4,260	4	2.0	4,259.0	4,260.0	4,260	4	2.1	4,259.0	4,260.0	4,260	4	1.3	4,260.0	4,260.0	4,260	0
rc202	2.4	3,380.0	3,380.0	3,380	0	2.7	3,380.0	3,380.0	3,380	0	2.6	3,380.0	3,380.0	3,380	0	2.8	3,380.0	3,380.0	3,380	0
rc203	6.3	3,269.0	3,269.0	3,269	0	9.3	3,269.0	3,269.0	3,269	0	6.5	3,269.0	3,269.0	3,269	0	6.3	3,269.0	3,269.0	3,269	0
rc204	8.9	2,997.0	2,997.0	2,997	0	9.3	2,997.0	2,997.0	2,997	0	10.3	2,997.0	2,997.0	2,997	0	10.1	2,997.0	2,997.0	2,997	0
rc205	5.5	3,380.0	3,380.0	3,380	0	6,069.0	3,380.0	3,380.0	3,380	0	6.4	3,380.0	3,380.0	3,380	0	8.1	3,380.0	3,380.0	3,380	0
rc206	4.0	3,240.0	3,344.0	3,344	4	2,324.0	3,344.0	3,344.0	3,344	0	2.4	3,344.0	3,344.0	3,344	0	3.1	3,344.0	3,344.0	3,344	0
rc207	10.0	2,983.0	2,983.0	2,983	0	10.3	2,983.0	2,983.0	2,983	0	11.5	2,983.0	2,983.0	2,983	0	12.3	2,983.0	2,983.0	2,983	0
rc208	–	2,929.8	2,929.8	–	2	–	2,929.8	2,929.8	–	2	–	2,929.8	2,929.8	–	2	6,266.5	2,929.8	2,931	2,931	2

Table 11 Instances with 50 Customers and $\rho = 1.05$

	$\mathcal{A}_2$					$\mathcal{A}_3$					$\mathcal{A}_4$					$\mathcal{A}_5$				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	171.4	12,031.6	12,173.0	12,173	658	224.7	12,031.6	12,173.0	12,173	780	174.8	12,031.6	12,173.0	12,173	644	188.1	12,031.6	12,173.0	12,173	606
r102	192.2	9,746.8	9,806.0	9,806	280	216.1	9,746.8	9,806.0	9,806	306	260.6	9,746.8	9,806.0	9,806	306	183.1	9,746.8	9,806.0	9,806	248
r103	3,117.2	8,057.2	8,203.0	8,203	1,694	2,390.7	8,057.2	8,203.0	8,203	1,302	2,372.1	8,057.2	8,203.0	8,203	1,016	965.2	8,067.5	8,203.0	8,203	546
r104	5,465.9	6,547.5	6,646.0	6,646	214	6,047.1	6,547.5	6,646.0	6,646	238	–	6,554.5	6,630.3	–	62	–	6,554.5	6,581.0	–	10
r105	150.6	10,290.4	10,379.0	10,379	282	183.1	10,290.4	10,379.0	10,379	286	156.2	10,290.4	10,379.0	10,379	220	100.5	10,290.4	10,379.0	10,379	146
r106	558.0	8,606.4	8,756.0	8,756	530	698.0	8,606.4	8,756.0	8,756	704	664.6	8,606.4	8,756.0	8,756	476	469.1	8,606.4	8,756.0	8,756	336
r107	3,973.9	7,571.2	7,738.0	7,738	1,432	4,435.6	7,571.2	7,738.0	7,738	1,642	4,141.8	7,571.2	7,738.0	7,738	878	2,586.92	7,571.2	7,738.0	7,738	684
r108	–	6,217.9	6,321.1	–	22	–	6,217.9	6,276.6	–	10	–	6,217.9	6,217.9	–	4	–	6,218.6	6,218.6	–	2
r109	3,442.5	8,254.8	8,523.0	8,523	2,740	3,422.9	8,254.8	8,523.0	8,523	2,882	2,936.8	8,254.8	8,523.0	8,523	1,576	1,837.4	8,254.8	8,523.0	8,523	1,432
r110	2,783.5	7,269.5	7,490.0	7,490	1,374	3,155.5	7,269.5	7,490.0	7,490	1,344	3,376.8	7,269.5	7,490.0	7,490	1,092	2,314.4	7,269.5	7,490.0	7,490	982
r111	678.9	7,389.7	7,524.0	7,524	198	749.8	7,389.7	7,524.0	7,524	212	841.3	7,389.7	7,524.0	7,524	202	832.3	7,389.7	7,524.0	7,524	212
r112	–	6,416.8	6,643.8	6,651	1,712	–	6,416.8	6,622.3	–	596	–	6,416.8	6,591.1	–	232	–	6,416.8	6,609.2	–	426
c101	421.6	4,696.7	4,995.0	4,995	628	725.5	4,696.7	4,995.0	4,995	832	920.8	4,696.7	4,995.0	4,995	728	557.0	4,696.7	4,995.0	4,995	518
c102	261.8	4,590.0	4,675.0	4,675	78	2,671.2	4,590.0	4,675.0	4,675	74	662.6	4,590.0	4,675.0	4,675	78	355.3	4,605.1	4,675.0	4,675	144
c103	–	4,465.7	4,500.1	–	8	–	4,465.7	4,500.2	–	8	–	4,465.7	4,500.2	–	8	–	4,468.6	4,505.1	–	8
c104	–	3,962.9	3,962.9	–	2	–	3,962.9	3,962.9	–	2	–	3,963.6	3,963.6	–	2	–	3,973.7	3,973.7	–	2
c105	50.3	4,587.4	4,666.0	4,666	64	137.2	4,587.4	4,666.0	4,666	136	143.2	4,587.4	4,666.0	4,666	112	500.7	4,587.4	4,666.0	4,666	354
c106	349.9	4,694.9	4,995.0	4,995	562	872.8	4,694.9	4,995.0	4,995	1,008	1,387.9	4,694.9	4,995.0	4,995	874	316.2	4,694.9	4,995.0	4,995	210
