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Practical Evaluation of Robust Control for a Class of Nonlinear Mechanical Dynamic Systems

Bram de Jager

Practical Evaluation of Robust Control for a Class of Nonlinear Mechanical Dynamic Systems

PRACTICAL EVALUATION OF ROBUST CONTROL FOR A CLASS OF NONLINEAR MECHANICAL DYNAMIC SYSTEMS

PROEFSCHRIFT

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aan de Technische Universiteit Eindhoven
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voor een commissie
aangewezen door het College van Dekanen
in het openbaar te verdedigen
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door

ABRAHAM GILLES DE JAGER

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prof. dr. ir. J. J. Kok
en
prof. ir. O. H. Bosgra

copromotor
dr. ir. F. E. Veldpaus

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Nous souhaitons la vérité, et ne trouvons
en nous qu'incertitude.
Nous cherchons le bonheur, et ne trouvons
que misère et mort.
Nous sommes incapables de ne pas souhaiter
la vérité et le bonheur
et sommes incapables ni de certitude
ni de bonheur.

(Blaise Pascal, Pensées)

Synopsis

The aim of the research reported in this thesis is threefold: to assess the robustness potential of controllers for a class of nonlinear mechanical systems, to get insight in the achievable performance of these control systems, and to grade several controller design methods according to the two aspects mentioned above.

We try to reach these goals by using a multi-stage evaluation strategy, with a literature search, numerical experiments, and laboratory experiments.

The control schemes studied are mostly based on the use of a nominal model of a more complicated nonlinear system. The model is used by the controller to linearize the nominal model, or to generate control inputs that achieve almost the same goal. This part of the controller can be combined with a stabilizing component, a robustifying component, and a parameter adaptation component. The use of acceleration signals to improve the estimation of state variables and to enhance the robustness is studied. Control concepts that adapt the structure of the controller, instead of only the parameters, are not studied.

We discuss the development of an appreciation strategy, using observation, manipulation, and experimentation techniques, to find and qualify robust controllers.

Our results show that an adaptive computed torque controller, or a computed torque controller combined with sliding mode control, should be preferred because they give smaller tracking errors than a PD controller, although the robustness properties are comparable. The use of acceleration feedback can further improve the tracking accuracy or robustness, but the acceleration signal should be relatively noise free to have some benefit. The use of an extended model, *e.g.*, for friction compensation, or higher sampling rates can also improve the control system performance. The use of extended models should be balanced with sample time requirements, to obtain an optimal mix between model accuracy and implementation accuracy.

These results are valid, both for a system that is alike the nominal design

model, and for a system with substantial unmodeled dynamics.

The quest for a robust controller design method is not finished. It is recommended to expand the results obtained in several directions, notably, to specific measures for improving the robustness of adaptive controllers and to the relation between model accuracy, model structure, performance requirements, and robustness characteristics of control systems.

Samenvatting

De doelstelling van het in dit proefschrift beschreven onderzoek is drieledig: het nagaan van de robuustheidseigenschappen van een klasse van niet-lineaire regelsystemen, het verkrijgen van inzicht in de prestaties van deze regelsystemen en het onderling vergelijken van een aantal ontwerpmethoden met aandacht voor de twee bovenvermelde eigenschappen.

De gevolgde methode om dit doel te bereiken bestaat uit drie elementen, een literatuur onderzoek, een numerieke en een experimentele evaluatie.

De onderzochte regelschemas gebruiken meestal een nominaal model van een gekompliceerder niet-lineair systeem. Dit model kan door de regelaar gebruikt worden om het niet-lineaire systeem exakt of bijna exakt te lineariseren. Het op een model gebaseerde deel van de regelaar kan aangevuld worden met een deel dat stabiliteit garandeerd, de robuustheid verhoogd, of modelparameters aanpast. Het gebruik van versnellingssignalen is ook onderzocht, waarbij de aandacht gericht was op het verbeteren van de schatting van de toestand en het verhogen van de robuustheid. Regelsystemen die niet alleeen de modelparameters maar ook de struktuur van de regelaar aanpassen vallen buiten het kader van dit onderzoek.

De ontwikkeling van een methode, gebaseerd op waarneming, manipulatie en experimenten, om de waarde van de regelschemas te beoordelen wordt besproken.

De resultaten tonen aan dat op een model gebaseerde adaptieve regelaars, eventueel gekombineerd met een schakelvlak methode, aanbeveling verdienen, omdat de prestaties het beste waren, alhoewel de robuustheid niet groter was als die van een standaard PD regelaar. Het gebruik van versnellingsterugkoppeling kan de robuustheid en/of volgfout verder verbeteren. Hiervoor moet het versnellingssignaal niet met teveel ruis verstoord zijn. Het gebruik van een verfijnder model, bijvoorbeeld voor de modellering van wrijving, of het toepassen van een kleinere bemonstertijd kan de prestatie van het regelsysteem verder doen toenemen. Het gebruik van een verfijnd model moet afgezet worden tegen de kleinere bemonstertijd, omdat beide maatregelen een grotere rekenkapaciteit vereisen, die niet altijd beschik-

baar is.

De resultaten zijn verkregen, zowel voor een nominaal model als voor een systeem dat daar sterk van kan afwijken.

De queest naar een ontwerpmethode voor robuuste regelaars is na dit onderzoek nog niet afgelopen. Het verdient aanbeveling om een aantal aspekten verder uit te zoeken. Met name kunnen worden genoemd maatregelen om de robuustheid van adaptieve regelaars te vergroten en een verder (kwantitatief) uitpluizen van de relatie tussen model nauwkeurigheid, model struktuur, prestaties van de regelaar en robuustheidseigenschappen.

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CHAPTER 1

Introduction

Let us, first of all, gently introduce the diligent reader to the subject of our investigation, and give some guidelines for reading this thesis. We do that by giving some cultural/historic and technical/economic background to motivate why and what we studied. Then we proceed with a short discussion of the methodologies that can be used for our study. To shed some more light on the subject and delineate the boundaries of our possible findings, we discuss also the limitations of this research. The thesis is not an amalgam of unrelated subjects, but has a structure. When you know this structure it is much easier to find the discussion of the subject of interest. So, some comments on this structure are included. Parts of the material discussed in this Chapter will be expanded in later Chapters, especially in Chapter 3.

To summarize, in this chapter we give

- the motivation of the research: why do we perform this specific investigation,
- the scope of the research: what do we investigate,
- the research methodology: how do we perform the investigation,
- the research limitations: what could be done another way and what is missing,
- a general outline of this thesis.

1.1. Motivation of research activities

A main reason to perform research is to satisfy human curiosity. Also, to satisfy some human needs, we want to known how our environment behaves, often with the goal of manipulation and control, to make it fulfill these needs. Research is performed by controlled manipulation of our environment (experiments), reasoning about our findings (deduction), and speculation about facts of nature which are really "Terra incognita" (generalization or induction).

This curiosity led, starting with the waning of the middle ages [1], to a growing knowledge of nature and an "Umwertung aller Werten" in our view of nature. This knowledge made it possible to control our environment in a predictable manner, with the aid of technical facilities, that again could be developed and manufactured due to growing knowledge and experience. The other way around, the technological progress had a profound influence in our view of the world [2, 3]. This manipulation of nature has often led to forceful resistance (Luddites), because the direction in which the society in general has to progress is not a "communis opinio".

Nowadays the development of technological skills and knowledge is no longer a task of individuals, but has been structured and is performed at research institutes and universities. Often researchers are working at very specialized subjects, where each research group tries to find its own niche market to fund their research. Often, this specialization has an adverse influence on the progress of technology, because researchers are even unable to speak and understand their neighbors jargon. This separation of research has led to a counter movement, the system theory view, where areas of research are connected by finding their, more abstract, commonalities. An offspring of the system theory point of view, is what is nowadays called control theory. In this theory one is concerned with analyzing and changing the (dynamic) behavior of systems, to enhance their usefulness and improve their performance.

In general, there is a (moving) boundary between technical concepts that can or cannot be applied because they are

- not yet cost effective in design, in production, in maintenance and/or in recycling,
- technological too advanced,
- conceptual too complicated to be handled by the current pool of workers,
- sensitive to variations in the production line making it difficult to step up the production from laboratory to production scale,
- not guaranteed to have a consistent quality level.

Most of these problems can be solved, but the costs can be prohibitive.

The main goal of the research performed in technical research groups is to shift the boundary between concepts that can and concepts that cannot be used to produce goods or deliver services. This should make it possible to satisfy customer needs with better fabricated products (constant quality, attractive price, safe usage, less waste, ...). New products can be introduced that satisfy (latent) needs of consumers. The question is how this thesis contributes to this shifting of the boundary. To answer this question, related to the usefulness of the research, we need to indicate the cost and the profit. The research costs are evident, but the profit has to be clarified. Therefore it is necessary to point out the specific problem, show how the problem can be solved and specify the cost, profit and profit margin of the proposed solution. The research is justified (in a narrow economical sense) if the profit margin of the proposed solution is larger then the research costs. We will not pursue this economic question, but rather elaborate the justification of the research from a technological point of view. Therefore we need a more detailed description of the research in relation to its application.

Control theory covers and is used in many application areas. The methodology commonly used in the design of control systems consist of several activities. It is often necessary to explicitly formulate the design objectives, or even to put them in some formal specifications. The system to be controlled should be known well, by using a model based on first principles (Newton's law, conservation principles, etc.), or by using model identification based on experimental data. The next step is the design of the controller itself, this requires an preliminary step where the model and formal specifications are put in some mathematical framework, *e.g.*, a state space model and weighting functions. This design is evaluated by simulation of the closed loop system. Often it is necessary to back track, because the translation to the mathematical framework does not give a one-to-one correspondence with the specifications, and repeat the controller computation with adapted weighting functions. It may be necessary to adapt the model or even the specifications because they cannot easily be obtained or perhaps they are even outside the limits of performance, to obtain a satisfactory behavior of the control system.

A short overview of some application area's and recent research topics follows. This enables us to find some common directions.

- Automotive industry:
  - Hydraulic steering for transport vehicles: makes positioning of components relative to each other (cabin ↔ chassis) much more flexible then when there is a mechanical steering system. Hydraulic steering necessitates a closed loop control system, that can be designed off-line by using a model of the system.
  - Continuous variable transmission: opens the possibility to use effectively an almost infinite number of transmission ratio that can be selected to fulfill spot needs. The selection can be performed automatic by a control system, using set points and a model of the system.
  - Advanced suspensions: improves the safety and comfort of vehicles by using a suspension that improves on the standard spring and damper type suspension. The design of the controller is model based and state space type controllers are used.

- Biochemical industry:
  - Optimal yield of bakers' yeast in fed-batch fermentation: the main problem is the lack of sensors for critical process conditions. By using available measurements and a model of the system, these conditions can then be reconstructed. Another problem is the determination of the optimal growth conditions and the optimal path to reach these conditions. These can be computed off-line with the aid of a model.
- Chemical industry:
  - Enhance dynamic behavior: damp sustained oscillations caused by closed circuit processes with long transport times during almost steady state operations.
  - Use smart sensors: the mentioned oscillation are difficult to counteract directly by process operators because the time period is often larger than the duration of the shift and the operators have limited knowledge of critical process conditions by lack of adequate sensors. So-called smart sensors, using available measurements and models of the system, can be used to present valuable information to process operators that can eventually also be used in closed loop control.
  - Transient control: optimize start-stop behavior (transients) caused by peak shaving in Cl production, where the conflicting requirements of safe and fast shut down/start up are limiting production during peak shaving. A new control system can be conceptualized by using a simulation model of the plant.
- Agricultural systems:
  - Robots for cow milking and sheep shearing: these have been developed to reduce human involvement in these activities.
  - Greenhouse climate control: to obtain a climate that stimulates the growth of plants.
- Mechanical systems:
  - Shorten cycle time: damp sustained oscillations in point to point control of robots because they enlarge the setup time. This is possible by using model based control, *e.g.*, computed torque, to get a fast response without overshoot and sustained oscillations when properly tuned.
  - Shorten cycle time: enable the use of light and flexible links in mechanism that can move faster with the same motors than mechanism with stiff and therefore heavy links. The additional vibrational modes can be suppressed by judiciously manipulating the motor torques, based on knowledge of a few low frequency modes.
  - Robustness enhancement: in the production of mechatronic mass production goods one tries to make the controlled system behav-

ior insensitive to variations in product quality, thereby enabling less tight product specifications.

Looking back at the projects mentioned above we can distinguish some trends. First, all solutions proposed for the control problems make use of a model of the system during the controller design phase. Moreover, this model is often used on-line and a main part of the control input may be based on this model, so a certain degree of confidence in the model is necessary. Also, we can remark that a large part of the time needed to design the control systems must be devoted to the derivation of the model equations or to model identification. The main problem is not the physical laws that must be used (except for the phenomenological equations), but the selection of the relevant aspects and equations from an overwhelming number of possibilities, and the sheer volume of work to manipulate the equations and put them in a suitable form for simulations and control system design. Model identification based on experimental data is rather laborious [4] and can only be used if the system is already in operation, making it difficult to modify the system with the aim of acceptable control system behavior. It requires determination and drive from a control engineer to spend so much time on the drudgery of the derivation of models. However, it is in essence only a sub task, and the mental powers used can better be directed to more profitable and interesting activities, *i.e.*, the main task of controller design. Often, there is no time slot left in the product design cycle to develop a comprehensive model, because the time-to-market is limited.

To shorten the design time it is therefore worth trying to reduce the time and work needed to set up an adequate model, although there are also other methods to achieve the same goal. A way used in the past is to design the system so that it can be easily controlled, and the controller does not need a model. An example is the design of airplanes that were stable without control. Nowadays unstable aircraft are used. Another example is the use of stiff and heavy mechanical structures to eliminate low frequency vibrations. Alternatively, it is possible to

- avoid the selection of relevant aspects and model the system in all its complexity; this leads to a complicated model that can perhaps be simplified using systematic methods,
- try to systematically select the relevant aspects and equations, aided by modeling software; this requires some additional knowledge, gained from previous experience, that may be acquired from a knowledge base,
- re-use models of key parts of the system, stored in a standard model data base by using modular or unit modeling concepts; a large number of standard models should be available and these models should be adequately parametrized and be equipped with well designed interfaces

to enable flexible model connections,
- develop a simple model, design a standard controller for the model, use the controller and check which part of the system limits performance, make a more detailed model of only this part and repeat; this requires a fast design and prototyping environment,
- de-emphasize the modeling, accept inaccurate and therefore erroneous models, and compensate for the errors in the model in a later stage of the design process, *e.g.*, the controller structure and design.

These solution strategies lead to a growing interest in computer assisted model building to obtain a comprehensive model, model reduction to simplify a complicated model, expert systems to aid during the setup of the model, databases of models used in earlier designs for repeated use, prototyping environments for rapid iterative improvements, and robust control to enable the use of erroneous models so they do not severely limit the achievable control system performance. Part of these activities is known under the name of "intelligent control", see, *e.g.*, [5].

The research described in this thesis is a contribution to the knowledge gathered in the control field. Its problem field is application of methods for robust control and the main theme of the thesis is therefore the use of erroneous models in control design and implementation. It derives its motivation from the last application area mentioned, mechanical systems, with the nonlinearity of the systems as most important aspect. This will be elaborated in more detail.

A somewhat disputable goal has always been to build anthropomorphic machines [6], called *robots* in their most recent incarnation. This term is also used to describe mechanical systems, equipped with sensors, actuators, and some form of intelligence, that can replace humans for more mundane or dangerous tasks. The main property of these systems is that they are nonlinear, making analysis and control more difficult.

There is a trend to use robots and other mechanical systems for less mundane and more difficult tasks, using vision, coordination, etc. This requires thigh bounds on the performance of the system. This drive to high performance mechanical systems, both for mass and limited size production, can be viewed as both a market pull and a technology push.

There is a demand for high performance electro-mechanical systems with low production numbers, *e.g.*, precision engineering production facilities like CNC machines, advanced industrial robots like wafer steppers, and with high production numbers, *e.g.*, cars, CD players, intelligent camera's, electronic watches. In both cases the aim is to reduce production costs to make the potential market larger. So, we have a dilemma between thigh production tolerances, to get a mechanical system with high bandwidths and therefore high performance, and low costs, to get a marketable product. A way to solve this is to accept a mechanical system with higher tolerances and larger variation between the individual products and attain the high bandwidth by sophisticated controllers.

The development of control theory and its application in technical disciplines has made rapid progress in the last decades, and has arrived at a level that makes it possible to design and implement controllers that can compensate for deficiencies in the mechanical design and variations in the production process.

The pull and push trends are stimulating further development of the control theory to solve more problems, quantitative and qualitative.

A question that comes to mind is: Can we expect to gain orders of magnitude in performance by using advanced controllers? Probably not. But how far can we go? A common conception or rule of thumb is that in general the performance enhancement of a sophisticated controller is not more than an order of magnitude. A larger increase in performance is only possible when the system to be controlled is changed. But is this common view perhaps common nonsense, too general or plain false? Answers to these questions at a high level of generality are perhaps impossible. How far have we to restrict this generality to get answers that have a certain degree of eloquence.

When the use of an advanced controller cannot achieve the desired performance, we have to resort to other modifications. In principle, an optimal design of a complete system is only possible when the design of all components of the system (mechanical structure, actuators, sensors, controllers) is integrated. A solitary design of separate components can produce disasters. When the performance is inadequate and cannot be achieved by better controllers, other parts of the system have to be redesigned: sensors, actuators or the mechanical structure.

This thesis presents an investigation to clarify some unsolved or not clearly solved problems in the application of control theory for a specific type of nonlinear systems. It contributes therefore to the technology push.

1.2. Scope of research activities

The control field, even when limited to mechanical systems with sloppy dynamics, also called *unknown systems* or *uncertain systems*, is broad. To avoid a shallow contribution we have to focus on a more specific field. The criteria for selecting this field are blurred and are based on ratio, experience, and intuition.

The main contribution in the past to the control of unknown systems has been for linear systems. This has not been fully extended to the nonlinear case. Some progress has been made, but several problems are unsolved, and other problems, which have been solved in the linear case, even don't have a solution in the nonlinear case. Hence, for mechanical systems, which often exhibit a nonlinear behavior, further development should focus on the control of nonlinear systems.

Unknown systems have as a nasty property that models of those systems invariable contain significant model errors. These errors can be characterized from several points of view

(1) type of error,
(2) approximate modeling of error,
(3) cause or purpose of error.

We first discuss the type of error we are interested in. To focus attention in this study, we select a specific type of error, namely the error caused by unmodeled dynamics. To give a more precise meaning to the term *unmodeled dynamics*, we will give a classification of model errors. We can define the following conceptual partition of model errors:

- the parameter error,
- the unmodeled statics error,
- the unmodeled dynamics error,
- the remaining error.

We are primarily interested in the unmodeled dynamics error.

During the design we incorporate this error as explicitly as possible, depending on the specific design method used. These methods will not encompass all possible error specifications, so this places a restriction on the type of model error they can handle, and on the specification of this error (the error model).

Model errors do no always stem from uncertainty, but can be caused also by deliberate manipulations or restrictions of the model. Two main causes are

- errors introduced by model reduction, needed to implement controllers in real time situations,
- deliberate model simplifications, because there is no budget to make a detailed model, although in principle that could be done.

Especially the first cause explicitly introduces unmodeled dynamics, although the model reduction algorithm strives to reduce this error as much as possible. For nonlinear systems the situation is more complicated than for linear ones, because linearity properties are not valid.

An example of the second cause is the modeling of mechanical systems, which is greatly simplified when elasticities of joints and links are neglected. The model is than a straightforward multi body model, described by ordinary differential equations, instead of a model with partial differential equations.

1.3. Research methodology

To select the most appropriate methodology for our research, several characteristics of possible methodologies are discussed.

When the field of research is not very well known, an exploratory study is the most appropriate one. Possibilities are a literature search or expert interviews.

A more ambitious type of study is a descriptive approach. Here, one does not only passively acquire knowledge, but also tries to gain more insight in a problem by controlled manipulations, *e.g.*, by experimentation, be it simulation, laboratory or field experiments. Often it is not possible to fully explain the findings because not all experimental conditions can be controlled, so one is not certain how to interpret the results, or an explanation for an observed causal relation is missing.

When one wants, and if it is possible, to fully explain the research findings, the most appropriate type of study is explanatory. For this type of study a fundamental problem is that the induction process, used to infer from fact to explanation, has not much validity.

The research methodology most appropriate for our research is not unique. The method we use depends on the stage of the research activities. Initially, an exploratory study, consisting mainly of a literature search, has to be performed. When more insight has been obtained, the methodology should be more strict, and a descriptive type of approach, with numerical and laboratory experiments or even an explanatory study, could be performed.

Because the results of the research are related to practical problems, with experimental conditions that cannot be completely controlled and with results that are marginally repeatable, a descriptive study seems the most appropriate one. Although one cannot expect to explain, in full glory, all observations or research findings, we will try to explain as much of our findings as possible, because that seems to be the most fruitful way to formulate new research questions.

1.4. Research limitations

The research reported in this thesis is certainly not a solution for all robustness problems in control systems. To be specific, our approach is hampered by the following problems

- our results depend on the trajectory chosen for evaluating the control schemes, this is caused by the nonlinearity of the control system,
- the results cannot be generalized with full confidence, because linearity properties are not applicable,
- because our results are not fully explained theoretically, it is sometimes difficult to give clear directives for improvements of the control schemes; although a theoretical foundation would hardly improve this, it could be a source of farther reaching conjectures,
- because we focus on a specific sub field, related problems in other subfields are untouched, and perhaps those fields are more effective for solving the robustness problem. Examples of related questions are
  - is it possible to solve the problem with probabilistic methods, *e.g.*, based on the variation between products coming from a production line,
  - when is it advisable to employ a more intelligent controller, with a detailed model, instead of a simple controller designed with robustness specifications,

  the last question will get some limited attention in the sequel.

1.5. Structure of thesis

The next Chapter gives an overview of relevant literature, mainly directed to robust control design methods for nonlinear mechanical systems. Several relevant methods for the design of robust linear systems are also presented. The problem statement and plan of action in Chapter 3 indicates the lacunae in the knowledge base found in the open literature, and, after confrontation with the general problem introduced in this chapter, gives the specific problem to be solved and the way in which we expect to solve it.

The three chapters that follow present the control design methods investigated and give an overview of the results for these methods, a numerical evaluation using simulation techniques and an experimental evaluation. Chapter 7 presents some results obtained with more carefully chosen models.

Finally, Chapter 8 gives the conclusions: can the problem posed in Chapter 3 be solved and is the way we followed to solve it properly chosen. Furthermore some recommendations are given: which part of the problem remains unsolved and badly needs a solution, are there better or alternative ways to solve the problem.

CHAPTER 2

Literature review of robust control

This chapter presents an overview of recent literature in the field of robust control of dynamical systems. It gives the results of an exploratory study, performed to get an overview of the work that has already been performed, and with the goal to determine a more specific area of research. It should lay the foundation for more clearly defining the research problem and developing the research design.

Robust control can be described succinctly as the control of plants with erroneous models available for the control system design, or plants that change during operation and those changes are not envisaged beforehand, where the control system can withstand these effects while maintaining the required level of performance.

Some nonlinear systems can be exactly linearized by state-feedback and coordinate transformation. This technique can be incorporated in an inner loop of a control system. So, for the design of an outer loop, a technique based on a linear model may be sufficient. Therefore, we discuss methods aimed at both linear and nonlinear systems.

Section 2.1 presents some background for the literature review. Here, the methodology and the criteria used for selecting the reviewed papers are discussed. It also gives a first level classification of the control schemes, to prevent a hodge-podge in the presentation of the control schemes in Section 2.2.

Section 2.2 gives a thorough but not exhaustive overview of the literature. It aims at providing a general view of the field and is also used to suggest a more detailed classification and some additional criteria for selecting promising attacks of the problem we would like to investigate.

A further discussion of papers selected according to the criteria developed in Section 2.2 is presented in Section 2.3. It treats the papers in more detail to show which papers are redundant, do not make significant progress, or are simply duplicates of earlier papers in different wordings. Furthermore, the selection criteria developed in Section 2.2 are applied. This makes the selection of a set of "core" papers possible. The resulting papers are used to get an impression or indication of the fields which are still open for research. Those fields are the most promising fields to attack, they add maximally to the knowledge base.

To summarize, we will

- discuss the methodology and give a first classification,
- give an overview of the literature consulted, develop additional selection criteria for our methodology and for a more detailed classification,
- elaborate articles that fit the scope of our research, and use these to get an indication of suitable fields of research, *i.e.*, open problems in robust control.

2.1. Review methodology and first classification

The methodology used to track articles in the field of robust control is quite standard. We started with the consultation of some abstracts and tried to determine some lines of research by backtracking articles based on references in more recent papers and by forward tracking using citation indices. This is just the classical snowball method. The abstracted material used comes mainly from the Computer and Control Abstracts, especially sections 13.40 k (control theory—specific systems—nonlinear systems) and 33.90 (control applications—robotics).

It appeared to be possible to distinguish several ways to attack the robust control problem. A unique classification of the relevant literature is not possible, because in some papers several approaches for enhancing robustness are combined, mostly to eliminate disadvantages of an approach, or to cancel disadvantages of an approach with advantages of another approach. Combination also arised to enhance some advantages by a kind of reinforcement approach, the approaches then do not cancel but amplify each other. Therefore, some papers are even mentioned twice.

A way to come up with a classification would be to discuss all relevant papers and based on this discussion discover or induce the lines of research that appear in the literature. This would lead to a hodge-podge of papers, difficult to digest. Also, according to Popper [7], inductive reasoning has no logical validity. The best it can be is a source of conjectures as to the forms of natural laws or structures. So a more practical and more tutorial like approach is to use some foresight, and to present the papers structured in a way we discuss below. This facilitates the reader, but it has the risk that another, and perhaps more appropriate, way of classification of the papers will be more difficult to discover, because it is hidden in the structure we have imposed. The papers are presented according to a classification based on the following characteristics of the control schemes presented

(1) optimal feedback
(2) linearization around trajectory
(3) exact linearization
(4) high-gain feedback
(5) variable structure
(6) adaptive control
(7) pole placement
(8) disturbance suppression
(9) state derivative feedback.

This classification is more or less an exhaustive enumeration of robust control design approaches. Later on this classification can be modified and some possible lines of research can be disregarded because they are not promising enough or they are already completely developed. The classification does not fulfill the requirements that it partitions the set of relevant papers in mutually exclusive and collectively exhaustive categories.

This classification can be confronted with the one proposed in [8, 9] for robotic systems. These surveys are also useful as a general overview of a part of the field of research. Another, but outdated, survey is [10].

In [8, 9] the following classification of approaches for robust control of rigid robots is proposed

(1) linear-multivariable or feedback-linearization approach
(2) passivity approach
(3) variable structure approach
(4) robust saturation approach
(5) robust adaptive approach.

This classification is not adopted in this review, based on the following arguments.

First, the classification in [9] is based on a distinction between two approaches for the *control of uncertain systems*, namely adaptive and robust. This distinction is based on an corresponding distinction for linear systems. There, robust controllers are linear and adaptive controllers are nonlinear, making a clear distinction possible. For nonlinear systems this distinction is no longer applicable since robust controllers can also be nonlinear. We therefore include adaptive control, and not only the specific kind of adaptive control meant in class (5) of the classification by [8, 9], as a method for robust control.

Second, the term robust should be interpreted as being more general than adaptive: the adaptive approach is one of many methods to robustly control systems with erroneous models, especially with parameter errors.

Third, the expression "control of uncertain systems" is misleading, because the subject is really the model based control of nonlinear systems where models with model errors are used, and the cause of those errors is not only uncertainty. Later on this term will still be used, but not with the usual connotation, but as an indication of systems with erroneous models.

Fourth, the classification in the five approaches is blurred, the approaches overlap each other and it is not clear whether the classification is exhaustive. Overlap is, however, difficult to avoid.

Fifth, the number of papers for each approach is unevenly distributed. A classification, more commensurate with the number of papers, is advisable.

Finally, [9] is targeted at the robust control of rigid robots. The target for our literature search is slightly larger, although rigid robots are an important subfield. and most of the literature cited is targeted at this application area.

Based on these arguments, we propose the more harmonious classification, given above.

2.2. Global presentation of literature

In this section we review literature on *uncertain systems*, *i.e.*, systems with models that are not exact. This overview of the literature can be conceived as a tree, where each branch or bough is a direction of research, manifested by a key concept, located at the connection between trunk and branch. We call those branches *research threads*, and discuss the threads represented by the key concepts presented in Section 2.1.

In this review we would like to address at least the following questions

- is implementation complicated or tedious,
- is the control algorithm suitable for on-line implementation,
- how general is the model and error model used,
- is explicit model of error needed or only norm bounds,
- is the conservatism of norm bound addressed,
- are problems, specific for the concept used, attacked,
- is only robust stability addressed or also robust performance,
- is convergence of tracking error guaranteed, is convergence exponential,
- how complicated is the underlying theory (*e.g.*, for MSc students)?

It is not possible to discuss these questions for each paper. This would make the review too lengthy and boring. Also, some questions are sometimes not relevant, obvious, or not discussed in the paper and it may be difficult to extend the discussion given in the paper to encompass all questions. So, the elaboration of these questions is postponed to the end of this section, in the discussion of the approaches and the selection of promising approaches. In Section 2.3, where a more detailed description of the literature is given, it is possible to enforce a more strict application of the given aspects, due to the reduced number of papers and a more general way of presentation.

We now start our global presentation of the papers, where we do not diverge too much from the literature itself, *i.e.*, interpretation of, and comments on, the papers are avoided.

Most of the literature cited is targeted at the control of robotic systems. A complete and exhaustive review of each of the nine approaches is not claimed.

2.2.1. Optimal feedback. In this concept, a controller is designed with the aid of an optimality criterion. In [11] and [12] a first attempt is presented to extend the classical optimal control theory from linear to nonlinear systems. In [13] and [14] the possibility is created to generalize robustness properties to nonlinear systems. In [15] a method is presented to asses the robustness properties before the optimal controller is designed. An enhancement of the optimal control theory is presented in [16]. This gives an improvement with respect to the linear quadratic (LQ) controllers. Discrete-time systems are the focus of [17].

Besides quadratic optimality criteria, recently several attempts are made to extend the H_∞ theory to nonlinear systems. See, *e.g.*, [18–22]. Three approaches can at least be distinguished, namely nonlinear interpolation, constrained optimization, and Hamiltonian vector field theory with a Hamilton-Jacobi equation.

Besides H_∞ theory, also structured singular values have been generalized to nonlinear systems [23]. An example for a rigid manipulator is given by [24].

2.2.2. Linearization around trajectory. After linearization around a trajectory, the well known design methods for linear systems can be applied. A review of linearization (not only around a trajectory, but also other types of linearization) is presented by [25]. An example is in [26]. In [27] several methods for acquiring linear models are compared. A disadvantage of this approach is that performance and stability can only be maintained in the neighborhood of the nominal trajectory. An approach with the so called pseudo-linearization is discussed by [28, 29].

Another approach is gain-scheduling. The nominal trajectory is then broken up in several static working points and controllers are designed for these points. Switching between the different controller settings is based on some strategy. The design of a suitable switching algorithm, that guarantees stability for all working points between and in the neighborhood of the set of points used to design the gains, is a major problem. See [30].

Sometimes a single working point is sufficient, as in [31], but the controller parameters had to be chosen carefully, and a linearization of a servo-valve characteristic had to be performed.

2.2.3. Exact linearization. The foundation of exact linearization is given in [32] and [33]. Their results are valid locally and sometimes also globally. Further research is presented in [34]. A review of several types of linearization control for rigid manipulators is given by [35] and for a restricted number of types, but in more detail, by [36]. Exact linearization has been a topic in current research, as can be seen in the series of papers [37–41] were several extensions of the problem are discussed. In [42] the demands on the system are reduced. In [43] the involutivity condition, required for input-state linearization, is circumvented by using an approximation that fulfills these conditions. For an early reference on approximate linearization see [44]. Examples of approximate linearization are given in [45] and [46]. In [47] a more general approach for linearization, in the context of a tracking task, is presented. In this paper the controller is considered. In [48] also the observer is taken into account. Small-gain feedback is based on the idea of restricting the loop gain to be smaller than one when no information of the phase is available. This is treated in [49]. In [50] the bounds on the model errors are investigated when a linear outer loop controller is used. Structured parameter variations are taken into account explicitly in [51]. They use a second level controller based on pole placement or LQ control. Partial (input-output) linearizing control is applied in [52]. Output feedback is also used in [53]. The linearization approach can also be combined with a variable structure controller. This combination with the linearizing state-feedback of [33] is made by [54] and [55] with good results.

Rigid mechanical systems have the advantage that they are passive. In [56] use is made of this property to show that linearizing state-feedback can tolerate large variations in the inertia matrix. In [57], a linearization is performed by a nonlinear state-feedback, and small-gain theory is used for a linear outer loop, where the linear loop is based on stable factorization theory. The approach of [57] is modified and extended in [58, 59]. These results are again extended by [60], to also apply to robots with elasticities in transmission elements. They also present a brief survey. The use of a linear outer loop, for a so-called practical tracking problem, is discussed by [61], in contrast with the outer loops proposed by [62] and [63], that are nonlinear. A linear robust servomechanism controller is used as outer loop in [64]. According to [65], a simple PD outer loop is often sufficient. Constrained control in robotics, *e.g.*, force control, is studied by [66].

The disadvantages of exact linearization, especially when the model used is inaccurate, *e.g.*, in process control, are addressed in [67]. The experiences with linearization control of chemical processes reported by [68, 69] are more positive.

2.2.4. High-gain feedback. In high-gain feedback the difference between model and reality is fed back with large gains. This is based on the property that positive real systems can tolerate infinite gains without stability problems. So, an approach is to make a system more or less positive real and use in an additional control loop high gains to counteract model errors. Sometimes the feedback is based on optimal control theory [70]. In [71] a comparison between model and reality is even performed twice, and therefore used in two loops. Large gains are also used in the stabilizing compensator of [72], with also a robustifying compensator in their control scheme. The use of high gain, combined with linearizing state-feedback, although theoretically possible, is severely restricted due to high frequency disturbances, see [73]. Disturbances are canceled by the high gain and stability problems are circumvented by positive real conditions. With model errors these conditions are not always satisfied.

