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K. van der Vliet, M.J. Reindorp, J.C. Fransoo

School of Industrial Engineering, Eindhoven University of Technology, Netherlands

Abstract

Reverse factoring –a financial arrangement where a corporation facilitates early payment of its trade credit obligations to suppliers– is increasingly popular in industry. Many firms use the scheme to induce their suppliers to grant them more lenient payment terms. By means of a periodic review base stock model that includes alternative sources of financing, we explore the following question: what extensions of payment terms allow the supplier to benefit from reverse factoring? We obtain solutions by means of simulation optimization. We find that an extension of payment terms induces a non-linear financing cost for the supplier, beyond the opportunity cost of carrying additional receivables. Furthermore, we find that the size of the payment term extension that a supplier can accommodate depends on demand uncertainty and the cost structure of the supplier. Overall, our results show that the financial implications of an extension of payment terms needs careful assessment in stochastic settings.

Keywords: Supply chain management, Supply chain finance, Reverse factoring, Payment delays, Simulation

1. Introduction

Trade credit is a short-term loan between firms that is linked both in terms of timing and value to the exchange of goods between them (Ferris, 1981). It is regarded as a common component of physical and service related transactions in the supply chain (Seifert et al., 2013). Recent estimates suggest that around 80-90% of the world trade is facilitated on trade credit (Williams, 2008). In the manufacturing sector, accounts receivable make up 20-25% of the total assets of firms (Mian & Smith, 1992; Fewings, 1992). The role of trade credit in our economy is extensive, and it has consequently been the topic of investigation of many studies (see Seifert et al., 2013, for an extensive review). These studies link the provision of trade credit to information asymmetry, transaction costs, hedging, moral hazard, quality assurance, and many other motives and market phenomena.

Regardless of the motive, the provision of trade credit is considered to be an investment on the microeconomic level. Its terms should therefore account for risk and opportunity cost of tying up capital in an asset, i.e., the receivable. Conventionally, financial management practices regard the provider's cost of capital as the basis for the opportunity cost (see, e.g., Brealey et al., 2011), but recent developments challenge this perspective. Indeed, the revelation of information on the risk of a specific asset can be the basis for improved financing offers, which reflect the risk of the asset concerned as opposed to that of the firm in general. Pfohl & Gomm (2009) illustrate the concept of inter-company financing transactions, which they term 'Supply Chain Finance' (SCF). The term SCF is also increasingly used by banks to denote to a range of payables financing or early payment services (Casterman, 2013). Reverse factoring is the prime example of such a service; it has received considerable recent interest from the business and research community (Tanrisever et al., 2012; Wuttke et al., 2013). Reverse factoring is essentially a development of conventional 'factoring' arrangements. In the latter, a firm independently sells one or more of its receivables to a financier - the factor - against a premium. In reverse factoring, the firm's client is also involved: the client makes an explicit payment guarantee to the factor (Klapper, 2006). This guarantee entails that the factor can offer the transaction with financing cost as low as when the client itself would apply for funds. Tanrisever et al. (2012) explain how reverse factoring entails a mitigation of information asymmetries in the capital market and thus allows for reduction of deadweight costs of borrowing. Investment grade firms, due to their credit worthiness and transparency, may therefore use reverse factoring to substantially improve the cost of credit for their suppliers.

The introduction of industry standards for reverse factoring and related services suggests that this a growing market for both intermediaries of credit and technology (Casterman, 2013). According to Hurtrez & Salvadori (2010), recent technological advances allow the service to be offered efficiently, and challenging economic conditions have accelerated adoption. Specifically, the crisis of 2008 increased the spread of short-term capital costs between large corporations and their SME suppliers; in some cases, the latter even saw their access to short-term capital cut. A study initiated by the Bank of England concludes that reverse factoring offers significant opportunities to rejuvenate lending to SME firms (Association of Corporate Treasurers, 2010). Nonetheless, many buyers also see reverse factoring as a means to reduce their own working capital costs: by offering competitively priced early payment options, they induce their suppliers to offer longer payment terms[1]. From a survey among executives, Seifert & Seifert (2011) find that buyers managed to reduce net working capital by 13% on average through reverse factoring. While literature suggests that payment terms can be reconfigured in a collaborative spirit, the approach of some buyers appears to neglect this perspective. For instance, Milne (2009) reports that a large corporation introduced reverse factoring as a 'sweetener' to an unpopular decision to move its payment terms to suppliers from 45 to 90 days. Wuttke et al. (2013) cites an executive of a major chemical firm: "We would say to our supplier, we will extend payment terms anyway. It is up to you to take our SCF[2] offer or leave it." These findings, together with Aberdeen's survey findings that 17% of its respondents experienced 'pressure' from trading partners to adopt supply chain finance, suggests that some members in these arrangements may doubt the chosen (re)configuration of contractual terms (Pezza, 2011).

Adjusting payment terms in the supply chain on the basis of a change in financing rates presumes an ability to assess the overall impact on financing costs for individual firms. Several studies make this assessment by considering the cost of capital and the (expected) funding tied up in receivables and/or payables (Randall & Farris, 2009; Hofmann & Kotzab, 2010). These studies posit that the configuration of trade credit can be made independently from operations. Others show, however, that lot-size or inventory decisions interact with the receipt and/or provision of trade credit (Gupta & Wang, 2009; Protopappa-Sieke & Seifert, 2010; Song & Tong, 2011). Following this latter group, we explore here the trade-off between the benefit of reverse factoring, i.e., cheaper financing rates, and the cost of extending payment terms. We examine this trade-off in a stochastic inventory setting. Beginning with a discrete time infinite horizon base stock inventory model of a supplier firm, we incorporate financial dimensions in the state description. Initially, we assume that the firm has only access to conventional short-term financing sources; subsequently, we extend this with the option to sell receivables through reverse factoring. In accordance with different scenarios for the opportunity cost of carrying receivables compared to the cost of factoring, we consider two discounting[3] policies

[1]Despite agreeing to longer *contractual* payment terms, the supplier can use reverse factoring to obtain early payment cheaply. The trade-off between 'longer' and 'cheaper' forms the core this study.

[2]Practitioners often use the general term Supply Chain Finance (SCF) to refer to reverse factoring in particular.

[3]We use the term 'discounting' to denote the activity of selling receivables. This term, commonly used in practice, is derived from the use of discount factors as a way to determine

for reverse factoring: manual or automatic discounting. Manual discounting is used when the opportunity cost of receivables is lower than the factoring cost, while auto discounting is used when the opportunity cost is equal to or higher than the factoring cost. In all cases our objective is the minimization of average cost per period, defined as the the sum of inventory and financing costs.

Within the operations management literature, our work contributes first of all to the relatively young research line in the area of supply chain finance (Pfohl & Gomm, 2009; Randall & Farris, 2009; Wuttke et al., 2013). In particular, we complement the work of Tanrisever et al. (2012) by examining the conditions under which reverse factoring is economically viable in a multi-period setting. We find that manual and auto discounting are to be treated as different type of systems with different accompanying trade-offs. While auto discounting allows for making a trade-off independent of inventory operations, manual discounting involves a more complex trade-off which is conditioned on demand uncertainty and the supplier firm's cost structure. These parameters affect the discounting cost, but they also impact the expected amount of receivables that needs to be carried; the overall impact on the payment term decision may consequently be difficult to predict. Furthermore, we show that the ability to extend payment terms in an economically justified fashion with manual discounting may be restricted to settings where the opportunity cost of holding receivables is low. In an extensive numerical study, we find a maximum opportunity cost of 0.5% per year for most of our settings. This corresponds to the short-term borrowing cost of investment-grade firms, which are unlikely to be the supplier in a reverse factoring arrangement.