c107	103.4	4,523.0	4,648.0	4,648	78	313.3	4,523.0	4,648.0	4,648	270	445.2	4,523.0	4,648.0	4,648	200	590.0	4,525.5	4,648.0	4,648	318
c108	428.6	4,503.2	4,624.0	4,624	174	1,078.0	4,503.2	4,624.0	4,624	392	1,499.5	4,503.2	4,624.0	4,624	180	1,061.8	4,505.4	4,624.0	4,624	146
c109	7,061.9	4,300.9	4,547.0	4,547	2,242	–	4,304.5	4,542.4	–	1,562	–	4,304.5	4,530.0	–	486	5,479.6	4,311.7	4,547.0	4,547	780
rc101	–	10,409.2	11,718.8	11,741	20,360	–	10,409.2	11,719.0	11,741	20,340	–	10,409.2	11,714.0	11,741	13,804	–	10,409.2	11,729.3	11,741	12,702
rc102	–	8,604.9	9,720.0	9,796	9,112	–	8,604.9	9,716.8	9,796	8,460	–	8,604.9	9,708.2	9,794	4,690	–	8,604.9	9,726.7	9,800	5,860
rc103	–	7,676.3	8,316.2	–	2,628	–	7,681.2	8,343.1	–	1,296	–	7,681.2	8,344.7	–	770	–	7,681.2	8,352.6	–	734
rc104	–	6,306.3	6,711.5	–	92	–	6,358.3	6,688.3	–	6	–	6,358.3	6,688.3	–	6	–	6,381.9	6,707.8	–	12
rc105	–	9,308.8	10,061.2	–	8,292	–	9,308.8	10,043.7	–	7,904	–	9,308.8	10,034.1	–	7,636	–	9,308.8	10,090.7	–	6,512
rc106	–	8,516.8	9,517.7	–	7,194	–	8,519.8	9,514.7	–	7,120	–	8,519.8	9,512.6	–	6,470	–	8,527.9	9,510.33	–	5,646
rc107	–	6,937.6	7,257.3	–	2,126	–	6,949.8	7,418.2	–	1,590	–	6,949.8	7,376.8	–	1,048	–	6,954.0	7,287.8	–	874
rc108	–	6,262.9	6,809.3	–	1,806	–	6,291.9	6,801.8	–	754	–	6,299.6	6,804.3	–	750	–	6,311.4	6,794.4	–	482
r201	1,773.7	8,292.5	8,441.0	8,441	920	1,672.3	8,292.5	8,441.0	8,441	816	2,164.9	8,292.5	8,441.0	8,441	898	1,863.1	8,292.5	8,441.0	8,441	760
r202	2,480.6	7,203.0	7,327.0	7,327	192	–	7,203.0	7,310.5	–	4	–	7,203.0	7,310.5	–	4	–	7,203.0	7,310.5	–	4
r203	–	6,156.2	6,170.0	–	4	–	6,160.5	6,190.1	–	6	–	6,160.5	6,190.1	–	6	–	6,180.7	6,207.9	–	6
r205	6,909.2	6,989.2	7,164.0	7,164	348	–	0.0	0.0	–	0	–	6,998.4	7,127.7	–	50	–	7,021.7	7,128.4	–	48
r206	–	6,356.3	6,408.3	–	10	–	6,998.4	7,134.8	–	90	–	6,379.5	6,379.5	–	0	–	6,379.5	6,379.5	–	0
r209	–	6,032.6	6,081.6	–	10	–	6,062.0	6,154.3	–	66	–	6,062.0	6,154.3	–	66	–	6,062.0	6,163.2	–	90
r210	–	6,424.9	6,520.9	–	10	–	6,428.0	6,428.0	–	2	–	6,428.0	6,428.0	–	2	–	6,428.0	6,428.0	–	2
c201	2,234.4	4,190.8	4,298.0	4,298	104	2,069.8	4,190.8	4,298.0	4,298	104	2,569.9	4,190.8	4,298.0	4,298	118	1,222.4	4,192.0	4,298.0	4,298	58
c202	–	4,091.1	4,122.4	–	8	–	4,091.1	4,122.4	–	8	–	4,091.1	4,122.4	–	4	–	4,099.7	4,153.4	–	18
c203	–	4,014.8	4,014.8	–	2	–	4,014.8	4,014.8	–	2	–	4,014.8	4,014.8	–	2	–	4,018.5	4,018.5	–	2
c205	–	4,135.4	4,250.8	–	282	–	4,135.4	4,251.5	–	302	–	4,135.4	4,245.5	–	146	7,055.8	4,210.0	4,278.0	4,278	186
c206	–	4,134.7	4,199.3	–	70	–	4,134.7	4,198.5	–	68	–	4,153.1	4,198.9	–	28	–	4,204.0	4,251.0	–	76
c207	–	4,082.3	4,138.7	–	12	–	4,082.3	4,138.7	–	12	–	4,082.3	4,110.6	–	6	–	4,127.2	4,154.7	–	6
c208	–	4,087.7	4,148.1	–	18	–	4,105.6	4,154.2	–	22	–	4,105.6	4,153.2	–	20	–	4,119.2	4,156.0	–	20
rc201	–	7,874.8	8,164.7	–	2,098	–	7,882.9	8,169.9	8,419	1,680	–	7,904.5	8,156.5	–	1,180	–	7,910.9	8,157.4	–	1176
rc202	–	6,864.8	7,059.0	7,612	720	–	6,888.6	6,960.4	–	10	–	6,888.6	6,958.8	–	8	–	6,914.4	7,017.5	–	32
rc203	–	6,251.8	6,284.9	–	4	–	6,160.5	6,227.1	–	6	–	6,284.4	6,348.5	–	2	–	6,297.1	6,303.8	–	4
rc205	–	7,022.9	7,138.6	–	168	–	7,044.3	7,141.9	–	384	3,649.2	7,055.2	7,177.0	7,177	74	2,367.6	7,048.7	7,177.0	7,177	52
rc206	–	6,284.0	6,595.7	–	362	–	6,428.7	6,636.8	–	342	–	6,428.7	6,634.7	–	138	–	6,459.3	6,637.5	6,691	148
rc207	–	5,611.2	5,871.0	–	4	–	5,777.3	5,829.9	–	2	–	5,777.3	5,791.7	–	4	–	5,802.8	5,852.3	–	2
rc208	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	4,870.3	4,870.3	–	2