2.2.5. Variable structure. The theory for variable structure control stems from the solution of differential equation with discontinuities, see [74] or the translation [75]. The same approach is used in [76] and [77]. The variable structure control approach can also be regarded as a high gain approach, where the gain is varying and may be infinite. A rudimentary form of variable structure control is bang-bang control as in [78] and [79]. A hypersurface (switching boundary) divides the state space in two parts. In one part the control input $u = u_{\text{high}}$ is applied, in the other part the control $u = u_{\text{low}}$. When a so-called *equivalent control* u_{eq} exist it is possible to keep the state trajectory on the switching boundary. A modified bang-bang controller is proposed in [80]. Here a switching zone (boundary layer) is introduced to replace the switching boundary. In this zone the control input varies linearly with the control error. This eliminates high frequency oscillations (chattering). This, and other, techniques to suppress chattering are also presented in [81–84]. The accompanying loss of the zero steady state property can be remedied by additional integral action, as in [85] and [86]. The sliding mode form of variable structure control in [87], is used as a basis for later developments, as discussed in, *e.g.*, [88] and [89]. For a recent account see [90]. In the sliding surface method the tracking error approaches zero exponentially when the state is on the sliding surface. The surface itself can be reached in finite time. In [91] the issue of balancing the robustness and performance is discussed and in [92] an "unbalanced" robust control design is presented. The sliding mode approach is taken by [93], but there not only controller but also observer aspects are discussed, as is done also in [94]. The use of a parabolic switching boundary to minimize the effect of stiction is the subject of investigation in [95]. In [96] recent results of input-output linearization are combined with a second order sliding mode control. A controller aiming at flexible manipulators is given by [97].

2.2.6. Adaptive control. This type of controllers tries to tackle uncertainty by estimating critical system parameters. In [98, 99] an overview is given for the field of robot control. In some cases the structure of the dynamics is neglected and black box models are used. This is the approach of [100], using a recursive least squares method to fit a second order time-varying model. The least squares method is also used by [101].

Other researchers use white or grey box models, and profit of some knowledge of the structure of the dynamics of the model of the system. This is done by [102–104]. Their example is followed by [105]. A combination of an adaptive controller with variable structure elements is presented by [106] (repeated in [107]) for mechanical systems and by [108] for a more general class of systems. The approach has been extended to control in end-effector space in [109]. The schemes of [104] and [106] are compared by [110]. An alternative for the scheme of [106] is proposed in [111] were a different measure of tracking accuracy is used. In [112] an adaptive version of a controller proposed by [113] is presented, that becomes a pure PD feedback if the adaptation and the + part of the controller are not used.

Mostly, the stability proof of adaptive controllers brakes down if unmodeled dynamics are present. The stability problem, in the presence of this type of model error, is analyzed by [114]. The robustness of adaptive control for the unmodeled dynamics error (and other types of errors) can be enhanced, as is shown in [115] and discussed by [99].

The problem of determining the convergence rate for the error in adaptive control is studied in [116]. An exponentially stable controller, that does not require persistent excitation, is presented by [117]. Sometimes a distinction is made between direct and indirect adaptive control. A unified approach based on passivity for direct adaptive motion control of robots can be found in [118]. In [119] a distinction is made between uncertainty-constrained schemes and nonlinearity-constrained schemes, and a not very restrictive uncertainty-constrained adaptive control scheme is proposed.

A general solution for an adaptive robot controller that solves the problems of motion, impedance, force, and dynamic hybrid control is presented in [120]. Adaptive control organized as independent joint control is the subject in [121, 122]. The control of flexible manipulators poses some special problems. These are addressed in [123] and [124]. They use a singular perturbation type technique and fast-slow control. Flexibility combined with friction is addressed in [125]. The use of a hybrid adaptive controller, addressing the problem of time delay and the discrete nature of the controller in implementations, is studied in [126]. Computation issues (especially efficient computations) are also addressed by [127], and are, slightly extended, repeated in [128].

2.2.7. Pole placement. By suitable placement of the closed loop poles of a linear system, a robust system may be obtained. Circle type criteria are often used. An early paper is [129]. This approach is expanded by [130–132]. A systematic approach for pole placement is presented in [133]. Another approach is in [134]. An adaptive exact linearization, combined with a robust pole assignment controller, is used by [135]. In [136] a robust pole assignment controller for flexible robots is proposed.

2.2.8. Disturbance suppression. In this approach, the difference between the real system and a simple model is viewed as a system disturbance. Singular perturbation theory is sometimes used. One often tries to bound the error, by choosing a suitable model, and to specify modest system requirements. This approach is presented in [137–139]. The results of a more extended research are presented by [140], were a decentralized approach is used. In [141] another approach is given, valid for a certain class of uncertain systems. Here, higher order dynamics is treated as disturbance and some matching conditions are necessary. Those conditions are removed in [142] for a so-called practical stability problem.

2.2.9. State derivative feedback. The use of state derivative feedback or, more limited, the use of an additional acceleration feedback loop for mechanical systems is discussed in, *e.g.*, [143]. In [144, 145] acceleration feedback is used to enhance the robustness for errors mainly in the inertia matrix.

2.2.10. Discussion. Looking back at the papers discussed above, and given the aim to reduce the number of approaches to be investigated, we should prune the research threads tree mercilessly. We also remark that most papers cited above are on robotics, so we limit our conclusions to this application area.

The optimal control approach is hampered by lengthy computation, that makes it unlikely to be implemented in full glory for real time control on something less than a high performance computer or other specialized hardware. This approach is therefore not further pursued. This computation argument is, however, only relevant for fast (mechanical) systems, and only for nonlinear optimal control where repeatedly solutions of nonlinear differential equations are required. In process control this argument is probably not valid.

Linearization around a trajectory has the disadvantage that stability and performance are difficult to guarantee. When controllers are designed, on-line, for each working point, lengthy computation are necessary. The gain scheduling approach is already being used in industry for a long time, especially in flight control systems, in a more or less ad hoc fashion. It seems not likely that in a short time pioneering progress will be made with regard to the application aspect of this approach. So, we do not discuss this approach any further.

Exact linearization is a nice approach, except when the errors between model and system become too large. It is then even possible that a working point controller may perform better than a controller based on exact linearization. Furthermore, only a linearizing controller is of no use, at least an additional control loop with stabilizing and robustifying properties should be added, and we should concentrate more on the relative merits of those loops than on the linearizing control loop itself. There are a large number of possibilities for this additional loop, some of which are discussed in the other approaches. When the progress is taken into account that has been made in the field of robust control of linear systems, it is an obvious suggestion to combine those new results with a linearizing state-feedback and extend the field to the robust control of nonlinear systems for which models exist that are feedback linearizable. So we keep the exact linearization approach, and concentrate on the use of robust linear controllers for the outer loop.

High-gain feedback is hampered by the possible occurrence of large input signals with high frequency components that may excite unmodeled dynamics. This is a serious disadvantage, because instability may result. There are approaches to eliminate this problem, like low pass filtering of the input, but then the proofs break down and not much can be said of the stability of the resulting control system.

In variable structure control high gains are not always necessary. This depends on the uncertainty, prevalent in the system. If the model error is large, large input signals are necessary to guarantee convergence to the switching boundary. Those signals are however more or less constant, only around the switching boundary high frequency oscillation can occur. Those oscillation can be suppressed by using smoother approximations of the switching functions in the vicinity of the switching hyperplane. This gives some reduced performance. Nevertheless, we rank the variable structure controller high, and will investigate it further.

In adaptive control, the use of models with structural properties has definite advantages. The disadvantage is that such a structure must be known. For mechanical systems this is often not a big problem. Furthermore, the results reported in literature are quite good. A possible problem is that this field is already quite developed, and a very active area of research for some researchers, see, *e.g.*, [146]. Nevertheless, we keep this approach for further research, to get a reference controller for comparison with other robust control approaches. One aspect of this control scheme that is not finally solved is robustness for unmodeled dynamics and unknown external forces. For some result of an application see [147, 148]. Some modifications are possible that enhance the robustness for unmodeled dynamics and unknown external forces, but their relative merits are unclear.

The pole placement method is hampered by conservative norm bounds that limit the performance too much to be of any value in practice. This is amply demonstrated in [149]. We will therefore dismiss this approach.

State derivative feedback can be applied when measurements or estimates of those derivatives are available. For mechanical systems this implies acceleration measurements. This type of measurements is quite cheap, and it is therefore likely that this sensor may become standard when the performance can be raised significantly. A disadvantage is that the performance is limited by the noise generated by the sensors and operational amplifiers used. When the noise is too high, no advantages are realized, see [150]. Still, this approach is promising enough to merit further investigation.

Concluding, we propose to discuss in the next section the following selection of robust control approaches for nonlinear systems

- exact linearization with linear controllers that should enhance the robustness,
- discontinuous input controllers, *e.g.*, variable structure or sliding mode controllers,
- adaptive computed torque controllers,
- state derivative (acceleration) feedback.

Most of these methods can only be applied to a restricted class of systems, *e.g.*, robotic systems.

2.3. Elaboration of literature

In this section we give a more detailed description of the selected approaches to solve the robust control problem. We first present an archetypal model, that is used to illustrate a possible distinction between several types of model errors, and is also useful as a starting point for the models needed in our detailed discussion of the literature. Then, we focus attention to a specific type of model error. Finally, we elaborate some literature, already glanced over in the previous section, and present a more precise analysis of its contributions and shortcomings. A detailed description of the control schemes proposed in some of these papers is deferred to Chapter 4.

2.3.1. Archetypal model. The model presented here is a basic model for nonlinear systems. We assume that the system can be described, with reasonable accuracy, by a nonlinear state-space model. We propose the following, quite general model

$$\dot{x} = f(x, \theta) + g(x, \theta)u, \qquad y = h(x, \theta) \tag{2.1}$$

where in the first (state) equation x is the n-dimensional state, f is a smooth vector field with model parameters θ, g has m columns g_i of smooth vector fields and u is the m-dimensional input, and in the second (output) equation y is the l-dimensional output and h is a column of l scalar-valued smooth functions h_i. Here we assumed that the state equation is affine in u and that there is no direct feed through from the input u to the output y. Both assumptions are easily circumvented, as follows.

INTERMEZZO 2.1. A model that is not affine in u

$$\dot{x} = f_u(x, u, \theta), \qquad y = h_u(x, \theta) \tag{2.2}$$

can be written as (2.1): connect m integrators to the system, one for each input, change the system boundary, *i.e.*, redefine the input of the system to the input v of the integrators and redefine the state x to include the original input u. See Fig. 2.1.

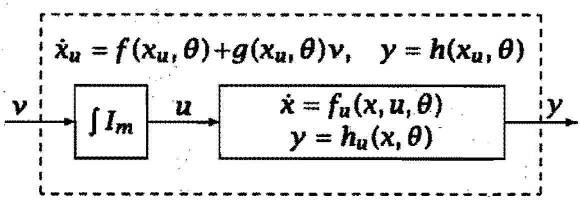

FIGURE 2.1. State augmentation at the input to force affine model

A model that contains direct feed through from u to y

$$\dot{x} = f_y(x, \theta) + g_y(x, \theta)u, \qquad y = h_y(x, u, \theta) \tag{2.3}$$

can be written as (2.1): connect l integrators to the system, one for each output, change the system boundary, *i.e.*, redefine the output of the system to the output z of the integrators and redefine the state x to include the original output y. See Fig. 2.2. Adding integrators to the input u is another option.

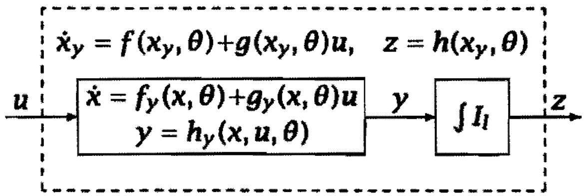

FIGURE 2.2. State augmentation at the output to force affine model

Of course, in both cases, the number of integrators may be lower than m or l respectively, depending on the form in which u enters g and h, and on which elements of u appear at the output y. It is possible to mix integrators for inputs and outputs, if the original model is not affine in u and there are also direct feed through terms. The number of integrators need not to be larger than $\min(m, l)$ for the direct feed through case.

There are disadvantages in adding integrators to a model, especially when the model is used to design controllers. We are then, effectively, giving a structure to our controllers, which may contain explicit integrators, a structure that may hamper some design goals. □

2.3.2. Classification of model errors. We are primarily interested in unmodeled dynamics. To give a more precise meaning to the term *unmodeled dynamics*, we will give a classification of model errors.

When we substitute in (2.1) noise free measurements x_m, $\dot{x}_m$, and u_m of quantities of the system that can be associated with the state x, its derivative $\dot{x}$, and the input u in (2.1), both sides of equation (2.1) do not equate and we obtain an error e, that we will call the *state equation error*, given by the following algebraic equations

$$e = \dot{x}_m - f(x_m, \theta) - g(x_m, \theta)u_m. \tag{2.4}$$

Also, because the operations of measuring and differentiation are not commutative, $\dot{x}_m \equiv (\dot{x})_m$ is not equal to $\frac{\mathrm{d}}{\mathrm{d}t}(x_m)$.

Now we can define the following conceptual partition of the equation error:

- the *parameter error* e_p: the difference between e and the "smallest" e (smallest in the sense of some suitable norm) that can be obtained in (2.4) by making an appropriate choice for the parameters θ;
- the *unmodeled statics error* e_s: the difference between $e - e_p$ and the "smallest" e that can be obtained in (2.4) by making an appropriate choice for θ and additionally by making appropriate choices for the smooth vector fields f and g_i;

- the *unmodeled dynamics error* e_d: the difference between $e - e_p - e_s$ and the "smallest" e that can be obtained in (2.4) by making appropriate choices for θ, f, and g, and by additionally augmenting the state x;
- the *remaining error* e_r: the error that remains after parameter optimization, function selection, and state augmentation; ideally this error should contain no information, *e.g.*, white noise.

So we can write for the equation error e

$$e = e_p + e_s + e_d + e_r. \tag{2.5}$$

To assess the unmodeled dynamics error e_d with reasonable accuracy, we should be confident that, in the model of the system, both the parameter and the unmodeled statics error can be reduced to a negligible level, and that e_r is small.

Next, we make some remarks on the available possibilities to model errors. From the linear theory we known that additive or multiplicative model errors are conservative, but powerful ways to encompass a broad class of more specific uncertainties. For unmodeled dynamics, this often leads to a transfer function type error model, with a high-pass character, because the unmodeled dynamics are often the dynamics at higher frequency (higher order modes). Another type of error modeling is by assuming some norm bounds on the vector fields f and g_i, $i = 1, \ldots, m$, but this is more directly related with parameter and unmodeled statics error and less with unmodeled dynamics. Also, the design of nonlinear controllers using these norm bounds is conservative, and often based on stability requirements, while performance requirements do not come in play.

For nonlinear systems the use of a transfer function type of model error is a more severe approximation, nevertheless, we will use it, in connection with linear design methods that will be used to design the outer loop of a multi-loop controller for nonlinear systems. The mean reason for this choice is convenience. A more accurate model of the error would be time consuming to produce.

Model errors do no always stem from uncertainty, but can be caused also by deliberate manipulations on or restrictions of the model. This has already been discussed in Section 1.2 and is included here to facilitate the reader. Two main causes are

- errors introduced by model reduction, needed to implement controllers in real time situations,
- deliberate model simplifications, because there is no budget to make a detailed model, although in principle that could be done.

Especially the first cause, generally only applied to linear models, explicitly introduces unmodeled dynamics, although the model reduction algorithm will strive to reduce this error as much as possible. An example of the second cause is the modeling of mechanical systems, which is greatly simplified when elasticities of joints and links are neglected. The model is than a straightforward multi body model, described by ordinary differential equations, instead of a model with partial differential equations.

For nonlinear systems the situation is more complicated. Errors in the parameters θ, in the model functions f, g, and in the order of the model do have an influence on the equation error e that does not only change proportionally with the trajectory of the state x, as for linear systems, but also depends on the specific trajectory itself. This is because linearity properties are not valid. This dependency on the trajectory further complicates the issue, because we should maximize a measure of e over all relevant trajectories, to get a (conservative) estimate of the effect of these errors. This implies that the results presented in this thesis are in principle only applicable to the chosen trajectories. A way to remedy this problem is to perform only comparative studies by using the same trajectories in our controller evaluation and generalize our results by invoking induction arguments based on a "representative trajectory" hypothesis. This generalization, however, has little predictive value.

2.3.3. Literature on unmodeled dynamics. This elaboration of the literature concentrates on papers discussing the analysis of systems in the presence of unmodeled dynamics errors and the corresponding design methods.

The following discussion is divided according to the selection proposed in Section 2.2. A further structuring of the literature will be incorporated in the discussion.

2.3.3.1. *Exact linearization with linear controllers.* This type of controllers is based on a control scheme that can be divided in at least two levels. Sometimes even three levels are proposed.

The first level is a nonlinear state-feedback and change of coordinates, that locally linearizes the system. This was first proposed by [32, 33] and has been extended by several other researchers. For an overview see [151–153].

The second level is a stabilizing and sometimes also robustifying controller for the linearized system. If only stabilization was an issue, a linear controller would always be sufficient. But because also robustness plays a role, nonlinear controllers could be of advantage. Here we restrict ourselves to linear controllers. Nonlinear controllers will be discussed later. An overview of several types of linear robust control methods is given by [154].

In principle all robust controllers proposed for linear systems can be used. Not all controllers aim at robustness for unmodeled dynamics, but controllers that enhance the robustness for parameter or unmodeled statics error can also increase the robustness for unmodeled dynamics. Robustifying linear controllers are often designed in the frequency domain, and based on the shaping of loop gains and the norm of weighted loop gains. Examples are the H_2 and H_∞ controllers. Other approaches are also possible, see, *e.g.*, [155].

A lot of research has been performed in the area of norm based linear controllers. This has been started by [156]. In this paper the authors propose a technique to make a standard H_2 norm based control design method, namely the optimal controller combined with the Kalman filter, robust for variations in the loop gain and phase. Computation of the controller data is extensively discussed in [157-159]. Doyle *et. al.* [159] discuss a two Riccati equations method for computing H_∞ controllers. The computational requirements are then the same as for LQ control. For these design methods, the plant, augmented with the weight functions, must satisfy some criteria. Some of these are artificial and can be removed by modifications of the design algorithm [160]. Software to compute H_∞ controllers is available [161].

The H_∞ control theory, when used for MIMO systems, leads to some difficult problems that cannot be solved with the corresponding techniques for SISO systems, *e.g.*, the robust performance problem, where performance is guaranteed for a class of systems that fits in an uncertainty structure. For these problems the μ controller design was developed. This design method was, and more or less still is, hampered by the lack of efficient and effective methods for computing μ, the structured singular value of a matrix, in our case of a matrix transfer function. Some approximation techniques have been developed, that work quite often, but are not guaranteed to always deliver the required results. This technique is based on the design of a sequence of H_∞ norm controllers, where the weights used in the design are adapted to solve the μ-synthesis problem. This iterative technique is called DK-iteration. Software to compute μ controllers is available [162].

Applications, combined with a linearizing state-feedback have, up to now, not shown up in the literature.

The third level can be a robustifying controller. This level is used if the second level controller was only aimed at stabilization and not at enhanced robustness.

2.3.3.2. *Discontinuous controllers.* The variable structure controllers are also often combined with a linearizing inner loop, to (approximately) linearize the nonlinear model of the system.

When explicit bounds on the range of possible values for model parameters are given, the controller parameters can often be chosen to guarantee stability, despite parameter variation. Model errors in the form of additional dynamics are more difficult to incorporate in the controller design, have not been given a full treatment in the literature, and offer therefore a potential field for further research.

Other, not completely resolved, problems in sliding mode control are the relative merits of several approaches to suppress chattering. The aim here is to reduce high frequency input signal oscillations, that lead to reduced robustness due to excitation of high frequency unmodeled dynamics, but maintain the performance characteristics of sliding mode control, *i.e.*, the guaranteed convergence of the tracking error. Several measures are proposed that are claimed to nicely solve this delicate balance between conflicting goals.

A common measure is to smooth the discontinuous input signal, *e.g.*, by a boundary layer method. In [163] even a time-varying boundary layer is proposed to reduce the conservatism of the controller. A comparison between several methods has, until now, not appeared in literature.

Most of these measures increase robustness but reduce performance. Several performance enhancements are proposed when smoothing of the input is applied. A possible enhancement is the use of additional integrators in the controller, to regain the zero steady state property. Other possibilities exist. A comparison of these possibilities and experimental tests are missing from the literature.

Besides first order differential equations to define the sliding surface, other types, *e.g.*, second order differential equations, can be used for this definition. The difference in performance between controllers based on several definitions of the sliding surface are hardly researched. Only in the recent paper [96] a second order sliding mode is used in combination with a linearizing inner loop. Also this point merits further investigation.

2.3.3.3. *Adaptive computed torque controllers.* The adaptive control field has been very active recently. This is apparent from the wealth of papers in this field of which only a fraction was presented in Section 2.2.

Here also, the main goal of this study is not to develop new theory, but to digest the theory already present and to investigate and review the relative merits of several approaches proposed. We will emphasize again the robustness for unmodeled dynamics, because this question is not completely answered in the literature.

Only recently several approaches to address this problem are proposed [115]. Earlier, only ad hoc solutions, such as the use of limiters on the adaptable parameters or the use of additional signals in the adaptation laws, are suggested for this problem.

Another point that needs further investigation is the use of adaptive computed torque like controllers that are closely related and only differ in some minor way, *e.g.*, in the definition of the adaptation law or the measure of tracking accuracy. The slight differences between the control laws, however, do not guarantee that the results obtained with these controllers, *i.e.*, robustness and performance, are also almost the same. Comparison of these variants are also missing from literature, and could be performed in this study.

2.3.3.4. *State derivative feedback controllers.* Although it is known that noise corrupted state derivative measurements will limit the usability of the corresponding feedback loop, see [163, Exercise 7.5], a thorough study of the effectiveness of derivative feedback is missing. Because for mechanical systems additional information of the state derivative, *i.e.*, the acceleration, is relatively easy to acquire, this merits further investigation. Especially the relation between measurement noise and robust performance is interesting, because this determines the difference between cost and profits and therefore the possibility to apply this technique on an industrial scale.

2.3.4. Discussion. The results of the review of the literature can be summarized as follows. There are several approaches for robust control that are promising, so we can expect to find among them design methods that will improve on the methods currently available and in use. Some of these approaches are not completely developed, *i.e.*, the theory is rudimentary, not complete, or not powerful enough, applications of these approaches on real world systems are missing, or the relative merits of the approaches are unknown.

Our literature review suggest at least four approaches that are suitable for further investigation, also because the theory is developed to such a state that application does not require a substantial effort, namely

- linearizing state-feedback with a linear controller in the outer loop,
- sliding mode controllers,
- adaptive computed torque methods,
- state derivative feedback based control.

Because we want to compare these four approaches, our field of application will be primarily limited to rigid robots. The adaptive computed torque controllers can only be applied rigorously on this kind of systems. The linearizing state-feedback is applicable to a slightly larger class of systems. Sliding model controllers and state derivative feedback can be applied to a much larger class of systems, but the main reason for state derivative feedback, the easy measurement of the acceleration, is only valid for mechanical systems.

Finally, we note that, strictly speaking, we do not have to restrict ourselves to systems, but only to models that are suitable for the design approaches. This means that a system that is not suitable, *e.g.*, it is not feedback linearizable, but can be approximated with a model that is, permits the use of the methods mentioned above, because they only need an appropriate model to work with.

CHAPTER 3

Statement of problem and plan of action

A basic problem in the statement of a problem is our inability to encompass all relevant aspects of a technical system and of its interaction with its environment, so we are unable to pose the real problems in a precise technical or mathematical statement. If we consider an aspect's enumeration of a problem, the mathematical and technical areas can be viewed as only two of many aspects.

Furthermore, there is always a gap between the technical problem we want to solve and the statement of the problem. The latter is shaped by the available technical and mathematical tools. Many theoretical works have some standard hypothesis. These hypothesis are rarely verified in applications, and are useful only for proving theorems.

Given this limitation, we will formulate a statement of the problem in a precise form, but do not anymore pretend that this is in all aspects the real problem we would like to solve.

3.1. Statement of problem

Given the state of the art, our aim is to contribute to a practical solution of the following research questions

- feasibility: can controllers, based on erroneous models, be used and can they improve the performance significantly?
  - model error bound: what is the model error that can be tolerated in the design of controllers that should achieve a specified performance?
  - performance bound: what is the performance that can be achieved in the design of controllers given a specified model error?
- constructibility: are there controller design methods that give a controller approaching or even attaining limits of uncertainty and performance?

The type of model errors we are interested in is unmodeled dynamics. The field of application is restricted to a subclass of the class of nonlinear systems affine in the input, *i.e.*, mechanical systems that can be linearized exactly. The type of control problems to be considered is tracking control, and the reference trajectory is known beforehand.

To solve the above problems, we have a multitude of possibilities. We further restrict our research by choosing beforehand a plan of action, although we are uncertain if the way chosen to answer the questions is the most elegant, effective, or efficient.

3.2. Plan of action

We attack the problem by several methods, each related to the others, but still independent enough to permit an answer if a method cannot give clear results. The methods are

- literature search,
- numerical experiments,
- laboratory experiments.

We can envisage these methods as a nested sequence of methods, or as a sieve, eliminating in successive stages the control design methods that have not much promise, or are hampered by too many disadvantages.

The first stage consists of a literature search in which analysis and design methods are qualified according to some criteria (generality, strongness, applicability, computability). This part of the research is explorative and descriptive.

In the numerical approach the methods selected in the first stage are used to control several simulation models. The robustness is assessed by making deliberate changes between the model used to design the controllers (the design model) and the simulation model on which the controllers are applied (the evaluation model). The model used to design the controllers is deliberately chosen simple. Robustness is assessed by changing the simulation model. This requires a single, or very limited number of, controller designs. Another approach, using a fixed simulation model to be controlled and changing the model used to design the controllers requires a more detailed model, and a controller design for each model used. This part of the research is descriptive.

The experimental approach is used to give a final qualification of the methods that passed the first and second stage. We cannot skip this stage, because laboratory experiments are of paramount importance to get an informed opinion about robust control design methods. This part of the research is descriptive with a causal or explanatory flavor.

We try to secure as much external validity by making experimental conditions as similar to conditions under which to apply our results. The experimental equipment used should therefore be capable to exhibit different types of dynamical behavior. It is then also possible to get insight in the robustness of the control schemes by applying them, without redesign, on the experimental system in several different configurations. One way to obtain these different types of behavior is to add dynamics to the system by replacing a stiff by a flexible connection. This results in an increase of the number of degrees-of-freedom. By manipulating the additional dynamics the unmodeled dynamics error can be, more or less, controlled.

Sometimes the effect of unmodeled dynamics on the equation error is small because

- the additional dynamics is not significant,
- the coupling between original and additional dynamics is weak, (this is related to the observability of the additional dynamics as seen from the original dynamics and to the controllability of the original dynamics as seen by the additional dynamics).

In general it is therefore not sufficient to be able to introduce additional dynamics in the experimental system, but it should be possible as well to change the effect of the additional dynamics. Based on the two causes mentioned above, this is possible by changing dynamic characteristics of the additional dynamics (eigen frequencies and damping for mechanical systems), and the coupling between the original and the additional dynamics. The experimental system should make this possible in a controlled way. The range in which the behavior of the additional dynamics should vary is limited by two criteria

- it should be representative for real applications,
- it should be large enough to get a good signal to noise ratio in our numerical and laboratory experiments.

Furthermore, if we want the numeric approach to have some predictive value for the laboratory experiments, we should be able to obtain an accurate evaluation model (possibly complicated) of the experimental system. The experimental system should therefore be chosen from an application field where the physical knowledge is sound. This precludes fields where only grey models are possible, although that are often the fields where robust control is needed the most: grey models are not very accurate when they are non-specific.

CHAPTER 4

Description of control schemes

In this chapter some of the control schemes sketched in Chapter 2 are presented in more detail. The schemes presented are

- input-output linearizing state-feedback with robustifying linear controller; the linear controllers are
  - LQG, H_2, and H_∞ controllers,
  - μ-synthesis controller,
  - PD controller for reference,
- model based controllers, not necessarily linearizing, combined with the following (nonlinear) controllers or control components
  - robust adaptive computed torque control,
  - sliding surface or VSS (Variable Structure System) control,
  - VSS, combined with adaptive computed torque, for those parameters that are not estimated by the adaptation mechanism.

All of the controllers above can be combined with acceleration feedback. The use of this additional feedback loop is investigated by combining it with an adaptive computed torque controller.

The controllers to be investigated should

- make the controlled system stable, preferably in a strict sense, *e.g.*, stable with exponentially convergent tracking error, perhaps only boundedness of signals can be assured,
- be capable to solve the tracking problem in the presence of disturbances, model errors etc., perhaps only approximately, *e.g.*, no point wise convergence of the tracking error but set wise,
- have provisions for robustness enhancements, preferably by using easy to derive bounds for the model error directly in the design method, and not by a trial and error method that may be necessary to suit a rigid mathematical framework.

The chosen controllers are expected to more or less fulfill these criteria.

We first give an overview of the control system design process, introducing several key notions that are repeatedly used. Then we elaborate, section wise, the control schemes enumerated above.

4.1. Introduction

We given an overview of the general methodology used in the design of control systems. In this overview we discuss the following aspect of the design process.

- The structure of the design process, iterative with repeated human intervention or fully automatic when the design data is available.
- The use of models of the system to be controlled.
- The specification used to express a favorable dynamical behavior of the control system.
- The mathematical framework which will "embody" the control design methods.
- The use of feedback and feedforward to modify the dynamic characteristics of a plant.
- The restrictions and freedom offered by the design methods that are used to generate the control structure and controllers.
- The interpretation of deviations from the specifications (are they acceptable or not?).

Then we give a short discussion of linearizing state-feedbacks, followed by a further specialization of the model structure, to be used in the sequel.

4.1.1. Control system design methodology. The methodology commonly used in the design of control systems requires to perform an interrelated set of activities that consist of several tasks and an information exchange. A flow chart of these tasks is in Fig. 4.1. We discuss the tasks, symbolically represented by the blocks, and the input and output of (or information required and produced by) these tasks, represented by the solid arrows. The dashed arrows represent relations that could be, but are not necessarily, present.

The system to be controlled should be known well, by using a model based on first principles (Newton's law, conservation principles, etc.), or by using model identification based on experimental data. Inevitably, some explicit or implicit assumptions are made when the model is set up. It is advantageous to make explicit as much assumptions as possible, because they enable us to get an estimate of the accuracy of the model. A knowledge of the accuracy of and errors contained in the model is mandatory for the formulation of a correct error model. It also largely determines the attainable performance of the control system. One should not expect to gain more than an order of magnitude in performance by using advanced control schemes. Normally, it is possible or even necessary to adapt the

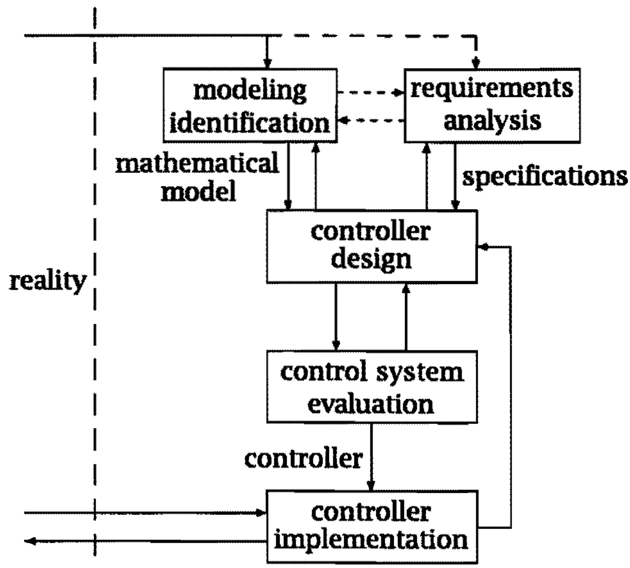

FIGURE 4.1. Structure of the control system design methodology

system or process to make it better suited for control. For instance, a correct placement of actuators and sensors is sometimes essential to obtain a stable controlled system, so these devices should be positioned with great care. Modification of a plant can be impeded, because of conflicting interest. In this case the designer or process engineer and the control engineer should generate a workable solution, arrived at after mutual consultation.

It is often advantageous to explicitly formulate the design objectives, or even to put them in some formal specifications, without regard to the formulation used by the design methods that are to be employed later on. This will clarify the needs to be solved. Mostly these requirements are put in the form of limits on pertinent signals, *e.g.*, no overshoot or a critically damped system (expressed in characteristics of a step response), dominant time constants or rise time (also related to the step response, but more directly but less accurately addressed by pole locations), tracking error (for the tracking problem, related to the rise time), suppression of periodic disturbances (commonly expressed in terms of frequency response). Often, the requirements are a mix of time domain and frequency domain requirements, expressed in some measure, in general a norm of a signal or a system. A problem is that design methods can only handle one type of requirements (time or frequency domain) and a translation between these domains is not always possible. Design methods that can handle mixed time and frequency domain requirements are virtually non existent. Also the type of goals a design method tries to reach does generally not overlap the type of the specifications. For example, an optimal controller will not automatically generate a critically damped control system, only by tedious manipulation of control design parameters can such a goal be reached. This makes it necessary to design by trial and error, or to employ the optimal controller algorithm as a subordinate layer of an optimization algorithm. Finally, the design specifications can be too tight, so no control systems can fulfill them, see [164]. In this case it is advantageous to be able to check this beforehand, instead of trying, in vain, to design a control system. Most design methods are not able to warn for this condition.

The next step is the design of the controller itself, this requires a preliminary step where the model and formal specifications are put in some mathematical framework, *e.g.*, a state space model and weighting functions. As remarked in the previous paragraph, the translation of the specification in terms that the control design method can handle is not automatic, and this is again an area where involvement of a control engineer is necessary. Based on experience and intuition, and guided by some rules of thumb, this translation has to be performed. Often this will not be a one shot operation, but some iterative refinement steps are necessary. The main item of the controller design is, however, the choice of an appropriate controller structure and a suitable design method. A lot of control structures are available, but they all employ two basic approaches, namely feedforward and feedback. On optimal mix of these two approaches is difficult to obtain, and a sound theory that will guide the innocent to an appropriate choice is lacking. Here, the control engineer has again to employ his skills to arrive at a satisfying solution. When the structure of the control system is fixed, one can choose from a forest of methods to fill in the remaining blanks, *e.g.*, the control system parameters. Often these methods are related, they have some aspects in common and it is possible to combine them to eliminate certain shortcomings of a particular method or to profit from a mutual impetus. Only three examples of well known design methods are mentioned here: optimal control, pole placement and adaptive control.