We contribute also to an emerging research area that considers interactions between inventory and financing in multi-period stochastic setting (Maddah et al., 2004; Gupta & Wang, 2009; Babich, 2010; Protopappa-Sieke & Seifert, 2010; Song & Tong, 2011; Luo & Shang, 2013). Our experiments suggest that that the optimal cash retention level to finance a base stock operation increases asymptotically in the payment term. The value of cash retention is therefore decreasing in the payment term. Viewing a payment term as lead time, this finding conforms with the intuition of Goldberg et al. (2012), that when the lead time is very large, the system is subject to so much randomness between an occurrence of an event and its manifestation that 'being smarter' provides almost no benefit.

The remainder of this research article is structured as follows. In section 2

the present value of future - possibly risky - cash flows.

we discuss our research questions and the literature relevant to our problem. In section 3, we describe the models we implement in our simulations. In section 4, we discuss the design of our experiments. In section 5, we present the results from the experiments. In section 6, we summarize our findings and draw final conclusions.

2. Research Questions and Literature

The cost of trade credit is conventionally determined as the expected value of outstanding receivables, multiplied first by the number of days outstanding and then by firm's weighted average cost of capital (Brealey et al., 2011). This approach implies that the cost of a payment term extension is independent of variability in the demand for the firm's goods. We investigate the limitations of this convention, by modelling the firm's financial flows in a stochastic inventory setting. We hypothesise that variability in demand will influence the amount of financing necessitated by a extension of payment terms. It is well known from stochastic inventory theory that longer replenishment lead times require higher levels of safety stock, in order to hedge against intervening demand uncertainty (Zipkin, 2000). Viewing a payment term as lead time, we expect that a firm's financial position is exposed to more variability when extending payment terms. Additional delay in payment entails the possibility of incurring more cash outlays and receipts between the moment of selling goods and collecting payment. As cash flow uncertainty is associated with the need to borrow money and/or hold more cash (Opler et al., 1999), we expect that financing cost increases as function of the payment term, regardless of the opportunity cost of receivables. We thus formulate a first research question:

(RQ 1) What impact does extending payment terms have on the cost of managing a stochastic inventory operation?

The presumption that the financing needed to support physical flows is a linear function of payment terms suggests that differences in the cost of short-term credit can be exploited in a rather straightforward fashion. Firms would benefit as long as the multiplier of the initial payment term is no greater than the inverse of the multiplier of the initial financing rate. If the initial financing rate were halved, for instance, the initial payment term could be doubled. Several studies illustrate the potential savings from adjusting payment terms in accordance with this view (Randall & Farris, 2009; Hofmann & Kotzab, 2010; Wuttke et al., 2013). In contrast, by considering explicitly the effect that demand uncertainty has on financial flows in a single period model, Tanrisever et al. (2012) find that this inverse-proportional relationship will generally underestimate the cost of an extension of payment terms. We explore this finding in a multi-period setting, where firms conduct transactions in an ongoing manner. Moreover, we incorporate facets from the practical application of reverse factoring. Specifically, firms in our model have two ways accessing their receivables: manual or auto discounting. In manual discounting[4] a firm can choose the moment to discount its receivables; in auto discounting the discounting is effected as soon as the receivable is available. The very existence of manual discounting suggests that some firms may assess the opportunity cost rate of receivables as equal to or lower than the available discount rate. Buzacott & Zhang (2004) shows that in a multi-period production setting it is not always optimal to borrow up to the loan limit. Bank overdrafts work on the same principle: firms borrow money only when they need it, and are charged interest only on the amount they borrow. If opportunity cost would be higher than the discount rate, however, firms should choose auto discounting. These considerations lead to our second research question:

(RQ 2) What payment term extension would allow a supplier to benefit from reverse factoring? Specifically, what is the maximum payment term extension when the receivables holding cost is:

(a) zero?
(b) equal to cost of factoring?
(c) positive but lower than the cost of factoring?

Looking further to relevant literature, we note first the relation of our study to the growing body of research the interface of operations and finance. Work in this area generally aims to identify conditions under which a tighter integration of the two disciplines creates value or allows improved risk management (Birge et al., 2007). Imperfections in capital markets are often assumed, since an interaction between investment and financing decisions is only then possible (Modigliani & Miller, 1958). Here we proceed on the basis that there is information asymmetry between financial intermediaries and firms; the mitigation of this asymmetry by reverse factoring gives rise to the option to exploit

[4] That reverse factoring in principle allows suppliers to use manual discounting is often not explicit in the marketing literature of finance providers. Our conversations with practitioners confirm that manual discounting is also always available. The Trade Facilitation Implementation Guide of the UNECE states that a supplier can discount an invoice or wait until the end of the agreed payment term: http://tfig.unece.org/contents/reverse-factoring.htm.

cheaper credit. Specifically, the payment guarantee from the buyer to the factor entails that the supplier can discount receivables at a cheaper rate than would otherwise be possible.

Three other research topics are particularly relevant to our study: inventory incorporating payment schemes, trade credit policy, and cash management. For inventory theorists, even in deterministic settings, payment schemes undermine the conventional assumption that funding needs are related to the average physical inventory level. Beranek (1967) was one of the first to address this issue and studied the implications of alternative payment practices on the economic lot size decision. Haley & Higgins (1973); Goyal (1985); Rachamadugu (1989) further enrich this stream. Kim & Chung (1990) proposes a model to combine the lot-size decision and the discount offered to customers for paying early. Schiff & Lieber (1974) use control theory to study the relationship between inventory and accounts receivable policy. More recently, scholars have explored the significance of payment schemes by means of stochastic inventory models. For instance, Maddah et al. (2004) investigate the effect of receiving trade credit in a periodic review (s, S) inventory model. Gupta & Wang (2009) show that a base-stock inventory policy continues to be optimal when a supplier gives trade credit, but requires adaptation of the base stock parameter. Protopappa-Sieke & Seifert (2010) develop a model to determine the optimal order quantity under working capital restrictions and payment delays. Song & Tong (2011) provide a new accounting framework that allows for evaluating key financial metrics under various inventory policies and payment schemes in serial supply chains. Luo & Shang (2013)) illustrate the value of cash pooling in multi-divisional supply chains.

Most research on trade credit itself is to be found in the economics literature. Given the existence of financial intermediaries, scholars have been interested to explain the role of trade credit (see Seifert et al., 2013, for an extensive review of this literature). In addition to this economic perspective, there is normative literature that explores the optimal credit policy to customers. Most of these studies consider the trade-off between lost sales when the policy is too tight and credit losses when policy is too easy. As one of the first, Davis (1966) presents the decision to grant credit as a trade-off between the marginal revenue and cost. Bierman & Hausman (1970) and Mehta (1970) formulate the credit decision respectively in a (finite) multi-period and an infinite horizon framework. Fewings (1992) obtains closed-form solution for the value of granting credit and an upper bound on the acceptable default risk. Another series illustrates the nuances of correctly evaluating a credit policy (Oh, 1976; Atkins & Kim, 1977; Dyl, 1977; Walia, 1977; Kim & Atkins, 1978; Weston & Tuan, 1980). Nonetheless, this literature invariably assumes that inventory policy and/or procurement does not interfere with the trade credit decision.

