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Introduction

In cardiovascular surgery patient specific data are needed for pre-operative decision making. When making proper assumptions on the local velocity profiles, 1D wave-propagation models may be well suitable to provide clinically relevant information (e.g. wall shear stress). Existing 1D wave-propagation models use rough estimates for velocity profile functions, resulting in poor estimates for the wall shear stresses. Velocity profiles based on boundary layer theory are believed to result in better friction estimates.

Objectives

- ☐ Revisit the 1D wave-propagation theory by Hughes and Lubliner[1] and modify it using a velocity profile function based on the boundary layer theory (BLT).
- ☐ Compare resulting profiles and wall shear stresses to Womersley's theory.
- ☐ Determine the influence of the friction term in comparison to Poiseuille friction.

Methods

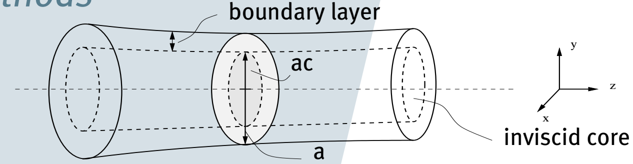

***Figure 1:** Part of a vessel (radius a) divided into a core (radius a_c) and a boundary layer (thickness δ_v).*

By dividing the flow in a vessel in an inertia dominated core and a friction dominated boundary layer (see Figure 1) and by assuming equilibrium between inertia and viscous forces at $r = a_c$, a velocity profile v is derived as a function of the flow q and the pressure gradient $\frac{\partial p}{\partial z}$.

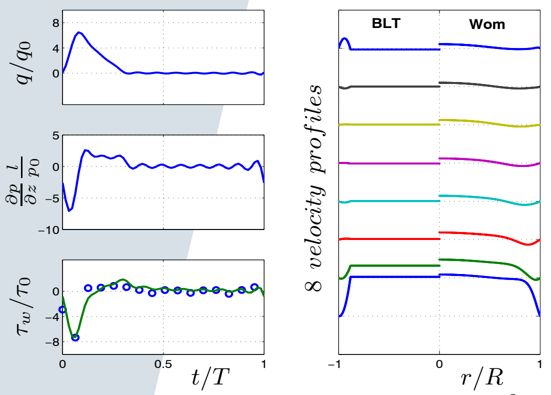

***Figure 2:** (left) Typical aortic flow q, pressure gradient $\frac{\partial p}{\partial z}$ and wall shear stresses (τ_w) derived by the BLT (○) and the Womersley theory (Wom) (-). (right) The resulting velocity profiles at 8 time steps.*

A comparison between BLT and the exact solution of the Navier-Stokes equations by Womersley is presented in Figure 2. The derived velocity profile function is used in the 1D wave propagation model to obtain an equation for mass conservation and momentum balance.

Numerical Results

An aorta-like vessel as well as an A.tibialis posterior-like vessel are modelled using 1D spectral elements. Physiological flow pulses are imposed at both inlets whereas resistances are prescribed at the outlets. Numerical results of the wave-equation using BLT and friction according to Poisseuille flow are shown below.

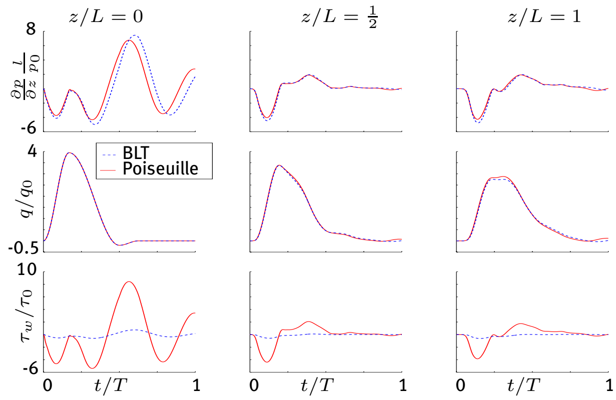

***Figure 3:** Normalized pressure gradient (top), flow (middle) and wall shear stress (bottom) as a function of time at 3 axial positions in an aorta-like tapered vessel using friction according to BLT (dashed) and Poiseuille theory (solid).*

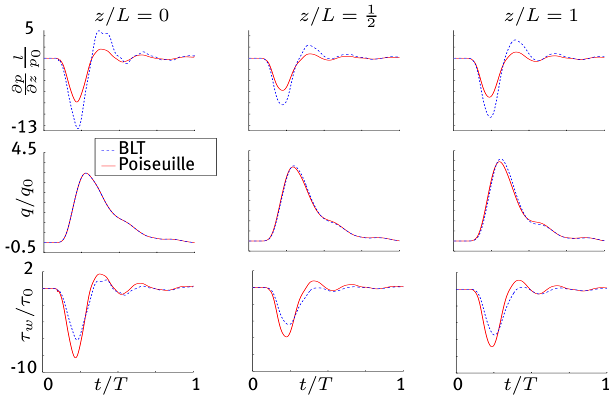

***Figure 4:** Normalized pressure gradient (top), flow (middle) and wall shear stress (bottom) as a function of time at 3 axial positions in an A.tibialis posterior-like tapered vessel using friction according to BLT (dashed) and Poiseuille theory (solid).*

Discussion

Velocity profiles obtained by BLT provide good approximations for wall shear stress when compared to Womersley's theory. Results of wave propagation simulations show that the choice of a proper velocity profile function is not only crucial in predicting the wall shear stress, but also in monitoring the pressure and flow wave characteristics.

References

1. Hughes, T. J. R. and Lubliner, J. (1973). On the one-dimensional theory of blood flow in large arteries. *Math. Biosciences*, **18**, 161-170.

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