To verify the design and to aid in the refinement of the choice of the control structure and the control system design parameters, the design is evaluated by simulation of the closed loop system. Often it is necessary to back track, because the translation to the mathematical framework does not give a one-to-one correspondence with the specifications, the selection of the control structure or the chosen control design method are not adequate, and repeat the controller computation with adapted weighting functions, changed structure or another design method. It may even be necessary to adapt the model or even the specifications because they cannot easily be obtained or perhaps they are even outside the limits of performance, to obtain a satisfactory behavior of the control system.

The final test of the design is the implementation of the controller in the control system and acceptance testing/commissioning. More often than not it will be necessary to tune some control system parameters to circumvent peculiarities that are not covered by the model and the evaluation of the control system by simulations. Only after successful acceptance testing we are sure that the control system can meet the specifications. The proof of the pudding is still the eating.

4.1.2. Exact linearization by state-feedback. The presentation of linearizing state-feedback is based on the *relative degree* and *normal forms* [151].

DEFINITION 4.1. The square nonlinear system, affine in the control input u

$$\dot{x} = f(x,\theta)+g(x,\theta)u, \qquad y = h(x,\theta) \tag{4.1}$$

is said to have a (vector) relative degree $\{r_1,\ldots,r_m\}$ at $x = x^o$ if

(1) $L_{g_j}L_f^k h_i(x) = 0$ for $i,j = 1,\ldots,m$, $k = 1,\ldots,r_i-2$, and for all x in a neighborhood of x^o,
(2) the $m \times m$ matrix

$$A(x) = \begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(x) & \cdots & L_{g_m}L_f^{r_1-1}h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1}L_f^{r_m-1}h_m(x) & \cdots & L_{g_m}L_f^{r_m-1}h_m(x) \end{bmatrix}$$

is nonsingular at x^o.

Here $L_f^k h_i(x)$ means the k^{th} successive Lie derivative of the scalar function $h_i(x)$ in the direction of the vector field f. The existence of a relative degree at x^o is a sufficient but not necessary condition for the linearization (in the input-output sense) of system (4.1) around x^o. This is based on the existence of a coordinate transformation $(\xi,\eta) = \Phi(x)$ that transforms (4.1) to the *normal form*

$$\begin{aligned} y_i = h_i(x,\theta) &= \xi_i^1 \\ \dot{\xi}_i^1 &= \xi_i^2 \\ \dot{\xi}_i^2 &= \xi_i^3 \\ &\vdots \\ \dot{\xi}_i^{r_i} &= b_i(\xi,\eta) + \sum_{j=1}^{m} a_{ij}(\xi,\eta)u_j \qquad \text{for } i = 1,\ldots,m \end{aligned} \tag{4.2}$$

and

$$\dot{\eta}_i = q_i(\xi, \eta) \qquad \text{for } i = 1 + \sum_{j=1}^{m} r_j, \ldots, n$$

where

$$\begin{aligned} a_{ij}(\xi,\eta) &= L_{g_j} L_f^{r_i - 1} h_i(\Phi^{-1}(\xi,\eta)) \\ b_i(\xi,\eta) &= L_f^{r_i} h_i(\Phi^{-1}(\xi,\eta)) \qquad \text{for } i, j = 1, \ldots, m \end{aligned}$$

so the terms a_{ij} are the entries of matrix A. We can therefore compactly write (with only equations containing the input u in (4.2))

$$\begin{aligned} \xi^{(r)} &= b(\xi,\eta) + A(\xi,\eta)u \\ \dot{\eta} &= q(\xi,\eta), \end{aligned} \tag{4.3}$$

where $\xi^{(r)}$ are the m elements of ξ on the $r_1, r_1 + r_2, \ldots, r$ places, with $r = \sum_{j=1}^{m} r_j$. Because A is nonsingular if the relative degree is well defined, the control

$$u = A^{-1}(v - b) \tag{4.4}$$

with the new input v is properly defined and linearizes the part of system (4.2) that is visible at the output of the system

$$\xi^{(r)} = v.$$

The nonlinear dynamics obtained when the output $y = h(x)$ is restricted to 0 by suitable initial conditions for ξ, *i.e.*, $\xi = 0$, and a suitable control v in (4.4), *i.e.*, $v = 0$, is

$$\dot{\eta} = q(0,\eta), \quad \eta(0) = \eta^o.$$

It is invisible at the output, and is called the *zero dynamics* of the system, because the dynamics is related to the zeros for linear systems, and also because it is related to the zero output.

REMARK 4.1. The given structure $\dot{\eta} = q(\xi,\eta)$ of the zero dynamics, with no explicit dependence on the input u, depends on a special form of the coordinate transformation $\Phi(x)$. When this specific Φ does not exist, it requires the involutivity of the distribution spanned by the vector fields $g_1, \ldots, g_m$, or cannot be found, which often happens because it may require the solution of some integrable partial differential equations that are difficult to solve, a more general structure can be derived, where $\dot{\eta}$ depends on u, *i.e.*, $\dot{\eta} = q(\xi,\eta) + p(\xi,\eta)u$. In that case the transformation Φ is only required to be invertible, and therefore its Jacobian should be nonsingular at x^o, and the zero dynamics is $\dot{\eta} = q(0,\eta) + p(0,\eta)u$ with $u = -A^{-1}(0,\eta)b(0,\eta)$.

REMARK 4.2. The linearization described above is a linearization from the new input v to the output y, an input-output linearization. With some regularity assumptions, the existence of an output map $y = h(x)$ for which there is no zero dynamics, *i.e.*, $r = n$, is equivalent with exact input-state linearization, *i.e.*, the model (4.1) can be transformed to a linear one with static or dynamic state-feedback and a change of coordinates [151, pp. 243–259]. The conditions on the output h for input-state linearization are therefore more restrictive than for input-output linearization.

REMARK 4.3. If local asymptotic stability of the state of the model is required, it is sufficient that there exists an output map $y = h(x)$ for which the zero dynamics is critically asymptotically stable. Under some regularity assumptions and with a more general definition of the zero dynamics – that can also be used when the relative degree is not defined – this condition is also necessary for the existence of a smooth locally stabilizing feedback law. If the relative degree is defined, the model can be stabilized using a state-feedback control law based on (4.4), but an output feedback may not be sufficient. For more information see [40]. If the zero dynamics is unstable, or if the relative degree is not defined, the control law based on (4.4) is not sufficient for asymptotic stabilization or does not exist. In these cases one has to find another control law, perhaps by using another approximate model, by defining another output function h, or by an approximate input-output linearization procedure. For some examples see [46, 165–167]. Stability requirements can also be combined with other structural control problems, *e.g.*, local disturbance decoupling with stability, see [168].

4.1.3. Model of mechanical systems. The discussion in this chapter will use the following model for a mechanical system as a basis for the presentation of the control schemes

$$M(q,\theta)\ddot{q} + h(q,\dot{q},\theta) = f \tag{4.5}$$

with $M(q,\theta) \in \mathbb{R}^{m\times m}$ the inertia matrix, defined such that $\frac{1}{2}\dot{q}'M\dot{q}$ is the kinetic energy, $f \in \mathbb{R}^m$ the control forces applied on the system, $h(q,\dot{q},\theta) \in \mathbb{R}^m$ all other internal or external forces acting on the system, *e.g.*, Coriolis, centrifugal, friction, and gravitational forces, $q \in \mathbb{R}^m$ the m degrees-of-freedom, and $\theta \in \mathbb{R}^p$ the p model parameters. Here it is assumed that the number of degrees-of-freedom of the model is equal to the number of control inputs, thereby making it easy to derive a linearizing state-feedback. The feedback linearization property is a consequence of the specific structure of model (4.5), that is not shared by all Hamiltonian systems, *e.g.*, nonholonomic mechanical systems. It can always be arranged that $M(q,\theta)$

is symmetric and positive definite. Then $\dot{q}'f$ has the units of power, which means that the degrees-of-freedom q are dual to the control force f. This is possible by proper definitions of the degrees-of-freedom q and of the control force f. When the equations of motion are derived with the Lagrange formalism, the mass matrix M will always be symmetric.

The mass matrix M is positive definite and therefore invertible and (4.5) can always be written as

$$\ddot{q} = -M(q,\theta)^{-1}h(q,\dot{q},\theta)+M(q,\theta)^{-1}f,$$

i.e., as a set of explicit second order differential equations. This can easily be put in state space form. This model is therefore an element of a subclass of the class of models affine in the input u as given by (4.1). It does not encompass the whole class, because (4.5), written like (4.1), has a special structure, and also because the input f is not only affine in (4.5) but, stronger, enters in (4.5) with a unit matrix as input matrix. This model fulfills the requirements for feedback linearization, mentioned earlier in this section, because it can easily be put in the form of (4.3) by taking $\xi = \{q_1, \dot{q}_1, \ldots, q_n, \dot{q}_n\}$ so

$$\xi^{(r)} = -M^{-1}h + M^{-1}f$$

with vector relative degree $\{2,\ldots,2\}$, $r = 2m$, and no zero dynamics. Comparing this with (4.3) we see that $b \equiv -M^{-1}h$, $A \equiv M^{-1}$, and $u \equiv f$.

4.2. Linearizing state-feedback and linear controllers

The controllers are based on the model (4.5) of a mechanical system. This model is used for a linearizing state-feedback only

$$f = M(q,\theta)(\ddot{q}_d + \nu)+h(q,\dot{q},\theta) \tag{4.6}$$

with q_d the desired trajectory. Substitution in the model equation (4.5) gives

$$M(q,\theta)(\ddot{q}_d - \ddot{q} + \nu)= 0$$

or simply

$$\ddot{\tilde{q}} + \nu = 0$$

with $\tilde{q} = q_d - q$ the tracking error of the system. This is simply the model for two integrators in series for each degree-of-freedom.

The new control input ν comes from another component of the control system. In general ν is generated by a linear dynamic system

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c \tilde{q} \\ \nu &= C_c x_c + D_c \tilde{q} \end{aligned} \tag{4.7}$$

where the choice of the control parameters A_c, B_c, C_c, D_c depends on the control design method and the specification used.

The robustness analysis is sometimes based on norm bounded uncertainties in the mass matrix and the nonlinear Coriolis and centrifugal terms. See [57], and [58] for some corrections. This method is proposed by [61] also. They stress the conservativeness of the design method, because the norm bounds are naturally global and not very specific. Therefore, they recommend simulation as a tool to judge the final design. See [62] for an introduction to this type of control schemes.

We discuss three classes of design methods used to generate (4.7)

- 2-norm or ∞-norm based controllers,
- μ-synthesis controller,
- a reference PD controller.

4.2.1. Norm based linear controllers. The control design methodology for linear systems used in this section is based on the definition of a norm type optimality criterion, on the parameterization of a representative class of controllers, and on the determination of the controller's parameters that minimize the criterion. For the sake of simplicity, the structure of the system (and controller) is limited to linear time invariant (LTI) dynamic systems for which a state space model exists.

The design problems can be cast in the form of the following *standard problem*. Given the abstract general system $G(s)$ with an exogenous input u_1, *e.g.*, disturbance and reference signals, a command input u_2, generated by the controller, a controlled output y_1 and measurement vector y_2, it is necessary to design a (dynamic) controller $F(s)$ with a controller input y_2 and controller output u_2 so that some criterion, related to the signals u_1 and y_1, is minimal and the system is internally stable. The general system $G(s)$ has to fit the following state space description

$$
\begin{aligned}
\dot{x} &= A_g x + B_{g1} u_1 + B_{g2} u_2 \\
y_1 &= C_{g1} x + D_{g11} u_1 + D_{g12} u_2 \\
y_2 &= C_{g2} x + D_{g21} u_1 + D_{g22} u_2
\end{aligned}
\tag{4.8}
$$

where x is the state of the system, A_g is the system, B_g the input, C_g the output, and D_g the connection matrix, see Fig. 4.2. Here, u_2 can be identified with v and y_2 with $\tilde{q}$ in (4.7).

The following short notation for the system $G(s)$ will be used

$$
\begin{bmatrix} A_g & B_{g1} & B_{g2} \\ C_{g1} & D_{g11} & D_{g12} \\ C_{g2} & D_{g21} & D_{g22} \end{bmatrix}.
$$

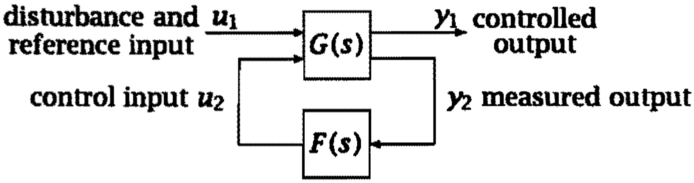

FIGURE 4.2. Standard problem setup

To fulfill the design criteria certain weighting factors or weighting transfer functions $W(s)$ are used to augment the plant transfer function $P(s)$, see Fig. 4.3.

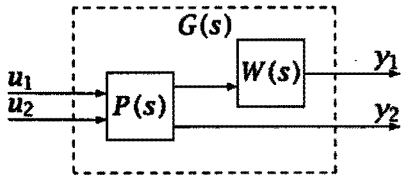

FIGURE 4.3. Augmented plant

Normally, a designer can specify his wishes in the time domain or in the frequency domain. In the first case, the choice of $W(s)$ will be based mainly on a trial and error procedure, although more direct methods are an active area of research [169]. In the last case, the choice of $W(s)$ is relatively straight forward.

Given the state space model for the plant $P(s)$

$$\begin{bmatrix} A_p & B_{p1} & B_{p2} \\ C_{p1} & D_{p11} & D_{p12} \\ C_{p2} & D_{p21} & D_{p22} \end{bmatrix} \tag{4.9}$$

and the frequency weighting filters $W(s)$

$$\begin{bmatrix} A_w & B_w \\ C_w & D_w \end{bmatrix}$$

the (not necessarily minimal) model for $G(s)$ can be written as

$$\begin{bmatrix} A_g & B_{g1} & B_{g2} \\ C_{g1} & D_{g11} & D_{g12} \\ C_{g2} & D_{g21} & D_{g22} \end{bmatrix} = \left[\begin{array}{cc|c|c} A_p & 0 & B_{p1} & B_{p2} \\ B_w C_{p1} & A_w & B_w D_{p11} & B_w D_{p12} \\ \hline D_w C_{p1} & C_w & D_w D_{p11} & D_w D_{p12} \\ \hline C_{p2} & 0 & D_{p21} & D_{p22} \end{array}\right]. \tag{4.10}$$

For the optimality criterion, three different criteria are considered

- A quadratic criterion in the time domain. This can result in the Linear Quadratic Regulator (LQR) control system design method, or, with a Kalman filter, in the Linear Quadratic Gaussian (LQG) design method. This criterion can be viewed also as an L_2 norm.
- A quadratic criterion in the frequency domain. The method based on this optimality criterion is called the H_2 method, where the H refers to the Hardy space (a space with stable transfer functions as its elements) and 2 refers to the quadratic criterion. Such a criterion can be viewed also as a norm for functions that are elements of the Hardy space.
- A supremum criterion in the frequency domain, which leads to the so-called H_∞ method, where ∞ comes from the infinity norm used for elements in the Hardy space.

The LQR and LQG methods are well-known and well documented [170, 171]. The H_2 and H_∞ methods are more recent. The solution algorithm for the H_2 problem is given in [159]. The LQG and H_2 methods are closely related. For the H_∞ design method, two generations of solution algorithms are known. The first generation is dated 1984–1987 [157], more recent algorithms are from 1988–1989 [158, 159].

4.2.1.1. *LQ design.* The LQ design method is based on the infinite time optimality criterion

$$J = \lim_{t\to\infty} \int_0^t \begin{bmatrix} x' & u_2' \end{bmatrix} Q \begin{bmatrix} x \\ u_2 \end{bmatrix} d\tau, \quad Q = \begin{bmatrix} Q_{xx} & Q_{xu} \\ Q_{xu}' & Q_{uu} \end{bmatrix} \tag{4.11}$$

and solves the following regulator problem

$$\min_{u_2}! \, J \quad \text{sub } \dot{x} = A_g x + B_{g2} u_2, \quad x_0 = x(0).$$

The weights $Q_{xx} = Q_{xx}' \geq 0$, $Q_{uu} = Q_{uu}' > 0$, and Q_{xu}, $Q \geq 0$, must be chosen by the designer so that certain design criteria are satisfied. In general, if the matrices Q_{xx} and Q_{uu} are chosen diagonal, an increase in a diagonal element of Q_{xx} will decrease the corresponding state and an increase in a diagonal element of Q_{uu} will decrease the corresponding input. Correct values for Q_{xx} and Q_{uu} can often only be found by trial and error. In our case, Q_{xx} and Q_{rr} are not chosen at all, but they depend on the weight functions $W(s)$, that can often be used more directly to obtain a controller that satisfies certain specifications.

A result of stochastic control theory is that the solution of the regulator problem can be used also as the solution of the standard problem, if the input signal u_1 is Gaussian white noise and the expectation of J is minimized.

The influence of u_1 can be neglected for the design of K_c, so

$$J = \lim_{t\to\infty} \int_0^t y_1' y_1 \, d\tau = \lim_{t\to\infty} \int_0^t \begin{bmatrix} x' & u_2' \end{bmatrix} \begin{bmatrix} C_{g1}' \\ D_{g12}' \end{bmatrix} \begin{bmatrix} C_{g1} & D_{g12} \end{bmatrix} \begin{bmatrix} x \\ u_2 \end{bmatrix} d\tau.$$

The weight matrix Q for use in (4.11) is then

$$Q = \begin{bmatrix} C_{g1}' C_{g1} & C_{g1}' D_{g12} \\ D_{g12}' C_{g1} & D_{g12}' D_{g12} \end{bmatrix}.$$

If the weight matrix Q does not satisfy the conditions given above, the standard problem is not properly defined, *e.g.*, the specifications expressed in $W(s)$ cannot be met, and $W(s)$ must be changed.

The optimal input signal u_2 is independent of the initial values x_0 of the state x, is a linear combination of state variables, $u_2 = -K_c x$, and stabilizes the system, if the triple $(\sqrt{Q_{xx}}, A_g, B_{g2})$ is detectable and stabilizable [171].

The solution K_c of the LQR problem follows from

$$K_c = Q_{uu}^{-1}(B_{g2}' P_c + Q_{xu}')$$

where P_c is the symmetric, (semi-)positive solution of the Riccati equation

$$P_c A_g + A_g' P_c - (P_c B_{g2} + Q_{xu}) Q_{uu}^{-1} (B_{g2}' P_c + Q_{xu}') + Q_{xx} = 0.$$

If the system and measurement noise intensities are non zero or if not all state variables can be measured, a Kalman filter

$$\dot{z} = A_g z + B_{g2} u_2 + K_f (y_2 - C_{g2} z - D_{g22} u_2)$$

can be designed to obtain an estimate z of the state x to form the feedback as $u_2 = -K_c z$.

The Kalman filter gain K_f follows from

$$K_f = (P_f C_{g2}' + V_{wv}) V_{vv}^{-1}$$

by solving the dual Riccati equation for $P_f = P_f' \geq 0$

$$P_f A_g' + A_g P_f - (P_f C_{g2}' + V_{wv}) V_{vv}^{-1} (C_{g2} P_f + V_{wv}') + V_{ww} = 0$$

where V_{ww} and V_{vv} are the intensities of the system and output Gaussian white noises and V_{wv} is the cross intensity between the system and output noise. The dual of the conditions for the LQR problem must be satisfied, to ensure the existence of a stabilizing Kalman filter gain K_f.

If not all state variables can be measured, but noise is not an issue, the state can be reconstructed with a Luenberger observer with the same structure as a Kalman filter. The noise intensity matrix

$$V = \begin{bmatrix} V_{ww} & V_{wv} \\ V'_{wv} & V_{vv} \end{bmatrix}$$

is then used as a design parameter, it should satisfy the same conditions as the matrix Q, and is tuned to get good dynamics characteristics of the observer. Both the Kalman filter and the Luenberger observer are dynamic state space systems, using the measured output.

A simple controller can be obtained with an output feedback $u_2 = -K_c C_{g2} x$. The corresponding fixed gain linear quadratic output feedback (LQOF) problem for calculating K_c is harder to solve. Because there is no known closed form of solution, the design can only be performed by numerical methods, and is often iterative. The solution also depends on the initial condition x_0. One method for solving the LQOF problem is to minimize the criterion J numerically. The criterion J can be expressed in the solution $P_c = P_c' \geq 0$ of the following Lyapunov equation

$$\begin{aligned} &P_c(A_g - B_{g2}K_cC_{g2}) + (A_g - B_{g2}K_cC_{g2})'P_c \\ &\qquad - Q_{xu}K_cC_{g2} - (Q_{xu}K_cC_{g2})' + C_{g2}'K_c'Q_{uu}K_cC_{g2} + Q_{xx} = 0 \end{aligned}$$

as

$$J = \operatorname{trace}(P_c \, \mathrm{E}(x_0 x_0'))$$

where E is the expectation operator. Then minimization of J can be accomplished with standard algorithms.

4.2.1.2. *H_2 design.* The H_2 design method is based on the following quadratic operator norm in the frequency domain, for a stable transfer function matrix T

$$\|T\|_p = \left[\lim_{\Omega \to \infty} \int_{-\Omega}^{\Omega} \sum_{i=1}^{n} (\sigma_i(T(j\omega)))^p \, d\omega \right]^{1/p} \tag{4.12}$$

where $p = 2$ and $\sigma_i(T(j\omega))$ is the i^{th} singular values of $T(j\omega)$, *i.e.*, the i^{th} square root of the sorted eigenvalues of $T(j\omega)'T(j\omega)$, evaluated for ω in $[-\Omega, +\Omega]$. The square root of the sum of the singular values squared is the matrix norm induced by the Euclidean vector norm. When the system has only one input or one output, the transfer function $T(s)$, evaluated at $j\omega$, is a row or a column and the single singular value is equivalent to the Euclidean vector norm.

For the standard problem the H_2 norm of the closed loop transfer function

$$T_{11} = G_{11} + G_{12}(I - FG_{22})^{-1}FG_{21}$$

from u_1 to y_1 must be minimized, where it is assumed that rank(D_{g12})= dim(u_2) and rank(D_{g21})= dim(y_2) in (4.8).

The solution of a standard H_2 problem is the same as the solution of an equivalent LQG problem where the weights Q_{xx}, Q_{xu}, and Q_{uu} for the LQR problem and the noise intensities V_{ww}, V_{wv}, and V_{vv} for the Kalman filter problem satisfy [159]

$$Q = \begin{bmatrix} C'_{g1}C_{g1} & C'_{g1}D_{g12} \\ D'_{g12}C_{g1} & D'_{g12}D_{g12} \end{bmatrix}, \qquad V = \begin{bmatrix} B_{g1}B'_{g1} & B_{g1}D'_{g21} \\ D_{g21}B'_{g1} & D_{g21}D'_{g21} \end{bmatrix}.$$

It is necessary that $D_{g11} = 0$ for the H_2 problem to be well posed, because when $D_{g11} \neq 0$ the 2-norm is not well defined. The triple (C_{g2}, A_g, B_{g2}) must be detectable and stabilizable for a solution of the H_2 problem to exist. Solving the H_2 problem requires the solution of two Riccati equations and certain matrix computations. The resulting controller $F(s)$ has the structure of a cascade of an observer of the same order as the system $G(s)$ and a state-feedback.

The LQ time domain and H_2 frequency domain criteria are closely related by Parseval's relation. Therefore, the LQG and H_2 design methods can be considered equivalent for all practical purposes.

4.2.1.3. *H_∞ design.* The H_∞ design method is similar to the H_2 design method, but the design is based on the supremum norm (the so-called H_∞ norm) instead of the H_2 norm.

The H_∞ norm is defined by the generalization of (4.12) with $p \to \infty$, and is given by

$$\|T\|_\infty = \sup_\omega \bar{\sigma}(T(j\omega))$$

where $\bar{\sigma}(T(j\omega))$ is the largest singular value of the transfer function $T(j\omega)$. For the standard problem $\|T_{11}\|_\infty$ must be minimized instead of $\|T_{11}\|_2$ and $D_{g11} \neq 0$ is permissible.

When the design is based on a state space formulation of the H_∞ problem [158], it is worth noting that solving the H_∞ problem, when a solution exists, requires the solution of two Riccati equations and some matrix computations, embedded within a one parameter search. The one parameter search is called γ iteration. This technique involves finding the optimal controller by specifying the H_∞ bound γ that the resulting closed loop system T_{11} should at least satisfy, and by reducing the value of γ, until no solution exists. Since iteration has to be repeated until there is no solution, this procedure is tricky numerically. Sometimes numerical problems can be circumvented if one is content with a suboptimal controller. The formulation given in [172] may alleviate this problem. The resulting controller has a lower order than the system G.

4.2.2. Linear controller synthesized with μ-specifications. For some design problems, *e.g.*, loop shaping, a design methodology based on ∞-norm or singular value specifications can often be used, although this methodology is not without critique [173]. For these specifications H_∞ controllers are adequate. However, not all design goals can be expressed in loop shaping and ∞-norm specifications [174]. Two examples are

- the robust performance problem for MIMO systems, that can be reformulated as a structured robust stability problem (with 2 blocks),
- the robust stability or robust performance with structured uncertainty.

For the first problem, an H_∞ design can be sufficient when the plant is not skew [175]. Otherwise, an H_∞ design, because only $\bar{\sigma}$ is considered, can give (too) conservative results. Some problems cannot be solved by $\bar{\sigma}$ specifications alone. Therefore structured singular values are introduced. In both problems mentioned above, design wishes can be put in the form of structured singular values, *i.e.*, μ-specifications. This is primarily based on the following theorems, which guarantee the desired behavior of the system, if the value of μ does not exceed a certain threshold [176].

- Robust stability: a feedback loop involving $T(s)$ and Δ is stable for all $\Delta \in \mathbf{\Delta}$ with $\bar{\sigma}(\Delta) < 1$ iff $\|T\|_{\mathbf{\Delta}} < 1$.
- Robust performance: by diagonally augmenting $\mathbf{\Delta}$ with an additional block for the performance specifications an equivalent robust stability problem has to be solved.

The structured singular value for a complex matrix M can be defined as

$$\mu_{\mathbf{\Delta}}(M) = \max_{\Delta \in B\mathbf{\Delta}} \rho(M\Delta)$$

or as

$$\mu_{\mathbf{\Delta}}(M) = \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(I - M\Delta) = 0\}}$$

where

$$B\mathbf{\Delta} = \{\Delta \in \mathbf{\Delta} : \bar{\sigma}(\Delta) \leq 1\}$$

because both definitions are equivalent [176]. Here the set of norm bounded matrices $B\mathbf{\Delta}$ has a specific structure $\mathbf{\Delta}$, that is defined as a set of matrices with a block structure, the reason why μ is called structured singular value. The structured singular value μ is not a norm because the triangle inequality is not satisfied.

For a transfer function T, the "μ" value is just defined as the maximum value of μ over all frequencies

$$\|T\|_{\Delta} = \sup_{\omega \in \mathbb{R}} \mu_{\Delta}(T(j\omega)).$$

As a function of frequency, the μ value of a transfer function, is not an analytic function, but belongs to the class of subharmonic functions [177], which makes the mathematical analysis more complicated.

A major problem with the controller synthesis based on μ-specifications is caused by the difficulties with the computation of μ. Techniques are known to efficiently compute an upper bound of μ based on the following property

$$\mu_{\Delta}(M) \leq \inf_{D \in \boldsymbol{D}} \bar{\sigma}(DMD^{-1}) \tag{4.13}$$

where $\boldsymbol{D}$ is a set of scaling matrices with a structure compatible with Δ, but this bound need not be close. An exact lower bound for μ can be computed based on the property

$$\max_{Q \in \boldsymbol{Q}} \rho(QM) = \mu_{\Delta}(M)$$

where $\boldsymbol{Q}$ is a set of matrices with the same structure as Δ and $Q^*Q = I$, but this computation is not effective nor efficient because the optimization problem is not convex.

For the design of controllers based on μ-specifications several methods are known, and one of them, the DK-iteration technique, is often used, but is not guaranteed to always deliver a linear controller satisfying the specifications on μ, when such a controller does exist. The DK-iteration is an intertwined computation between an H_∞ controller design and the synthesis of so-called D-scalings, it is thus based on (4.13), to appropriately modify the weighting functions used in the H_∞ design. The iteration has to be performed a number of times, until no further improvements, *i.e.*, a lower value for μ, can be obtained. So one can conclude that the design of μ controllers is more or less a solved problem, although some details are still not satisfactory. Software to perform the required computations is available [162].

The analysis and design of μ related problems is often treated starting from the formulation of a standard problem. This problem setup is shown in Fig. 4.4. The signals in this figure are associated with

- the input and output of the controller $F(s)$ to be designed,
- the input and output of the structured uncertainty Δ, modeled as a feedback,

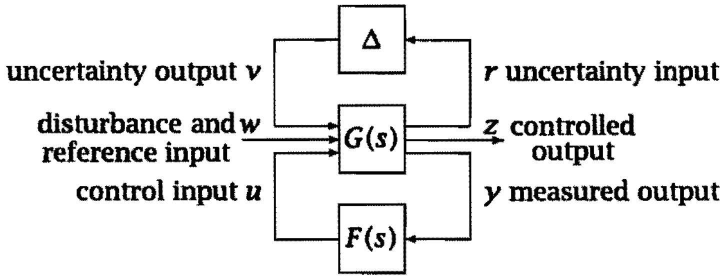

FIGURE 4.4. Standard problem setup for μ-synthesis

- the input and output for which the μ value is specified, after closing the loop by $F(s)$, the controller to be designed, and for all possible choices of Δ.

The uncertainty Δ has a specific structure $\mathbf{\Delta}$, but is assumed to be unspecified further, except for a norm bound

$$\|\Delta\|_\infty < \beta$$

so it is not useful, without further modifications, to handle real parameter uncertainty. Variations in the magnitude of the uncertainty are put in weighting functions that are combined with the plant P in the general system G. Also additional phase information of the uncertainties can only be handled by incorporating it in the weighting functions. This setup can be used to solve different types of problems. The type of problem solved is expressed by making a choice for the signals w and z, so by changing the definition of the input and output several design problems, *e.g.*, noise suppression, performance, robust stability and robust performance, can be put in the same framework.

Besides the computational problem, the main problem, however, is the specification of the weighting functions to be used in the design, *i.e.*, the setup of the standard problem, to express all the wishes the designer has with regard to the dynamic behavior of the controlled system. In our case, where the μ-synthesis controller is used as an outer loop, the problem stems from the specifications of the structured uncertainty. The nominal model for the outer loop design is simply a cascade of integrators. The problem of selecting a structure for the uncertainty and the corresponding weights, that fits the errors in the model used for the linearizing state-feedback, is largely unsolved. Up to now, one can only approximate, based on some simplifying assumptions, *e.g.*, a local linearization of the problem, a solution for the uncertainty modeling problem. This will be presented in more detail in Section 5.3.4, where a controller synthesized with μ-specifications is discussed.

The form of the μ-synthesis controller is the same as the form of the H_∞ controller, it is just a dynamic system given in state space form and can replace simply an existing H_∞ controller. The number of states depends on the number of states of the system to be controlled, of the states of the weighting functions and of the D-scalings. If the weights are improper, additional states are necessary if the improperness cannot be accommodated for by "plant state tapping" [178]. In general, the number of states of the μ controller will be larger then that of a corresponding H_∞ controller, because of the additional D-scalings.

4.2.3. PD control. A simple exponent of the class of linear controllers is a PD controller, acting on the controlled output y and its derivative $\dot{y}$

$$v = K_v\dot{\tilde{y}} + K_p\tilde{y},$$

for use in (4.4), with K_v and K_p positive definite matrices, *e.g.*, diagonal matrices with positive diagonal elements. When velocity measurements are available as part of the measured state, the PD controller can be implemented as a partial static state-feedback controller, *i.e.*, as (4.7) but without controller states. When only position measurements are available, an observer is needed to reconstruct the velocities and the PD feedback can be implemented as (4.7). A PD controller can be used without linearizing state-feedback also. Then

$$u = K_v\dot{\tilde{y}} + K_p\tilde{y}, \tag{4.14}$$

generating u instead of v, for use in (4.1).

Controllers which consist of three levels: a linearizing state-feedback, a stabilizing PD controller, and a robustifying outer loop are also proposed.

4.3. Controllers for rigid robots

These controllers are sometimes based on a model that is not necessarily used to linearize the system. Stability is often proved by the second method of Lyapunov. Much variation is possible.

A premier example of adaptive computed torque control is a scheme proposed by Slotine and Li [106, 107]. We selected this method because of its simplicity and elegance. Also, measurements of the joint accelerations and inversion of the inertia matrix M are not necessary for its implementation, and the structure of the model is fully exploited. The exploitation of the model structure is also a disadvantage, because it limits the area of applicability of this controller.

A related control scheme is proposed by Kelly [111]. In both schemes uncertain model parameters are estimated. For an overview of these and related methods see [98, 99].

A VSS or sliding surface control scheme is based on quite another approach. First one defines a manifold in state space, a *sliding surface*, with nice properties and designs a controller, acting in this manifold, which realizes those properties. Second, one adds a component to the controller that assures the arrival of the state on this manifold in a finite time, despite parameter errors in the model.

We also discuss PD control, just an application of position and velocity feedback for robots, that can be used in an outer loop as part of a computed torque scheme or directly as a decentralized joint controller.

Adaptive computed torque control and VSS control can be merged easily. Some parameters are then estimated, other parameters have a fixed value, but based on known parameter error bounds, the VSS part of the controller assures convergence to the sliding surface, see [179].

Before we give a short description of those schemes (see also [180] for the adaptive controllers) we give a slightly more detailed model of a mechanical system, used to present the control schemes:

$$M(q,\theta)\ddot{q} + C(q,\dot{q},\theta)\dot{q} + g(q,\dot{q},\theta) = f \tag{4.15}$$

where $M(q,\theta)$ is the $m \times m$ positive definite inertia matrix, with model parameters θ, $C(q,\dot{q},\theta)\dot{q}$ is the m vector of Coriolis and centripetal forces, $g(q,\dot{q},\theta)$ the m vector of gravitational forces, Coulomb, and viscous friction, f the m vector of generalized control forces (forces or torques). In this model each degree-of-freedom has its own motor. Here, we neglect the dynamics of the motors and amplifiers, stiction, backlash, and flexibility of the joints and links. Compared with (4.5) the forces acting on the manipulator are given in more detail. That is needed in the stability proof.