Cash management is concerned with how firms (should) manage their cash. The economics literature identifies four capital market frictions as possible motives for holding cash (Bates et al., 2009): (1) transaction costs, (2) precaution to adverse shocks and/or costly access to capital markets, (3) taxes, and (4) agency problems. Many models for optimal cash management exist, of which the ones by Baumol (1952) and Miller & Orr (1966) are considered to be seminal. Both models propose cash control policies to balance liquidity with the foregone opportunity of holding cash. While the interaction between cash flow uncertainty and cash management has been studied already, the interaction between a stochastic inventory control, cash management, and trade credit combined is still relatively unexplored terrain. The work of (Song & Tong, 2011) is to our best knowledge the only work in this area. Similarly to us, they assume a cash retention policy based on a single parameter.

3. Models

We consider a periodic review inventory model of a firm selling to a creditworthy buyer that offers reverse factoring. Periods are indexed by the variable t. At the end of each period, the supplier firm will receive a stochastic demand $D_t \geq 0$ from the buyer. In order to have its products ready before demand is revealed, the supplier orders stock at start of each period. We assume that inventory is controlled by a base stock policy of I units. The supplier buys stock at price c per unit and sells it at price $p > c$ per unit. The ordered items are delivered immediately prior to the end of the period and the supplier pays for them upon delivery. Once demand is revealed, if it cannot be fully met from inventory, the unmet portion is back-ordered until the next period. Each backlogged unit entails a penalty cost b. For each unsold unit, the supplier incurs a storage cost $h < b$. The supplier grants the buyer a payment term of $k \in \mathbb{N}^+$ periods. The payment term starts to count from the moment that a demanded unit is met from inventory. Once revenue from prior demands are collected and costs are paid, the supplier may at the end of each period release cash to shareholders. The cash management policy releases any cash above a threshold level $T \geq 0$.

In the initial version of our the model, the supplier meets periodic expenses with cash retained from previous periods or by borrowing from a bank. Borrowing only occurs to the extent that cash is insuffiicent. We assume unlimited borrowing capacity. The model can impose a credit limit, but this forces much of our focus to lie on default events instead of purely on the change in financing needs that results from a payment period extension. In every period the supplier receives the money from the sales realised k periods ago. The supplier's total payment periodic payment is P_t, which includes a fixed cost f and variable expenses for the replenishment of its stock, inventory (holding and shortage) costs, and interest for debt outstanding during the period t. The annual interest charge for bank borrowing is $\beta\%$ per monetary unit per period. As cash retained in the firm could have been invested elsewhere by the owner, an opportunity cost of $\alpha\%$ per period is assessed on each monetary unit retained. While in a perfect capital market $\alpha = \beta$, we assume that capital market frictions may entail $\alpha < \beta$ or $\alpha > \beta$ (Myers & Majluf, 1984). Furthermore, analogous to the opportunity cost of cash, an opportunity cost of $\eta\%$ per year is assessed on each monetary unit of accounts receivable that result from the payment term. We assume $\eta < \alpha$ as the risk of investing in an account receivable is lower than the risk of investing in the firm itself. Indeed, while settlement of the account receivable is due after a known delay, the timing of cash dividends from the firm depends on demand and realised profits, and is consequently uncertain.

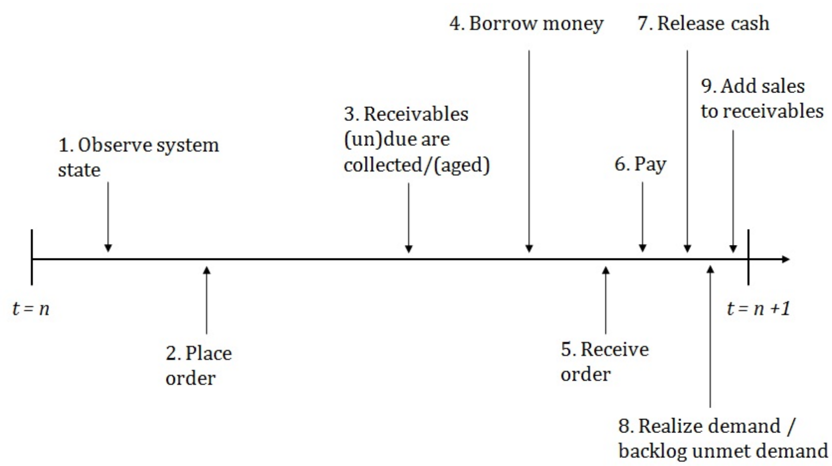


Figure 1: The sequence of events within a single period.

The system state at the start of period t is $S_t = (x_t, m_t, \boldsymbol{r}_t)$. The scalar x_t represents the inventory position and the scalar m_t represents the cash position. A tangible cash balance or a bank overdraft is represented by $m_t > 0$ or $m_t < 0$ respectively. The k-dimensional vector $\boldsymbol{r}_t$ with components $r_{t,i}$ for $i = 1, 2, \ldots, k$ represents the outstanding accounts receivable, i.e., the payments to be collected at the end of periods $t, t+1, \ldots, t+k-1$. The vector S_t conveys all information needed to implement the ordering and cash retention policies in period t: how many units needed to reach base stock, the associated cash payment and the amount of cash that will be received.

Figure 1 summarises the sequence of events in a period. At the start of period S_t is observed (1) and the order is placed (2). At the end of the period, cash is collected from the oldest accounts receivable and the position of others is decremented (3). Money is borrowed if needed (4), the units ordered are received (5) and the cash payment is made (6). If policy allows, cash is be released from the firm (7). Finally, demand is received and met to the extent that inventory allows. This creates a new account receivable (9).

Since the initial version of our model includes only a conventional source of short-term financing, bank borrowing, we henceforth refer to this as the 'conventional financing' model (CF). The mathematical formulation of this model is given in Section 3.1 below. In Section 3.2 we describe extensions to CF that model the application of reverse factoring. These extensions represent, respectively, the 'manual discounting' model (MD), where discounting occurs as soon cash deficits arise (Section 3.2.1), and the 'auto discounting' model (AD), where discounting is always applied as soon as possible (Section 3.2.2).

3.1. Conventional financing model (CF)

The transition equations for inventory, cash, and receivables are as follows:

$$x_{t+1} = I - D_t \tag{1}$$

$$r_{t+1,i} = \begin{cases} ((-x_t)^+ + I \vee D_t)p & i = k \\ r_{t,i+1} & i = 1, \ldots, k-1 \end{cases} \tag{2}$$

$$m_{t+1} = \begin{cases} T & m_t + r_{t,1} - P_t \geq T \\ m_t + r_{t,1} - P_t & m_t + r_{t,1} - P_t < T \end{cases} \tag{3}$$

where

$$P_t(I, x_t, m_t) = f + (I - x_t)c + h(x_t)^+ + b(-x_t)^+ + \beta(-m_t)^+,$$
$$(a)^+ = max\{0, a\},$$
$$\text{and } a \vee b = min\{a, b\}.$$

For a specific joint base stock and cash management policy $Z = (I, T)$, we define $G_{CF}(Z)$ to be the long-run average cost per period.

$$\mathrm{G}_{CF}(Z) = \lim_{t \to \infty} \frac{1}{t} \sum_{t=1}^{\infty} h(x_t)^+ + b(-x_t)^+ + \beta(P_t(I, x_t, m_t) - m_t - r_{t,1})^+$$
$$+ \alpha((m_t + r_{t,1} - P_t(I, x_t, m_t)) \vee T)^+ + \eta \sum_{i=1}^{i=k} r_{t,i}. \tag{4}$$

This includes direct costs for inventory and borrowing and opportunity costs assessed on cash management and receivables. We wish to find the policy $Z^* = (I^*, T^*)$ that minimises $G_{CF}(Z)$.