Table 12 Instances with 50 Customers and $\rho = 1.2$

	$\mathcal{A}_2$					$\mathcal{A}_3$					$\mathcal{A}_4$					$\mathcal{A}_5$				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	82.4	11,277.3	11,364.0	11,364	352	79.9	11,277.3	11,364.0	11,364	352	67.5	11,277.3	11,364.0	11,364	342	100.9	11,277.3	11,364.0	11,364	358
r102	174.3	9,286.6	9,337.0	9,337	216	122.6	9,286.6	9,337.0	9,337	160	108.3	9,286.6	9,337.0	9,337	154	208.4	9,286.6	9,337.0	9,337	214
r103	2,839.5	7,814.3	7,929.0	7,929	1,550	2,231.6	7,814.3	7,929.0	7,929	1,136	2,537.2	7,814.3	7,929.0	7,929	1,206	1,121.6	7,815.6	7,929.0	7,929	554
r104	4,640.5	6,305.4	6,500.0	6,500	738	3,172.3	6,310.0	6,500.0	6,500	356	2,845.3	6,310.0	6,500.0	6,500	284	2,503.5	6,311.1	6,500.0	6,500	302
r105	171.2	9,668.4	9,820.0	9,820	256	155.0	9,668.4	9,820.0	9,820	256	157.9	9,668.4	9,820.0	9,820	240	122.2	9,668.4	9,820.0	9,820	218
r106	827.3	8,264.7	8,410.0	8,410	590	409.5	8,264.7	8,410.0	8,410	290	339.6	8,264.7	8,410.0	8,410	222	336.8	8,264.7	8,410.0	8,410	250
r107	961.0	7,270.1	7,417.0	7,417	260	1,096.7	7,270.1	7,417.0	7,417	286	794.8	7,270.1	7,417.0	7,417	214	695.1	7,270.3	7,417.0	7,417	182
r108	6,992.2	6,028.7	6,282.0	6,282	418	7,037.9	6,029.2	6,282.0	6,282	496	6,897.5	6,029.2	6,282.0	6,282	426	6,470.1	6,029.3	6,282.0	6,282	406
r109	496.0	7,982.1	8,179.0	8,179	348	503.1	7,982.1	8,179.0	8,179	358	439.0	7,982.1	8,179.0	8,179	256	379.1	7,982.1	8,179.0	8,179	272
r110	1,789.2	7,125.1	7,339.0	7,339	792	1,822.4	7,125.1	7,339.0	7,339	768	1,852.2	7,125.1	7,339.0	7,339	760	1,514.2	7,125.1	7,339.0	7,339	716
r111	536.5	7,181.7	7,331.0	7,331	106	547.9	7,181.7	7,331.0	7,331	108	567.6	7,181.7	7,331.0	7,331	108	436.3	7,181.7	7,331.0	7,331	94
r112	2,621.8	6,310.5	6,494.0	6,494	446	2,712.1	6,310.5	6,494.0	6,494	464	2,416.1	6,310.5	6,494.0	6,494	386	1,984.5	6,310.5	6,494.0	6,494	354
c101	374.6	4,356.5	4,551.0	4,551	378	309.0	4,356.5	4,551.0	4,551	304	382.3	4,356.5	4,551.0	4,551	286	364.9	4,367.9	4,551.0	4,551	294
c102	6,176.6	4,281.9	4,421.0	4,421	3,016	4,771.3	4,281.9	4,421.0	4,421	2,256	1,886.1	4,281.9	4,421.0	4,421	778	1,967.5	4,306.7	4,421.0	4,421	536
c103	–	4,147.7	4,229.9	–	1,514	3,762.9	4,147.7	4,243.0	4,243	512	2,280.0	4,147.7	4,243.0	4,243	258	1,725.1	4,147.7	4,243.0	4,243	184
c104	–	3,794.2	3,794.2	–	2	–	3,801.3	3,801.3	–	2	–	3,801.3	3,801.3	–	2	–	3,806.4	3,806.4	–	2
c105	112.6	4,217.2	4,289.0	4,289	106	105.5	4,217.2	4,289.0	4,289	72	106.2	4,217.2	4,289.0	4,289	68	222.3	4,217.2	4,289.0	4,289	142
c106	63.2	4,279.8	4,313.0	4,313	30	40.5	4,279.8	4,313.0	4,313	32	38.4	4,279.8	4,313.0	4,313	32	11.7	4,296.4	4,313.0	4,313	10
c107	51.3	4,171.3	4,257.0	4,257	46	48.8	4,171.3	4,257.0	4,257	36	45.9	4,171.3	4,257.0	4,257	36	66.4	4,171.3	4,257.0	4,257	34
c108	179.5	4,162.7	4,245.0	4,245	74	233.8	4,162.7	4,245.0	4,245	80	210.5	4,162.7	4,245.0	4,245	80	189.4	4,162.7	4,245.0	4,245	78
c109	–	4,043.6	4,216.8	–	2,500	2,356.2	4,043.6	4,218.0	4,218	676	1,688.0	4,043.6	4,218.0	4,218	510	1,151.6	4,043.6	4,218.0	4,218	240
rc101	1,247.7	9,722.9	10,986.0	10,986	2,536	1,336.0	9,722.9	10,986.0	10,986	2,582	1,176.7	9,722.9	10,986.0	10,986	2,590	1,027.3	9,722.9	10,986.0	10,986	2,394
rc102	–	7,999.1	9,086.3	9,148	7,872	–	7,999.1	9,088.5	9,148	7,762	–	7,999.1	9,087.0	9,148	8,430	–	7,999.1	9,082.8	9,148	5,680
rc103	–	7,172.1	7,826.9	–	1,646	5,767.0	7,172.1	7,834.0	7,834	1,764	5,708.7	7,172.1	7,834.0	7,834	1,796	3,109.2	7,173.1	7,834.0	7,834	958
rc104	–	5,797.9	6,041.1	–	160	–	5,865.1	6,071.7	–	90	–	5,865.1	6,067.8	–	96	–	5,865.1	6,096.9	–	126
rc105	–	8,567.6	9,547.0	–	6,206	–	8,567.6	9,565.0	–	6,344	–	8,567.6	9,571.3	–	6,874	–	8,567.6	9,577.7	–	7,728
rc106	–	7,776.8	8,435.1	–	3,894	–	7,790.5	8,452.5	–	3,574	–	7,790.5	8,455.8	–	4,060	–	7,790.5	8,468.5	–	4,668
rc107	–	6,542.7	7,078.4	–	1,930	–	6,550.7	7,077.3	–	1,738	–	6,550.7	7,088.2	7,470	1,826	–	6,550.7	7,108.2	–	2,184
rc108	–	5,969.0	6,374.8	–	326	–	5,992.4	6,427.0	–	180	–	5,992.4	6,431.6	–	186	–	5,992.4	6,496.2	–	144
r201	246.0	8,108.3	8,197.0	8,197	62	217.1	8,108.3	8,197.0	8,197	56	204.0	8,108.3	8,197.0	8,197	56	141.7	8,108.3	8,197.0	8,197	54
r202	–	7,079.7	7,081.3	–	6	–	7,079.7	7,081.3	–	4	–	7,079.7	7,081.3	–	4	–	7,079.7	7,081.3	–	4
r203	–	6,063.6	6,095.5	–	2	–	6,064.4	6,083.4	–	4	–	6,064.4	6,097.3	–	2	–	6,064.4	6,097.3	–	2
r205	–	6,879.4	6,919.6	–	14	–	6,886.0	6,927.9	–	12	–	6,886.0	6,961.4	-	30	–	6,886.0	7,004.0	–	78
r206	–	6,306.6	6,325.2	–	6	–	6,316.7	6,323.5	–	4	–	6,316.7	6,350.1	–	2	–	6,320.8	6,351.1	–	2
r207	–	–	–	–	–	–	–	–	–	–	–	5,705.6	5,733.4	–	2	–	5,709.6	5,736.4	–	2
r209	–	6,004.7	6,004.7	–	2	–	6,036.5	6,084.6	–	44	–	6,036.5	6,084.6	–	58	–	6,043.5	6,043.5	–	0
c201	216.6	4,068.3	4,123.0	4,123	8	271.6	4,068.3	4,123.0	4,123	8	278.9	4,068.3	4,123.0	4,123	8	26.3	4,078.5	4,123.0	4,123	2
c202	–	3,929.5	3,929.5	–	2	–	3,929.5	3,929.5	–	2	–	3,929.5	3,929.5	–	2	–	3,929.5	3,929.9	–	8
c205	5,814.1	3,999.1	4,119.0	4,119	146	5,905.1	3,999.1	4,119.0	4,119	146	7,092.2	3,999.1	4,119.0	4,119	98	4,739.4	3,999.1	4,119.0	4,119	54
c206	–	3,997.9	4,098.4	–	70	–	3,997.9	4,068.5	–	22	–	3,997.9	4,098.4	–	46	5,336.3	4,034.0	4,119.0	4,119	46
c207	–	3,900.2	3,914.6	–	4	–	3,900.2	3,914.6	–	4	–	3,900.2	3,914.6	–	4	–	3,900.2	3,914.6	–	4
c208	–	3,963.4	3,963.4	–	2	6,813.2	3,983.4	4,026.0	4,026	22	6,416.5	3,983.4	4,026.0	4,026	22	–	4,002.0	4,023.8	–	14
rc201	–	7,509.1	7,574.5	8,220	664	–	7,512.4	7,554.2	8,096	688	–	7,512.7	7,808.9	7,932	1,936	–	7,531.6	7,801.9	7,955	1,976
rc202	–	6,513.2	6,648.4	7,139	416	–	6,540.5	6,540.5	–	2	–	6,540.5	6,587.4	–	36	–	6,548.5	6,678.1	6,767	804
rc203	–	5,926.3	5,939.1	–	6	–	5,949.6	5,949.6	–	2	–	5,949.6	5,949.6	–	0	–	5,949.6	5,949.6	–	0
rc204	–	–	–	–	–	–	4,790.7	4,790.7	–	2	–	–	–	–	–	–	–	–	–	–
rc205	4,555.3	6,656.4	6,803.0	6,803	310	5,407.9	6,690.3	6,803.0	6,803	294	–	6,690.3	6,780.0	7,032	204	3,700.8	6,690.3	6,803.0	6,803	180
rc206	–	6,100.0	6,108.6	6,687	4,408	–	6,100.0	6,112.3	6,687	3,838	–	6,100.0	6,214.0	6,811	974	–	6,173.0	6,272.1	–	614
rc207	–	5,601.5	5,601.5	–	2	–	5,601.5	5,601.5	–	2	–	5,616.5	5,659.3	–	16	–	5,622.8	5,662.5	–	22
rc208	–	4,753.3	4,753.3	–	0	–	4,753.3	4,753.3	–	0	–	4,753.3	4,753.3	–	0	–	4,753.3	4,753.3	–	0