4.3.1. Adaptive control scheme of Slotine and Li. The adaptive control scheme of Slotine and Li [163] has a feed forward component, based on an estimate of the manipulator dynamics, and a PD component. The generalized control force is just the sum of these components

$$f = \hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{g}(q,\dot{q}) + K_v s \tag{4.16}$$

where $\hat{M} = M(q,\hat{\theta})$, $\hat{C} = C(q,\dot{q},\hat{\theta})$, and $\hat{g} = g(q,\dot{q},\hat{\theta})$ are the same as the corresponding terms in (4.15), with $\hat{\theta}$ an estimate of the model parameters θ, $\dot{q}_r = \dot{q}_d + \Lambda\tilde{q}$ a virtual reference trajectory, $s = \dot{\tilde{q}} + \Lambda\tilde{q}$ a measure of tracking accuracy and certainly not a filtered q_d, $\tilde{q} = q_d - q$ the tracking error and $q_d(t), \dot{q}_d(t), \ddot{q}_d(t)$ the desired trajectory. We will make a few remarks about this control scheme. First, the feed forward is based on a virtual reference trajectory q_r and not on the desired trajectory q_d. This is equivalent to a feedback loop. There are a number of reasons for this choice of which we mention two

- the trajectory q will catch up the desired trajectory q_d faster,
- asymptotic tracking is assured.

Second, the component $K_v s$ is a genuine PD control, because it is equal to $K_v(\dot{\tilde{q}} + \Lambda\tilde{q}) = K_v\dot{\tilde{q}} + K_p\tilde{q}$ with $K_p = K_v\Lambda$. Putting the PD component in this form makes it easy to extend the class of controllers for the tracking error from PD to, *e.g.*, sliding motion controllers, based on the sign of s. The measure of tracking accuracy s is used also in the adaptation part of the controller. Third, unmodeled external forces acting on the manipulator are not compensated. Finally, in VSS control $s = 0$ would be used as the equation for a first order sliding surface.

Adaptation of the model parameters used in $\hat{M}$, $\hat{C}$, and $\hat{g}$ is based on the reasonable assumption that, with an appropriate choice of parameters, the generalized control force (4.16) is linear in the parameters $\hat{\theta}$ and can be expressed as

$$f = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\theta} + K_v s. \tag{4.17}$$

Then the adaptation proceeds according to

$$\dot{\hat{\theta}} = \Gamma^{-1} Y'(q, \dot{q}, \dot{q}_r, \ddot{q}_r)s. \tag{4.18}$$

The convergence of the tracking error of the closed loop system, using this adaptive controller, can be proved with the second method of Lyapunov [107]. In the proof some properties of the model (4.15) are used: by a suitable parametrization the control force (4.16) is linear in the parameters and the matrix $\dot{M} - 2C$ is skew symmetric for a suitable choice of C. It is assumed that the controller parameters K_v, Λ, and Γ are positive definite [179]. Furthermore, stability in the sense of Lyapunov can also be proved, see [181]. The adaptation scheme can exhibit parameter drift, because under certain conditions the right hand side of (4.18) may contain only quadratic terms, and $\hat{\theta}$ will grow without bound. These conditions include the case where the reference trajectory q_d is not persistently exciting and the measurements are noisy [112].

A general approach to prove the stability of this control scheme and several variations thereof is presented in [99]. They also discuss stability in the presence of

- bounded disturbances,
- actuator dynamics,

- joint flexibility,
- friction.

For bounded disturbances [114] gives an example of a disturbance which causes the parameter estimate to diverge. They propose a switching σ scheme to counteract this. They also discuss the influence of actuator dynamics.

If joint flexibility is present, stability is preserved if a control signal proportional with the difference between the link and motor speeds is added to (4.16), see [182]. With a suitable chosen additional controller parameter, and with (4.16) solely based on the rigid model with the link positions as degrees-of-freedom, stability can be proved. A disadvantage of this modification is that the link position, link speed and motor speed must be available. Normally only motor position (and speed) are measured.

If friction is present, stability is preserved if the friction is dissipative, see the remarks in [99].

Of course, these proofs break down if the model (4.15) cannot faithfully reproduce the dynamic behavior of the system and if the model error is not one of the four types of error mentioned above. In practice one can always choose the controller parameters such that the closed loop system will be unstable.

4.3.2. Adaptive control scheme of Kelly. The adaptive control scheme of Kelly has a computed torque component, based on an estimate of the manipulator dynamics, and a compensation component. The generalized control force is just the sum of these components

$$f = \hat{M}(q)(\ddot{q}_d + K_v\dot{\tilde{q}} + K_p\tilde{q}) + \hat{C}(q,\dot{q})\dot{q} + \hat{g}(q,\dot{q}) + \hat{C}(q,\dot{q})\nu \tag{4.19}$$

where $\hat{M}$, $\hat{C}$, and $\hat{g}$ are the estimates of the corresponding terms in (4.15), ν a measure of tracking accuracy, defined by $\dot{\nu} + \lambda\nu = \ddot{\tilde{q}} + K_v\dot{\tilde{q}} + K_p\tilde{q}$, a first order filtered second order error dynamics equation, $\tilde{q} = q_d - q$ the tracking error, and $q_d(t), \dot{q}_d(t), \ddot{q}_d(t)$ the desired trajectory.

We will make a few remarks about this control scheme. First, the computed torque term $\hat{M}\ddot{q}_d$ is based on the desired trajectory q_d, and not on a virtual reference trajectory. Second, the computed torque term $\hat{M}(K_v\dot{\tilde{q}} + K_p\tilde{q})$ is not a genuine PD control, because the inertia matrix $\hat{M}(q)$ is also involved, so the product of $\hat{M}$ with K_v, respectively K_p, is not a constant matrix, but K_v and K_p are still required to be positive definite. Third, apart from the computed torque component a compensation component $\hat{C}\nu$ is present, but unmodeled forces are not compensated. Fourth, the controller is slightly more complicated than the previous one and it has one additional parameter matrix, λ, that must be tuned. Fifth, measurement or estimation of the acceleration is necessary. Finally, in VSS control $\nu = 0$ would be used as the equation for a second order sliding surface.

Adaptation of the model parameters used in $\hat{M}$, $\hat{C}$, and $\hat{g}$ is based on the reasonable assumption that the generalized control force (4.19) is linear in the parameters $\hat{\theta}$ and can be expressed as

$$f = \Phi(q, \dot{q}, \ddot{q}_d + K_v\dot{\tilde{q}} + K_p\tilde{q}, \nu)\hat{\theta}. \tag{4.20}$$

Then the adaptation proceeds according to

$$\dot{\hat{\theta}} = \Gamma^{-1}\Phi'(q, \dot{q}, \ddot{q}_d + K_v\dot{\tilde{q}} + K_p\tilde{q}, \nu)\nu. \tag{4.21}$$

The function matrix Φ is an explicit function of the measure of tracking accuracy ν.

The remarks on the stability proof of the previous control scheme are also valid for this control scheme. In addition, the parameter matrix λ must be positive definite.

The control laws of the two adaptive controllers are almost equivalent. The main differences are the terms $\hat{C}s$ and $\hat{C}\nu$, and the controller parameters K_v and K_p, that are multiplied by $\hat{M}$ in the scheme of Kelly, but not in the one of Slotine and Li.

4.3.3. VSS control. The VSS (Variable Structure System) control concept is proposed for the control of systems for which it is difficult to obtain accurate models. It is often used when the structure of the model is inaccurate, or if the model parameters itself are unknown, but upper and lower bounds can be determined. VSS controllers are often used as sliding mode controllers. For an overview, see [87] or the recent translation [90].

The design of sliding mode controllers goes roughly as follows. Assume we want to solve a tracking problem, so design a controller that makes the tracking error $\tilde{y} = y_d - y$ small, where $y(t)$ is the variable which we want to follow a desired trajectory $y_d(t)$. The synthesis of the controller can be divided in three steps. First, choose a sliding surface, *i.e.*, a manifold in the tracking error space defined by the function $s(\dot{\tilde{y}}, \tilde{y}) = 0$. Second, compute the control input to force the tracking errors to this surface, despite errors in the model of the system and unmeasurable disturbances acting on the system, by assuring that the sliding condition $\lim_{s\to 0} s\dot{s} < 0$ is fulfilled. Last, compute the so-called equivalent control, which assures that the tracking error stays on the manifold $s = 0$, by making $\dot{s} = 0$ when $s = 0$ for the nominal system. Then the tracking error $\tilde{y}$ will converge to 0 along the sliding surface for $t \to \infty$. This zero steady state (or asymptotic state) error property also holds if there are persistent disturbances or model errors if the controller parameters are chosen properly, see [76, Chapter 5].

To assure that $s\dot{s} < 0$ one uses a discontinuous control input. Due to non-zero switching time and hysteresis in continuous time controllers or time delays caused by the finite sampling rate in discrete time controllers, high-frequency oscillations occur around the sliding surface: chattering. For some systems, *e.g.*, power electronics, this is not a problem, but often chattering is undesirable because it causes excessive control action, leading to increased wear of the actuators, and excitation of high-frequency unmodeled dynamics.

To eliminate chattering one makes, in one way or another, the control input continuous in a region (the boundary layer) around the sliding surface. The sampling rate of the controller implementation has a direct influence on the required width of the boundary layer and therefore on the tracking accuracy. As a result, the steady state error may no longer be zero when there are constant disturbances. It is even possible that limit cycles occur when the error reaches 0.

REMARK 4.4. Only a non-chattering sliding mode controller can have the zero steady state property. When chattering occurs there is by definition no steady state and so the tracking error is never zero.

To obtain again the zero steady state error property of the original sliding mode controller one adds integral action to the control schemes. This introduces an additional tunable parameter. Another possibility to avoid chattering is to design a discrete version of a continuous sliding mode controller, see [95]. Then the compromise between tracking accuracy and sampling rate becomes explicit.

It is not always clear which values should be chosen for the controller parameters. Some parameters depend on uncertainty bounds, and when these bounds are not tight the control action will be unnecessarily large. Other parameters are left to the discretion of the designer, and only global guidelines are given, *e.g.*, positive definiteness. Consequently, the design is by trial and error, and therefore time consuming.

In recent literature several methods are proposed to

- make the control input continuous, see [88],
- eliminate steady state errors, see [85],
- give guidelines for tuning the controller parameters, see [86],
- make the choice of some key parameters not critical, see [84].

Often the feasibility of these methods is shown by numerical examples. Experimental validation and a comparison with other methods is missing.

We investigate and compare several methods, namely the control schemes presented in [82–86, 88]. All schemes include provisions to avoid chattering. Some include integral action to eliminate steady state errors and propose tuning rules.

We will not discuss all intricacies of the sliding controller concept. Our interest is mainly in the modifications of the original sliding controller that avoid chattering and nonzero steady state errors. Only a short overview of the design of sliding controllers is necessary. To avoid a complicated notation, the formulae given assume a single-input single-output plant. The modifications proposed are only sketched.

The standard control input u in VSS tracking control of a system with state x is of the form

$$u = u_e + K \operatorname{sgn} s$$

where u_e is the equivalent control, $K \operatorname{sgn} s$ is the switching term, $s(e) = 0$ is the definition of the sliding surface, $e = x_d - x$ is the state tracking error, and x_d is the desired state corresponding to the desired output y_d to be tracked. The equivalent control u_e assures that $\dot{s} = 0$ if $s = 0$. The gain K is designed to achieve that $s\dot{s} < 0$ in the reaching phase. It is based on uncertainty bounds and is not necessarily constant.

All schemes investigated aim at replacing the term $\operatorname{sgn} s$ in the proximity of the sliding surface by a continuous approximation. They can be characterized as

- use parallel boundary layer with
  - linear interpolation inside boundary layer, see [86, 88]

    $$\operatorname{sgn} s \rightarrow \operatorname{sat} \frac{s}{\sigma} \tag{4.22}$$

    where $\rightarrow$ means replace by, and the saturation function sat is

    $$\operatorname{sat} \frac{s}{\sigma} = \begin{cases} \operatorname{sgn} s & \text{if } \|s\| \geq \sigma \\ \frac{s}{\sigma} & \text{if } \|s\| < \sigma \end{cases}$$

    the parameter $\sigma > 0$ defines the width of the boundary layer,
  - power law interpolation inside boundary layer, given by [83]

    $$\operatorname{sgn} s \rightarrow \frac{s\sigma^{q-1}}{\|s\|^q}$$

    with $q \in [0, 1)$ for $\|s\| < \sigma$ and $q = 1$ for $\|s\| \geq \sigma$, $q = 0$ gives linear interpolation and $q = 1$ gives switching,
  - fractional interpolation with constant offset, used by [85]

    $$\operatorname{sgn} s \rightarrow \frac{s}{\|s\| + \delta}$$

with δ small; with this modification there is no boundary layer persé, $\|s\| = \delta$ gives only half of the value of sgn s, but an equivalent boundary layer can be defined,

- fractional interpolation with state dependent offset, used by [85]

$$\operatorname{sgn} s \to \frac{s}{\|s\| + \delta(x)}$$

this represents a slight change of the previous modification,

- use cubic error feedback, proposed by [84] for the regulator problem,

$$\operatorname{sgn} s \to s p' b e' \Delta e \tag{4.23}$$

for a linear system $\dot{x}(t) = A(t)x(t) + b(t)u(t)$, with $s = p'e$ and $\Delta > 0$ a diagonal matrix of design parameters; here the control action outside the equivalent boundary layer can be much larger,

- use an integral transformation with a cone like boundary layer, proposed by [83]

$$\operatorname{sgn} s \to \operatorname{sat}\left(\int_{t_0}^{t} k_t \operatorname{sgn} s \, \mathrm{d}\tau\right) \tag{4.24}$$

with k_t the integration gain design parameter; some anti reset windup measures are necessary,

- use sliding condition of higher order, compatible with the plant, and periodically redefine the sliding surface, so the reaching phase is avoided, suggested by [82].

For a detailed discussion of the modifications of the straight sgn type of sliding controller, the original literature should be consulted.

An application of these control schemes is in a two level controller. The inner loop of the controller consists of an input-output linearizing state-feedback as presented in Section 4.1. Due to model errors the resulting system will be nonlinear. The outer loop of the controller is a sliding mode controller, that should correct the effects of the imperfect cancellation of the nonlinearities in the system and give the system its desired dynamics. The analysis of the closed loop system when the model errors fulfill the *matching condition* is described in [96]. This condition implies that the model errors are inside the kernel of the map defined by the vector fields

$$\{\mathrm{d}h_i, \mathrm{d}L_f h_i, \ldots, \mathrm{d}L_f^{r_i-2} h_i\} \qquad \text{for } i = 1, \ldots, m$$

with $\mathrm{d}h_i = \partial h_i / \partial x$, etc., which assures that the effects of the model errors do not appear too fast in the output y_i, *i.e.*, only after a number of differentiations of the output at least equal to that needed for the control input, the relative degree r_i.

4.3.4. PD control. The simplest exponent of this class is a PD controller as outer loop for a linearizing state-feedback, acting on the tracking error $\tilde{q}$ and its derivative $\dot{\tilde{q}}$

$$\nu = K_v\dot{\tilde{q}} + K_p\tilde{q},$$

for use in (4.6), with K_v and K_p positive definite matrices, *e.g.*, diagonal matrices with positive diagonal elements. When velocity measurements are available the PD controller can be implemented as a static state-feedback controller. When only position measurements are available, an observer is needed to reconstruct the velocities. A PD controller can be used without linearizing state-feedback also. Then

$$f = K_v\dot{\tilde{q}} + K_p\tilde{q}, \tag{4.25}$$

generating f instead of ν, for use in (4.5).

4.4. Acceleration feedback based control

Acceleration feedback can be combined with the controllers presented in the previous sections. Some benefits of using acceleration measurement are

- the acceleration measurement can replace more expensive "no structure mounted" measurements, the acceleration sensor can be attached to the structure easily, and has low costs [183],
- use of the acceleration can improve the estimates of position and velocities, *i.e.*, reduce the contamination with noise by filtering the measurements, or raise the bandwidth of the measurements [150, 184],
- the acceleration can give an indication of the equation error, simply by filling in the measurements in the model equation; the resulting residue is an indication of the equation error (2.5) (but then for (4.15)) and there are several ways to reduce it, using acceleration feedback, as will be discussed in the following.

A simple method to reduce the equation error is using the acceleration as an additional input to the controller. If the controller output is a linear combination, with suitable chosen factor, of the output of the original controller and the acceleration, the residue can be reduced by this factor, see [143]. We will explain this further.

First, define the equation error for (4.15) as

$$e = M(q_m, \theta)\ddot{q}_m + C(q_m, \dot{q}_m, \theta)\dot{q}_m + g(q_m, \dot{q}_m, \theta) - f_m \tag{4.26}$$

where q_m, $\dot{q}_m$, $\ddot{q}_m$ and f_m come from measurements. The control force $f = f(q, \dot{q}, t)$ can be extended to $f^* = f^*(q, \dot{q}, \ddot{q}, t)$ when acceleration measurements are available. As shown by [143], when the acceleration enters linearly in the feedback law as

$$f^*(q,\dot{q},\ddot{q},t) = (1+\alpha)f(q,\dot{q},t) + \alpha\ddot{q}, \tag{4.27}$$

it is possible to reduce the equation error e to

$$\frac{e}{1+\alpha}. \tag{4.28}$$

A large α may reduce the equation error considerably. The main limitation of this method is the fact that the acceleration signal is contaminated with noise, see [185], and is fed back with some time delay. Therefore, the choice of α is limited, *e.g.*, $\alpha < 0.5$. Also relation (4.28) does not hold exactly.

Another approach for using the acceleration signal is sketched in [185]. A term $\alpha\dot{s}$ is added to the control input signal of (4.16). A reduction of the influence of parametric uncertainty on performance by a factor $1+\alpha/\beta$, with β the gain margin, is claimed. So a large α is desired to improve the tracking performance. However, the influence of noise $n_{\ddot{q}}$ on the acceleration measurement will diminish this improvement. A relative error Δ_r in this measurement is claimed to have the same influence on tracking performance as a disturbance signal of relative size $\frac{\alpha}{\alpha\beta+1}\Delta_r$. For small α this influence is negligible ($\approx \alpha\Delta_r$), but for large α it is proportional with Δ_r/β. So, a good conditioning of the acceleration signal, by using filters, and an accurate sensor are necessary.

Berlin *et. al.* [144, 145] propose still another approach. They compare several methods for using acceleration feedback. These methods are based on a norm bound γ for the error in the mass matrix M

$$\gamma = \max_{q,\theta} \|M^{-1}\hat{M} - I\|.$$

It is advisable that γ is as small as possible to make the controller not too conservative. To use their stability proof the condition $\gamma < 1$ is necessary. They also compare several methods for computing the "gain" for the acceleration feedback loop and propose an optimal choice, depending on the uncertainty in the system. Noise in the acceleration measurements is not taken into account, although this is believed to be a major limitation for the effectiveness of acceleration feedback.

Based on the available literature, there is presently no readily available recipe to design the acceleration feedback gain, let alone some guidelines for the use of acceleration in a more complex control scheme than a simple feedback loop (besides using it in a state estimator). Studying several approaches for this problem and testing them for their effectiveness seems therefore a fruitful objective.

CHAPTER 5

Description and results of numerical experiments

In this chapter we discuss numerical experiments for the control schemes presented in Chapter 4.

We use three types of models

- the model of an RT-robot, *i.e.*, a mechanical system with a chain of links, connected to a fixed base by a rotational (R) and a translational (T) joint,
- the model of an XY-table, just a big plotter type machine, with two prismatic joints,
- a linear mass-damper-spring system, that could result from the use of a linearizing state-feedback inner loop and a PD outer loop.

The RT model has nonlinear Coriolis and centrifugal terms, the XY model has Coulomb friction as nonlinearity. By using these systems we study two different types of nonlinearities. The mass-damper-spring system is linear.

The type of unmodeled dynamics to be introduced is also different. For the RT-robot, the unmodeled dynamics are caused by the neglected actuator dynamics. For the XY-table a flexible bar, that is modeled as a stiff connection, is used as a source for unmodeled dynamics. For the mass-damper-spring model we study the effects of persistent disturbances.

For the RT-robot we study two types of control tasks, *i.e.*, position control and hybrid (position and force) control. The hybrid control task is more challenging. For the XY-table and mass-damper-spring model only position control is considered.

For the RT-robot position control problem four types of controllers are investigated, a simple PD controller for each degree-of-freedom (DOF), a model based controller with additional PD for each DOF (a computed torque type control) and two adaptive computed torque controllers. Linear, VSS, and acceleration based controllers have not been applied for this problem because the linear controllers proved to be not very effective in the hybrid control task and it is not our aim to check each combination of control problem and controller because that would introduce redundancy in our results. For the RT-robot hybrid control a computed torque type controller with PD, with some norm based, or with a μ-synthesis controller is used. These controllers, used in an outer loop, are all linear. Adaptive, VSS, and acceleration based control have not been used for this problem. For the XY-table position control problem the same controllers are used as for the RT-robot position control, plus VSS and acceleration based controllers. From the linear controllers only PD has been applied. For the mass-damper-spring model only VSS control is used.

We first discuss the nominal and simulation models. A description of the four control problems and a presentation of the results obtained follows. A discussion and a summary of our findings mark the end of this chapter.

5.1. Simulation models

We give a short description of the models used for the design of the controllers and used in the simulations.

5.1.1. RT-robot model. To assess the robustness of the controllers, different models are used for the design and the evaluation. So, a nominal *design model* and an *evaluation model* are introduced. The controllers are designed for the design model and evaluated for the evaluation model. The evaluation model is based on the design model with parametrized unmodeled dynamics added.

The design model chosen is a model for a two degrees-of-freedom RT-robot, moving in the horizontal plane, with a rotational and a prismatic joint. The model equations for the RT-robot of Fig. 5.1 are

$$\begin{aligned} \theta_1 \ddot{r} - (\theta_1 r - \theta_2)\dot{\varphi}^2 &= F + F_{x_1} \cos\varphi + F_{x_2} \sin\varphi \\ (\theta_1 r^2 - 2\theta_2 r + \theta_3)\ddot{\varphi} + 2(\theta_1 r - \theta_2)\dot{r}\dot{\varphi} &= M - F_{x_1} r \sin\varphi + F_{x_2} r \cos\varphi \end{aligned} \tag{5.1}$$

where r and φ are the prismatic and rotational degree-of-freedom, F_{x_1} and F_{x_2} are the components of the external force F_e in x_1 and x_2 direction, M and F are the motor torque and force acting on the manipulator, and θ_1, θ_2, and θ_3 are related to the physical parameters by

$$\begin{aligned} \theta_1 &= m + m_l \\ \theta_2 &= \frac{1}{2} m l \\ \theta_3 &= I + \frac{1}{3} m l^2. \end{aligned} \tag{5.2}$$

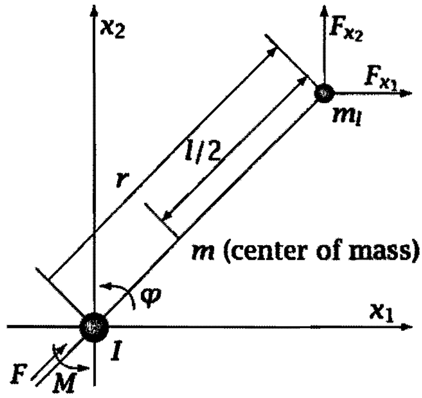

FIGURE 5.1. Schematic drawing of RT-robot

To rewrite (5.1) as (4.15) define the following quantities:

$$q = \begin{bmatrix} r \\ \varphi \end{bmatrix},$$

$$M(q,\theta) = \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_1 q_1^2 - 2\theta_2 q_1 + \theta_3 \end{bmatrix},$$

$$C(q,\dot q,\theta) = \begin{bmatrix} 0 & -(\theta_1 q_1 - \theta_2)\dot q_2 \\ (\theta_1 q_1 - \theta_2)\dot q_2 & (\theta_1 q_1 - \theta_2)\dot q_1 \end{bmatrix},$$

$$g(q,\dot q,\theta) = \begin{bmatrix} \theta_4 \operatorname{sgn} \dot q_1 + \theta_5 \dot q_1 - F_{x_1}\cos q_2 - F_{x_2}\sin q_2 \\ F_{x_1} r \sin q_2 - F_{x_2} r \cos q_2 \end{bmatrix},$$

$$f(q) = \begin{bmatrix} F \\ M \end{bmatrix}.$$

In the term g gravitational forces are absent because the manipulator moves in the horizontal plane, but a Coulomb friction term and a viscous damping term have been added, that can be used if need arises. For the parameter values used in the computations see Table 5.1.

The model to evaluate the design is equal to the design model plus first order models for the dynamics of the motors. So instead of choosing the torque f delivered by the motors as $f = \tau$, with f proportional to the controller output vector τ, we obtain

$$\begin{bmatrix} \tau_f & 0 \\ 0 & \tau_m \end{bmatrix} \dot f + f = \tau$$

with τ_f, τ_m the motor time constants. The motor time constants are chosen equal, between $\frac{1}{200}$ [s] and $\frac{1}{800}$ [s], with a nominal value of $\frac{1}{400}$ [s]. In our

Parameter	Value	Unit
m	10	kg
m_l	5	kg
I	5	kg m^2
l	1	m
θ_1	15	kg
θ_2	5	kg m
θ_3	$5 + \frac{10}{3}$	kg m^2
θ_4	20	N
θ_5	5	N s m^{-1}

TABLE 5.1. Nominal parameters of the RT-robot design model

examples, the motor dynamics will often only be applied for the rotational degree-of-freedom. For some simulations a second order motor model is used.

5.1.2. XY-table model. A complicated model of the XY-table [186] has been used for numerical experiments. It will not be elaborated here. For the design computations, a complicated model of the XY-table might be overkill, so a much simpler model has been used.

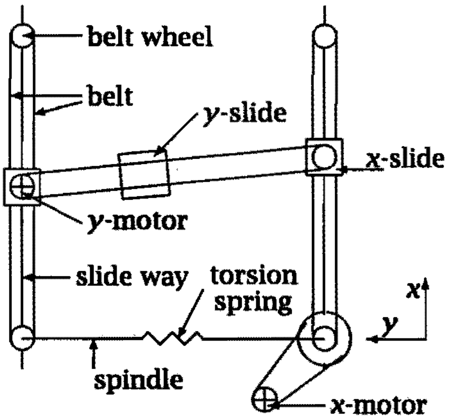

FIGURE 5.2. Schematic drawing of XY-table

The equations for the simple model of the XY-table of Fig. 5.2 are

$$\begin{aligned} \theta_1 \ddot{x} + \theta_3 \,\mathrm{sgn}\, \dot{x} &= f_x \\ \theta_2 \ddot{y} + \theta_4 \,\mathrm{sgn}\, \dot{y} &= f_y \end{aligned} \tag{5.3}$$

where x and y are the two prismatic degrees-of-freedom, f_x and f_y the control forces in x and y direction, and θ_i, $i = 1, \dots, 4$, the model parameters: θ_1 and θ_2 are the equivalent masses in x and y direction, θ_3 and θ_4 are the coefficients of the Coulomb friction in x and y direction.
To rewrite (5.3) as (4.15) define the following quantities:

$$\begin{aligned} q &= \begin{bmatrix} x \\ y \end{bmatrix}, \\ M(q,\theta) &= \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix}, \\ C(q,\dot{q},\theta) &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ g(q,\dot{q},\theta) &= \begin{bmatrix} \theta_3 \operatorname{sgn} \dot{q}_1 \\ \theta_4 \operatorname{sgn} \dot{q}_2 \end{bmatrix}, \\ f &= \begin{bmatrix} f_x \\ f_y \end{bmatrix}. \end{aligned}$$

Coriolis and centrifugal forces are absent, because there is almost no coupling between movements in x and y direction. In the term g gravitational forces are absent because the manipulator moves in the horizontal plane. For the nominal parameter values used in the design computations see Table 5.2.

Parameter	Value	Unit
θ_1	46.5	kg
θ_2	4.3	kg
θ_3	50.0	N
θ_4	15.0	N

TABLE 5.2. Nominal parameters of the XY-table design model

5.1.3. Mass-damper-spring model. This system can be described by the following model in state space notation

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{b}{m} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

where $x \in \mathbb{R}^2$ is the state, $u \in \mathbb{R}$ the input. The model has the following parameters: m the mass, b the damping constant, and c the spring constant. For the nominal values of these parameters see Table 5.3.

Parameter	Value	Unit
m	10	kg
b	100	$\mathrm{N\,s\,m^{-1}}$
c	1000	$\mathrm{N\,m^{-1}}$

TABLE 5.3. Nominal parameters of the mass-damper-spring design model

5.2. RT-robot position control

In this section the results are presented of the two adaptive controllers, discussed in Section 4.3, when they are used to control a desired trajectory of the RT-robot.

We discuss the control task, the design of the controllers, the controller evaluation setup, and finally present and discuss some simulation results.

The focus is mainly on the robustness characteristics of the controllers, in comparison with PD and non-adaptive computed torque like controllers, that are used as reference. Some other aspects will also be discussed.

5.2.1. Control task. The goal of the controllers is to track a desired end-effector position by judiciously manipulating the input to the model. The trajectory is specified in joint space (r, φ), so no inverse model computation is needed, and is defined by the following skew sinusoids

$$
\begin{aligned}
r_d &= \frac{3}{4}t - \frac{3}{8\pi}\sin(2\pi t) + \frac{1}{4} && \text{for } 0 \le t \le 1,\\
\dot{r}_d &= 0 && \text{for } t > 1,\\
\varphi_d &= \frac{\pi}{2}t - \frac{1}{4}\sin(2\pi t) && \text{for } 0 \le t \le 1,\\
\dot{\varphi}_d &= 0 && \text{for } t > 1.
\end{aligned}
$$

This trajectory has been chosen as representative for a pick and place task, with smooth trajectory derivatives. The desired trajectory in end-effector space is, however, not a straight line, see Fig. 5.3, but is more likely to be an optimal time like trajectory, although that property is not a part of the formal trajectory specifications. When the need arises, this trajectory can be extended easily for $t > 1$, because it is of a periodic nature.

5.2.2. Controller design. The controller design aims at selecting the controller parameters so the controlled model is stable, despite parameter and

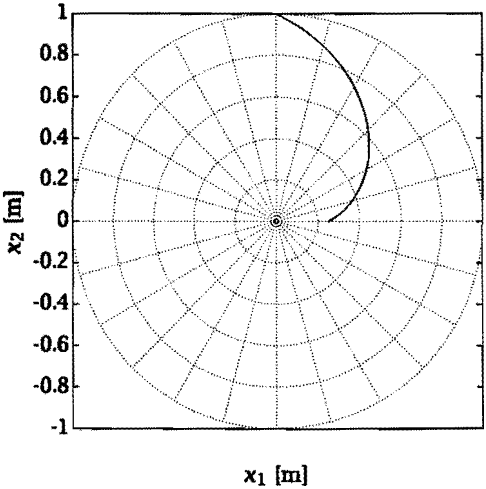

FIGURE 5.3. Desired trajectory in end-effector space

unmodeled dynamics errors, and the tracking error is small. Here, it is assumed that the parameter error will be canceled by the adaptation mechanism of the controllers, so the only relevant model error is the unmodeled dynamics error. Further, it is assumed that the unmodeled dynamics is not completely unknown, but that it consists of a high frequency type of unmodeled dynamics and that a lower bound for the frequency influence (the characteristic frequency) is known. In our case, the unmodeled dynamics is synthesized in the simulation model, so this assumption is not restrictive. In cases where the unmodeled dynamics are completely unknown, a certain degree of tuning of the controller parameters, *i.e.*, adapting the parameters based on "in situ" tests, seems to be unavoidable.

The controller is designed so the bandwidth of the model, in a sense to be specified later, will never exceed this lower bound, or, more conservative, will never approach the bound. The design of the controller parameters has been performed as follows. In a suitable working point r_0 the model is linearized, leading to the following equations of motion

$$\theta_1 \ddot{r} = F, \qquad J\ddot{\varphi} = M$$

with $J = \theta_1 r_0^2 - 2\theta_2 r_0 + \theta_3$. Use of the PD controller (4.25) leads to the following closed loop equation

$$\ddot{q} + 2\beta\omega_0\dot{q} + \omega_0^2 q = 2\beta\omega_0\dot{q}_d + \omega_0^2 q_d.$$

With the diagonal matrices ω_0 and β the required bandwidth and damping can be specified. These matrices are related to the PD controller parameters by

$$K_p = \omega_0^2 \begin{bmatrix} \theta_1 & 0 \\ 0 & J \end{bmatrix}, \qquad K_v = 2\beta\omega_0 \begin{bmatrix} \theta_1 & 0 \\ 0 & J \end{bmatrix}.$$

A suitable working point is defined by $r_0 = \frac{1}{3}$, because $J(r_0) = 20/3$ has a minimum for this value of r_0 with the model parameters in Table 5.1.

For example, specifying $\omega_0 = 10I$ and $\beta = I$ gives

$$K_p = \begin{bmatrix} 1500 & 0 \\ 0 & 2000/3 \end{bmatrix}, \qquad K_v = \begin{bmatrix} 300 & 0 \\ 0 & 400/3 \end{bmatrix}.$$

The same procedure leads to the following expressions for the controllers of Slotine/Li

$$\Lambda K_v \doteq K_p = \omega_0^2 M(q_0), \qquad K_v = 2\beta\omega_0 M(q_0),$$

so $\Lambda = 5I$, and for Kelly

$$K_p = \omega_0^2, \qquad K_v = 2\beta\omega_0,$$

so the last two controller parameter matrices are independent of the model parameters.

As will be explained in Section 5.3.3 this choice of controller parameters gives a bandwidth of $\approx 2\omega_0$, depending on the definition of bandwidth used.