Note that the cash management cost is linked to uncertainty in the match between incoming and outgoing cash flows. If demand were constant, the firm would always be able to match these flows and would not need to borrow money and/or retain cash. Furthermore, note that there is an interaction between the base stock level I and the cash retention level T. The replenishment cost in period t depends on I and D_{t-1}, the demand of the preceding period. The cash available to meet the replenishment cost depends on T and the size of the met demand in period $t-k$. Even when the payment term is only one period a deficit can arise, since backlogged demand is included in the immediate replenishment cost but revenue is delayed.

3.2. Reverse factoring model extensions

Reverse factoring allows a firm to advance receipt of cash from receivables through discounting. The scalar $0 < \gamma < 1$ is the discount rate applicable when advancing cash from a receivable that is due in one period . Following the discussion above we set $\gamma < \beta$, so reverse factoring is preferred over borrowing cash for one period. The static k-dimensional vector $\boldsymbol{\gamma}$ with components γ_j represents the rates applicable for discounting receivables that are otherwise due j periods from the beginning of current period . We set $\gamma_1 = 0$ since the receivable $r_{t,1}$ is due anyway at the end of period t. For $j > 1$ we set $\gamma_j = \gamma_{j-1} + \gamma$. The discount factor $1 - \gamma_j$ is applied to the receivable amount at the moment the receivable is discounted. As the discount percentage increases in the time remaining until a receivable is due, the firm discounts receivables in order of increasing maturity, i.e., the younger ones first. If the firm discounts all receivables but still needs more cash, conventional borrowing is used. The sequence of events with reverse factoring is the same as in Figure 1, except that at (4) the firm discounts receivables as needed and available, before resorting to borrowing.

Next we discuss the further model extensions that accommodate the two ways of applying reverse factoring: manual discounting or auto discounting.

3.2.1. Manual discounting (MD)

We assume that receivables can be partially discounted, in order close exactly the cash deficit that may arise in a period. In practice the factor may require that the full value of a receivable must be discounted, which would reduce the value of reverse factoring mechanism. With manual discounting, the transition equations for receivables (2) and cash (3) are changed as follows.

$$r_{t+1,i} = \begin{cases} ((-x_t)^+ + I \vee D_t)p & i = k \\ (1-\varphi_{t,i})r_{t,i+1} & \text{otherwise} \end{cases} \tag{5}$$

where

$$\varphi_{t,i} = \frac{(1-\gamma_n)r_{t,i+1} \vee (P_t - m_t - \sum_{n=1}^{n=i}(1-\gamma_n)r_{t,n})^+}{(1-\gamma_n)r_{t,i+1}}$$

is the fraction of $r_{t,i+1}$ that needs to be discounted to meet the cash need.

$$m_{t+1} = \begin{cases} T & m_t + r_{t,1} - P_t > T \\ m_t + r_{t,1} - P_t & 0 \leq m_t + r_{t,1} - P_t \leq T \\ 0 & m_t + \sum_{n=1}^{n=k}(1-\gamma_n)r_{t,n} - P_t \geq 0 \\ m_t + \sum_{n=1}^{n=k}(1-\gamma_n)r_{t,n} - P_t & \text{otherwise.} \end{cases} \tag{6}$$

The there are four possible outcomes for the cash position that are captured by the respective cases in (6):

(a) After paying P_t and without discounting any receivables, the firm's cash position exceeds T. Excess cash is released to shareholders and the cash position returns to T.

(b) After paying P_t and without discounting any receivables, the firm's cash position is non-negative, but less then or equal to T. No cash is released to shareholders.

(c) The firm must discount some receivables to meet P_t. Receivables are discounted so that the cash position is equal to zero. No cash is released to shareholders.

(d) Even after discounting all receivables, the firm has insufficient cash to meet P_t. Borrowing occurs, so the cash position is negative. No cash is released to shareholders.

Again we wish to find the policy $Z^* = (I^*, T^*)$ that minimises the long run average cost per period, but the new objective function $\mathrm{G}_{MD}(Z)$ includes factoring as well as conventional borrowing:

$$\mathrm{G}_{MD}(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} h(x_t)^+ + b(-x_t)^+ + \beta(P_t(I, x_t, m_t) - m_t - r_{t,1})^+$$

$$+ \alpha((m_t + r_{t,1} - P_t(I, x_t, m_t)) \vee T)^+ + \sum_{i=1}^{k} (\gamma_i \varphi_{t,i} + \eta(1 - \varphi_{t,i})) r_{t,i}. \tag{7}$$

3.2.2. Auto discounting (AD)

In this setting the supplier discounts the full value of any receivable as soon as it is possible to do so. Due to sequence of events (Figure 1), holding costs for receivables are still incurred for one period. With auto discounting, the transition equations for receivables(2) and cash (3) are changed as follows.

$$r_{t+1,i} = \begin{cases} ((-x_t)^+ + I \vee D_t)p & i = k \\ 0 & i = i, .., k-1 \end{cases} \tag{8}$$

$$m_{t+1} = \begin{cases} T & m_t + (1 - \gamma_k) r_{t,k} - P_t > T \\ m_t + (1 - \gamma_k) r_{t,k} - P_t & \text{otherwise.} \end{cases} \tag{9}$$

We aim to find the policy $Z^* = (I^*, T^*)$ that minimises the long run average cost per period, as defined by the objective function $\mathrm{G}_{AD}(Z)$:

$$\mathrm{G}_{AD}(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} h(x_t)^+ + b(-x_t)^+ + \beta(P_t(I, x_t, m_t) - m_t - \gamma_k r_{t,k})^+ +$$

$$\alpha((m_t + \gamma_k r_{t,k} - P_t(I, x_t, m_t)) \vee T)^+ + (\eta + \gamma_k) r_{t,k}. \tag{10}$$

4. Algorithm and Experimental Design

The objective function of a base stock policy in a pure inventory setting is convex (Porteus, 2002), but the inclusion of cash and receivables in the state description of our model precludes a comparable analytic insight. Besides the increased size of the state space, the interaction between the base stock parameter and cash retention parameter complicates analysis. In an initial exploration of the solution space by means of simulation, we find the objective function to exhibit convexity in both decision variables for all system configurations (CF, MD and AD) and a range of parameter values. Specifically, in all cases we find a unique policy Z^* that yields globally minimal average cost, and no policy that yields a local extreme point. Based on this insight, we utilize a 3-stage algorithm in our subsequent simulation experiments, in order to find the globally optimal policy efficiently. In this section we describe first this algorithm; see also Algorithm 1. Afterwards we describe in detail the design of our experiments. The results of the experiments are given in Section 5.