Table 13 Instances with 50 Customers and $\rho = 1.5$

	$\mathcal{A}_2$					$\mathcal{A}_3$					$\mathcal{A}_4$					$\mathcal{A}_5$				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	16.1	10,435.0	10,490.0	10,490	24	16.0	10,435.0	10,490.0	10,490	24	24.1	10,435.0	10,490.0	10,490	44	22.9	10,435.0	10,490.0	10,490	20
r102	12.7	9,028.0	9,028.0	9,028	8	15.4	9,028.0	9,028.0	9,028	8	15.6	9,028.0	9,028.0	9,028	8	2.0	9,028.0	9,028.0	9,028	0
r103	1,903.8	7,557.7	7,606.0	7,606	614	1,971.6	7,557.7	7,606.0	7,606	666	1,286.1	7,557.7	7,606.0	7,606	422	450.2	7,560.4	7,606.0	7,606	118
r104	–	6,115.4	6,228.8	–	850	–	6,115.4	6,228.7	–	758	–	6,115.4	6,245.4	6,311	746	–	6,115.4	6,250.8	6,294	746
r105	78.6	9,161.7	9,261.0	9,261	70	76.9	9,161.7	9,261.0	9,261	70	75.0	9,161.7	9,261.0	9,261	70	58.5	9,161.7	9,261.0	9,261	66
r106	179.8	7,835.6	7,916.0	7,916	88	81.6	7,835.6	7,916.0	7,916	48	125.6	7,836.9	7,916.0	7,916	62	90.7	7,836.9	7,916.0	7,916	48
r107	606.9	6,934.5	7,083.0	7,083	80	499.0	6,934.5	7,083.0	7,083	76	438.9	6,934.5	7,083.0	7,083	60	344.5	6,936.8	7,083.0	7,083	56
r108	–	5,868.2	5,944.7	–	18	–	5,868.2	5,934.9	–	16	–	5,868.2	5,951.0	–	22	–	5,868.5	5,957.0	–	26
r109	386.4	7,753.4	7,919.0	7,919	276	417.7	7,753.4	7,919.0	7,919	284	330.2	7,753.4	7,919.0	7,919	278	358.0	7,753.4	7,919.0	7,919	284
r110	308.3	6,951.0	7,052.0	7,052	58	313.2	6,951.0	7,052.0	7,052	58	281.0	6,951.0	7,052.0	7,052	58	285.5	6,951.0	7,052.0	7,052	64
r111	–	6,962.9	7,134.1	7,177	3,694	–	6,962.9	7,138.1	7,140	3,640	–	6,962.9	7,135.0	7,171	3,360	–	6,962.9	7,128.4	7,162	2,932
r112	4,386.7	6,149.3	6,371.0	6,371	916	3,897.9	6,149.3	6,371.0	6,371	830	4,798.4	6,149.3	6,371.0	6,371	920	4,484.3	6,149.3	6,371.0	6,371	884
c101	10.3	3,976.0	4,003.0	4,003	6	13.4	3,976.0	4,003.0	4,003	6	12.7	3,976.0	4,003.0	4,003	6	11.1	3,991.0	4,003.0	4,003	6
c102	69.5	3,966.0	3,993.0	3,993	6	52.6	3,966.0	3,993.0	3,993	6	53.6	3,966.0	3,993.0	3,993	6	54.2	3,981.0	3,993.0	3,993	6
c103	–	3,614.0	3,661.1	–	4	–	3,614.0	3,671.1	–	4	–	3,614.0	3,671.1	–	4	–	3,684.5	3,733.7	–	4
c104	–	3,580.0	3,621.0	–	4	–	3,580.0	3,621.0	–	4	–	3,580.0	3,621.0	–	4	–	3,629.8	3,629.8	–	2
c105	92.8	3,624.0	3,845.0	3,845	40	99.1	3,624.0	3,845.0	3,845	44	74.3	3,624.0	3,845.0	3,845	28	51.0	3,701.3	3,845.0	3,845	6
c106	84.3	3,624.0	3,845.0	3,845	46	83.1	3,624.0	3,845.0	3,845	38	73.0	3,624.0	3,845.0	3,845	38	12.9	3,700.7	3,845.0	3,845	4
c107	162.1	3,624.0	3,845.0	3,845	46	153.2	3,624.0	3,845.0	3,845	42	130.8	3,624.0	3,845.0	3,845	28	420.9	3,704.1	3,845.0	3,845	4
c108	211.4	3,624.0	3,845.0	3,845	40	826.9	3,624.0	3,845.0	3,845	44	727.0	3,624.0	3,845.0	3,845	32	561.8	3,690.8	3,845.0	3,845	4
c109	527.6	3,624.0	3,845.0	3,845	60	698.1	3,624.0	3,845.0	3,845	94	694.8	3,624.0	3,845.0	3,845	78	656.5	3,708.7	3,845.0	3,845	6
rc101	–	9,341.8	10,465.5	10,670	9,234	–	9,341.8	10,465.7	10,798	9,370	–	9,341.8	10,461.3	11,212	8,796	–	9,341.8	10,462.5	10,792	9,000
rc102	–	7,099.0	8,178.8	8,418	4,482	–	7,099.0	8,177.8	8,396	4,442	–	7,099.0	8,177.4	8,396	4,280	–	7,099.0	8,198.9	8,390	3,612
rc103	–	6,298.2	6,669.0	7,152	1,398	–	6,320.0	6,654.8	7,184	1,082	–	6,320.0	6,655.8	7,188	1,028	–	6,415.2	6,685.1	7,152	434
rc104	–	5,295.0	5,545.8	–	4	–	5,455.6	5,455.6	–	2	–	5,455.6	5,455.6	–	2	–	5,517.4	5,517.4	–	2
rc105	–	7,624.4	8,562.6	–	5,952	–	7,624.4	8,554.9	–	5,864	–	7,624.4	8,553.0	–	5,520	–	7,624.4	8,556.5	–	4,832
rc106	–	6,644.3	7,319.0	–	3,454	–	6,644.3	7,295.0	–	3,250	–	6,644.3	7,303.4	–	3,210	–	6,684.0	7,329.2	–	2,108
rc107	–	6,011.8	6,481.1	–	2,136	–	6,011.8	6,495.7	–	1,808	–	6,011.8	6,495.1	–	1,770	–	6,034.7	6,485.2	–	1,534
rc108	–	5,411.7	6,053.8	–	678	–	5,458.9	6,043.6	–	476	–	5,458.9	6,044.6	–	496	–	5,585.1	6,063.8	–	712
r201	1,489.1	7,919.9	8,006.0	8,006	884	1,214.6	7,919.9	8,006.0	8,006	588	–	7,922.7	7,978.3	–	9,982	1,152.3	7,922.7	8,006.0	8,006	684
r202	–	6,985.6	7,002.7	–	346	–	6,985.6	7,003.8	–	462	–	6,996.1	7,027.7	–	630	–	6,996.3	7,042.0	7,622	858
r203	–	5,986.2	6,103.5	–	12	–	5,986.2	6,103.5	–	12	–	5,986.2	6,103.5	–	8	–	5,986.2	6,103.5	–	12
r205	–	6,828.5	6,854.5	–	8	–	6,828.5	6,854.5	–	8	–	6,828.5	6,854.5	–	8	–	6,828.5	6,854.5	–	8
r206	–	6,263.5	6,283.2	–	2	–	6,263.5	6,283.2	–	2	–	6,263.5	6,283.2	–	2	–	6,263.5	6,283.2	–	2
r207	–	5,641.4	5,654.1	–	2	–	5,641.4	5,719.4	–	4	–	5,641.4	5,719.4	–	4	–	5,647.3	5,653.4	–	2
r209	–	5,998.3	5,998.3	–	4	–	5,998.3	5,998.3	–	4	–	5,998.3	5,998.3	–	4	–	5,998.3	5,998.3	–	4
c201	2,279.7	3,887.7	3,959.0	3,959	32	4,962.9	3,887.7	3,959.0	3,959	46	3,692.3	3,887.7	3,959.0	3,959	36	39.4	3,924.9	3,959.0	3,959	2
c202	130.4	3,832.0	3,832.0	3,832	0	144.6	3,832.0	3,832.0	3,832	0	153.4	3,832.0	3,832.0	3,832	0	122.4	3,832.0	3,832.0	3,832	0
c203	331.6	3,769.0	3,769.0	3,769	0	370.4	3,769.0	3,769.0	3,769	0	386.1	3,769.0	3,769.0	3,769	0	321.7	3,769.0	3,769.0	3,769	0
c205	6,561.9	3,869.4	3,894.0	3,894	42	5,596.3	3,869.4	3,894.0	3,894	36	1,662.4	3,869.4	3,894.0	3,894	20	1,049.8	3,869.4	3,894.0	3,894	14
c206	–	3,848.0	3,868.8	–	22	–	3,848.0	3,875.8	3,978	26	4,995.0	3,848.0	3,894.0	3,894	28	4,465.0	3,848.0	3,894.0	3,894	20
c207	1,217.3	3,771.0	3,771.0	3,771	0	1,508.5	3,771.0	3,771.0	3,771	0	1,558.5	3,771.0	3,771.0	3,771	0	1,537.5	3,771.0	3,771.0	3,771	0
c208	–	3,777.5	3,777.5	–	2	–	3,792.3	3,792.3	–	2	–	3,792.3	3,792.3	–	2	–	3,792.3	3,792.3	–	2
rc201	64.6	6,848.0	7,166.0	7,166	16	62.8	6,963.5	7,166.0	7,166	14	63.0	6,963.5	7,166.0	7,166	14	82.0	7,035.0	7,166.0	7,166	14
rc202	551.8	6,136.0	6,363.0	6,363	42	734.4	6,209.8	6,363.0	6,363	42	812.9	6,209.8	6,363.0	6,363	40	791.1	6,238.2	6,363.0	6,363	28
rc203	–	5,553.0	5,553.0	–	2	–	5,553.0	5,587.3	–	18	–	5,553.0	5,589.6	–	20	–	5,553.0	5,577.5	–	12
rc205	–	6,302.0	6,527.0	6,575	10	929.8	6,408.0	6,575.0	6,575	34	843.8	6,408.0	6,575.0	6,575	32	1,054.9	6,408.0	6,575.0	6,575	38
rc206	8.2	6,100.0	6,100.0	6,100	0	10.3	6,100.0	6,100.0	6,100	0	10.3	6,100.0	6,100.0	6,100	0	10.6	6,100.0	6,100.0	6,100	0
rc207	1,742.1	5,586.0	5,602.0	5,602	14	1,664.0	5,586.0	5,602.0	5,602	12	1,953.6	5,586.0	5,602.0	5,602	12	1,063.1	5,586.0	5,602.0	5,602	10
rc208	–	4,722.8	4,725.5	–	2	–	4,725.5	4,727.0	4,792	4	–	4,725.5	4,725.5	4,792	2	–	4,725.5	4,727.0	4,792	4