The gain matrix Γ^{-1} for the adaptation is initially taken to be

$$\Gamma^{-1} = \begin{bmatrix} 100I & \\ & 500I \end{bmatrix}$$

with a gain of 100 for the mass and a gain of 500 for the friction parameters. This choice was based on simulations that revealed, as could be expected, that when the model structure is exact, the adaptation gains should be chosen large, so the parameters will quickly converge. Only when some sufficient richness conditions are satisfied is convergence to the model parameters guaranteed [187]. Otherwise, the converged values for the adapted parameters will only guarantee an asymptotically convergent tracking error.

When unmodeled dynamics is present the adaptation gain must be chosen carefully. There are at least two criteria possible

- the adaptation dynamics should not introduce poles that could increase the bandwidth of the controlled design model,
- the gains are chosen along the lines of [127, 128].

The specific choice for the adaptation gain, system bandwidth, and damping will therefore depend on the system to be controlled and is given with the presentation of the simulation results.

5.2.3. Controller evaluation. The evaluation of the four control schemes under investigation, namely a PD controller (PD), a computed torque without adaptation (CT), and the adaptive computed torque controllers proposed by Slotine and Li (SL) and Kelly (K), aims at assessing the tracking error when the controllers are based on an erroneous model, and is performed along the following lines

- a reference result is created by controlling a model with exactly known structure, but with largely unknown parameters,
- friction is added to the model to test if the controllers can handle this type of nonlinearity,
- additional dynamics with parametrized characteristics is introduced in the model to be controlled, but not incorporated in the model used for the design, torque computation, and adaptation.

We will now give a more detailed description of the models used. The design model is already presented in Section 5.1.1. The friction is simply a standard Coulomb friction model and viscous damping, only added to the r degree-of-freedom of the model. The additional dynamics used here is not a linear first order model, but a more realistic second order model, used to represent the neglected motor and power electronics dynamics. It is only applied for the φ degree-of-freedom. As remarked in [188, 189] this type of neglected dynamics has often an important effect on the dynamic behavior. It is given by the following transfer function

$$\frac{\omega_m^2}{s^2 + 2\beta_m \omega_m s + \omega_m^2}$$

where the parameters ω_m and β_m are used to change the behavior of the additional dynamics.

Another aspect of the evaluation is the choice of the controller parameters. The main consideration is stability in relation to unmodeled dynamics. We can expect an unstable behavior of the control system if the gains are chosen too high, but only if unmodeled dynamics is present. Before the onset of instability we can expert a tracking error that decreases for increasing feedback gains. On the other hand, when there are no model errors there is no need to choose large gains for the model based controllers, because the tracking error will converge and will be small after an initial transient. Only for the PD controller, without model based feedforward, will the tracking error be reduced further if the feedback is more prominent.

5.2.4. Simulation results. A limited number of results is presented. The results can be divided in three groups. The first group is to validate the controller implementations, to check the adaptation mechanism of the adaptive controllers, and to show the effects of parameter errors. The second group gives the results with Coulomb friction in r-direction. Because Coulomb friction is included in the model based controllers, except for the computed torque controller, this is also a parameter type model error, like the first group. The last group gives the results with unmodeled dynamics in φ-direction. These dynamics can be associated with neglected dynamics of the power electronics, motor, and transmission. This is representative for unmodeled dynamics errors. The results presented are not very suitable to assess the effects of unmodeled statics errors.

The control system parameters used for each group of simulation are given in Table 5.4, with the corresponding figure numbers as main entry. In this table, PD means a PD controller, CT a computed torque controller, SL the controller proposed by Slotine and Li, and K the controller proposed by Kelly. As can be seen, the parameters are chosen similar for all controllers. So we can expect the PD controller to be worse than the other ones, because no feedforward is employed. The difference in performance between the CT and the adaptive controllers will depend on the accuracy of the model, and will therefore be a function of the choice for the model error, friction and/or unmodeled dynamics.

Figures	Parameter	PD	CT	SL	K
5.4–5.9	K_p	$\begin{bmatrix}1500 & \\ & 666\end{bmatrix}$	$\begin{bmatrix}1500 & \\ & 666\end{bmatrix}$	$\begin{bmatrix}1500 & \\ & 666\end{bmatrix}$	$100I$
	K_v	$\begin{bmatrix}300 & \\ & 133\end{bmatrix}$	$\begin{bmatrix}300 & \\ & 133\end{bmatrix}$	$\Lambda = 5I$	$20I$
	Γ^{-1}			$\begin{bmatrix}100I & \\ & 500I\end{bmatrix}$	$\begin{bmatrix}100I & \\ & 500I\end{bmatrix}$
	λ				$25I$
5.10–5.12	K_p	$\begin{bmatrix}1500 & \\ & 666\end{bmatrix}$	$\begin{bmatrix}1500 & \\ & 666\end{bmatrix}$	$\begin{bmatrix}1500 & \\ & 666\end{bmatrix}$	$50I$
	K_v	$\begin{bmatrix}300 & \\ & 133\end{bmatrix}$	$\begin{bmatrix}300 & \\ & 133\end{bmatrix}$	$\Lambda = 5I$	$15I$
	Γ^{-1}			$\begin{bmatrix}3I & \\ & 500I\end{bmatrix}$	$\begin{bmatrix}3I & \\ & 500I\end{bmatrix}$
	λ				$25I$

TABLE 5.4. Controller parameters for RT-robot position control problem

The first group, Figs. 5.4–5.6, gives the tracking error results for the exactly known system structure but largely unknown parameters. There is no Coulomb friction, no unmodeled statics, and no unmodeled dynamics in the evaluation model. The initial estimate for the adaptive controller parameters was taken equal to 70% of the nominal values, and for the CT controller 100% was used. The initial estimates cannot be chosen equal to zero, because the control scheme of Kelly does not tolerate zero estimates for the inertia parameters. Because the CT controller uses exact model parameters, we can expect the error to be very small. The adaptive controllers should be able to estimate the parameters quite well, and their tracking performance should be, after an initial transient, a copy of the performance of the CT controller.

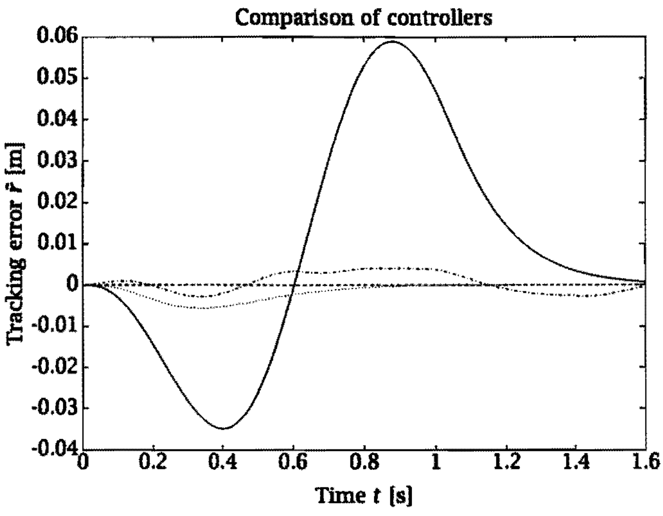

FIGURE 5.4. Tracking error in r-direction, no friction, no unmodeled dynamics, (—) PD, (- -) CT, (· · ·) SL, (- ·) K

An overview of the tracking error, expressed in the MATE (Mean Absolute Tracking Error), is in Fig. 5.6. We see that our expectations with respect to the tracking performance of the CT and adaptive controllers are fulfilled. The tracking error in r-direction for the controller of Kelly has not been completely converged within the duration of the transient. For the SL controller this aspect is OK. The error in φ-direction, for the PD controller, is almost twice as large as in r-direction, but in a Cartesian reference frame that difference is not important.

REMARK 5.1. The choice of the MATE, *i.e.*, the 1-norm of the tracking error, as a measure of performance is not critical. We could also use the RMS of the tracking error or the maximum value, corresponding, respectively, with

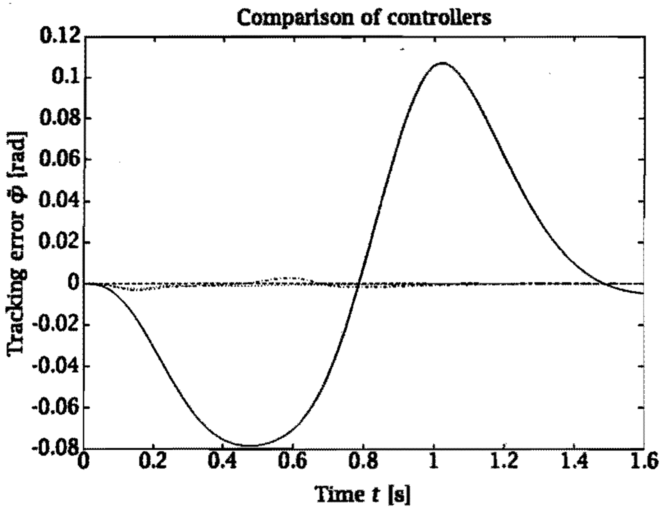

FIGURE 5.5. Tracking error in φ-direction, no friction, no unmodeled dynamics, (—) PD, (- -) CT, (· · ·) SL, (- ·) K

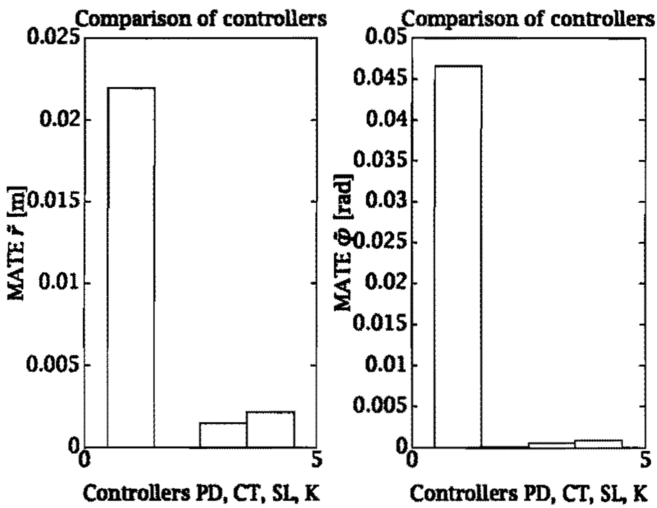

FIGURE 5.6. MATE in r and φ directions, no friction, no unmodeled dynamics

the 2- and ∞-norm of the tracking error. The ranking of the controllers is not sensitive to the measure of tracking accuracy used, because the tracking error for a controller often has a form similar with that of the other ones. Also, a single figure of merit is easier to comprehend than a complete time response. Furthermore, this measure fulfills the conditions for internal validity, *i.e.*, relevance, freedom from bias, reliability.

The second group of results, presented in Figs. 5.7–5.9, gives the results with Coulomb friction added to the evaluation model and design model. For the PD controller, results are presented employing static friction compensation with constant parameter values of 50% of the nominal ones, so it is assumed that the friction can be estimated within 50% accuracy, which is likely to be too inaccurate, so only an lower bound of the achievable performance is obtained. For the adaptive controllers, the initial estimates of the Coulomb friction parameters are also taken at 50% of the nominal values. For the CT controller no friction compensation is used, so the effects of unmodeled statics can by studied. The inertia parameters of the adaptive controllers are again chosen at 70% of their nominal values.

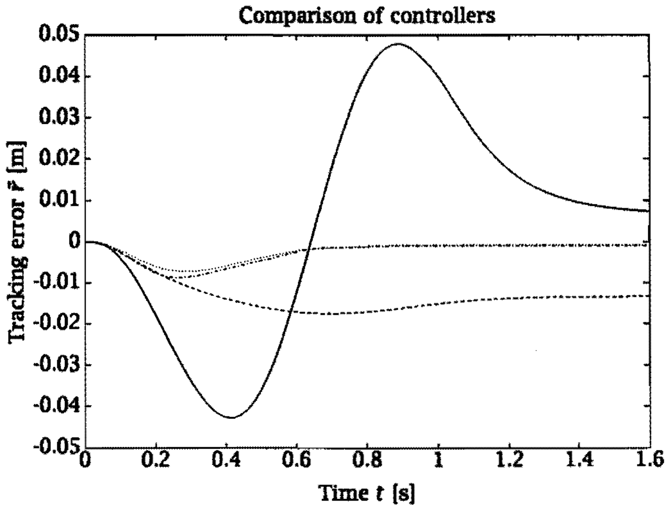

FIGURE 5.7. Tracking error in r-direction, friction, no unmodeled dynamics, (—) PD, (- -) CT, (· · ·) SL, (- ·) K

The MATE results are in Fig. 5.9. Comparison with Fig. 5.6 shows that the errors in r-direction are only slightly larger, except for the CT controller that uses no compensation. This is an indication that it is not easy to cope with unmodeled statics errors in model based controllers. The use of high feedback gains could improve the performance of the CT controller, but that would probably endanger the robustness. Figure 5.7 shows that the PD and CT controllers have a static error in r-direction. This is caused by the incomplete friction compensation. This could be remedied by using integral action in the controllers. For both adaptive controllers the static

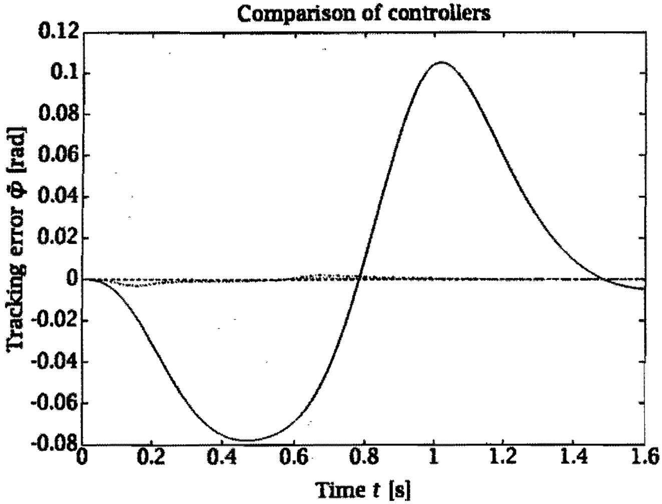

FIGURE 5.8. Tracking error in φ-direction, (no) friction, no unmodeled dynamics, (—) PD, (- -) CT, (· · ·) SL, (- ·) K

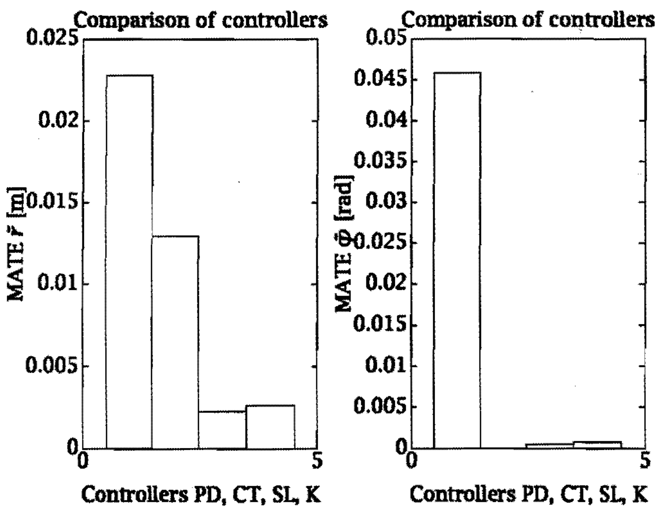

FIGURE 5.9. MATE in r and φ-directions, friction, no unmodeled dynamics

error is negligible, due to correctly adapted friction parameters. Differences in performance for the φ-direction, whose dynamics are not modified, are not perceptible, so the two DOF are completely decoupled.

The last and most interesting group of results, in Figs. 5.10–5.12, is for the model with additional dynamics. As can be seen in Table 5.4, the adaptation gain matrices had to be reduced to avoid increasing the bandwidth of the controlled system, that should now be lower than the lower bound of the characteristic frequency of the additional dynamics, chosen here to be $\omega_m = 25$, $\beta_m = 1$. Also, the feedback gains of the controller proposed by Kelly are reduced, to prevent stability problems.

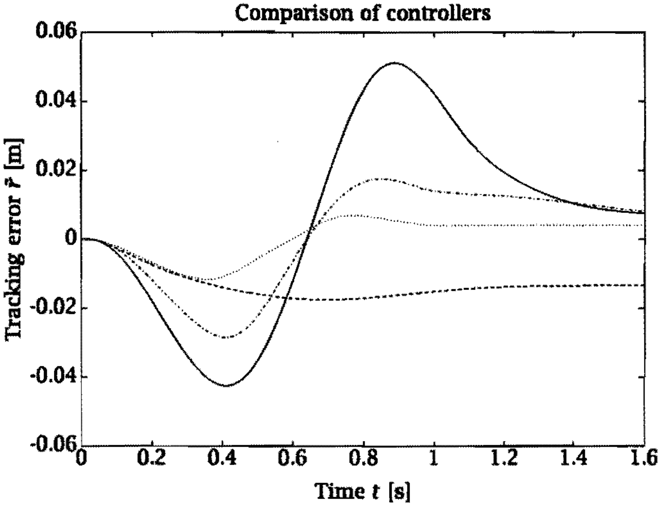

FIGURE 5.10. Tracking error in r-direction, friction, (no) unmodeled dynamics, (—) PD, (- -) CT, (· · ·) SL, (- ·) K

The MATE results are in Fig. 5.12. As could be expected, the errors in φ-direction are much larger then before, except for the PD controller, although the tracking error of the PD controller is still the largest. Especially the performance of the CT controller is unexpected, it is better than the adaptive controllers, mainly due to the correct choice of the model parameters. This is a rare case where adaptation has disadvantages. It also means that unmodeled dynamics has an adverse influence on the adaptation mechanism and therefore on the tracking performance. Also, as remarked before, with unmodeled dynamics present the stability of the control system is not guaranteed, as is also clear because some controller parameter are detuned to avoid stability problems. Figure 5.11 shows that the convergence of the tracking error is not good, especially for the model based controllers. Even after 1.6 [s] the steady state errors are not obtained. Comparison with Fig. 5.8 makes clear that this is also due to the unmodeled dynamics, perhaps because of the change in controller parameters. The effect of the controller parameters is clear from the tracking error in

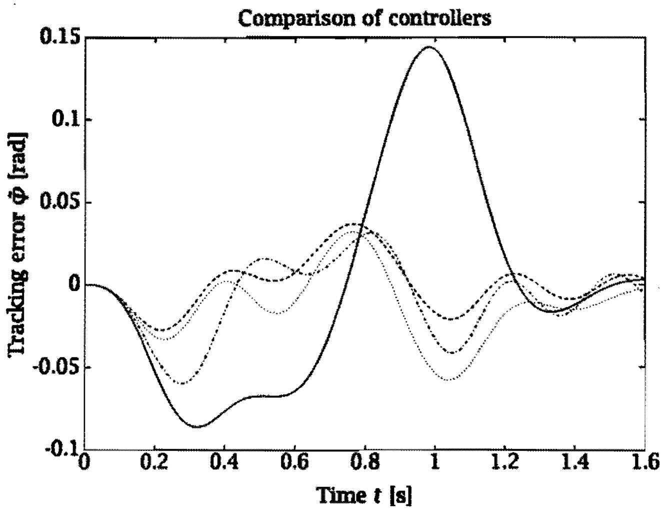

FIGURE 5.11. Tracking error in φ-direction, (no) friction, unmodeled dynamics, (—) PD, (- -) CT, (· · ·) SL, (- ·) K

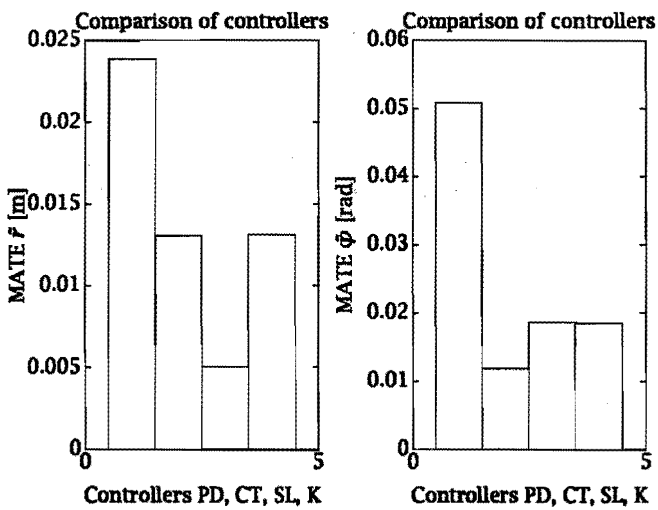

FIGURE 5.12. MATE in r and φ directions, friction, unmodeled dynamics

r-direction for the controller proposed by Kelly. It is now as large as the error for the CT controller, and much larger than in Fig. 5.9. Figure 5.10 shows that all controllers have a static error in r-direction, also the adaptive controllers. So, the adaptation of the friction parameters is not correct, although the adaptation gains for the friction did not change. The tracking errors in r-direction are also larger. Both phenomena indicate that, due to the unmodeled dynamics, the two DOF are no longer completely decoupled.

5.2.5. Summarizing remarks. The results presented in the previous section give rise to the following remarks

- PD is at least as robust as adaptive,
- adaptive gives much smaller tracking error,
- adaptive gives smaller tracking errors than computed torque with exactly known parameters if no unmodeled dynamics is present,
- the parameter adaptation does not perform well in the presence of additional dynamics,
- the controller of Kelly might be better than the one of Slotine and Li due to the higher order character of the dynamical equation for the measure of tracking accuracy v, that is then used in the adaptation law (4.21),
- extensive tuning of the parameter λ appearing in the equation for v may be necessary, which makes the control scheme of Kelly not very attractive.

At this point there is no reason to eliminate one of the adaptive controllers, the differences in performance and robustness are too small, both will therefore be used for more extensive simulation and experimental tests.

5.3. RT-robot hybrid (position and force) control

5.3.1. Control task. The control task is to follow a circular object with position control along the circumference and force control in radial direction. The stiffness K_e of the object is assumed to be known precisely and is equal to 10^6 [N/m]. We select this task because it is a challenging application for adaptive control schemes, when the object stiffness must be estimated. In this research, see also [190], the stiffness is assumed to be known and constant, justified by the aim to assess the robustness for unmodeled dynamics.

The desired trajectory in Cartesian end-effector space is

$$x_d(t) = \begin{bmatrix} a + R_d \cos \psi_d \\ b + R_d \sin \psi_d \end{bmatrix}$$

with $R_d = r_e = 0.25$ [m] the radius of the object, $\psi_d = 2\pi t - \frac{\pi}{2}$ [rad] the desired angular position and $a = 0$ [m], $b = 0.5$ [m] the center of the object, see Fig. 5.13. The desired force F_n, with positive direction towards the center of the object, is equal to 100 [N]. The periodic nature of the task makes it easy to compute accurate error statistics, without influence of initial transients.

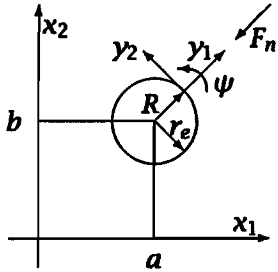

FIGURE 5.13. Desired trajectory

REMARK 5.2. Three coordinates systems were introduced, joint space (q-coordinates), end-effector space (x-coordinates) and task space (y-coordinates). The robot Jacobian J is used to transform from end-effector space to joint space. To transform from task space to end-effector space the transformation R, introduced in the next section, is used.

5.3.2. Hybrid control scheme. The scheme proposed in [191] is used for the hybrid control task. For the structure of the controller see the block diagram in Fig. 5.14. The meaning of the symbols in Fig. 5.14 is specified later.

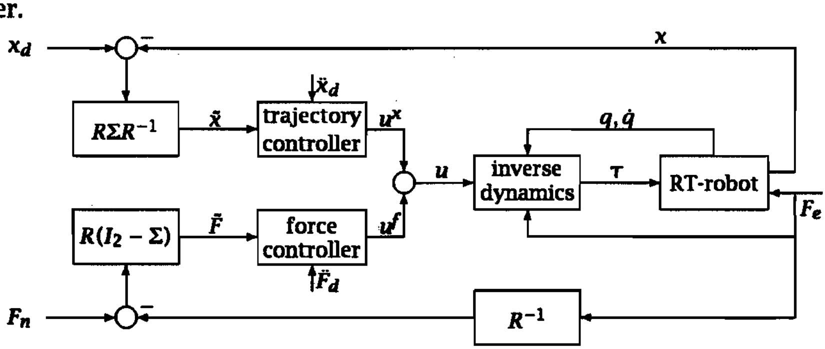

FIGURE 5.14. Hybrid control scheme

Three main blocks can be distinguished. First, the computed torque part, using the inverse dynamics of the design model. Second, a trajectory control loop, driven by the projection of the position error on the tangent to the object at the actual position. Third, a force control loop, driven by the radial projection of the force error.

The inverse dynamics of the design model is [191]

$$\tau = M(q)J^{-1}(q)(u - \dot{J}(q,\dot{q})\dot{q}) + C(q,\dot{q})\dot{q} + g(q) - J^T(q)F_e \tag{5.4}$$

with u the sum of the outputs u^x and u^f of the trajectory and force controller.

The trajectory controller is a standard PD controller, with a feedforward loop for the desired acceleration $\ddot{x}_d$, so, with the inverse dynamics block it is in essence a computed torque controller with PD component. The controller has constant parameters and is given by

$$u^x = \ddot{x}_d + K_v^x \dot{\tilde{x}} + K_p^x \tilde{x} \tag{5.5}$$

with $\tilde{x} = x_d - x$. The controller parameters are equal to

$$K_p^x = \begin{bmatrix} k_p^x & 0 \\ 0 & k_p^x \end{bmatrix} = \begin{bmatrix} 140^2 & 0 \\ 0 & 140^2 \end{bmatrix},$$

$$K_v^x = \begin{bmatrix} k_v^x & 0 \\ 0 & k_v^x \end{bmatrix} = \begin{bmatrix} 2 \cdot 140 & 0 \\ 0 & 2 \cdot 140 \end{bmatrix}$$

to get approximately critically damped dynamics with an undamped radial frequency of 140 [rad/s], well below the inverse of the smallest motor time constant.

The force controller is the subject of this investigation. Here, H_2, H_∞, μ-synthesis, and a reference PD controller are used, with a feedforward path for $\ddot{F}_d$, so

$$u^f = -K_e^{-1}(\ddot{F}_d + K_v^f \dot{\tilde{F}} + K_p^f \tilde{F}) \tag{5.6}$$

for the PD controller, with $\tilde{F} = F_d - F_e$. The term $\ddot{F}_d$, in end-effector coordinates, is equal to $\ddot{x}_d F_n / R_d$, when the desired force F_n in task space is constant. This expression can be derived by expressing F_d in the desired position and differentiating it twice with respect to time. The term $K_v^f \dot{\tilde{F}} + K_p^f \tilde{F}$ is replaced by the output of the H_2, H_∞, or μ-synthesis controller, both driven by $\tilde{F}$ only.

The matrix R^{-1} appearing in the control scheme is equal to

$$R^{-1} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}$$

and is used to transform from Cartesian end-effector space (x-coordinates) to Cartesian task space (y-coordinates). The projection of the position and force errors is performed by the matrix

$$\Sigma = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

By this projection we obtain the force and position errors needed for the controllers.

REMARK 5.3. Because the controllers are working in Cartesian end-effector space, the control of position and force, although one dimensional tasks, requires for each of these controllers two inputs and two outputs. This could perhaps be circumvented by placing the controllers in task space, but then the generality and flexibility of the control scheme will probably be reduced.

REMARK 5.4. Because the linearized model is decoupled, both the trajectory and force controllers are themselves decoupled. The controllers for the two diagonal blocks of the force controller are identical, so the design is performed for one block only, and the resulting controller is diagonally augmented to obtain the complete force controller. Refer to [191] for further details of the control scheme.

REMARK 5.5. The computed torque part of the controller, using exact parameters, will linearize and decouple the design model. Because in the evaluation model motor dynamics is also included, the controlled evaluation model will be nonlinear and it is also not decoupled. That is why we modify the outer feedback loop, to increase the robustness of the overall control system.

5.3.3. Controller design. Because the stiffness of the environment K_e is constant, the force controller can be designed as a position controller, the controllers designed should only be scaled with $-K_e^{-1}$, as in (5.6). To avoid a more complicated notation, in the following, therefore, a pure trajectory controller design is discussed and the superscript ^{x} is dropped.

We design the controllers for the design model, augmented with the inverse dynamics and acceleration feedforward part of the control scheme. Because the controllers are designed in end-effector space, the design model equations in end-effector space are needed. They are

$$M^*(q)\ddot{x} + J^{-T}(q)(C(q,\dot{q})\dot{q} + g(q)) - M^*(q)\dot{J}(q,\dot{q})\dot{q} = J^{-T}(q)f + F_e \tag{5.7}$$

where $M^*(q) = J^{-T}(q)M(q)J^{-1}(q)$. When the inverse dynamics (5.4) block is applied to the design model of the manipulator (5.7), for which $f = \tau$, there results

$$\ddot{x} = u, \tag{5.8}$$

a decoupled system of two second order differential equations. After transformation to state space we obtain

$$\dot{v}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} v_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i \tag{5.9}$$

for each degree-of-freedom x_i, with $v_i = [x_i, \dot{x}_i]'$. This is a model for two integrators in series. After applying PD feedback $u_{pd} = K_v\dot{\tilde{x}} + K_p\tilde{x}$ the controlled system has a transfer function matrix, where the diagonal entries are second order models with additional zeros

$$\frac{k_v s + k_p}{s^2 + k_v s + k_p}, \tag{5.10}$$

with the desired states x_d as inputs. The sum of the feedforward parts of the controllers (5.5,5.6) is the input to a transfer function matrix with diagonal entries

$$\frac{1}{s^2 + k_v s + k_p}.$$

The structure of the controlled system (5.8) in block diagram form is given in Fig. 5.15.

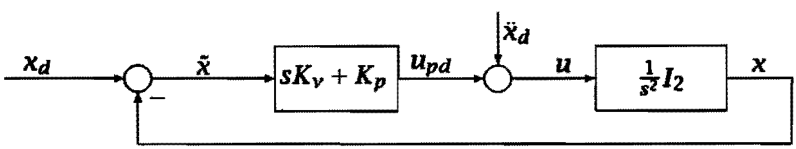

FIGURE 5.15. Block diagram of trajectory control loop with PD controller

When the influence of the zero $s_z = -k_p/k_v$ in (5.10) is neglected, the undamped radial frequency ω_0 is $\sqrt{k_p}$ [rad/s] and the damping factor β is $\frac{k_v}{2\omega_0}$. The design parameters for the PD controller are selected to get a prescribed ω_0 and a damping factor $\beta = 1$, *i.e.*, a critically damped second order system when s_z is neglected, so $k_p = \omega_0^2$ and $k_v = 2\omega_0$. For this choice of parameters, the actual bandwidth ω_b of the controlled model (5.10), depending on the definition of bandwidth, will be $\sqrt{3+\sqrt{10}}\omega_0 \approx 2.4824 \cdot \omega_0$ for $\alpha_b = \frac{1}{\sqrt{2}}$ and $\sqrt{2+\frac{3}{2}\sqrt{2}}\omega_0 \approx 2.0301 \cdot \omega_0$ for $\alpha_b = \frac{1}{\sqrt{2}}\max_\omega \alpha = \sqrt{\frac{2}{3}}$, where α_b is the magnitude α of the transfer function (5.10) at the bandwidth frequency ω_b. In the following discussion, we use the second definition of the bandwidth ω_b.

To bring the reference PD controller in the same state space frame as the dynamic linear controllers, the D part of the PD controller is approximated by a tame differentiator by using a series connection with a first order system with time constant $\frac{1}{\delta}$, so only the position error signal is used as input to the controller. The state space form of the PD controller becomes

$$\dot{z} = -\delta I_2 z + \tilde{x}$$

$$u_{pd} = \left[-\delta^2 K_v + \delta K_p\right] z + \delta K_v \tilde{x}.$$

In expanded form

$$\dot{z} = \begin{bmatrix} -\delta & 0 \\ 0 & -\delta \end{bmatrix} z + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tilde{x}$$

$$u_{pd} = \begin{bmatrix} -\delta^2 k_v + \delta k_p & 0 \\ 0 & -\delta^2 k_v + \delta k_p \end{bmatrix} z + \begin{bmatrix} \delta k_v & 0 \\ 0 & \delta k_v \end{bmatrix} \tilde{x}.$$

To make the influence of the first order system small, choose the factor δ much larger, 10^5, than the largest design frequency ω_0.

The design of the H_2 and H_∞ controllers follows the lines given in [157]. For the design of the μ-synthesis controller see [162]. The actual computation of the controllers is elaborated in the next section. We design the H_∞ controller $F(s)$ in such a way that the bandwidth ω_b of the resulting controlled system can be specified with the weight function $W_3(s)$ for the complementary sensitivity function $T(s)$. Select the weight function $W_1(s)$ for the sensitivity function $S(s) = 1 - T(s)$ so that the sensitivity function is as small as possible, within the requirement that the H_∞ norm of the transfer function

$$\begin{bmatrix} W_1(s)S(s) \\ W_2(s)F(s)S(s) \\ W_3(s)T(s) \end{bmatrix} \tag{5.11}$$

is smaller than or equal to 1. We achieve this by choosing

$$W_1^{-1}(s) = \frac{1}{\rho} \frac{(\frac{\omega_1}{\omega_0})^{m_1} (\frac{s}{\omega_1} + 1)^{m_1}}{(\frac{s}{2\omega_0} + 1)^{m_1}} I_2, \quad m_1 = 1, \ldots, 3$$

$$W_2 = 0$$

$$W_3^{-1}(s) = \frac{\alpha_3}{(\frac{s}{\omega_0} + 1)^{m_3}} I_2, \quad m_3 = 1, 2$$

with $\omega_1 = \omega_0 / \sqrt[m_1]{10^{3+m_1}}$, and performing a search for maximal ρ (often called γ iteration), within the constraint that the H_∞ norm of (5.11) is ≤ 1. Use the factor α_3 in W_3^{-1} to force the amplitude α of W_3^{-1} through the point (ω_b, α_b). The amplitude of (5.10) coincides at this point. For $m_3 = 1$ the factor $\alpha_3 \approx 2$. For $m_3 = 2$ the factor $\alpha_3 = \sqrt{9+6\sqrt{2}}$. Use the factors m_1 and m_3 to tune the slopes of S and T. The best results were obtained with $m_1 = 2$, $m_3 = 1$. This choice of the weight functions W_1^{-1} and W_3^{-1} gives S a slope of $+2$ for $\omega < \omega_0$, like a PD controller and T a slope of -1 for $\omega > \omega_0$, also like a PD controller. Although a slope of -2 for T, corresponding with $m_3 = 2$, should make the system more robust for high frequency unmodeled dynamics, because those dynamics are excited less, the results did then not match those of the selected weights.