4.1. Solution Algorithm

In its first two stages, the algorithm determines a truncated search interval, $[I_l, I_u] \times [T_l, T_u]$, through iterative gradient estimations in each policy dimension. In the third stage, stochastic approximation is used to find the optimal policy. Stochastic approximation is an iterative scheme that attempts to find a zero of the gradient of the objective function. It has been widely studied since the pioneering works of Robbins & Monro (1951) and Kiefer & Wolfowitz (1952) (Fu, 2006; Broadie et al., 2009). We use a multidimensional version of the Kiefer-Wolfowitz algorithm, which was first introduced by Blum (1954). As algorithms of this type are prone to poor finite-time performance, we make two improvements to reach faster convergence, following the proposals of Broadie et al. (2009). First, we use different tuning sequences in each dimension, in order to adapt better to the different convexity characteristics of each. Second, to avoid long oscillatory periods, we check in each iteration whether the next policy would be located within the truncated search interval; if it goes outside, the tuning sequence is amended to ensure that the next policy policy lies again within the search interval.

4.2. Experimental design and parameter settings

We answer our research questions by means of four sets of simulation experiments: Experiment 1 and Experiment 2(a) - Experiment 2(c), corresponding to the numbering of the research questions in Section 2. In each experiment, we explore 3×3 basic settings: all combinations of three possible levels for the expected net profit margin, $\omega = (\mu_D(p-c) - f)/\mu_D p$, and three possible levels for operating leverage, $\psi = f/(\mu_D c + f)$. Operating leverage is a measure of the relationship between fixed cost and total cost for a firm (Brealey et al., 2011). The specific values of p, c and f that underlie the nine basic settings are shown in Table 1.

In all experiments we take demand to be log-normally distributed with mean $\mu_D = 10$ and coefficient of variation $c.v. = \mu_D/\sigma_D$ equal to either 0.25 or 0.50. Full detail of demand and cost parameters for each experiment appears in Table 2. In all experiments, one period corresponds to one week. Table 2 shows annual financing rates, which are converted to weekly rates under the assumption of simple interest.

Algorithm 1 Stochastic approximation algorithm for determination of Z^*

Step 0: Choose algorithm parameters

- initial step sizes a_0^k for $k = 1, 2$; default values $a_0^1 = 1, a_0^2 = 2$;
- initial policy $Z_0 = (I_0, T_0)$; default value $Z_0 = (\mu_D, 0)$;
- stopping condition υ; default value $\upsilon = 1 \times 10^{-6}$;

Step 1: Localise $[I_l, I_u]$, the search interval for the base stock parameter
Set $Z_n = (\mu_D + na_0^1, 0)$ for $n \in \mathbb{N}^+$. Evalute iteratively the gradient estimation $\tilde{G}'(n) = (\tilde{G}(Z_{n+1}) - \tilde{G}(Z_n))/a_0^1$. In each iteration, increase the number of replications dynamically until the confidence interval of the estimation, $[\tilde{G}_{LB}, \tilde{G}_{UB}]$, indicates a statistically significant direction, i.e., $\tilde{G}'_{LB}(n) > 0$ or $\tilde{G}'_{UB}(n) < 0$. If $\tilde{G}'_{UB}(n) < 0$, then $\{n\} \leftarrow \{n+1\}$; if $\tilde{G}'_{LB}(n) > 0$, store values as indicated below and move to step 2.

- initial policy for step 2: $\{Z_0\} \leftarrow \{Z_n\}$;
- base stock search interval for step 3: $[I_l, I_u] = [I_n - a_0^1, I_n + a_0^1]$.

Step 2: Localise $[T_l, T_u]$, the search interval for the cash retention parameter
Starting at policy Z_0, evaluate iteratively the gradient $\tilde{G}'(n)$ with $Z_n = (I_0, 0 + na_0^2)$ until $\tilde{G}'_{LB}(n) > 0$ or $\tilde{G}'_{UB}(n) < 0$. If $\tilde{G}'_{UB}(n) < 0$, then $\{n\} \leftarrow \{n+1\}$; if $\tilde{G}'_{LB}(n) > 0$ store values as indicated below and move to step 3.

- the initial policy for step 3: $\{Z_0\} \leftarrow \{Z_n\}$;
- the cash retention search interval $[T_l, T_u] = [T_n - a_0^2, T_n + a_0^2]$.

Step 3: Determine the joint optimal policy: Z^*

- Set $\{a_0^k\} \leftarrow \{0.1a_0^k\}$, $\{\tau^k\} \leftarrow \{0.1a_0^k\}$, and $\{\lambda^k\} \leftarrow \{0\}$ for $k = 1, 2$.
- Evaluate $\tilde{G}' = (\tilde{G}(I_0 + a_0^1, T_0) - \tilde{G}(Z_0))/a_0^1$ and set $\{\theta^1\} \leftarrow \{1/|\tilde{G}'_1|\}$.
- Evaluate $\tilde{G}' = (\tilde{G}(I_0, T_0 + a_0^2) - \tilde{G}(Z_0))/a_0^2$ and set $\{\theta^2\} \leftarrow \{1/|\tilde{G}'_2|\}$.

Use the following recursion to calculate Z_{n+1}:

$$Z_{n+1} = Z_n - \left(a_n^1 \frac{\tilde{G}(Z_n + c_n^1) - \tilde{G}(Z_n)}{c_n^1}, a_n^2 \frac{\tilde{G}(Z_n + c_n^2) - \tilde{G}(Z_n)}{c_n^2} \right),$$

where

- $\boldsymbol{c}_n$ with $c_n^k = \tau^k / n^{\frac{1}{4}}$ for $k = 1, 2$ is the sequence of finite difference widths,
- $\boldsymbol{a}_n$ with $a_n^k = \theta^k / (n + \lambda^k)$ for $k = 1, 2$ is the sequence of step sizes.

In each iteration, check:

- if $|\tilde{G}(Z_{n+1}) - \tilde{G}(Z_n)| < \upsilon$, return Z_{n+1} and terminate search;
- if $I_{n+1} < I_l$ or $I_{n+1} > I_u$, adapt λ^1 such that $I_l \leq I_{n+1} \leq I_u$;
- if $T_{n+1} < T_l$ or $T_{n+1} > T_u$, adapt λ^2 such that $T_l \leq T_{n+1} \leq T_u$.

$\psi \backslash \omega$	0.1	0.2	0.3
0	10, 9, 0	10, 8, 0	10, 7, 0
0.3	10, 6.3, 27	10, 5.6, 24	10, 4.9, 21
0.6	10, 3.6, 54	10, 3.2, 48	10, 2.8, 42

Table 1: Values of unit selling price p, unit cost c, and total fixed cost f underlying the experimental settings of net profit margin ω and operating leverage ψ.

Experiment	1	2(a)	2(b)	2(c)
Scenario	$CF_{\eta=0\%}$	$CF_{\eta=0\%}$ vs. $MD_{\eta=0\%}$	$CF_{\eta=8\%}$ vs. $AD_{\eta=\gamma\%}$	$CF_{\eta>0\%}$ vs. $MD_{\eta>0\%}$
μ_D	10	10	10	10
$c.v.$	0.25, 0.5	0.25, 0.5	0.25, 0.5	0.25
h	0.02	0.02	0.02	0.02
b	0.1,0.2,0.4	0.2	0.2	0.2
α	4%,8%,12%	8%	8%	8%
β	8%	8%	8%	8%
η	0%	0%	8%	0.5%,1%,2%,4%
k	1-10	n.a.	n.a.	n.a.
k_0	n.a.	2,4	2,4	2,4
k_e	n.a.	2-12,4-14	2-12,4-14	2-12,4-14

Table 2: Demand and cost parameter settings.