Table 14 Instances with 100 Customers and $\rho = 1.05$

	$\mathcal{A}_2$					$\mathcal{A}_3$					$\mathcal{A}_4$					$\mathcal{A}_5$				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	1,903.1	18,299.0	18,406.0	18,406	1478	1,832.0	18,299.0	18,406.0	18,406	1478	1,808.1	18,299.0	18,406.0	18,406	1478	1,710.9	18,299.0	18,406.0	18,406	1518
r102	1,068.4	15,322.2	15,354.0	15,354	202	1,050.4	15,322.2	15,354.0	15,354	202	1,038.8	15,322.2	15,354.0	15,354	202	714.0	15,322.2	15,354.0	15,354	144
r103	–	12,214.8	12,235.9	–	324	–	12,214.8	12,235.9	–	324	–	12,214.8	12,235.9	199,900	328	2,115.0	12,215.1	12,237.0	12,237	86
r104	–	9,925.8	9,987.1	–	28	–	9,925.8	9,990.9	–	32	–	9,925.8	9,991.6	–	34	–	9,930.4	10,003.4	–	54
r105	1,964.0	14,531.5	14,652.0	14,652	610	1,959.7	14,531.5	14,652.0	14,652	610	1,892.0	14,531.5	14,652.0	14,652	576	1,718.3	14,531.5	14,652.0	14,652	596
r106	–	12,784.0	12,892.3	–	438	–	12,784.0	12,902.6	–	584	–	12,784.0	12,904.1	–	606	–	12,784.0	12,916.2	–	862
r109	–	12,103.9	12,264.5	–	842	–	12,103.9	12,265.5	–	878	–	12,103.9	12,264.9	–	844	–	12,103.9	12,274.7	–	1174
r110	–	11,019.1	11,149.3	–	268	–	11,019.1	11,151.5	–	284	–	11,019.1	11,140.8	–	202	–	11,019.1	11,162.7	–	396
r111	–	10,812.6	10,927.3	–	258	–	10,812.6	10,925.5	–	232	–	10,812.6	10,918.5	–	186	–	10,812.6	10,932.2	–	308
c101	908.88	11,019.5	11,145.0	11,145	296	881.13	11,019.5	11,145.0	11,145	296	872.5	11,019.5	11,145.0	11,145	296	730.9	11,019.5	11,145.0	11,145	274
c102	4,547.9	10,190.7	10,363.0	10,363	464	4,752.5	10,190.7	10,363.0	10,363	498	4,856.3	10,190.7	10,363.0	10,363	498	4,456.3	10,190.7	10,363.0	10,363	468
c103	–	9,964.4	10,080.6	–	244	–	9,964.4	10,081.8	–	260	–	9,964.4	10,080.9	–	256	–	9,964.4	10,087.4	–	342
c104	–	9,153.7	9,153.7	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
c105	3,919.3	10,506.4	10,716.0	10,716	988	3,637.5	10,506.4	10,716.0	10,716	988	3,882.3	10,506.4	10,716.0	10,716	988	3,734.0	10,506.4	10,716.0	10,716	1006
c106	–	10,516.0	10,768.5	–	1464	–	10,516.0	10,769.1	–	1490	–	10,516.0	10,770.3	–	1524	–	10,516.0	10,770.0	–	1490
c107	1,232.8	10,273.7	10,306.0	10,306	140	1,284.4	10,273.7	10,306.0	10,306	152	1,303.1	10,273.7	10,306.0	10,306	152	799.5	10,273.7	10,306.0	10,306	118
c108	4,582.8	10,180.1	10,293.0	10,293	502	3,764.8	10,180.1	10,293.0	10,293	444	3,723.0	10,180.1	10,293.0	10,293	444	3,621.3	10,180.1	10,293.0	10,293	434
c109	–	9,565.6	9,825.5	–	420	–	9,565.6	9,821.2	–	378	–	9,565.6	9,821.8	–	386	–	9,565.6	9,833.4	–	580
rc101	–	18,186.2	18,720.7	–	3522	–	18,186.2	18,718.9	–	3420	–	18,186.2	18,719.5	–	3454	–	18,186.2	18,733.3	–	4284
rc102	–	15,288.3	15,660.5	–	1216	–	15,288.3	15,661.5	–	1236	–	15,288.3	15,660.8	–	1222	–	15,288.3	15,684.3	–	1612
rc103	–	13,196.8	13,403.6	–	190	–	13,196.8	13,389.9	–	148	–	13,196.8	13,396.7	–	162	–	13,200.1	13,453.4	–	322
rc104	–	11,719.6	11,774.4	–	10	–	11,720.8	11,774.1	–	8	–	11,720.8	11,774.1	–	8	–	11,720.8	11,812.4	–	24
rc105	–	16,605.3	17,000.3	–	1730	–	16,605.3	17,001.8	–	1766	–	16,605.3	17,003.9	–	1800	–	16,605.3	17,018.4	–	2254
rc106	–	14,725.9	14,919.3	–	1190	–	14,725.9	14,919.5	–	1198	–	14,725.9	14,920.6	–	1214	–	14,725.9	14,931.0	–	1618
rc107	–	12,917.0	13,080.6	–	384	–	12,917.0	13,081.0	–	392	–	12,917.0	13,081.0	–	392	–	12,917.0	13,095.4	–	530
rc108	–	11,593.1	11,707.3	–	78	–	11,593.1	11,693.7	–	70	–	11,593.1	11,693.7	–	70	–	11,593.1	11,693.7	–	68
r201	–	11,782.2	11,881.3	–	346	–	11,782.2	11,881.0	–	348	–	11,782.2	11,881.0	–	330	6,659.5	11,782.2	11,885.0	11,885	380
r203	–	8,754.0	8,754.0	–	2	–	8,754.0	8,754.0	–	2	–	8,754.0	8,754.0	–	2	–	8,754.6	8,754.6	–	2
r205	–	–	–	–	–	–	9,513.8	9,560.1	–	14	–	9,513.8	9,560.1	–	14	–	9,513.8	9,560.2	–	16
r209	–	8,477.1	8,491.0	–	6	–	8,478.0	8,478.0	–	2	–	8,478.0	8,478.0	–	2	–	8,479.7	8,479.7	–	2
r210	–	8,952.6	8,963.9	–	4	–	8,952.6	8,964.4	–	4	–	8,952.6	8,964.4	–	4	–	–	–	–	–
c201	–	6,616.6	6,782.4	–	22	–	6,622.6	6,622.6	–	2	–	6,633.9	6,633.9	–	2	–	6,660.3	6,660.3	–	2
c202	–	6,584.1	6,584.1	–	2	–	6,585.2	6,585.2	–	2	–	6,585.2	6,585.2	–	2	–	6,590.0	6,590.0	–	2
c205	–	6,546.6	6,610.9	–	8	–	6,553.1	6,615.5	–	8	–	6,553.1	6,626.5	–	8	–	6,597.3	6,666.4	–	14
c206	–	6,475.1	6,549.5	–	8	–	6,489.3	6,555.1	–	12	–	6,489.3	6,553.8	–	10	–	6,519.6	6,568.2	–	6
c207	–	6,427.2	6,503.4	–	4	–	6,443.3	6,514.7	–	4	–	6,460.0	6,460.0	–	2	–	6,495.0	6,495.0	–	2
c208	–	6,339.9	6,421.9	–	4	–	6,356.4	6,435.2	–	8	–	6,356.4	6,433.1	–	6	–	6,444.7	6,499.1	–	4
rc201	–	12,830.1	12,928.6	–	84	–	12,830.1	12,930.6	–	86	–	12,830.1	12,928.6	–	78	–	12,830.1	12,928.6	–	76
rc202	–	11,059.6	11,076.9	–	18	–	11,060.1	11,104.9	–	36	–	11,060.1	11,091.8	–	24	–	11,060.1	11,100.9	–	30
rc205	–	11,582.8	11,676.0	–	64	–	11,582.8	11,692.2	–	96	–	11,582.8	11,663.5	–	48	–	11,582.8	11,688.4	–	76
rc206	–	10,540.7	10,582.6	–	22	–	10,540.7	10,595.2	–	36	–	10,540.7	10,588.9	–	24	–	10,540.7	10,595.2	–	36
rc207	–	9,484.8	9,486.6	–	4	–	–	–	–	–	–	9,484.9	9,486.6	–	4	–	–	–	–	–