We designed the H_2 and μ-synthesis controller employing the same weight functions as used for the H_∞ design.

5.3.4. Controller computation. For the computations of the H_∞, H_2, and μ-synthesis controllers commercial software is available, see [161, 162]. This software has been used for the controller design, although some problems with the computation of solutions for the Riccati equations surfaced.

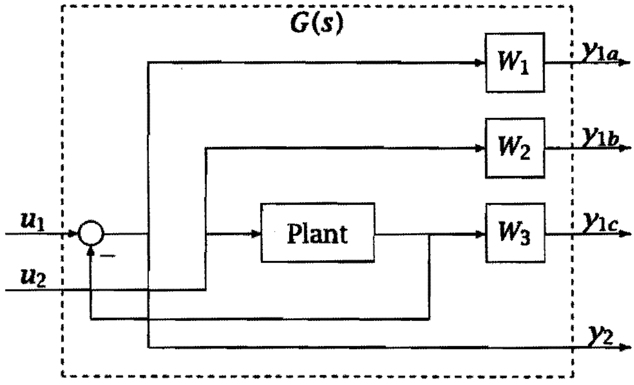

FIGURE 5.16. Problem setup for the H_∞ control problem

To see how our problem fits the standard H_∞ setup, see Fig. 5.16 and compare it with Fig. 4.2. A few modifications of the above problem are necessary to enable a solution for the H_∞ computations. Shift the poles of the decoupled linearized model (5.9) to the right slightly. Also assign a small value to the weight W_2. Give both factors a value so small to enable the computation, but without perceptible influence on the resulting controllers. The algorithm used for the H_∞ controller computation is the one given in [172].

Three modifications of the software proved to be advantageous. First, a bisection method to determine the optimal, *i.e.*, largest, ρ was used. Second, in the algorithm a transformation from descriptor form to state space form is used. This transformation did not always give the correct state space model, because the order of the resulting controller was not the same for all designs. This was modified to obtain a controller of specified order. Finally, for the solution of the two Riccati equations appearing in the H_∞ design two methods are available, one based on eigenvector decomposition, the other on Schur decomposition. It was only possible to specify the same method for both equations. A modification to specify a method for each equation separately was implemented. This improved the accuracy of the computations.

The H_2 design is performed according to the formulae given in [159], in essence formulae for the calculation of an H_∞ controller, but with their parameter $\gamma = 10^3$, because when $\gamma \to \infty$ the H_∞ controller approaches an H_2 controller for the same problem with the same weight functions. This strategy is chosen to prevent numerical problems, which occur in an algorithm for a direct H_2 design based on a combination of an optimal controller and an optimal Kalman filter (LQG) design. The same modifications of the design problem as for the H_∞ design were used.

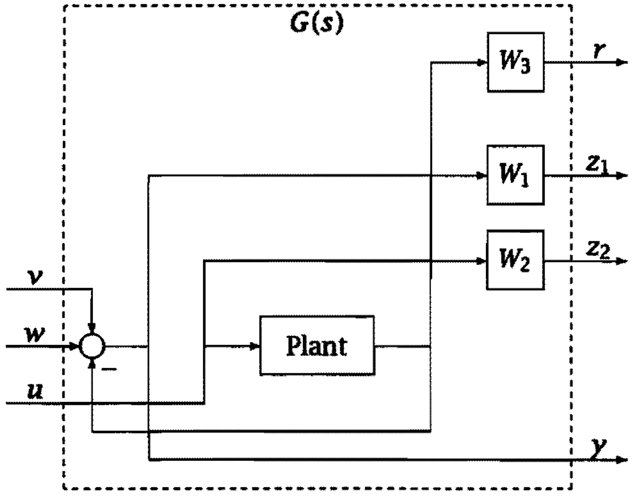

FIGURE 5.17. Problem setup for the μ-synthesis problem

To see how our problem fits the standard μ-synthesis setup, see Fig. 5.17 and compare it with Fig. 4.4. The same modifications of the design problem as for the H_∞ design were used. The difference with the setup for the H_∞

problem is slight. An additional perturbation input is included, identical with the reference input, so the model error is modeled as a multiplicative output perturbation. In essence, the difference with the H_∞ controller lies in the structure of the block structured matrix Δ for the robust performance problem. For the H_∞ controller this matrix has the following structure

$$\begin{bmatrix} * & * & * \end{bmatrix},$$

i.e., a single block, and for the μ-synthesis controller it is

$$\left[\begin{array}{c|cc} * & & \\ \hline & * & * \end{array}\right],$$

i.e., two blocks, where the first block corresponds with the multiplicative output perturbation, and the second block with the performance specifications. For this block structure the μ-synthesis controller was able to reduce the μ value to ≈ 1.35, and the μ value with the H_∞ controller, designed for the same specifications, was only slightly larger. The software for the μ-synthesis does not allow non proper weighting functions. This problem was solved by using code of the H_∞ design software. The resulting controllers for the μ-synthesis are of larger dimension due to the frequency dependent D-scalings used. Model reduction was used to obtain a controller of approximately the same dimension as the H_2 controller. The nominal closed loop with the μ-synthesis controller is often marginally stable. After removing the D-scaling the loop can be even unstable. This is probably due to the large difference between the largest and smallest eigenvalue, typically of the order 10^{10}, for the closed loop poles (still including weighting functions). When the controller order is reduced this problem becomes more prominent. So, to assure stability of the closed loop, an additional P action is added to the controller by modifying its D matrix. This modification is large enough to shift the unstable closed loop poles to the left half plane, but so small there is no perceptible influence on the Bode magnitude plot of the controller.

The dimensions of the controllers are 2, 6, 8, and (6–10) for the PD, H_∞, H_2, and μ-synthesis design, respectively.

5.3.5. Controller evaluation. The controllers are evaluated by simulating the evaluation model, with the hybrid controller, for 2 [s]. The simulation data for the first second is disregarded, because of transients originating from the incorrect settings of the initial conditions of the dynamic controller. From the simulation data of the second part of the simulation run the root mean square (RMS) values of the position and force error are used for evaluation, to prevent a difficult interpretation based on all the abundant simulation data.

To assess the robustness of the controller two system parameters are varied. First, the controllers are designed for different design frequencies ω_0. Second, the time constants of the unmodeled dynamics, included in the evaluation model are varied.

The transfer functions of the force controllers designed for $\omega_0 = 400$ [rad/s] are in Fig. 5.18. It is a bit surprising that the controllers generated are not able to mimic a PD controller over a large frequency range, although the specifications were chosen with that objective. The values of ρ used in the weight function W_1 are given, as function of the design frequency ω_0, in Fig. 5.19. The non smooth appearance of the line in this plot is due to numerical problems that sometimes cause a premature breakdown of the iteration for optimal ρ. When a suboptimal ρ is used the controller generated is almost the same at the crossover frequency, but deviates from the optimal one at higher and lower frequencies.

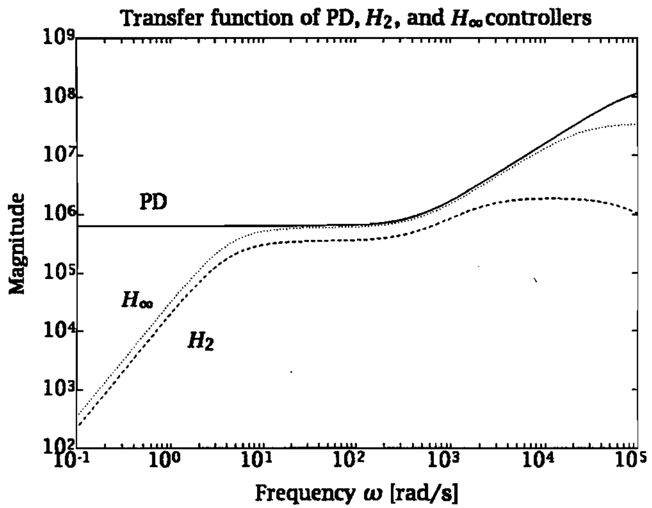

FIGURE 5.18. Transfer function amplitude of H_∞, H_2, and PD controllers

5.3.6. Simulation results. The results are presented in four parts. First, a sample of the time responses calculated during one of the simulations. Second, an overview of the influence of the design frequency on the RMS errors, with fixed motor time constants. Third, another overview but now for a fixed design frequency and varying motor time constants. Finally, the influence of unmodeled dynamics and design frequency is presented in combined form.

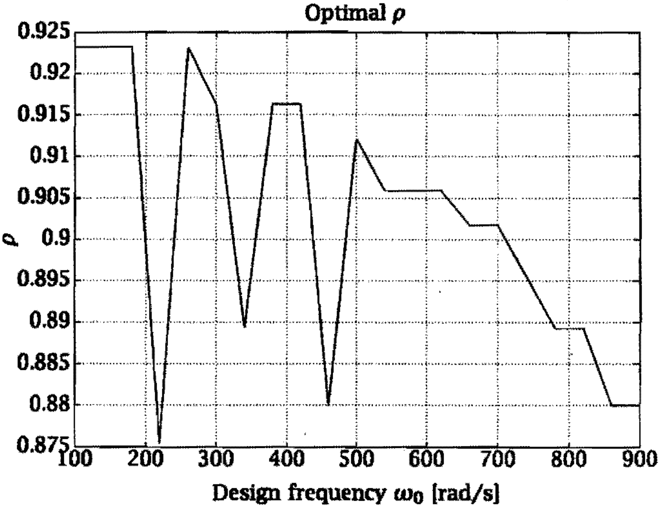

FIGURE 5.19. Design parameter ρ

A sample of the time responses is in Figs. 5.20–5.22. The results are obtained with an H_∞ controller designed for the nominal motor time constants and a nominal design frequency ω_0 of 400 [rad/s]. In Fig. 5.20 the position error for $t = 1 - 2$ [s] is projected on the reference trajectory. The position error is scaled with a factor of 400, but the position error is still very small, of the order of several [μm]. The tracking error $\tilde{q}$ is largest for small x_2, because then $\dot{\varphi}$ is large. In Fig. 5.21 the force error for $t = 1 - 2$ [s] is presented, also projected on the reference trajectory. The length of the lines is a measure of the force error. The scaling is such that a line from the circumference to the center corresponds to 100 [N], so the figure shows that the force error is also small, of the order of several [N].

Figure 5.22, force error against time, shows that the system is still far from the stability limit, because of the fast decay of the oscillations, induced by incorrect initial conditions of the controllers.

The results for the H_∞ controller are given in Fig. 5.23.

The RMS values of position and force error against design frequency ω_0 for nominal motor time constants of $\frac{1}{400}$ [s] are presented, for comparison purposes, in Fig. 5.24 and Fig. 5.25. The errors are large for small design frequencies, caused by loose control and diminishing with larger design frequencies because the control becomes more tight. The levels of the RMS values for the three control designs are different. Also, for higher design frequencies the controllers are unable to stabilize the system, so for $\omega_0 >$ 900 [rad/s] no RMS results are given. The onset of instability for the H_2

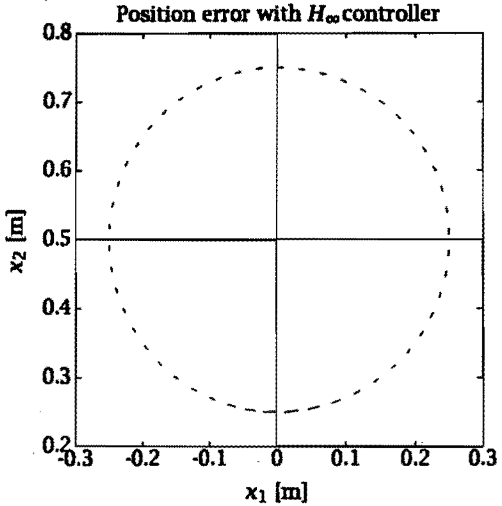

FIGURE 5.20. Position error

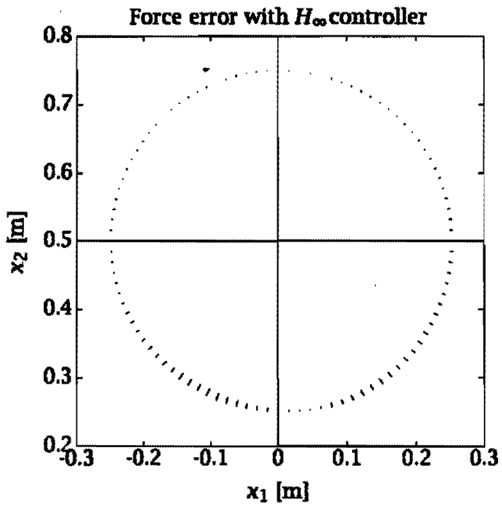

FIGURE 5.21. Force error

controller is at a much lower design frequency ω_0 as for the PD and H_∞ controllers.

The RMS values of position and force error against motor time constant, for a nominal design frequency of 400 [rad/s] are presented in Figs. 5.26 and 5.27. The errors increase monotonically, and almost linearly, for larger time constants, because the unmodeled dynamics are excited at a lower

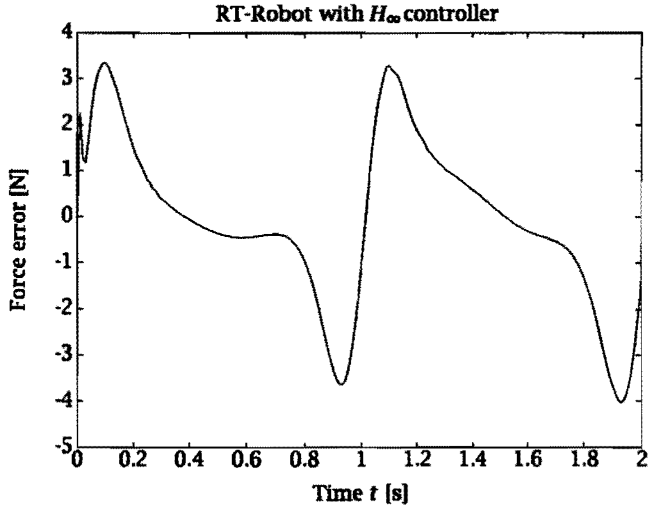

FIGURE 5.22. Force error against time

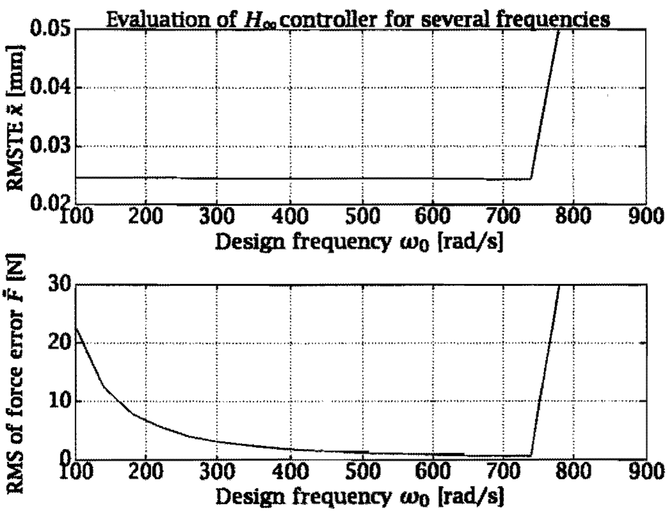

FIGURE 5.23. RMS values of position and force error against design frequency for H_∞ controller

frequency, until the onset of instability is reached.

The results of further simulations, relating the influence of design frequency, motor time constants, and position and force errors, are given in the truncated surface diagrams, presented in Figs. 5.28–5.29, for the H_∞ controllers. The truncation is necessary to prevent blow up of the figure,

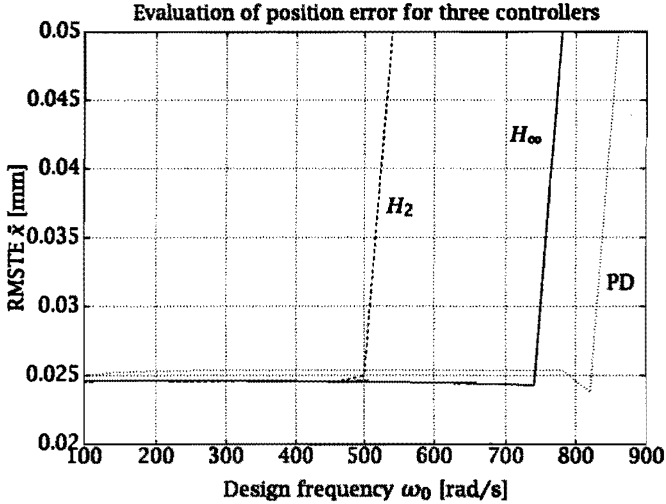

FIGURE 5.24. RMS values of position error against design frequency

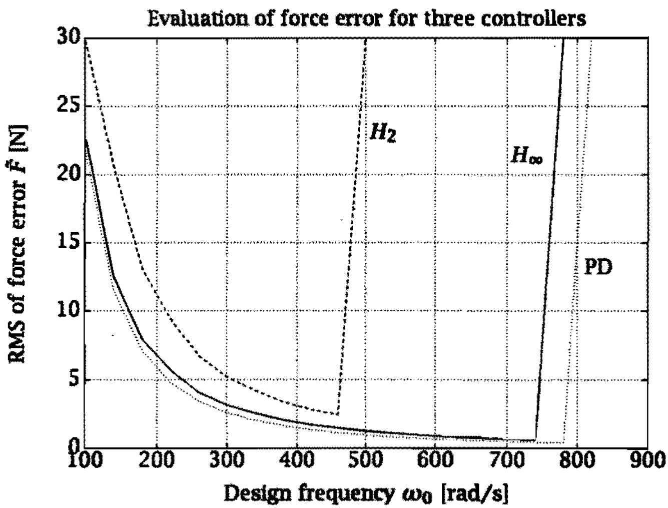

FIGURE 5.25. RMS values of force error against design frequency

which causes diminishing details.

In the truncated part of these two figures the force control loop is (almost) unstable. We remark that the control system for the position is a fixed PD controller with acceleration feedforward, and the design frequency mentioned is the design frequency for the force control loop. This explains the almost constant tracking error as a function of frequency. There is a

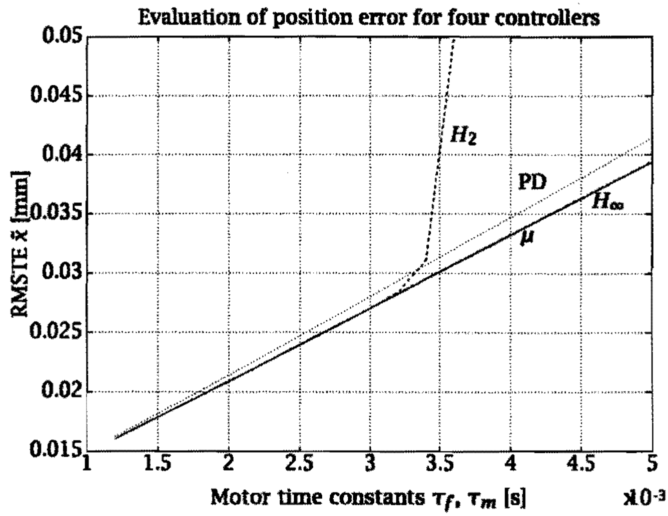

FIGURE 5.26. RMS values of position error against motor time constants

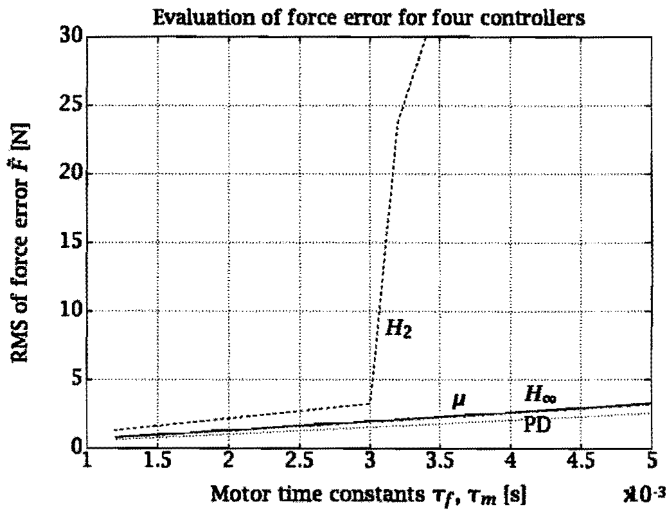

FIGURE 5.27. RMS values of force error against motor time constants

slight influence of the force control loop, because the two loops are not completely decoupled due to the motor dynamics that is the source of model errors. The force error is almost proportional with the motor time constants. An interesting feature is the onset of the instability. The force error is monotonically decreasing with increasing design frequency, up to

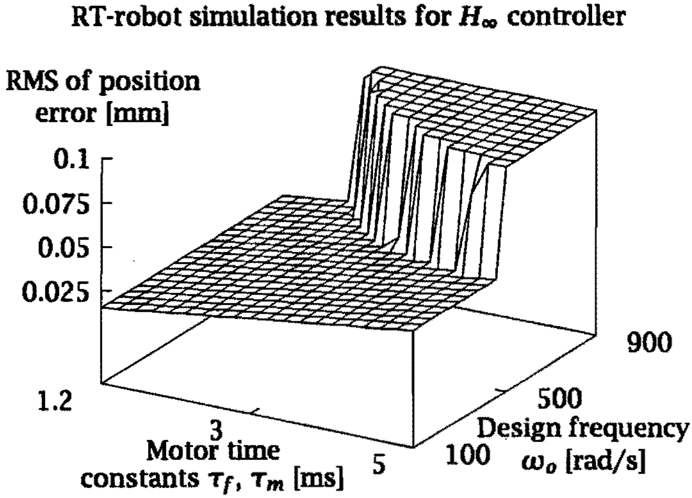

FIGURE 5.28. RMS of position error against design frequency and motor time constants

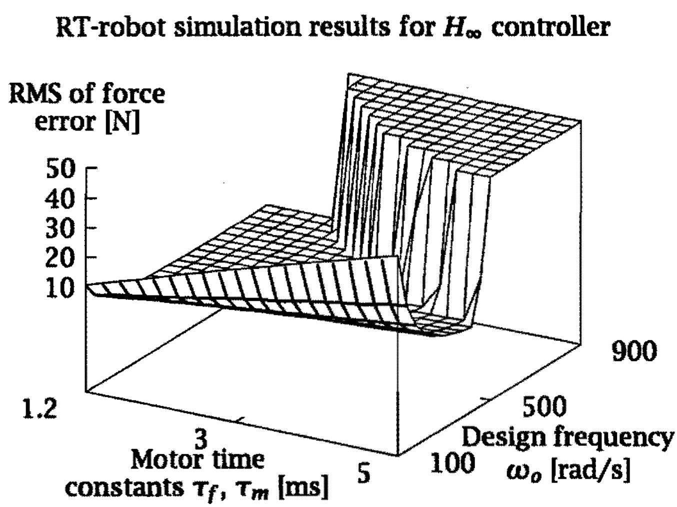

FIGURE 5.29. RMS of force error against design frequency and motor time constants

the point where the control system becomes unstable. There is (almost) no region where the force error does increase smoothly before instability occurs.

5.3.7. Summarizing remarks. From the results presented we can observe that the linear dynamic controllers do not improve on the PD controller. The performance of the H_2 controller is inferior when the same weight functions as for the H_∞ design are chosen. The μ-synthesis controller does not improve on the H_∞ controller with the quite general model error model (multiplicative output perturbation) used here.

The most promising way to improve this situation is to use a more detailed error model, *e.g.*, based on differences in the model parameters for a set of linearized versions of the nonlinear model. This will pinpoint the model error more directly and will therefore give better robustness results. The quantitative implications of this qualitative statement are to be determined.

Besides the disappointing performance of the linear controllers, the general trend of the results is as expected.

The time needed to compute a μ-synthesis controller is almost an order of magnitude larger due to the additional work for the DK-iteration, with the selection of the D-scaling weighting functions. This could be a reason to avoid this controller design method.

5.4. XY-table position control

We discuss the control task, the design of the controllers, the controller evaluation setup and finally present and discuss some simulation results when the controllers are used to track a desired trajectory of the XY-table, presented in Section 5.1.2. Because they have also been used for the RT-robot position control problem, the design, evaluation, and results of the two adaptive controllers of Section 4.3 are presented separate from the VSS and acceleration based controllers. This permits a short analogous treatment.

Again, the focus is mainly on the robustness characteristics of the controllers, in comparison with a PD controller for each degree-of-freedom and a computed torque like controller, that are used as reference.

5.4.1. Control task. The goal of the controllers is to precisely track a desired end-effector trajectory by judiciously manipulating the input to the model. The trajectory is specified in end-effector space (x, y). Because the XY-table is a Cartesian type of robot, the inverse model computation to get the desired trajectory in joint space is trivial. The desired trajectory is a circle or ellipse, defined by

$$x_d = x_c - r\cos(\omega t),$$
$$y_d = y_c - r\cos(\omega t + \varphi).$$

Here x_c and y_c represent the center of the working area of the XY-table, r is the "radius" of the trajectory, and φ is the phase shift between the cosine in x and y direction. When $\varphi = \frac{\pi}{2}$ the trajectory is a circle, for other values of φ the trajectory deforms to an ellipse or even a straight line. This trajectory has been chosen because it is periodic and has smooth derivatives. For an example of a desired trajectory (in this case an ellipse with $\varphi = \frac{\pi}{4}$) see Fig. 5.30. When the need arises, *e.g.*, to investigate the

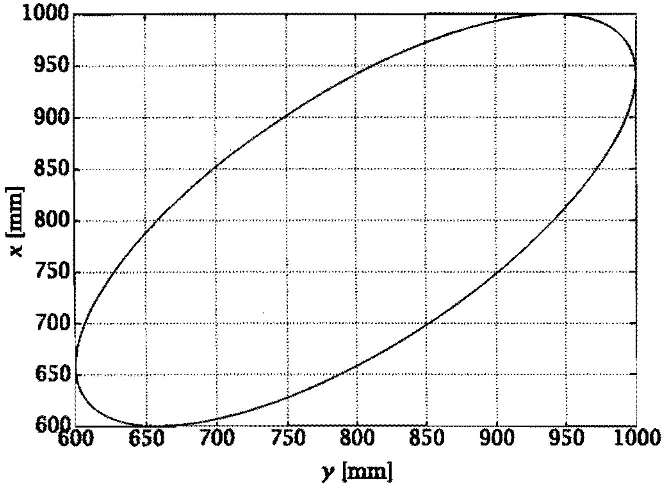

FIGURE 5.30. Desired trajectory in end-effector space

convergence of parameter estimates, this trajectory can be extended easily, because it is periodic.

5.4.2. Adaptive controllers. For the two adaptive controllers investigated, the one proposed by Slotine and Li, and the one proposed by Kelly, we first discuss the controller design and evaluation. This is followed by a presentation of the results.

5.4.2.1. *Controller design.* The same aims as for the RT-robot position control problem are in force. It is assumed that the unmodeled dynamics is not completely unknown, but that the flexible bar only introduces a high frequency type of unmodeled dynamics and that a lower bound for the frequency where the influence of the unmodeled dynamics becomes significant (dependent on the stiffness of the bar) is known.

The controllers are designed following the same procedure as for the RT-robot position control problem, but a linearization procedure is not necessary, because a linear model with two DOF for the XY-table can be directly formulated, so a suitable working point is not necessary.

The specific choice for the adaptation gain matrices depends on the system to be controlled. Here, the adaptation gains are chosen such that the experimental system (to be discussed in the next Chapter) does not become unstable with a reasonable margin. This tuning of the parameters has been performed only with one order of magnitude accuracy.

5.4.2.2. *Controller evaluation.* Basically, the same methodology to evaluate the controllers as for the RT-robot position control has been used.

We will now give a more detailed description of the models used. The design model is already presented in Section 5.1.2. The friction is simply a standard Coulomb friction model, added to each degree-of-freedom of the model, sometimes extended with a position dependent component, and viscous friction. The additional dynamics is a flexible bar that introduces an additional degree-of-freedom. It is used to represent the joint flexibility. As remarked in [124] this type of neglected dynamics has often an important effect on the dynamic behavior.

As stated before, an extensive model for the XY-table with three DOF is given in [186]. This model is nonlinear. A linearized version of this model (except for the mass matrix that depends on the DOF), has been derived and used for the simulations. It was obtained by neglecting the Coriolis and centrifugal forces, and it is valid when the wind up of the torsion spring is small because then those forces are small also. This model can be extended further with position dependent friction, viscous friction, and torque ripple. It also includes the effects of quantization of the position measurement and of discretization due to a finite sampling rate.

5.4.2.3. *Simulation results.* A limited number of results is presented. The results can be divided in three groups. The first group is used to investigate the effects of parameter errors. The results of the second group show the effects of the unmodeled statics error. The effects of unmodeled dynamics are evident from the last group of results.

The control system parameters used for each group of simulation are given in Table 5.5, with the corresponding figure numbers as main entry, for each controller used. In this table, PD indicates a PD controller, CT a computed torque controller, SL the controller proposed by Slotine and Li, and K the controller proposed by Kelly. The computed torque controller is implemented as the controller of Slotine and Li with adaptation not enabled.

The first group, Figs. 5.31–5.33, gives the tracking error results, for the second of two cycli of 3.5 [s] each, so the first 3.5 [s] of the response is omitted

Figures	Param	PD	CT	SL	K
5.31-5.33	K_p	$(8\pi)^2 M$	$(8\pi)^2 M$	$(8\pi)^2 M$	$(8\pi)^2$
	K_v	$11.2\pi M$	$11.2\pi M$	$11.2\pi M$	11.2π
	Γ^{-1}			$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$	$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$
	λ				$\begin{bmatrix} 10 & \\ & 10 \end{bmatrix}$
5.34-5.36	K_p	$(8\pi)^2 M$	$(8\pi)^2 M$	$(8\pi)^2 M$	$(8\pi)^2$
	K_v	$11.2\pi M$	$11.2\pi M$	$11.2\pi M$	11.2π
	Γ^{-1}			$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 10^3 & \\ & & & 10^2 \end{bmatrix}$	$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 10^3 & \\ & & & 10^2 \end{bmatrix}$
	λ				$\begin{bmatrix} 10 & \\ & 10 \end{bmatrix}$
5.37-5.39	K_p	$(6\pi)^2 M$	$(6\pi)^2 M$	$(6\pi)^2 M$	$(6\pi)^2$
	K_v	$8.4\pi M$	$8.4\pi M$	$8.4\pi M$	8.4π
	Γ^{-1}			$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 10^3 & \\ & & & 10^2 \end{bmatrix}$	$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 10^3 & \\ & & & 10^2 \end{bmatrix}$
	λ				$\begin{bmatrix} 10 & \\ & 10 \end{bmatrix}$

TABLE 5.5. Controller parameters for XY-table position control problems

to avoid transients originating from the incorrect initial conditions for the degrees-of-freedom and their derivatives, and for the exactly known system structure but unknown parameters. Coulomb friction is not included in the evaluation model, but is still included in the model used for the model based controllers (CT, SL, and K), but with zero parameters. The initial estimate for the other adaptive controller parameters was taken equal to 80% of the nominal parameters, and also for the computed torque controller 80% was used. The inertia parameters cannot be chosen equal to zero, because the control scheme of Kelly does not tolerate zero estimates for them. The adaptation gains for the friction parameters had to be chosen equal to zero to avoid overparametrization problems, *i.e.*, larger tracking errors for the adaptive controllers.

An overview of the tracking error, expressed in the MATE (Mean Absolute Tracking Error), is in Fig. 5.33. The errors for the adaptive controllers are

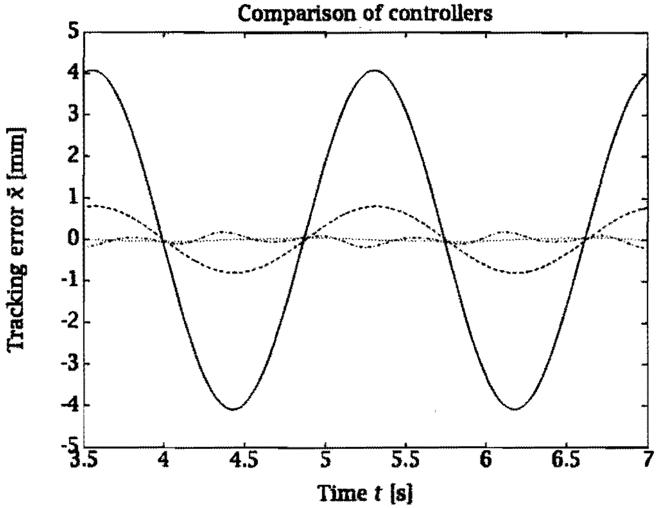

FIGURE 5.31. Tracking error in x-direction, no friction, no unmodeled dynamics, (—) PD, (- -) CT, (· · ·) SL, (- ·) K

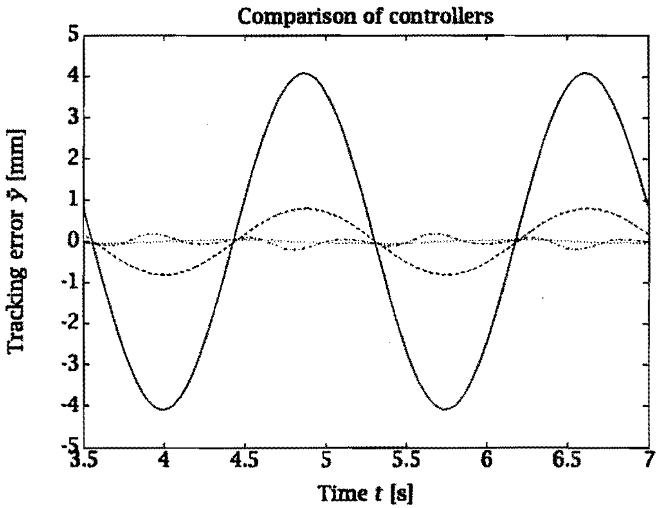

FIGURE 5.32. Tracking error in y-direction, no friction, no unmodeled dynamics, (—) PD, (- -) CT, (· · ·) SL, (- ·) K

quite small, a partial proof of the correctness of the controller implementations. The errors do not become zero due to the quantization error in the position measurement, the prediction error of the Kalman filter, and the discrete time implementation of the control algorithms.