The next paragraphs describe explicitly our four experiments. Although we determine the optimal policy Z^* for every experimental instance, we are generally most interested to compare policies or the performance of the system across different payment terms. Consequently, in order to facilitate the presentation, we use $Z^*(k) = (I^*(k), T^*(k))$ to denote the optimal policy for payment term k, and we rewrite the objective functions as $G_{(.)}(k)$, supressing the immediate dependence on Z^*.

Experiment 1: *The impact of payment terms with conventional financing and no opportunity cost of holding receivables.* Here we explore how payment terms impact total financing cost for the supplier firm when the opportunity cost of receivables is neglected, i.e., $\eta = 0\%$. In addition to the experimental settings shown in Table 1, we test the sensitivity of our findings to changes in other factors: the relative magnitude of inventory backlog cost b to inventory holding cost h, and the relative magnitude of cash opportunity cost α to conventional financing cost β.

Experiment 2(a): *Maximum payment term extension with no opportunity cost of holding receivables.* We explore the trade-off between cheaper credit and extended payment terms in reverse factoring when there is no opportunity cost of holding a receivable, i.e., $\eta = 0\%$. For each initial payment term k_0, we determine $Z^*(k_0)$ when only conventional financing at rate β is used. Then, with reverse factoring at rate $\gamma \leq \beta$ also available, we determine the maximum extended payment term $k_e \geq k_0$ such that $G_{MD}(k_e) \leq G_{CF}(k_0)$.

Experiment 2(b): *Maximum payment term extension with greatest opportunity cost of holding receivables.* We set the opportunity cost of holding receivables equal to the cost of factoring, $\eta = \gamma$, but otherwise explore the same trade-off as in Experiment 2(a). Accordingly, we seek the maximum extended payment term $k_e \geq k_0$ for which $G_{AD}(k_e) \leq G_{CF}(k_0)$.

Experiment 2(c): *Maximum payment term extension with intermediate opportunity cost of holding receivables.* Again we explore the same basic trade-off as in Experiment 2(a), but now the opportunity cost of holding receivables is less than reverse factoring rate, $0 < \eta < \gamma$. We determine the maximum extended payment term $k_e \geq k_0$ such that $G_{MD}(k_e) \leq G_{CF}(k_0)$.

In all experiments we let the system start with zero cash, zero inventory, and zero receivables. We begin to assess performance after a warm-up of 500 periods, which is determined based on Welch's procedure (Welch, 1983; Law & Kelton, 2000). We calculate 95% confidence intervals from 30 independent replications, each with total run-length of 20,000 periods (including warmup). Relative relative error is approximately 0.5% (Law & Kelton, 2000). After some initial global calibration, we were able to locate the optimal policy and cost for each setting on a ordinary personal computer within two or three minutes.

5. Numerical results

Here we present and discuss the results from each experiment. Section 5.1 covers Experiment 1, the impact of a payment term extension on the firm's financing cost, and the accompanying sensitivity analysis for changes in inventory and cash management cost parameters. Sections 5.2 - 5.4 cover respectively Experiments 2(a) - 2(c), the trade-off between payment term extension and reverse factoring for the three different scenarios of opportunity cost of holding receivables.

5.1. The impact of payment terms with conventional financing and no opportunity cost of holding receivables.

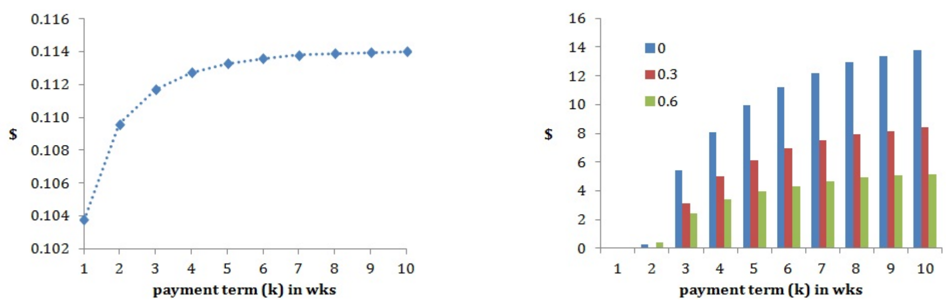
In all configurations of this experiment, we observe the following general relationship between financing cost and payment term:

The optimal financing cost $G_{CF}(k)$ increases asymptotically in the payment term k.

Figure 2a illustrates this finding. While the optimal cash retention level $T^*(k)$ increases asymptotically, the optimal base stock level $I^*(k)$ decreases slightly or remains constant. Changes in the base stock level occur because a backlog event delays the receipt of cash, which may entail a financing need. The relative impact of this is greater when the payment term is short, as the base stock then tends to be higher. Despite changes in the base stock level, changes in inventory cost appear statistically insignificant across the different payment term settings. The increase in cost from a payment term extension can thus be entirely attributed to greater variability in cashflow. As there is no opportunity cost for holding receivables in this experiment ($\eta = 0$), we conclude that a payment term extension entails greater financial costs than such opportunity costs alone.

The apparent concavity of the objective function implies that the relative cost of extending payment terms decreases with the pre-existing payment term. This makes intuitive sense. In a system with arbitrarily long payment terms, the incoming and outgoing cashflows become essentially independent. Additional delay in cash receipts resulting from a payment term extension should have negligible impact. The optimal cash retention level thus increases asymptotically. Being 'smarter' with cash provides little benefit when payment terms are very long. This same argument has already been used to explain the asymptotic optimality of constant-order policies for lost sales inventory models with large lead times (Goldberg et al., 2012).

Since we set $\alpha = \beta$ in this experiment, the cash retention level is the result of a trade-off that minimises the amount of capital needed for running the base stock operation. According to conventional finance literature, capital market frictions form the main motivation to retain and/or optimise cash (Myers & Majluf, 1984; Bates et al., 2009). Frictions can cause $\alpha \neq \beta$. In the last part of this section we therefore present a sensitivity study on the impact of these frictions.



(a) $G_{CF}(k)$ with fixed parameters $c.v. = 0.25, \omega = 0.2, \psi = 0.3$

(b) $T^*(k)$ with $c.v. = 0.25, \omega = 0.2$, and varing levels of operating leverage ψ.

Figure 2: The impact of trade credit on system cost and cash retention

Turning to the basic parameters that define our experimental scenarios, we examine their effect on the relative cost of a payment term extension. Specifically, if $G_{CF}(k)$ is the cost of an initial payment term k, then $\Delta G(k) \equiv (G_{CF}(k+1) - G_{CF}(k))/G_{CF}(k)$ is the relative cost of extending the payment term by one week. The results of Experiment 1 then support the following assertion:

The firm's relative cost of a payment term extension $\Delta G(k)$ is increasing in the coefficient of variation for demand, but decreasing in the initial payment term k, the net profit margin ω, and the operating leverage ψ.