Table 15 Instances with 100 Customers and $\rho = 1.2$

	$\mathcal{A}_2$					$\mathcal{A}_3$					$\mathcal{A}_4$					$\mathcal{A}_5$				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	769.3	17,087.6	17,216.0	17,216	396	701.2	17,087.6	17,216.0	17,216	396	714.4	17,087.6	17,216.0	17,216	396	774.9	17,087.6	17,216.0	17,216	418
r102	1,466.2	14,984.2	14,995.0	14,995	266	1,431.3	14,984.2	14,995.0	14,995	266	1,453.7	14,984.2	14,995.0	14,995	266	1,707.2	14,984.2	14,995.0	14,995	342
r103	3,423.3	11,970.6	12,015.0	12,015	180	3,397.1	11,970.6	12,015.0	12,015	180	3,486.7	11,970.6	12,015.0	12,015	180	1,732.1	11,970.6	12,015.0	12,015	92
r104	–	9,676.4	9,685.3	–	6	–	9,676.4	9,685.3	–	6	–	9,676.4	9,685.3	–	6	–	9,676.4	9,709.2	–	18
r105	788.7	13,866.3	13,913.0	13,913	184	741.0	13,866.3	13,913.0	13,913	184	732.9	13,866.3	13,913.0	13,913	180	657.4	13,866.3	13,913.0	13,913	160
r106	–	12,508.9	12,640.4	–	636	–	12,508.9	12,641.5	–	652	–	12,508.9	12,641.5	–	652	–	12,508.9	12,647.1	–	834
r107	–	10,602.2	10,728.1	–	224	–	10,602.2	10,730.2	–	236	–	10,602.2	10,730.2	–	234	–	10,602.2	10,741.0	–	322
r108	–	9,227.1	9,283.4	–	22	–	9,227.1	9,279.6	–	22	–	9,227.1	9,283.4	–	24	–	9,227.1	9,283.4	–	24
r109	–	11,716.3	11,846.6	–	702	–	11,716.3	11,849.3	–	738	–	11,716.3	11,850.1	–	762	–	11,716.3	11,857.9	–	972
r110	–	10,740.5	10,878.0	–	334	–	10,740.5	10,880.0	–	370	–	10,740.5	10,878.3	–	354	–	10,740.5	10,887.0	–	496
r111	–	10,567.1	10,650.4	–	254	–	10,567.1	10,650.4	–	268	–	10,567.1	10,650.1	–	240	–	10,567.1	10,651.5	10,655	268
r112	–	9,461.6	9,511.3	–	22	–	9,461.6	9,511.3	–	22	–	9,461.6	9,511.3	–	22	–	9,461.6	9,513.7	–	28
c101	4,511.7	9,728.7	9,918.0	9,918	1336	4,701.5	9,728.7	9,918.0	9,918	1372	4,829.9	9,728.7	9,918.0	9,918	1384	4,349.1	9,728.7	9,918.0	9,918	1260
c102	–	9,447.0	9,556.3	–	506	–	9,447.0	9,559.0	–	518	–	9,447.0	9,557.9	–	478	6,515.4	9,451.8	9,563.0	9,563	332
c103	–	9,284.8	9,422.9	–	220	–	9,284.8	9,421.6	–	194	–	9,284.8	9,419.1	–	184	–	9,284.8	9,425.5	–	220
c104	–	8,755.5	8,755.5	–	2	–	8,755.5	8,755.5	–	2	–	8,755.5	8,755.5	–	2	–	8,755.5	8,755.5	–	2
c105	5,504.8	9,509.1	9,646.0	9,646	1358	5,328.9	9,509.1	9,646.0	9,646	1356	5,346.4	9,509.1	9,646.0	9,646	1360	2,952.6	9,509.1	9,646.0	9,646	682
c106	–	9,571.3	9,700.2	–	1420	–	9,571.3	9,695.1	–	1504	–	9,571.3	9,701.2	–	1552	–	9,571.3	9,708.4	–	1346
c107	2,047.7	9,389.3	9,546.0	9,546	310	1,766.8	9,389.3	9,546.0	9,546	262	1,339.21	9,389.3	9,546.0	9,546	262	1,898.4	9,389.3	9,546.0	9,546	256
c108	–	9,377.3	9,540.1	–	948	–	9,377.3	9,539.6	–	942	–	9,377.3	9,542.2	–	1158	–	9,377.3	9,539.3	–	776
c109	–	8,975.6	9,092.7	–	360	–	8,975.6	9,098.0	–	482	–	8,975.6	9,098.0	–	438	–	8,975.6	9,093.6	–	396
rc101	–	17,116.0	17,565.2	–	2984	–	17,116.0	17,578.5	–	3694	–	17,116.0	17,566.0	–	3030	–	17,116.0	17,579.9	–	3666
rc102	–	14,630.8	14,938.0	–	1080	–	14,630.8	14,962.4	–	1420	–	14,630.8	14,942.0	–	1128	–	14,630.8	14,962.0	–	1408
rc103	–	12,637.5	12,833.7	–	154	–	12,638.4	12,850.7	–	238	–	12,638.4	12,831.9	–	154	–	12,640.0	12,859.8	–	272
rc104	–	11,256.4	11,324.9	–	16	–	11,256.4	11,343.0	–	20	–	11,256.4	11,300.3	–	10	–	11,281.1	11,299.1	–	8
rc105	–	15,858.7	16,223.4	–	1536	–	15,858.7	16,248.3	–	1992	–	15,858.7	16,225.0	–	1552	–	15,858.7	16,252.1	–	2026
rc106	–	14,005.3	14,183.8	–	1012	–	14,005.3	14,197.9	–	1340	–	14,005.3	14,184.8	–	1030	–	14,005.3	14,197.8	–	1338
rc107	–	12,425.8	12,562.0	–	328	–	12,425.8	12,572.1	–	440	–	12,425.8	12,561.1	–	322	–	12,425.8	12,575.3	–	462
rc108	–	11,189.5	11,286.3	–	80	–	11,189.5	11,297.3	–	114	–	11,189.5	11,288.3	–	82	–	11,189.5	11,284.8	–	110
r201	4,451.8	11,599.5	11,677.0	11,677	206	3,749.5	11,599.5	11,677.0	11,677	210	4,735.9	11,599.5	11,677.0	11,677	210	4,036.4	11,599.5	11,677.0	11,677	220
r202	–	10,253.5	10,276.5	–	22	–	10,253.5	10,276.5	–	24	–	10,253.5	10,275.6	–	22	–	10,253.5	10,276.8	–	24
r203	–	8,697.4	8,697.4	–	2	–	8,697.4	8,697.4	–	2	–	8,697.4	8,697.4	–	2	–	8,697.4	8,697.4	–	2
r205	–	9,448.3	9,476.4	–	24	–	9,448.3	9,484.7	–	28	–	9,448.3	9,476.4	–	24	–	9,448.3	9,494.0	–	30
r206	–	8,694.8	8,694.8	–	2	–	–	–	–	–	–	8,694.8	8,694.8	–	2	–	–	–	–	–
r209	–	8,438.3	8,466.1	–	12	–	–	–	–	–	–	8,438.3	8,470.1	–	12	–	–	–	–	–
r211	–	–	–	–	–	–	6,332.6	6,569.1	–	50	–	–	–	–	–	–	–	–	–	–
c201	–	6,332.6	6,568.9	–	48	–	6,325.4	6,335.2	–	4	–	6,348.6	6,586.4	–	32	–	6,437.1	6,611.4	–	28
c202	–	6,322.4	6,335.2	–	4	–	6,229.2	6,229.2	–	2	–	6,325.8	6,335.2	–	4	–	–	–	–	–
c205	–	6,299.9	6,476.6	–	20	–	6,299.9	6,470.2	–	12	–	6,304.2	6,357.4	–	4	–	6,367.6	6,388.7	–	4
c206	–	6,261.3	6,390.6	–	8	–	6,261.3	6,309.2	–	4	–	6,261.3	6,309.2	–	4	–	6,269.6	6,273.8	–	4
c207	–	6,228.8	6,228.8	–	2	–	6,228.8	6,228.8	–	2	–	6,228.8	6,228.8	–	2	–	6,299.1	6,299.1	–	2
c208	–	6,147.6	6,147.6	–	2	–	6,147.6	6,147.6	–	2	–	6,147.6	6,147.6	–	2	–	6,228.7	6,229.1	–	4
rc201	–	12,680.8	12,793.6	–	324	–	12,680.8	12,793.6	–	326	–	12,680.8	12,794.7	12,798	328	6,390.2	12,680.8	12,798.0	12,798	346
rc202	–	10,956.6	10,997.4	–	34	–	10,958.4	10,997.4	–	32	–	10,958.4	10,996.0	–	30	–	10,958.4	11,013.6	–	42
rc203	–	–	–	–	–	–	0.0	0.0	–	0	–	–	–	–	–	–	–	–	–	–
rc205	–	11,494.4	11,537.5	–	46	–	11,494.4	11,525.1	–	34	–	11,494.4	11,532.9	–	44	–	11,494.4	11,529.3	–	40
rc206	–	10,445.1	10,478.5	–	24	–	10,445.1	10,476.5	–	26	–	10,445.1	10,475.3	–	24	–	10,445.1	10,489.3	–	36
rc207	–	9,474.6	9,474.9	–	6	–	9,474.6	9,474.9	–	8	–	9,474.6	9,474.9	–	8	–	9,474.6	9,486.4	–	10