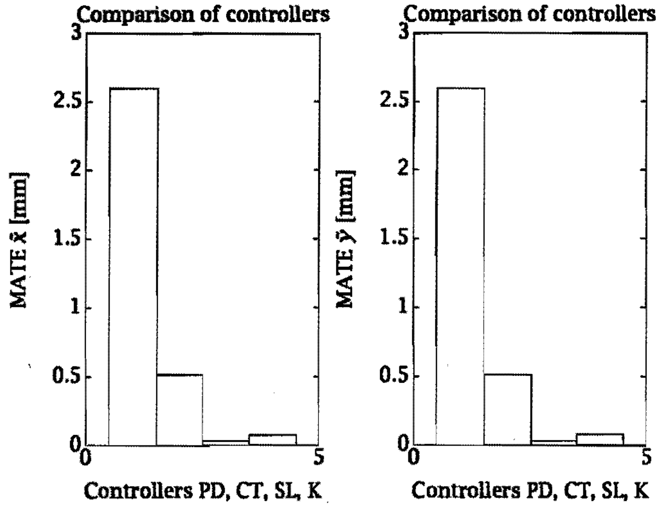

FIGURE 5.33. MATE in x and y directions, no friction, no unmodeled dynamics

The second group of results, presented in Figs. 5.34–5.36, gives the results with Coulomb friction added to the evaluation model. The evaluation model also includes viscous friction, position dependent friction, and torque ripple. These last effects are not included in the model used by the model based controllers. This makes it possible to investigate the effects of unmodeled statics. For the PD controller, results are presented without and with (PDW) friction compensation, with parameters having values of 80% of the nominal parameters. For the CT controller, results are presented with 80% of the nominal parameters. These values are also used for the initial estimate of the parameters for the adaptive controllers.

The MATE results are in Fig. 5.36. Comparison with Fig. 5.33 shows that the errors are substantially larger with friction than without, especially in y-direction. The differences between Figs. 5.33 and 5.36 are an indication of that part of the unmodeled statics that cannot be coped with by the controllers. There is a marked difference between the results for the PD and PDW controllers in x and y direction. This is because the inertia force in x-direction is more substantial, in comparison with the friction force, than in y-direction, so the improvement due to friction compensation is also less in x-direction than in y-direction.

The third and most interesting group of results, in Figs. 5.37–5.39, is for the model with additional dynamics, *i.e.*, an additional degree-of-freedom in x-direction, introduced by a flexible spring in the link connecting the two belt wheels with the x-motor belt wheel. This shows the effects of unmodeled

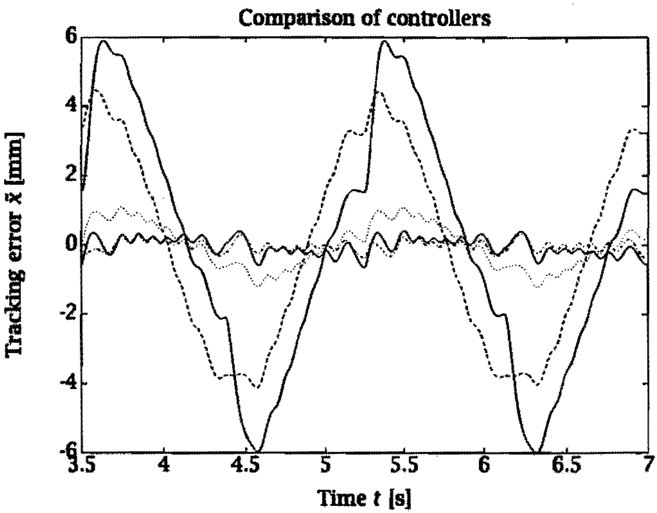

FIGURE 5.34. Tracking error in x-direction, friction, no unmodeled dynamics, (—) PD, (- -) PDW, (· · ·) CT, (- ·) SL, (—) K

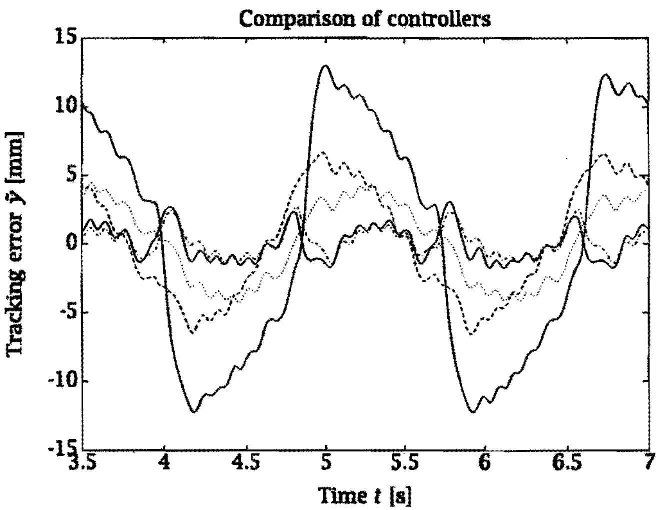

FIGURE 5.35. Tracking error in y-direction, friction, no unmodeled dynamics, (—) PD, (- -) PDW, (· · ·) CT, (- ·) SL, (—) K

dynamics. As can be seen in Table 5.5, the adaptation gain matrix had to be detuned. This was to avoid a too large bandwidth of the controlled system. The bandwidth should now be lower than the lower bound of the characteristic frequency of the additional dynamics, chosen here to correspond

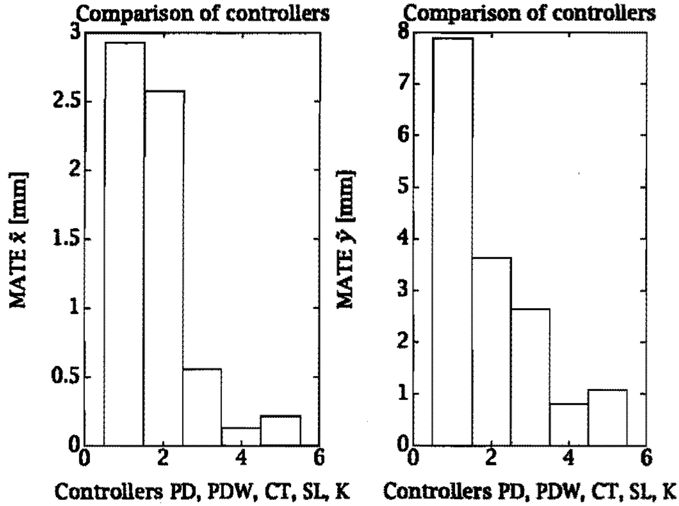

FIGURE 5.36. MATE in x and y directions, friction, no unmodeled dynamics

with a stiffness coefficient of the bar of 0.214 [Nm/rad] or 2140 [mN/mm]. Coulomb and viscous friction, corresponding to 20% of the nominal values, are assigned to the third DOF in the evaluation model.

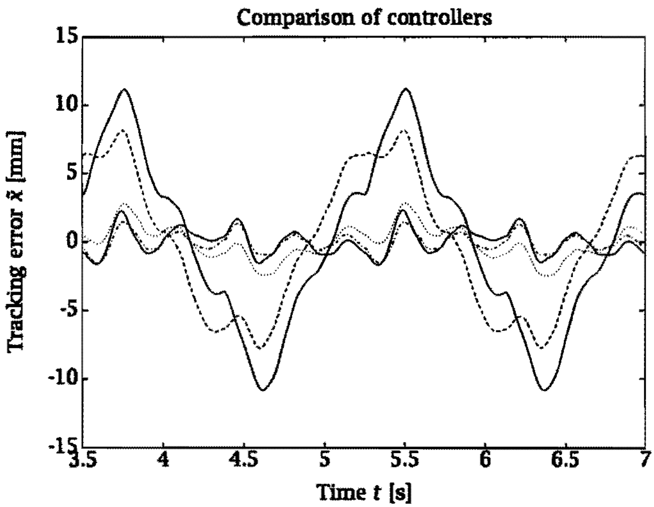

FIGURE 5.37. Tracking error in x-direction, friction, unmodeled dynamics, (—) PD, (- -) PDW, (· · ·) CT, (- ·) SL, (—) K

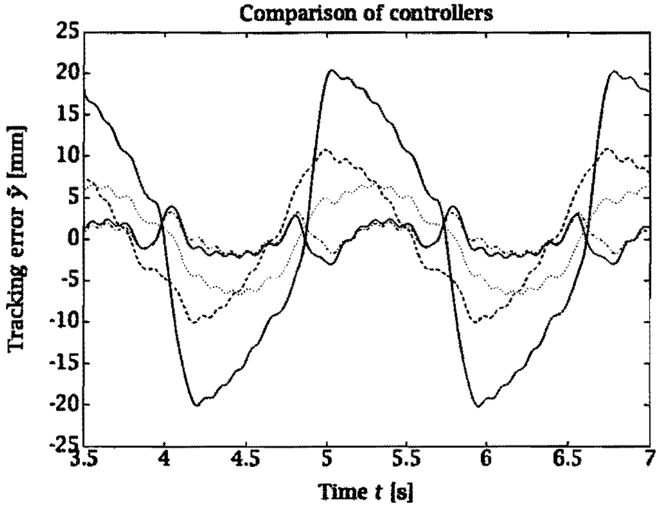

FIGURE 5.38. Tracking error in y-direction, friction, unmodeled dynamics, (—) PD, (- -) PDW, (· · ·) CT, (- ·) SL, (—) K

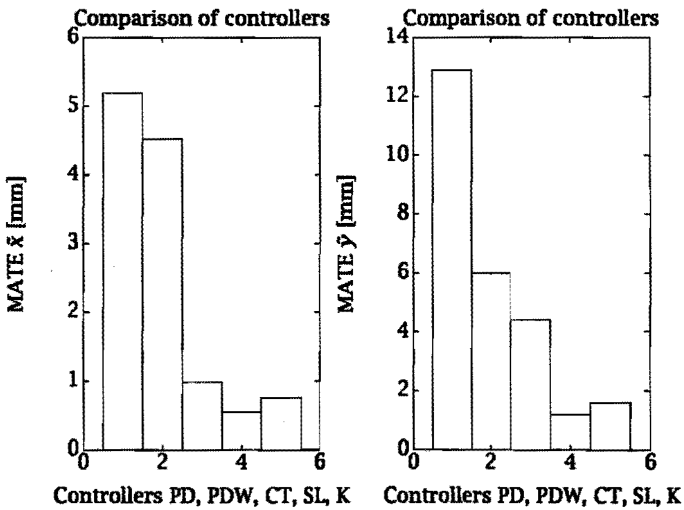

FIGURE 5.39. MATE in x and y directions, friction, unmodeled dynamics

The MATE results are in Fig. 5.39. The errors are again larger than those of the previous MATE results. The errors in x-direction for the CT and adaptive controllers are quite close, so the adaptive controllers are not very good at canceling the effects of unmodeled dynamics. The differences between Figs. 5.36 and 5.39 are an indication of the unmodeled dynamics error and the influence of the reduced bandwidth of the controlled system. In y-direction there is no unmodeled dynamics. The difference in performance is only caused by the choice of the bandwidth of the controlled system, which is reduced by 25%. This leads to an increase in tracking error of ≈30% for the PD controller. Assuming the same increase in x-direction leaves another ≈40% increase, due to the influence of unmodeled dynamics. However, because the reduction in controller gain in x-direction is necessary, due to the unmodeled dynamics, the total increase in tracking error in x-direction by almost a factor of 2, and for the adaptive controllers even more, should be attributed to the unmodeled dynamics.

5.4.3. VSS and acceleration based controllers. For the VSS and acceleration feedback controllers investigated, we first discuss the controller design and evaluation. This is followed by a presentation of the results.

5.4.3.1. *Controller design.* The same aims as for the RT-robot position control problem are in force.

The VSS controller is just a computed torque controller with a discontinuous switching term added. The only additional controller parameters to be designed are the amplitudes of this switching term. The design of the other parameters is copied from the corresponding design in the previous section.

The same is true for the acceleration based controller. It is just a computed torque controller with additional parameters that determine how much of the acceleration signal is fed back.

The specific choice for these additional control parameters depends on the system to be controlled. The VSS parameter is determined by the uncertainty and amplitude of the additional dynamics and must be estimated. This estimate should not be too large, because otherwise the amplitude of the switching term becomes large, and saturation of the actuators can be a problem. The parameters α in the acceleration based controller have to be chosen as large as possible, but are limited by the noise in the acceleration measurements, not only but primarily measurement noise.

The choice of both parameters can only be done correctly if more knowledge of the dynamics of the system is available. We will use values for the simulations that proved to be useful for the XY-table experiments given in the next Chapter.

5.4.3.2. *Controller evaluation.* The same methodology used to evaluate the adaptive controllers and the same models used during the evaluation of the adaptive controllers are used here.

5.4.3.3. *Simulation results.* A limited number of results is presented. The results can be divided in three groups. The first group is used to investigate the effects of parameter errors. The results of the second group show the effects of unmodeled statics error. The effects of unmodeled dynamics are evident from the last group of results.

Figures	Parameter	CT	VSS	AF
5.40–5.42	K_p	$(8\pi)^2M$	$(8\pi)^2M$	$(8\pi)^2M$
	K_v	$11.2\pi M$	$11.2\pi M$	$11.2\pi M$
	λ		0.1	
	σ [mm/s]		20	
	α			1/3
5.43–5.45	K_p	$(8\pi)^2M$	$(8\pi)^2M$	$(8\pi)^2M$
	K_v	$11.2\pi M$	$11.2\pi M$	$11.2\pi M$
	λ		0.1	
	σ [mm/s]		20	
	α			1/3
5.46–5.48	K_p	$(6\pi)^2M$	$(6\pi)^2M$	$(6\pi)^2M$
	K_v	$8.4\pi M$	$8.4\pi M$	$8.4\pi M$
	λ		0.1	
	σ [mm/s]		20	
	α			1/3

TABLE 5.6. Controller parameters for XY-table position control problems

The control system parameters used for each group of simulations are given in Table 5.6, with the corresponding figure numbers as main entry. In this table, VSS indicates the sliding mode controller and AF the acceleration feedback based controller. Both controllers are extensions of the adaptive controller proposed by Slotine and Li, but without adaptation, like the CT controller which is used as comparison. The controller parameter λ, the gain of the saturation part of the VSS controller, is expressed in fractions of the maximal allowable control input, and is therefore different in x and y directions. The fraction α of the acceleration signal used for the feedback cannot be chosen much larger, due to the torque ripple and due to the measurement noise that make the acceleration signal not very reliable.

The first group, Figs. 5.40–5.42, gives the tracking error results, for the second of two cycli of 3.5 [s] each, and for the exactly known system structure but unknown parameters. Coulomb friction is not included in the evaluation model, but is still included in the model used for the model based controllers (VSS and AF). For the model parameters, used in the model based part of the control schemes, 80% of the nominal values are used. The error in the model is therefore of parameter error type.

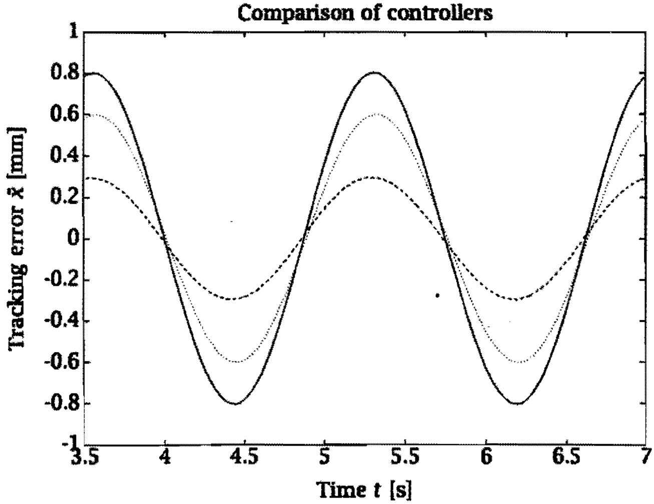

FIGURE 5.40. Tracking error in x-direction, no friction, no unmodeled dynamics, (—) CT, (- -) VSS, (· · ·) AF

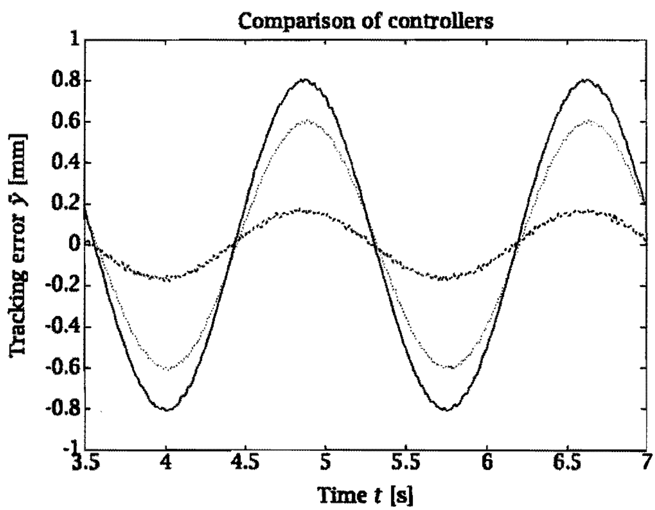

FIGURE 5.41. Tracking error in y-direction, no friction, no unmodeled dynamics, (—) CT, (- -) VSS, (· · ·) AF

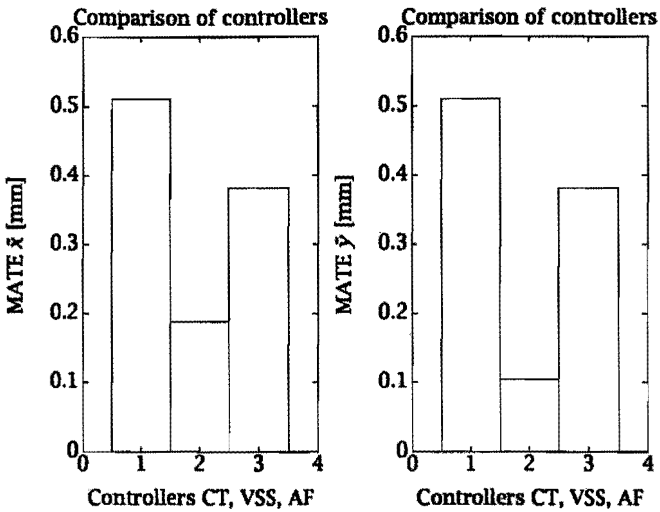

FIGURE 5.42. MATE in x and y directions, no friction, no unmodeled dynamics

An overview of the tracking error, expressed in the MATE, is given in Fig. 5.42. It is evident that the VSS controller can cope better with the model parameter error than the AF controller. The improvement of the VSS controller, with respect to the CT controller, is a factor of 2, but the adaptive controllers perform much better, see Fig. 5.33. This is to be expected, because parameter errors can be cancelled completely by the adaptation mechanism.

The second group of results, presented in Figs. 5.43–5.45, gives the results with Coulomb friction added to the evaluation model. The evaluation model also includes viscous friction, position dependent friction, and torque ripple. These last effects are not included in the model used by the model based controllers. This makes it possible to investigate the effects of unmodeled statics. For all three controllers, results are presented with values of 80% of the nominal values of the Coulomb friction for its compensation. Also the inertia parameters are set at 80% of the nominal values.

The MATE results are in Fig. 5.45. Again, the VSS controller gives the best results. The mean absolute tracking error in x-direction has hardly changed, but the tracking error itself, in Fig. 5.43, is less smooth, so the average additional effects of the unmodeled statics are nicely cancelled by the VSS controller, at least in x-direction. Comparison with Fig. 5.42 shows that the error is increased, but not as much as in the case of the adaptive controllers in Section 5.4.2.3. A comparison with Fig. 5.36 shows that the adaptive controllers perform still better, but the VSS controller performance is already

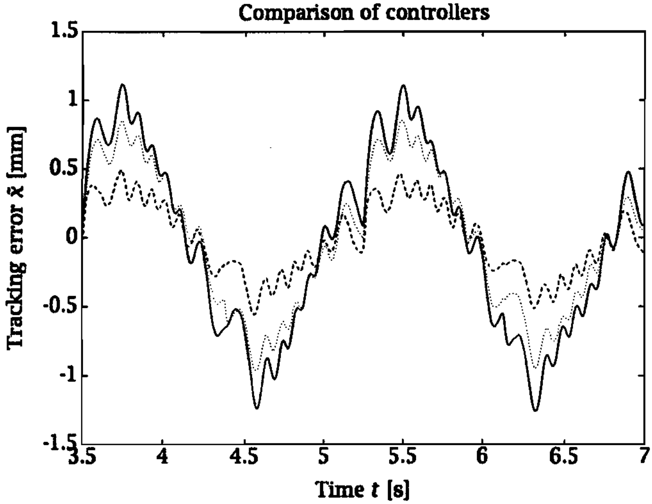

FIGURE 5.43. Tracking error in x-direction, friction, no unmodeled dynamics, (—) CT, (- -) VSS, (· · ·) AF

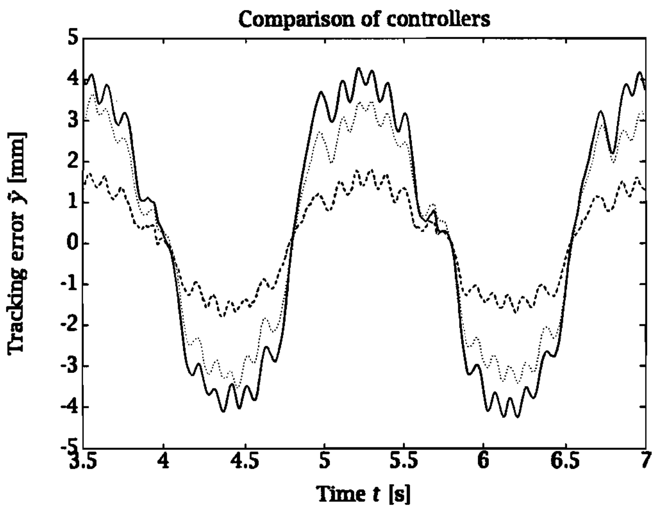

FIGURE 5.44. Tracking error in y-direction, friction, no unmodeled dynamics, (—) CT, (- -) VSS, (· · ·) AF

quite close.

The last and most interesting group of results, in Figs. 5.46–5.48, is for the model with additional dynamics. The model parameters for the additional dynamics were chosen equal to those used for the initial parameter values

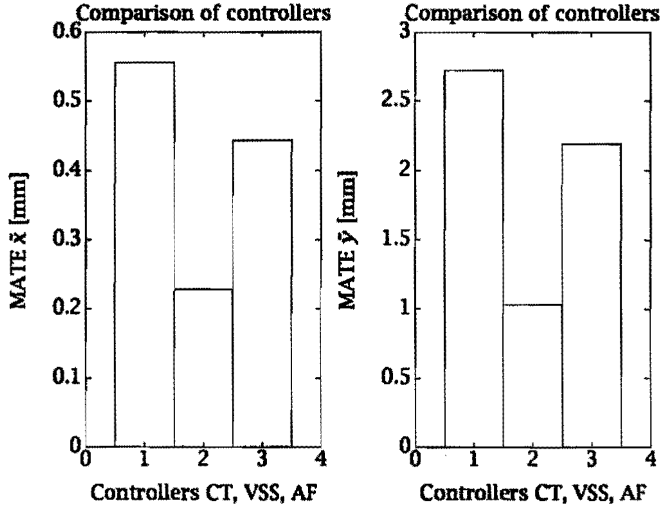

FIGURE 5.45. MATE in x and y directions, friction, no unmodeled dynamics

in the evaluation of the adaptive computed torque controllers, *i.e.*, 80% of the nominal values. This group of results, again, is to assess the influence of unmodeled dynamics on the control system performance.

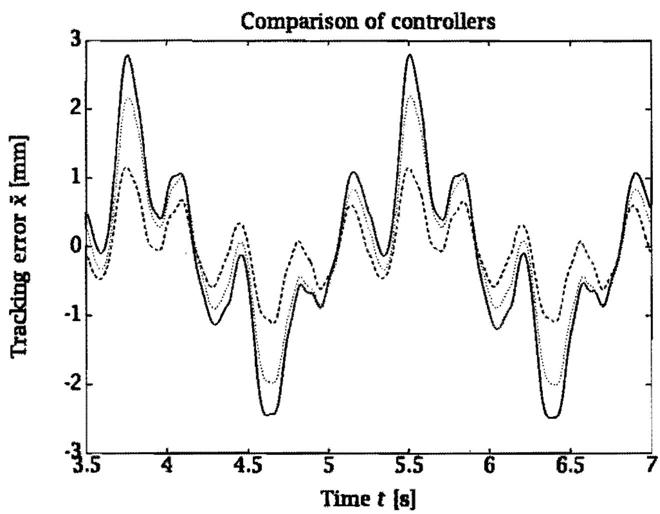

FIGURE 5.46. Tracking error in x-direction, friction, unmodeled dynamics, (—) CT, (- -) VSS, (· · ·) AF

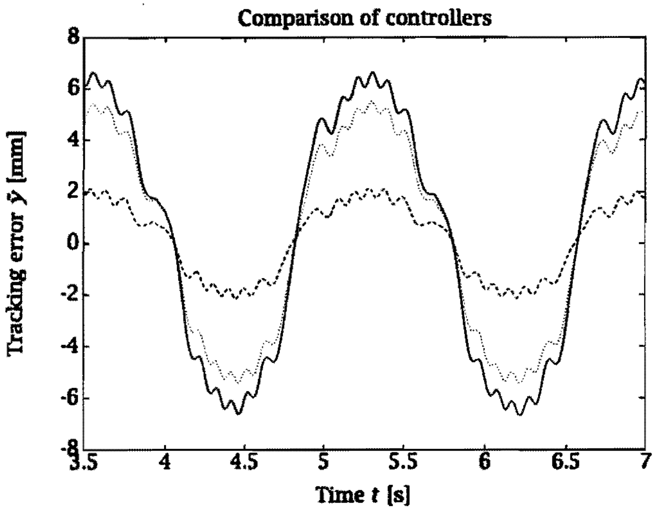

FIGURE 5.47. Tracking error in y-direction, friction, unmodeled dynamics, (—) CT, (- -) VSS, (· · ·) AF

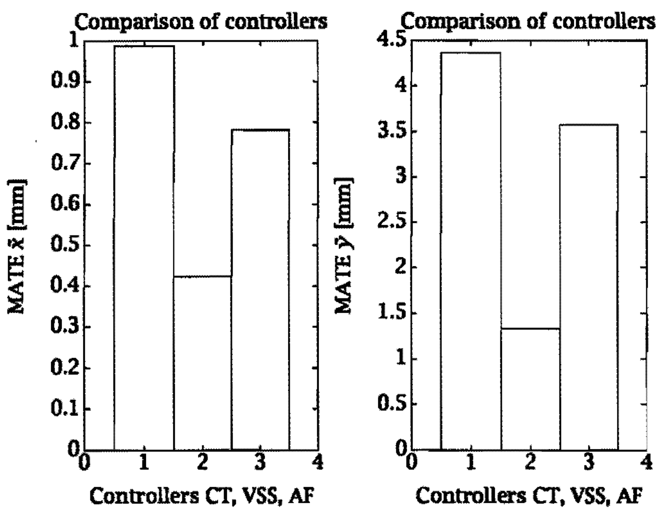

FIGURE 5.48. MATE in x and y directions, friction, unmodeled dynamics

The MATE results are in Fig. 5.48. Comparison with Fig. 5.45 shows that the error is increased, in x-direction by a factor of 2 and in y-direction slightly less. It is therefore clear that both VSS and AF controllers can cope better with unmodeled statics than with unmodeled dynamics errors. Compare with Fig. 5.39 to see the difference with the adaptive controller results. This shows that the VSS controller performance is comparable with, and in x-direction even slightly better than, those of the adaptive controllers. Again an indication that VSS controllers are better suited for systems with unmodeled dynamics, here also in comparison with adaptive controllers.

5.4.4. Summarizing remarks. The results presented in the previous sections give rise to the following remarks (some remarks are the same as for the RT-robot position control problem results)

- PD is at least as robust as adaptive,
- adaptive gives a much smaller tracking error,
- adaptive gives smaller tracking errors than computed torque with values for the parameters at 80% of the nominal ones,
- with unmodeled dynamics, computed torque can be better than adaptive,
- the parameter adaptation does not perform well in the presence of additional dynamics,
- the controller of Kelly is perhaps better than the one of Slotine and Li due to the higher order character of the dynamical equation for the measure of tracking accuracy ν, that is then used in the adaptation law (4.21),
- from a user's point of view, the control scheme of Kelly is worse, due to the additional parameter λ, appearing in the equation for ν, that has to be tuned,
- VSS control can eliminate effects of parameter uncertainty and to some degree of unmodeled dynamics,
- acceleration based control can decrease the tracking error.

At this point there is no reason to eliminate one of the adaptive controllers, the VSS or the acceleration based controller, the differences in performance and robustness are too small. All controllers will therefore be used for more extensive experimental tests.

5.5. Mass-damper-spring position control

In this section the results are presented of the VSS controllers, discussed in Section 4.3.3, when they are used to control a desired trajectory of the mass-damper-spring system presented in Section 5.1.3, see also [192].

We discuss the control task, the design of the controllers, the controller evaluation setup, and finally present and discuss some simulation results.

The focus is mainly on the characteristics of the controllers, compared with each other, with respect to their ability to avoid chattering and to nullify the effects of persistent disturbances. Some other aspects will also be discussed.

5.5.1. Control task. This system is required to track the following trajectory

$$x_d = \begin{bmatrix} a\cos\omega t \\ -a\omega\sin\omega t \end{bmatrix}$$

a sinusoid with amplitude a and frequency ω. For their values, see Table 5.7.

5.5.2. Controller design. The control input has three components

(1) the nominal input u_d required to track the reference signal x_d in the absence of model errors, disturbances, and errors in the initial state,

$$u_d = a\left((c - m\omega^2)\cos\omega t - b\omega\sin\omega t\right),$$

(2) the equivalent control u_e to ensure $\dot{s} = 0$ when $s = 0$, where the first order sliding surface is parametrized as

$$s = \begin{bmatrix} L & 1 \end{bmatrix} e$$

with $e = x_d - x$; then

$$u_e = m\left((-\frac{c}{m} + \frac{b}{m}L - L^2)e_1 + (L - \frac{b}{m})s\right),$$

(3) the switching control u_s to force the tracking error to the sliding surface $s = 0$, despite model errors and disturbances

$$u_s = m\lambda \operatorname{sgn} s \tag{5.12}$$

or one of the modifications listed in Section 4.3.3.

The total control used is just the sum of the three control components

$$u = u_d + u_e + u_s.$$

The second order sliding surface modification, suggested by [82], was implemented as follows. The sliding surface, defined by

$$s = \ddot{x}_d - \ddot{x} + 2\beta_e\omega_e e_2 + \omega_e^2 e_1$$

is set equal to 0, and solved for the input u, that appears in the expression for $\ddot{x}$. As implemented, this has not much resemblance to VSS control.[1] In fact, it is a PD feedback plus an additional acceleration error feedback, to force a desired error dynamics.

[1]The controller used by [82] is not the same as our implementation.

The integral action, for all controllers except for the integral transformation modification (4.24), acts on u_s

$$u = u_d + u_e + u_s + k_i \int_{t_0}^{t} u_s \, d\tau \tag{5.13}$$

with gain k_i.

The discrete time implementation of the continuous time sliding mode controllers is characterized by a zero-order hold on the control u and a sample time t_s.

Although it is known that for discrete time systems a continuous controller, not containing a sgn term, is sufficient to attain sliding motion, we will choose the sample time t_s small enough, related to the reference trajectory and control system bandwidth, to use (5.13) at discrete points in time. For the nominal control system parameters and the sample time t_s used, see Table 5.7.

Parameter	Value	Unit
a	0.1	m
ω	20.0	rad s^{-1}
L	50.0	s^{-1}
λ	44.0	m s^{-2}
σ	0.5	m s^{-1}
q	0.5	-
Δ	$\begin{bmatrix} L^2 & \\ & 1 \end{bmatrix}$	
k_t	50.0	s^{-1}
ω_e	20.0	rad s^{-1}
β_e	0.8	-
k_i	5.0	s^{-1}
t_s	0.001	s

TABLE 5.7. Nominal values for the simulation parameters

The initial conditions for the plant are $x^0 = \begin{bmatrix} -a & -a\omega/2 \end{bmatrix}'$. The control parameters λ and L are chosen such that the sliding surface is reached in 0.25 [s], and the tracking error is negligible or constant after 0.3 [s], *i.e.*, within one period of the desired trajectory. The parameter σ bounds the steady state tracking error to a value of $\sigma/L = 0.01$ [m], which seems reasonable for a pick and place task, if a parallel boundary layer is used. The value used for the Δ of (4.23) makes the cubic feedback proportional with

s^3. The integration gain k_t is chosen so the sign of the modified function (4.24) will follow sgn s within $2/k_t$ [s], or within 40 samples. If k_t is much larger chattering will not be diminished and if k_t is much smaller the control system will behave sluggish. The values of ω_e and β_e are chosen so the dynamics of the control system with the second order sliding surface does not differ too much from the first order one. The integration gain k_i is chosen so the steady state error will normally be small within 0.5 [s].

5.5.3. Simulation results. Three series of simulations are presented. The first without modifications to eliminate chattering. The second with modifications to avoid chattering but without integral action. The last with measures to avoid chattering and with integral action.

To test the effectiveness of the different approaches the simulations were performed with an additional persistant disturbance, *i.e.*, a constant force, to verify the zero steady state property. The disturbance is chosen equal to 66% of the value that would prevent s to become 0.

Figure 5.49 gives the result of a reference simulation, without smoothing of the control input u and integral action. The trajectory starts in the right upper corner of the plot. The solid line is the sliding surface. Because chattering occurs, the zero steady state property is not valid, although that is difficult to see in this plot.