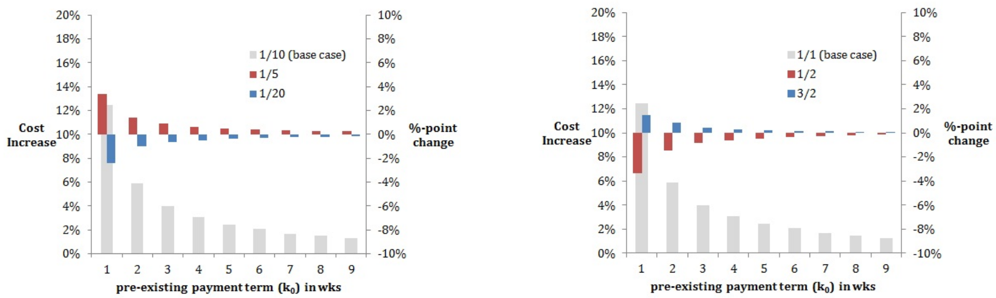
Sample paths show that a higher demand uncertainty causes higher uncertainty in the incoming and outgoing cash flows, exacerbating the impact of the payment term extension. The cash deficits or excesses accumulated will each tend to be greater in magnitude. A lower net profit margin or a lower operating leverage also increases the firm's sensitivity to an extension of payment terms. The effect of a higher net profit margin is intuitively reasonable, since it provides a greater buffer against the potential deficits that arise from a mismatch between incoming and outgoing cash flows. The effect of operating leverage is less obvious, since fixed cost are often considered to be burdensome. Firms with high operating leverage are even considered to more risky by investors (Brealey et al., 2011). While we indeed find that a higher operating leverage may imply a higher absolute cost, higher operating leverage makes the firm less sensitive to payment term extension. This appears to result from the relative stability of the outgoing cash flows for firm with higher operating leverage. Moreover, the optimal cash retention level decreases with the operating leverage. Figure 2b illustrates this. Firms that rely more heavily on external purchases need to keep more cash to competitively sustain their payment term to customers than firms that rely more on internal production with fixed costs. While having a variable cost structure may seem attractive as a means to handle with lower demand realisations in a seasonal setting, it has negative implications for the ability to match incoming and outgoing cash flows when demand patterns are more stationary.

Sensitivity Study for Other Parameters

As the cost of a payment term extension is the basis of exploration in our subsequent experiments, we explore the sensitivity of our results to changes in the relative magnitude of inventory costs, h and b, and the relative magnitude of financing costs, α and β. These studies support the following assertion:

> *The relative cost of a payment term extension is increasing in the ratio h/b and in the ratio α/β.*

Changes in the inventory or financing cost ratios have a significant effect on the optimal base stock and cash retention level, but a minor effect on the the increase in financing cost that results from a payment term extension. Figure 3 provides illustration. Figure 3a shows the impact of varying the inventory cost ratio. As the value h/b increases, the sensitivity of costs to an increase in payment terms also increases. An increase in h/b entails a decrease in the optimal base stock level, which increases the probability of backlog. As explained earlier, backlog increases the probability of incurring a cash deficit, as it simultaneously delays cash receipts while additional cash is needed for stock replenishment. Figure 3b shows the impact of varying the financing cost ratio. As the value α/β increases, the relative cost of a payment term extension also increases. An increase in α/β means that the cost of holding cash becomes relatively expensive in comparison to borrowing, which limits the ability to use retained cash as a protection against cashflow uncertainty. If α/β is large, the firm may completely stop holding cash, i.e., $T^* = 0$ becomes the optimal cash retention threshold.



(a) Percentage cost increase from a payment term extension of 1 week for base case $h/b = 1/10$ (left axis) and incremental change for $h/b = 1/5$ and $h/b = 1/20$ (right axis)

(b) Percentage cost increase from a payment term extension of 1 week for base case $\alpha/\beta = 1/1$ (left axis) and incremental change for $\alpha/\beta = 1/2$ and $\alpha/\beta = 3/2$ (right axis)

Figure 3: Cost sensitivity to a payment term extension of 1 week for varying levels of inventory cost ratio h/b and financing cost ratio α/β. Fixed parameters $c.v. = 0.5, \omega = 0.1,$ and $\psi = 0.5$.

5.2. Maximum payment term extension with no opportunity cost of holding receivables.

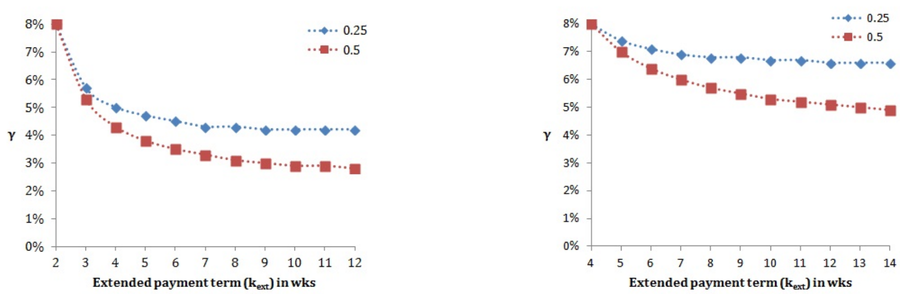
Building on the insights provided by Experiment 1, we explore the maximum payment term extension that allows a firm to benefit from reverse factoring when the opportunity cost of holding receivables is negligible. Although the payment term extension increases the total value of outstanding receivables, the firm only considers the direct cost of financing its inventory operation. We define k_e, the maximum payment term extension with reverse factoring, to be the longest payment term such that the firm no greater financing cost as it did with conventional financing and no payment term extension:

$$k_e \equiv \max k \\ \text{subject to } G_{MD}(k) - G_{CF}(k_0) \leq 0. \quad (11)$$

This experiment supports the following assertion:

When the opportunity cost of holding receivables is negligible and the firm uses manual discounting, the maximum payment term extension k_e for a given reverse factoring rate γ is decreasing in the coefficient of variation for demand, but increasing in the initial payment term k_0, the net profit margin ω, and the operating leverage ψ.

The significance of our main experimental parameters for the maximum payment term extension with reverse factoring appears consistent with their significance for the relative cost of a payment term extension in the conventional financing setting. Where previously we saw greater relative costs for an extension, here we see a smaller maximum possible extension. Figure 4 shows, for different values of initial payment term and demand uncertainty, the set of (γ, k_e) values for which the financing cost with reverse factoring and manual discounting case is equivalent to the initial cost with conventional financing. With reverse factoring and manual discounting, borrowing activity is generally reduced to negligible levels: factoring substitutes for borrowing. In some cases, most particularly when the experimental setting allows only a minimal payment term extension, borrowing may still occur. The optimal cash retention level may be positive in manual discounting ($T^* > 0$), but in contrast to the case with conventional financing, it decreases with the extended payment term. As payment terms get longer the firm acquires enough receivables to finance operations without borrowing or retaining cash.



(a) $k_0 = 2$ (b) $k_0 = 4$

Figure 4: Trade-off between factoring rate γ and maximum extended payment term k_e with manual discounting. Fixed parameters $\eta = 0, \omega = 0.2, \psi = 0; c.v. = 0.25$ or 0.50.

5.3. Maximum payment term extension with greatest opportunity cost of holding receivables.

Here we explore the maximum payment term extension that allows a firm to benefit from reverse factoring when the opportunity cost of holding receivables η is taken to be equal to γ, the cost of discounting them. The definition of maximum payment term extension k_e in this experiment is analogous to (11), but with $G_{AD}(.)$ in place of $G_{MD}(.)$. When $\eta = \gamma$, the supplier is indifferent between manual and auto discounting. Although a greater opportunity cost of holding receivables is in principle possible, the choice for auto discounting is constant at this point and beyond. The experiments here support the following assertion.