Table 16 Instances with 100 Customers and $\rho = 1.5$

	$\mathcal{A}_2$					$\mathcal{A}_3$					$\mathcal{A}_4$					$\mathcal{A}_5$				
Instance	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**	**Time**	**Root LB**	**Best LB**	**UB**	**Tree**
r101	55.3	16,739.5	16,794.0	16,794	24	52.3	16,739.5	16,794.0	16,794	24	56.4	16,739.5	16,794.0	16,794	24	71.7	16,739.5	16,794.0	16,794	28
r102	63.7	14,699.5	14,700.0	14,700	2	78.0	14,699.5	14,700.0	14,700	2	77.9	14,699.5	14,700.0	14,700	2	70.2	14,699.5	14,700.0	14,700	2
r103	1,904.7	11,839.4	11,857.0	11,857	50	1,870.3	11,839.4	11,857.0	11,857	50	1,872.6	11,839.4	11,857.0	11,857	50	1,233.1	11,841.6	11,857.0	11,857	32
r104	–	9,426.3	9,459.4	–	22	–	9,426.3	9,457.2	–	20	–	9,426.3	9,457.2	–	20	–	9,426.3	9,471.7	–	34
r105	942.2	13,514.0	13,614.0	13,614	226	898.0	13,514.0	13,614.0	13,614	226	899.6	13,514.0	13,614.0	13,614	226	804.0	13,514.0	13,614.0	13,614	226
r106	3,505.8	12,207.0	12,280.0	12,280	172	3,502.3	12,207.0	12,280.0	12,280	172	3,503.9	12,207.0	12,280.0	12,280	172	3,120.1	12,207.0	12,280.0	12,280	178
r107	–	10,339.2	10,434.5	–	86	–	10,339.2	10,434.8	–	90	–	10,339.2	10,434.8	–	90	–	10,339.2	10,431.8	–	106
r108	–	8,984.5	9,033.4	–	26	–	8,984.5	9,033.4	–	26	–	8,984.5	9,033.4	–	26	–	8,984.5	9,034.6	–	36
r109	–	11,340.1	11,436.2	–	626	–	11,340.1	11,438.3	–	658	–	11,340.1	11,437.6	–	650	–	11,340.1	11,444.4	–	828
r110	–	10,554.9	10,626.2	–	264	–	10,554.9	10,627.0	–	274	–	10,554.9	10,627.0	–	274	–	10,554.9	10,634.7	–	356
r111	–	10,345.6	10,428.3	–	162	–	10,345.6	10,428.9	–	168	–	10,345.6	10,428.5	–	166	–	10,345.6	10,432.9	–	214
r112	–	9,264.7	9,264.7	–	2	–	9,264.7	9,264.7	–	2	–	9,264.7	9,264.7	–	2	–	9,264.7	9,279.4	–	10
c101	80.5	8,625.0	8,625.0	8,625	6	78.9	8,625.0	8,625.0	8,625	6	78.8	8,625.0	8,625.0	8,625	6	43.31	8,625.0	8,625.0	8,625	2
c102	678.9	8,625.0	8,625.0	8,625	4	656.3	8,625.0	8,625.0	8,625	4	657.2	8,625.0	8,625.0	8,625	4	196.5	8,625.0	8,625.0	8,625	0
c103	–	8,263.0	8,263.0	–	24	–	8,263.0	8,263.0	–	26	–	8,263.0	8,263.0	–	26	–	8,263.0	8,266.3	–	32
c105	–	8,273.0	8,304.4	8,475	740	–	8,273.0	8,304.0	8,475	728	–	8,273.0	8,303.5	8,475	726	–	8,273.0	8,327.4	8,475	730
c106	–	8,273.0	8,285.9	8,475	598	–	8,273.0	8,285.9	8,475	602	–	8,273.0	8,285.9	8,475	596	–	8,273.0	8,310.8	8,475	626
c107	–	8,273.0	8,291.9	8,475	614	–	8,273.0	8,292.0	8,475	620	–	8,273.0	8,292.2	8,475	624	–	8,273.0	8,310.5	8,475	642
c108	–	8,273.0	8,277.0	–	226	–	8,273.0	8,277.0	–	230	–	8,273.0	8,277.0	–	230	–	8,273.0	8,288.8	8,475	376
c109	–	8,273.0	8,273.0	–	106	–	8,273.0	8,273.0	–	114	–	8,273.0	8,273.0	–	114	–	8,273.0	8,279.3	–	236
rc101	–	16,213.2	16,622.7	–	3230	–	16,213.2	16,623.8	–	3280	–	16,213.2	16,623.9	–	3282	–	16,213.2	16,635.5	–	3916
rc102	–	14,090.6	14,307.0	–	860	–	14,090.6	14,311.6	–	894	–	14,090.6	14,310.7	–	884	–	14,090.6	14,331.7	–	1218
rc103	–	12,038.3	12,213.4	–	166	–	12,038.3	12,213.4	–	166	–	12,038.3	12,213.4	–	166	–	12,038.8	12,213.4	–	144
rc104	–	10,756.0	10,756.0	–	2	–	10,756.0	10,756.0	–	2	–	10,756.0	10,756.0	–	2	–	10,761.1	10,761.1	–	2
rc105	–	15,331.7	15,610.4	–	1732	–	15,331.7	15,610.2	–	1730	–	15,331.7	15,610.2	–	1728	–	15,331.7	15,623.6	–	2194
rc106	–	13,181.3	13,367.2	–	970	–	13,181.3	13,369.5	–	1014	–	13,181.3	13,369.3	–	1008	–	13,181.3	13,378.9	–	1278
rc107	–	11,833.7	11,907.0	–	58	–	11,833.7	11,894.9	–	40	–	11,833.7	11,894.9	–	40	–	11,833.7	11,933.7	–	132
rc108	–	10,733.4	10,777.7	–	58	–	10,733.4	10,777.1	–	56	–	10,733.4	10,777.1	–	56	–	10,733.4	10,785.3	–	76
r201	–	11,403.0	11,432.2	–	116	–	11,403.0	11,432.2	–	116	–	11,403.0	11,432.2	–	112	–	11,403.0	11,434.0	–	144
r202	–	10,222.3	10,232.4	–	26	–	10,222.3	10,232.4	–	22	–	10,222.3	10,232.4	–	22	–	–	–	–	–
r205	–	9,389.3	9,411.0	–	14	–	9,389.3	9,392.9	–	10	–	9,389.3	9,392.9	–	10	–	9,389.3	9,413.3	–	24
r206	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	8,668.7	8,668.7	–	2
r209	–	8,414.0	8,440.5	–	8	–	8,414.0	8,426.1	–	6	–	8,414.0	8,426.1	–	6	–	–	–	–	–
r210	–	8,893.7	8,893.7	–	2	–	–	–	–	–	–	–	–	–	–	–	8,893.7	8,902.3	–	4
c201	–	5,891.0	5,963.3	–	4	–	6,035.4	6,035.4	–	2	–	6,035.4	6,035.4	–	2	–	6,140.8	6,140.8	–	2
c202	–	5,891.0	5,963.3	–	4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
c205	–	5,864.0	5,936.3	–	4	–	6,005.9	6,005.9	–	2	–	6,005.9	6,005.9	–	2	–	6,119.3	6,119.3	–	2
c206	–	5,860.0	5,932.3	–	4	–	6,003.8	6,003.8	–	2	–	6,003.8	6,003.8	–	2	–	6,099.1	6,099.1	–	2
c207	–	5,858.0	5,858.0	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
rc201	–	12,559.4	12,618.0	12,624	184	–	12,559.4	12,608.2	12,664	146	–	12,559.4	12,609.9	12,664	134	5,570.6	12,559.4	12,618.0	12,618	174
rc202	–	10,880.8	10,892.5	–	10	–	10,880.8	10,892.5	–	8	–	10,880.8	10,892.5	–	8	–	–	–	–	–
rc205	–	11,476.1	11,479.3	–	16	–	11,476.1	11,479.2	–	10	–	11,476.1	11,479.2	–	10	–	11,476.1	11,479.3	–	20
rc206	–	10,386.0	10,402.5	–	26	–	–	–	–	–	–	–	–	–	–	–	10,386.0	10,402.5	–	26
rc207	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	9,473.1	9,476.3	–	16

Table 17 Aggregate Comparison Between Pricing Algorithms with Different Cuts

Algorithm	No. of instances	Avg. rool LB	Avg. Best LB	Avg. time (s)	Avg. Tree
$\mathcal{A}_1$	241	6,870.2	7,026.0	1,899.5	744.1
$\mathcal{A}_2$	246	6,870.3	7,032.8	1,877.6	746.3
$\mathcal{A}_3$	254	6,881.1	7,032.5	1,841.1	780.9
$\mathcal{A}_4$	258	6,881.9	7,028.7	1,806.8	549.6
$\mathcal{A}_5$	267	6,905.4	7,048.2	1,536.7	462.6

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Appendix.

References

Balas, E. 1975. Facets of the knapsack polytope. *Mathematical Programming* **8** 146–164.

Baldacci, R., A. Mingozzi, R. W. Calvo. 2011a. An exact method for the capacitated location-routing problem. *Operations Research* **59**(5) 1284–1296.

Baldacci, R., A. Mingozzi, R. Roberti. 2011b. New route relaxation and pricing strategies for the vehicle routing problem. *Operations Research* **59**(5) 1269–1283.

Baldacci, R., A. Mingozzi, R. Roberti. 2012. Recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints **218**(1) 1–6.

Baldacci, R., A. Mingozzi, R. Roberti, R. W. Calvo. 2013. An exact algorithm for the two-echelon capacitated vehicle routing problem. *Operations Research* **61**(2) 298–314.

Bräysy, O., M. Gendreau. 2005a. Vehicle routing problem with time windows, Part I: Route construction and local search algorithms. *Transportation Science* **39**(1) 104–118.

Bräysy, O., M. Gendreau. 2005b. Vehicle routing problem with time windows, Part II: Metaheuristics. *Transportation Science* **39**(1) 119–139.

Chabrier, A. 2006. Vehicle routing problem with elementary shortest path based column generation. *Computers and Operations Research* **33**(10) 2972–2990.

COIN CLP. 2011. COIN-OR linear programming solver. `https://projects.coin-or.org/Clp`.

Contardo, C., R. Martinelli. 2014. A new exact algorithm for the multi-depot vehicle routing problem under capacity and route length constraints. *Discrete Optimization* **12** 129–146.

Cook, W., J. L. Rich. 1999. A parallel cutting plane algorithm for the vehicle routing problem with time windows. *Technical Report TR99-04, Computational and Applied Mathematics, Rice University, Housten, USA* .

Degraeve, Z., R. Jans. 2007. A new dantzig-wolfe reformulation and branch-and-price algorithm for the capacitated lot-sizing problem with setup times. *Operations Research* **55**(5) 909–920.

Desaulniers, G, J. Desrosiers, M. M. Solomon. 2005. *Column Generation*. Springer.

Desaulniers, G., F. Lessard, A. Hadjar. 2008. Tabu search, partial elementarity, and generalized k-path inequalities for the vehicle routing problem with time windows. *Transportation Science* **42**(3) 387–404.

Desrochers, M. 1986. La fabrication d'horaire de travail pour les conducteurs d' autobus par une methode de generation de colonnes.

Desrochers, M., J. Desrosiers, M. Solomon. 1992. A new optimization algorithm for the vehicle routing problem with time windows. *Operations Research* **40**(2) 342–354.

Feillet, D., P. Dejax, M. Gendreau, C. Gueguen. 2004. An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems. *Networks* **44**(3) 216–229.

Gendreau, M., C.D. Tarantilis. 2010. Solving large-scale vehicle routing problems with time windows: The state of the art. Tech. Rep. 2010-04, Montreal, QC, Canada, CIRRELT.

Gromicho, J., J. J. van Hoorn, A. L. Kok, J. M. J. Schutten. 2012. Vehicle routing with restricted loading capacities. *Beta Working Paper, Eindhoven University of Technology* (804).

Irnich, S., D. Villeneuve. 2006. The shortest path problem with resource constraints and k-cycle elimination for $k \geq 3$. *INFORMS Journal on Computing* **18**(3) 391–406.

Jespen, M., B. Petersen, S. Spoorendonk, D. Pisinger. 2008. Subset-row inequalities applied to the vehicle-routing problem with time windows. *Operations Research* **56**(2) 497–511.

Kallehauge, B. 2008. Formulations and exact algorithms for the vehicle routing problem with time windows. *Computers and Operations Research* **35**(7) 2307–2330.

Kaparis, K., A. N. Letchford. 2008. Local and global lifted cover inequalities for the 0-1 multidimensional knapsack problem. *European Journal of Operational Research* **186** 91–103.

Kohl, N., J. Desrosiers, O. B. G. Madsen, M. M. Solomon, F. Soumis. 1999. 2-path cuts for the vehicle routing problem with time windows. *Transportation Science* **33**(1) 101–116.

Laporte, G. 1992. The vehicle routing problem: an overview of exact and approximate algorithms. *European Journal of Operational Research* **59**(3) 345–358.

Laporte, G. 2007. What you should know about the vehicle routing problem. *Naval Research Logistics* **54**(8) 811–819.

Lübbecke, M. E., J. Desrosiers. 2005. Selected topics in column generation. *Operations Research* **53**(6) 1007–1023.

Mourgaya, M., F. Venderbeck. 2007. Column generation based heuristic for tactical planning in multi-period vehicle routing. *European Journal of Operational Research* **183**(3) 1028–1041.

Muter, I., J.F. Cordeau, G. Laporte. 2014. A branch-and-price algorithm for the multidepot vehicle routing problem with interdepot routes a branch-and-price algorithm for the multidepot vehicle routing problem with interdepot routes. *Transportation Science, Articles in Advances* 1–17.

Ren, Y., M. Dessouky, F. Ordóñez. 2010. The multi-shift vehicle routing problem with overtime. *Computers and Operations Research* **37** 1987–1998.

Righini, G., M. Salani. 2006. Symmetry helps: Bounded bi-directional dynamic programming for the elementary shortest path problem with resource constraints. *Discrete Optimization* **3**(3) 255–273.

Righini, G., M. Salani. 2008. New dynamic programming algorithms for the resource constrained elementary shortest path problem. *Networks* **51**(3) 155–170.

Solomon, M. M. 1987. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research* **35**(2) 254–265.

Toth, P., D. Vigo. 2002. *The vehicle Routing Problem*, vol. 9. SIAM Monographs on Discrete Mathematics and Applications. SIAM, Philadelphia.

Wolsey, L.A. 1975. Faces for linear inequalities in 0-1 variables. *Mathematical Programming* **8** 367–372.

Zonghao, G., G. L. Nemhauser, M. W. P. Savelbergh. 1998. Lifted cover inequalities for 0-1 integer programs: Computation. *INFORMS Journal on Computing* **10**(4) 427–437.

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