With reverse factoring and auto discounting, the maximum extended payment term k_e for a given reverse factoring rate γ is not affected by the coefficient of variation for demand, net profit margin, or operating leverage.

Figure 5 shows, for different values of initial payment term and demand uncertainty, the set of (γ, k_e) values for which the financing cost with reverse factoring and auto discounting case is equivalent to the initial cost with conventional financing. Note that the maximum payment term extension and the reverse factoring rate are inverse-proportionally related. Since the firm discounts all of its receivables in every period, periodic expenses can almost always be met. Borrowing activity is negligible in this setting, even when operating leverage and demand uncertainty are both high. When the factoring rate is equal to (or less than) the opportunity cost of holding a receivable, a decision-maker can evaluate a proposed reverse factoring arrangement independently of the stochastic and economic aspects of inventory operations.

(a) $k_0 = 2$ (b) $k_0 = 4$

Figure 5: Trade-off between factoring rate γ and maximum extended payment term k_e with auto discounting. Fixed parameters $\eta = \gamma, \omega = 0.2, \psi = 0; c.v. = 0.25$ or 0.50.

5.4. Maximum payment term extension with intermediate opportunity cost of holding receivables.

When the cost of receivables is higher than zero but below the cost of discounting, the cost of additional capital tied in receivables and the cost of additional capital required to fund cash deficits need to be accounted for in the trade-off. We wish to determine the maximum payment term extension that allows a supplier to benefit from reverse factoring. In this case the simulation results support the following assertion:

There exists an opportunity cost $\eta_{max} < \gamma$ such that no economically viable payment term extension is possible when $\eta_{max} < \eta < \gamma$. When $0 < \eta < \eta_{max}$, the maximum extended payment term k_e for reverse factoring is decreasing in η. The maximum extended payment term k_e is decreasing in the net profit margin ω and operating leverage ψ.

The maximum extended payment term appears to be highly sensitive to the opportunity cost of receivables. In this setting, the firm pays a dual premium for extended payment terms: the cost of carrying additional receivables and the cost of additional cashflow uncertainty. The possibility to extend payment terms and still realize a lower expected cost appears limited to settings where the opportunity cost of receivables is low. Table 3 shows the values of η_{max} that result from our basic experimental settings. In many cases, the opportunity cost of holding receivables must be below 0.5% if the firm is to extend payment terms and realize a reduction in financing cost. The presumption that a firm assesses it opportunity cost of holding receivables at a rate equal to or higher than the reverse factoring rate can therefore be deceiving in terms of value creation.

$\psi\backslash\omega$	0.1	0.2	0.3
0	1%	0.5%	0.5%
0.3	1%	0.5%	0.5%
0.6	1%	0.5%	0.5%

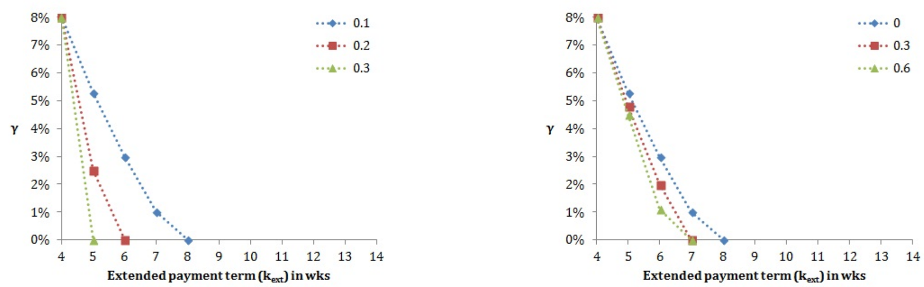
(a) $k_0 = 2, c.v. = 0.25$

$\psi\backslash\omega$	0.1	0.2	0.3
0	4%	1%	0.5%
0.3	2%	0.5%	0.5%
0.6	2%	0.5%	0.5%

(b) $k_0 = 4, c.v. = 0.25$

Table 3: Maximum opportunity cost of holding receivables η_{max} such that a payment term extension $k_e > k_0$ is possible.

In contrast to the case of $\eta = 0$ examined in Experiment 2(a), the maximum extended payment term for reverse factoring appears here to be *decreasing* in net profit margin and operating leverage. Figure 6 illustrates this. While a lower net profit margin or lower operating leverage make an extension of payment terms more costly in terms of cash flow variability, this same variability leads the firm to discount a greater proportion of its receivables; the average amount of outstanding receivables and the corresponding opportunity cost is thus ultimately lower. The latter effect tends to dominate when the opportunity cost of holding receivables increases towards η_{max}, so lower net profit margin and lower operating leverage then both facilitate longer payment terms. As $\eta \to 0$ the cost of cashflow variablity becomes more important, and direction of significance for net profit margin and operating leverage tends to reverse. This contrast shows that a good estimation of the opportunity cost of holding receivables is essential if managers are to evaluate any payment term extension proposed in a reverse factoring arrangement.



(a) Different levels of ω; $\psi = 0$ fixed. (b) Different levels of ψ; $\omega = 0.1$ fixed.

Figure 6: Trade-off between γ and k_e with manual discounting. Fixed parameters $\eta = 0.05, k_0 = 4,$ and $c.v. = 0.25$.

6. Conclusions

The market for approved receivables financing and early payment services has grown significantly over the last years (Casterman, 2013). In addition, banks and technology providers continue to invest in transactions to facilitate competitively priced credit between supply chain members (Hurtrez & Salvadori, 2010).

Despite growing business interest for supply chain finance, little is yet scientifically known about the optimal management and benefits of such innovative financing arrangements. Our study focuses on reverse factoring, an arrangement that promises improvement of working capital financing for investment-grade buyers and their suppliers. The buyer facilitates cheaper short term financing for the supplier, and the latter in return may be asked to grant longer payment terms. We couple a periodic review, infinite horizon base stock inventory model with financing by either conventional or reverse factoring, which allows us to explore the effect of a payment term extension.

The classical paradigm in literature holds that the cost of extending trade credit is related to the foregone opportunity and risk of carrying a receivable. We show first that even without considering such opportunity cost, a payment term extension will generally entail a higher cost to the supplier. Costs increase because more financing is necessary to cope with more variable cash-flows. Introducing reverse factoring, we identify settings that allow a decision maker to make a payment term decision independently of inventory, and other settings where the maximum viable payment term extension depends on demand uncertainty, net profit margin, and operating leverage. We show that these significance of these parameters for financing costs may be complex and interrelated. Correspondingly, a decision about a payment term extension may be challenging.

Based on the data of a large provider of supply chain finance services, Klapper et al. (2012) finds that creditworthy buyers receive contracts with the longest maturities from their smallest, least creditworthy suppliers. While it is known that large buyers may use their strong bargaining position in extending trade credit terms with their suppliers (Wilson & Summers, 2002), this finding can also be seen as an indication of how emerging services impacts the trade credit landscape. Features of reverse factoring suggest that the facilitation of trade credit has become easier: financing rates are low and receivables can be discounted any time during the trade credit period. Our results suggest, however, that making payment term decisions based on the expected working capital changes will not account for the dynamics of stochastic inventory operations and their interaction with financing requirements.

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