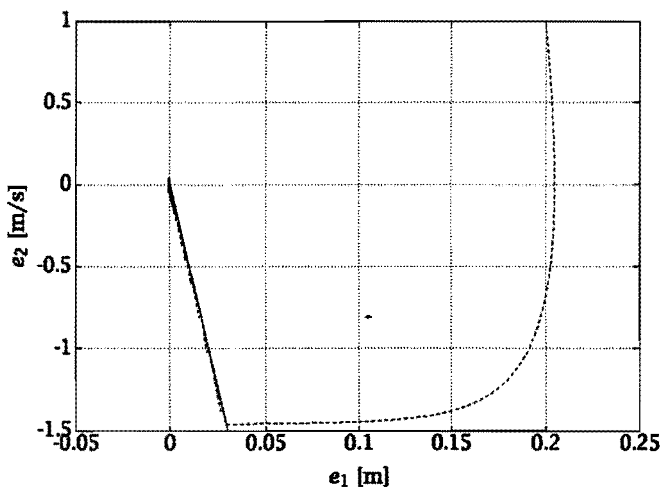

FIGURE 5.49. Phase plot of e for standard controller

Figures 5.50–5.53 give the results when a parallel boundary layer is used, with respectively linear, power law, and fractional interpolation, and the results for cubic error feedback.

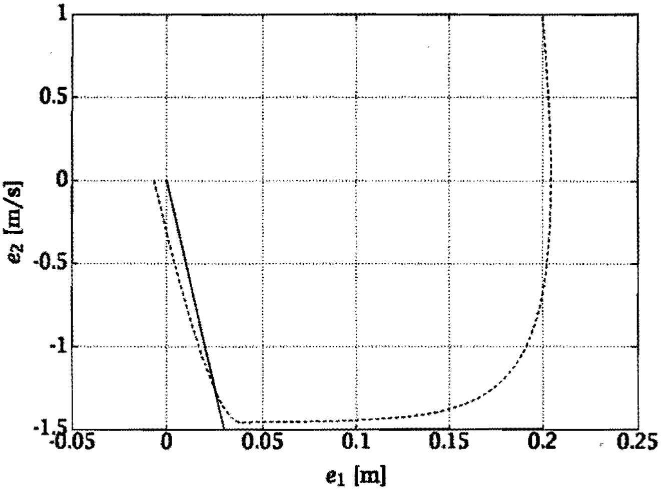

FIGURE 5.50. Phase plot of e for linear interpolation

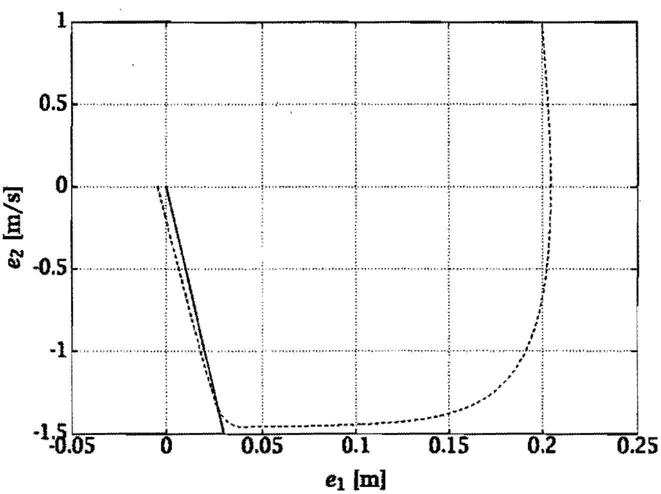

FIGURE 5.51. Phase plot of e for power law interpolation

It is evident from the plots that there is not much difference between the three modifications of the standard sliding mode controller based on boundary layer interpolation. In all cases there is some remaining steady state error, due to the constant disturbance. The steady state error for the

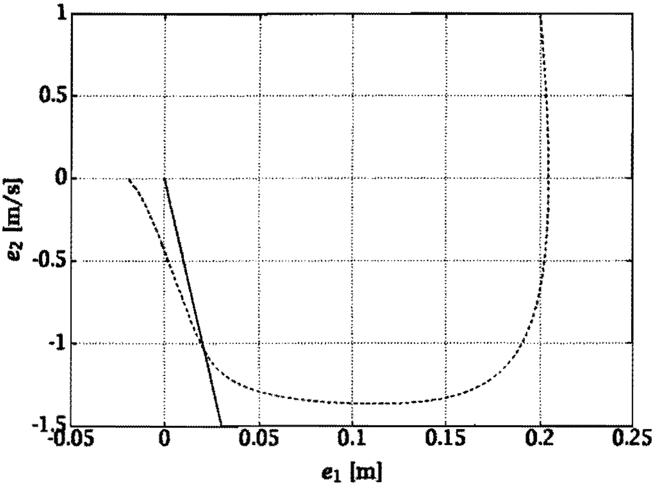

FIGURE 5.52. Phase plot of e for fractional interpolation

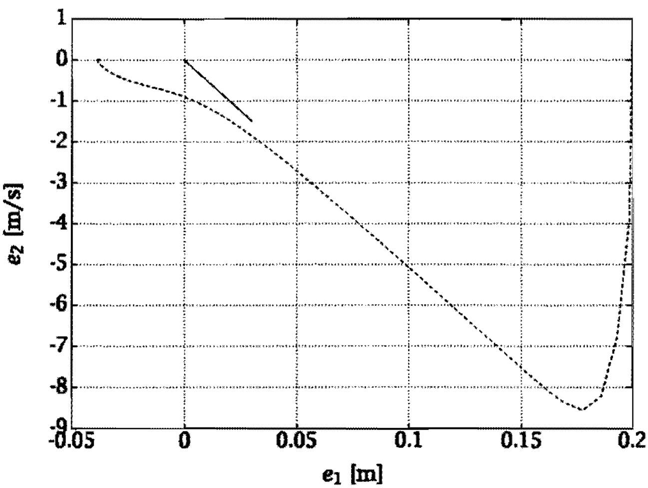

FIGURE 5.53. Phase plot of e for cubic error feedback

power law interpolation is smaller due to a larger gain in the boundary layer. The cubic feedback has some disadvantages. The control input is very large when $s \gg 1$, so the system is forced to the sliding surface with large control authority. When $s \ll 1$ the control action becomes sluggish, leading to a relatively large steady state tracking error. The second order sliding surface approach also gives a large steady state error, according to

Fig. 5.54.

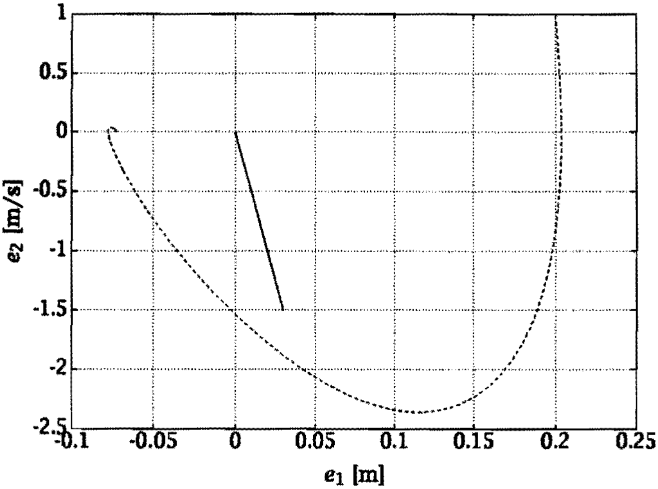

FIGURE 5.54. Phase plot of *e* for second order sliding surface

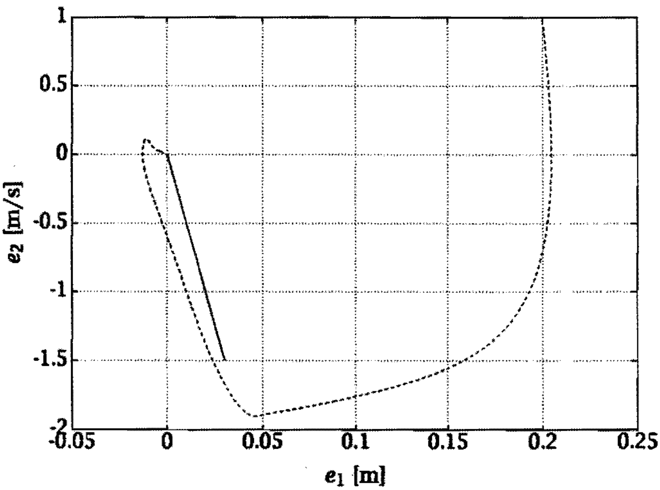

FIGURE 5.55. Phase plot of *e* for linear interpolation with I action

Figures 5.55–5.60 give the results when also integral action is used. Here, none of the three approaches for a parallel boundary layer, in Figs. 5.55-5.57, can distinguish itself by giving uniform better results, the results are similar. The result for the cubic feedback with integral action in Fig. 5.58

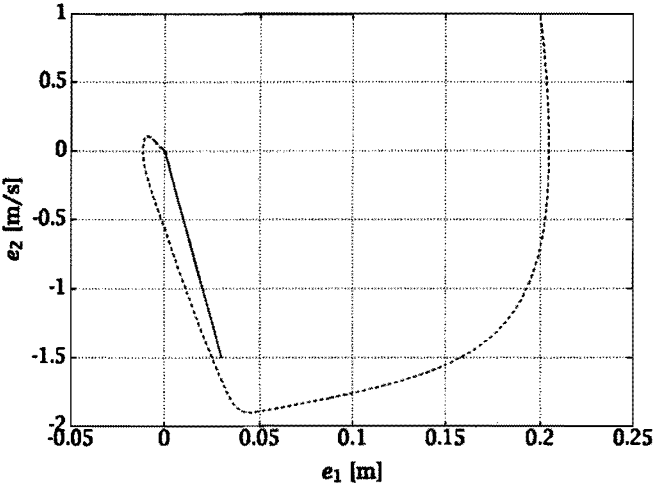

FIGURE 5.56. Phase plot of e for power law interpolation with I action

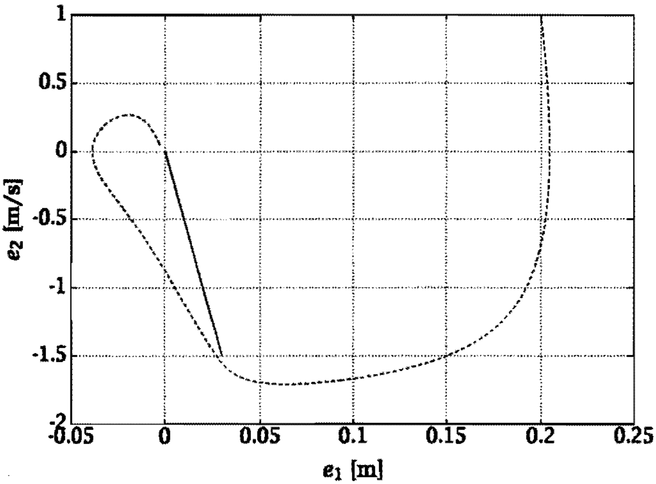

FIGURE 5.57. Phase plot of e for fractional interpolation with I action

shows the same weaknesses as without integration. The steady state error, however, becomes small. In the result for the integral transformation, see Fig. 5.59, a cycling phenomenon occurs. The results given by [83] show the same behavior, and they remedied it by an additional, smaller, boundary layer with linear interpolation. Our example could also benefit from this

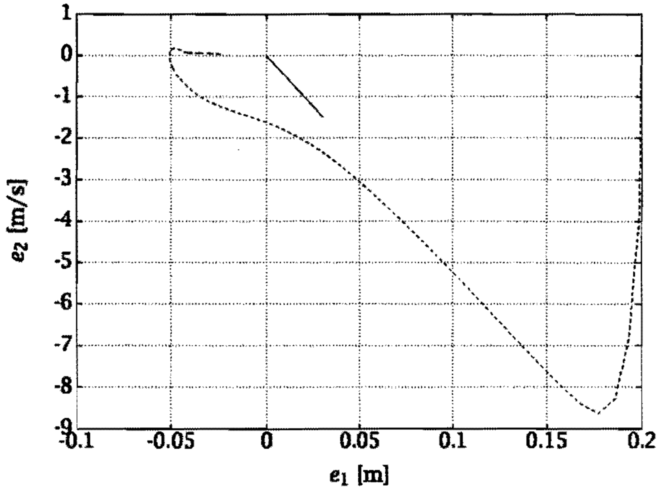

FIGURE 5.58. Phase plot of e for cubic feedback with I action

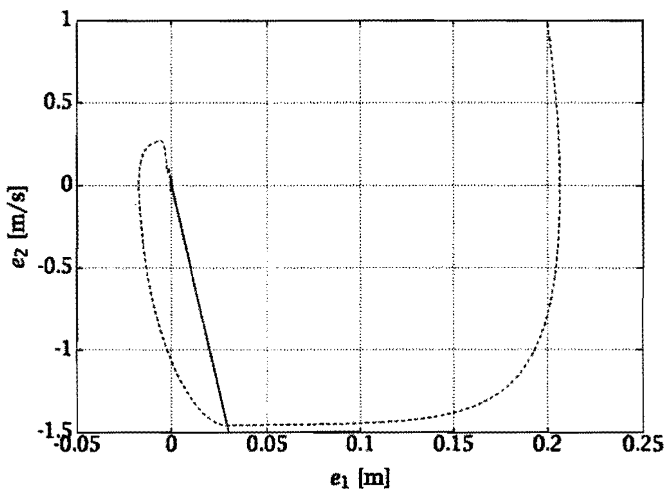

FIGURE 5.59. Phase plot of e for integral transformation

additional modification. Figure 5.60 shows the result for the second order sliding surface controller. It is not very attractive, but the steady state error is removed.

5.5.4. Summarizing remarks. The properties of the control system we emphasize in our summary of the simulation results are the elimination of

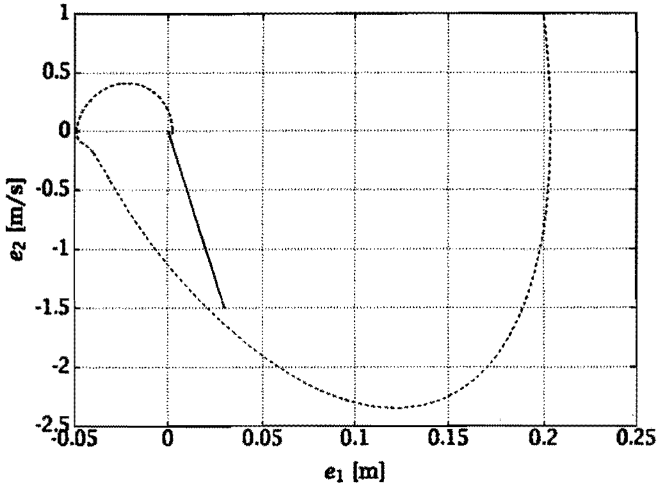

FIGURE 5.60. Phase plot of e for second order sliding surface with I action

chattering and the rejection of unknown persistent disturbances.

From the results presented in Section 5.5.3 the following observations are made:

- all controllers can avoid or diminish chattering,
- with integral action added, they can assure the zero steady state property in the presence of persistent disturbances,
- there is not much difference between the dynamic behavior of the control system using controllers based on a parallel boundary layer modification, the other controllers have some performance disadvantages.

5.6. Discussion of results

We first discuss the results of the four groups of simulations separately. A more general statement follows.

5.6.1. RT-robot position control results. We refer to the remarks made in Section 5.2.5, that are more or less also the conclusions for this part of the simulation study.

5.6.2. RT-robot hybrid control results. The results that were presented in Section 5.3 give rise to the following remarks (the remarks for the H_∞ controller are also valid for the μ-synthesis controller, because they are virtually identical)

- Performance
  - The PD and H_∞ controllers have the best performance with respect to tracking and force error.
  - The robustness of the controllers is comparable. This can be attributed to the fact that the design problem is a SISO problem, so the only potential advantage left for the H_∞ and H_2 controllers are the loop shaping capabilities, which, for this problem could not improve on the PD controller. For the μ-synthesis controller a more detailed error model seems to be necessary to outperform the PD controller.
- Implementation
  - The computational complexity of the PD controller is the lowest, of the H_2 controller the largest. This is related to the controller order squared.
  - The computational complexity of the position and force control loop can be neglected compared with the computed torque and position and force error calculations, which require time consuming trigonometric function evaluations.
- Design
  - The PD controller is the easiest to compute, but with the arrival of control system design (CSD) programs that already incorporate design algorithms for H_2 and H_∞ controller design [161] this is no longer an important issue, except for the μ-synthesis controller design that can be tedious.
  - The PD controller is the easiest to design, only the design frequency ω_0 is used as design parameter. For the H_∞ and H_2 controllers much more effort is needed to select suitable weight functions.

The main conjecture that can be drawn from the computational experiments and results for the RT model hybrid control is that there are no advantages in using H_2, H_∞, or μ-synthesis controllers instead of a PD controller, for the system, unmodeled dynamics, control task, and controller structure investigated. This conjecture probably can be generalized to control problems where the controllers are designed for a decoupled system, resulting in a SISO design problem, and when PD control is equivalent with state-feedback. The number of problems that fits in this class is not negligible.

5.6.3. XY-table position control results. The results for the adaptive controllers are in general the same as for the RT-robot position control. Some additional remarks are related to the influence of unmodeled dynamics, that can be problematic for the parameter adaptation, so switching off the adaptation after some time may even have some advantages.

The VSS controller is able to improve the tracking performance, and is especially effective in combination with adaptive control in the presence of unmodeled dynamics.

The acceleration feedback based computed torque like controller can reduce the tracking error, but is sensitive to measurement noise. Perhaps one of the other approaches mentioned in Section 4.4 can improve this.

5.6.4. Mass-damper-spring position control results. Based on the remarks in Section 5.5.4 our conclusion with respect to the elimination of chattering, achievable performance, and rejection of persistent disturbances is clear: all controllers can reach their claimed goals, the controllers based on a parallel boundary layer perform well and there is no reason to prefer one of the other approaches. Better performance can only be obtained by more rigorous modifications of the basic sliding mode controller, *e.g.*, by the use of second order sliding modes [96], state augmentation [193, 194], and asymptotic observers [195].

5.7. Summary of results

Here we discuss the relative merits of the controller we evaluated by simulations, and decide if some of them should be deleted from the list of controllers to be implemented on the experimental system. Based on the results we obtained we will eliminate some controllers that are less promising.

From the four classes of controllers selected for simulation tests we eliminate three of the controllers in the linearizing state-feedback class. The H_2, H_∞, and μ-synthesis controllers will not be investigated further, because their potential seems not to be better than PD control. The other controllers, (adaptive) computed torque, VSS with parallel boundary layer modification, AF and PD control have passed the simulation sieve, and qualify themselves for the experimental stage. So, we try the final stage, *i.e.*, evaluate these controllers on the experimental system, it can be worthwhile. In the next Chapter the results of this stage are elaborated.

CHAPTER 6

Description and results of XY-table experiments

In this chapter we discuss test bed experiments, using some of the control schemes presented in Chapter 4 that have also been used for simulation experiments in Chapter 5 and did perform well.

Section 6.1 presents the experimental setup. The next section elaborates the Kalman filter implementation, used for velocity estimation. Then the controller design and implementation are discussed in Section 6.3. Section 6.4 shows the results obtained. Finally, Section 6.5 contains a discussion of the results, primarily to compare the controllers.

6.1. Experimental setup

In this section the experimental equipment and the simple model used for the design of the controllers are discussed. In Chapter 5, a model of the XY-table has been introduced, and some characteristics of the system have been presented. To make this chapter more self contained, the description of the model of the XY-table is repeated.

6.1.1. Experimental system. The main specifications for the design of the experimental system are

- low cost,
- easy to access,
- easy to modify the dynamic behavior,
- easy to implement a range of controllers.

The system chosen is a two degrees-of-freedom manipulator, moving in the horizontal plane, with two prismatic joints. It is a so-called TT-robot or, emphasizing the Cartesian coordinates and the horizontal plane in which the end-effector is moving, an XY-table. For a schematic drawing of the XY-table, see Fig. 6.1.

The choice for this system is based on usability, availability, and adaptability. Disadvantages of this choice are that the system is specific, it does not exhibit effects of Coriolis, centrifugal, and gravitational forces. On the

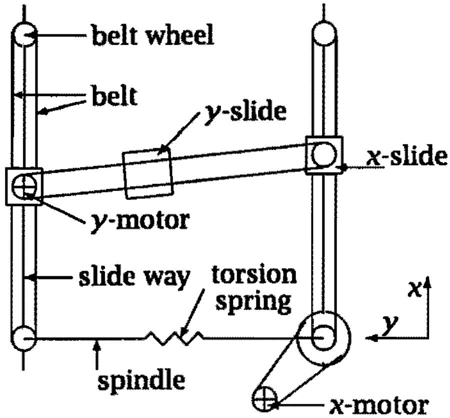

FIGURE 6.1. Schematic drawing of XY-table

other hand, this makes it easier to compare the control schemes, because the design model is simple. Furthermore, the system contains substantial Coulomb friction, which is a major source of tracking errors in mechanical systems. The main characteristics of the system are as follows

- working area 1×1 [m],
- two permanent magnet DC-motors,
- two current amplifiers,
- optical encoders for the motor positions,
- accelerometers fixed to the end-effector,
- operational amplifiers for the acceleration signals,
- laser based measurement system with optical encoders for the end-effector position,
- flexible spindle with adaptable stiffness,
- microcomputer based control.

See Fig. 6.2 for a view of the controller hardware [143]. In this drawing the acceleration measurement subsystem is not included.

6.1.2. Design model. The equations for the simple model of the XY-table of Fig. 6.1 are

$$\begin{aligned} \theta_1 \ddot{x} + \theta_3 \operatorname{sgn} \dot{x} &= f_x \\ \theta_2 \ddot{y} + \theta_4 \operatorname{sgn} \dot{y} &= f_y \end{aligned} \tag{6.1}$$

where x and y are the two prismatic degrees-of-freedom, f_x and f_y the control forces in x and y direction, and θ_i, $i = 1, \dots, 4$, the model parameters: θ_1 and θ_2 are the equivalent masses in x and y direction, θ_3 and θ_4 are the coefficients of the Coulomb friction in x and y direction.

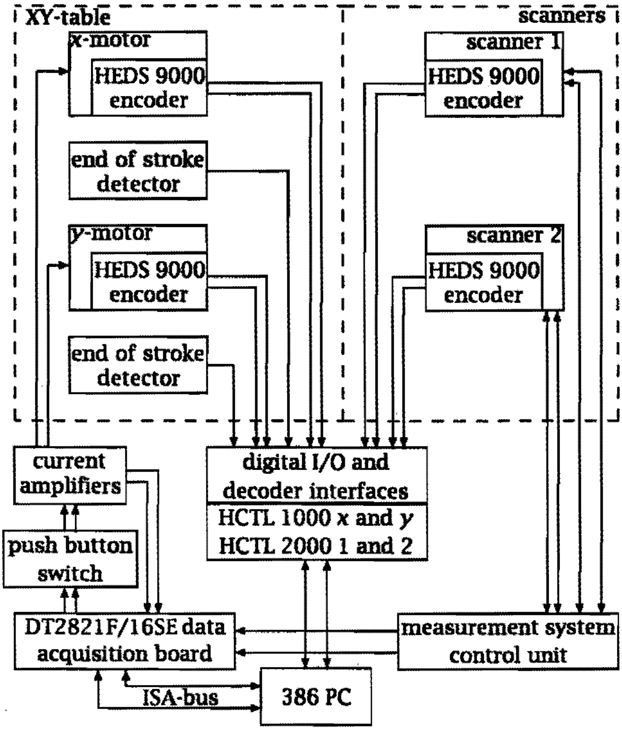

FIGURE 6.2. Schematic drawing of XY-table controller hardware

To rewrite (6.1) as (4.15) define the following quantities:

$$q = \begin{bmatrix} x \\ y \end{bmatrix},$$

$$M(q,\theta) = \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix},$$

$$C(q,\dot{q},\theta) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$g(q,\dot{q},\theta) = \begin{bmatrix} \theta_3 \operatorname{sgn} \dot{q}_1 \\ \theta_4 \operatorname{sgn} \dot{q}_2 \end{bmatrix},$$

$$f = \begin{bmatrix} f_x \\ f_y \end{bmatrix}.$$

Coriolis and centripetal forces are absent, because there is almost no coupling between movements in x and y direction. In the term g gravitational forces are absent because the manipulator moves in the horizontal plane. This results in expressions for Y and Φ in (4.17) and (4.20) as follows

$$Y = \begin{bmatrix} \ddot{x}_r & 0 & \operatorname{sgn}\dot{x} & 0 \\ 0 & \ddot{y}_r & 0 & \operatorname{sgn}\dot{y} \end{bmatrix}$$

and

$$\Phi = \begin{bmatrix} \ddot{x}_d + K_{v_x}\dot{\tilde{x}} + K_{p_x}\tilde{x} & 0 & \operatorname{sgn}\dot{x} & 0 \\ 0 & \ddot{y}_d + K_{v_y}\dot{\tilde{y}} + K_{p_y}\tilde{y} & 0 & \operatorname{sgn}\dot{y} \end{bmatrix}.$$

For the nominal parameter values used in the design computations see Table 6.1.

Parameter	Value	Unit
θ_1	46.5	kg
θ_2	4.3	kg
θ_3	50.0	N
θ_4	15.0	N

TABLE 6.1. Nominal parameters of the design model

To assess the robustness of the control schemes, the XY-table is used in two configurations. One for the design and tuning of the control schemes, the other to assess the robustness. So, a nominal *design system* and an *evaluation system* are introduced. The evaluation system is equivalent to the design system with additional dynamics. This is achieved by changing the stiffness of one of the links of the XY-table drastically. This link is the spindle connecting the two belt wheels, driving the belts for the left and right x-slide. So, the y-slide way does not always line up with the y-axis. With this an additional degree-of-freedom is introduced, that is not accounted for in the model based part of the control schemes.

6.2. Kalman filter (velocity estimate) implementation

As can be seen from the drawing of the control system hardware in Fig. 6.2, there are no provisions in the hardware for velocity measurements. All controllers proposed make use of the state of the model, *i.e.*, position and velocity of the end-effector in x and y direction. Only motor position measurements are available however. Initially, velocity data was acquired by taking finite differences of the position data. This proved to be too inaccurate due to quantization errors of the position encoders. The effects of the quantization errors can be diminished or smoothed by taking finite differences over an interval larger than one sample time, but this introduces an unacceptable phase shift [147].

The velocity error contributed for a large part to the unmodeled dynamics, thereby preventing an assessment of the robustness of the controller caused only by deliberate changes to the system. It was therefore necessary to eliminate, as far as possible, the errors in the computation of the velocity. A Kalman filter was used as observer for this purpose. Because our model is nonlinear due to the friction model, we have to use a nonlinear observer as well. This has some implications for the stability of the closed loop system, see, *e.g.*, [196], because the separation principle is not valid for nonlinear systems. A sliding mode type of observer has been suggested by [197], repeated in [198]. Other types of observers are proposed by [199]. This work has been extended in [200].

The Kalman filter used for estimating the motor velocities (and for filtering the motor positions) was implemented as a discrete one-step-ahead predictor to compensate for the time delay in the control loop. The design was based on system and measurement noise covariances. The statistical properties of the noise signals were derived from the model and measurement errors. The model error was computed by comparing the required input signal for the design model, for which the model output exactly followed a representative response of the experimental system, with the actual input signal applied for this response. The measurement error was derived from the quantization error of the motor position code wheels. During the implementation the Kalman filter was detuned to get a better state estimate. This was due to additional noise signals, not accounted for during the Kalman filter design.

6.3. Controller design and implementation

Several of the controllers, presented in Chapter 4, are implemented in the control system of the XY-table. The design of the controllers is initially based on the design model. The designs are then tuned during initial test runs on the experimental system to get a small tracking error (in a norm based sense) without undesirable fluctuations in the control input due to chattering or small stability margins. A distinction is therefore made between design and tuning. We first discuss the initial design and then the modifications and tuning made to get good performance on the actual system. Finally, in the last part of this section, some implementation issues are discussed.

6.3.1. Controller design. We discuss the design for the control schemes of Slotine/Li and Kelly, already presented in Chapter 4. Using the design model (6.1), while neglecting the Coulomb friction, the design of the PD part of the controllers, *i.e.*, the matrices K_v and K_p, aims at a closed loop system behavior equivalent with a second order system with prespecified eigen frequencies and damping. The desired closed loop transfer function denominator polynomial has the form

$$d(s) = s^2 + 2\beta\omega_0 s + \omega_0^2.$$

The controller parameters are given the values summarized in Table 6.2. The values of β and ω_0 in x and y direction may differ.

Parameter	PD, Slotine/Li	Kelly
K_p	$\begin{bmatrix} \omega_0^2\theta_1 & 0 \\ 0 & \omega_0^2\theta_2 \end{bmatrix}$	$\begin{bmatrix} \omega_0^2 & 0 \\ 0 & \omega_0^2 \end{bmatrix}$
K_v	$\begin{bmatrix} 2\beta\omega_0\theta_1 & 0 \\ 0 & 2\beta\omega_0\theta_2 \end{bmatrix}$	$\begin{bmatrix} 2\beta\omega_0 & 0 \\ 0 & 2\beta\omega_0 \end{bmatrix}$

TABLE 6.2. Design parameters for the controllers

The adaptation gain Γ^{-1} is selected such that the dominant eigenvalues of the closed loop system are those corresponding to the adaptation, the eigenvalues of the second order system are more to the left. In other words, the adaptation should not increase the "bandwidth" of the closed loop system, and the adaptation should be slower than the tracking control loop with fixed parameters.

The additional parameter in the scheme of Kelly, λ, was only tuned and not designed.

REMARK 6.1. When using this design of K_v and K_p, the schemes of Slotine and Li and of Kelly are very alike. Only the adaptation law is different (based on s or v), and the scheme of Kelly has the additional compensation term $\hat{C}v$ instead of $\hat{C}s$, but this is irrelevant in our case because $C = 0$. Using the design parameters $K_v = M\tilde{K}_v$ and $K_p = M\tilde{K}_p$ the generalized control force for the control scheme of Slotine and Li can be written as

$$f = \hat{M}(q)(\ddot{q}_r + \tilde{K}_v\dot{\tilde{q}} + \tilde{K}_p\tilde{q}) + \hat{C}(q,\dot{q})\dot{q}_r + \hat{g}(q,\dot{q})$$

where we assumed $\hat{M} = M$. Using $\ddot{q}_r = \ddot{q}_d + \Lambda\dot{\tilde{q}}$ this yields

$$f = \hat{M}(q)(\ddot{q}_d + (\tilde{K}_v + \Lambda)\dot{\tilde{q}} + \tilde{K}_p\tilde{q}) + \hat{C}(q,\dot{q})\dot{q}_r + \hat{g}(q,\dot{q}).$$

This again is equal to

$$f = \hat{M}(q)(\ddot{q}_d + (\tilde{K}_v + \Lambda)\dot{\tilde{q}} + \tilde{K}_p\tilde{q}) + \hat{C}(q,\dot{q})\dot{q} + \hat{g}(q,\dot{q}) + \hat{C}(q,\dot{q})s$$

because $\dot{q}_r = \dot{q} + s$. This expression only differs from the control scheme of Kelly (4.19) by the term $\hat{M}\Lambda\dot{\tilde{q}}$ in the computed torque part and by using $\hat{C}s$ instead of $\hat{C}v$ in the compensation part. But $\Lambda = K_v^{-1}K_p < \tilde{K}_v$ with a choice of $\beta > \frac{1}{2}$, as can be easily verified. The two control schemes differ therefore mainly in their definition of the measure of tracking accuracy s and v respectively. Because here $C = 0$, the only consequence of this definition is in the adaptation laws (4.18) and (4.21), driven by s in the control scheme of Slotine and Li, and by v in that of Kelly. Also, the expressions for Y and Φ are not equal.

6.3.2. Controller tuning. The tuning of the controllers should give specific values for the controller parameters β, ω_0, Γ^{-1}, and λ. It was necessary to estimate the motor speed because no tachometers were used. Friction compensation was also necessary.

The values of β, ω_0, Γ^{-1}, and λ may depend on the actual configuration of the XY-table (stiff or flexible link) or on the desired trajectory. They will be given in the experimental results section.

A Kalman filter, as described in Section 6.2, was applied.

Friction compensation was implemented using the desired velocity $\dot{q}_d$, and not the velocity estimate of the Kalman filter, to prevent problems when the velocity changes sign, but the estimated velocity did not yet, or vice versa. This also improves the convergence of the tracking error [107]. Further aspects of the friction compensation are discussed in Chapter 7.

6.3.3. Controller implementation issues. The two main issues of the controller implementation are

- the limited sampling rate,
- the discrete time nature of the controllers.

The difference in the type of controller that is used in the theoretical derivation of stability properties and for the initial design (a continuous time type) and the type of controller implemented on the available hardware (a discrete time type) can be regarded as an additional source of unmodeled dynamics. By choosing the sample time low (or equivalently the sampling rate high) the contribution of this type of model error can be made arbitrary small. Unfortunately, the computing speed of the hardware is not infinite, so during most experiments a sample time $t_s = 0.007$ [s] has been used. With this sample time, a marked influence of the discrete time nature of the controller is evident. When a smaller sample time is chosen, a lower tracking error can be realized.

Besides the discrete time nature of the controller, also the time delay incurred between the measurements and the actual application of the computed torque has a marked influence. This influence is diminished by the use of a predictor type of Kalman filter. However, because of the prediction step in the filter, the filter estimation error is relatively large. So, prediction of states is inevitable coupled with larger state estimation error. The use of prediction type filters is therefore no panacea to solve the time delay incurred loss of tracking performance.

To substantiate our views expressed above, simulation and experimental results for some representative cases are presented in Chapter 7.

6.4. Experimental Results

This section gives the results of the experiments. We present only a subset of the results obtained, enough to substantiate our main points. First we discuss the control task. Then the results of a comparison between the two adaptive control schemes are given and discussed, the implications of VSS control and acceleration feedback are presented, followed by the results of the robustness investigation.

6.4.1. Control task. The control task is to follow a periodic trajectory in a plane, with position control in both coordinate directions.

The desired trajectory in Cartesian end-effector space is

$$\begin{bmatrix} x_d(t) \\ y_d(t) \end{bmatrix} = \begin{bmatrix} a - R_d \cos \psi_d \\ b - R_d \cos(\psi_d + \psi_0) \end{bmatrix}$$

where $R_d = 0.15$ [m] is the "radius" of the trajectory, $\psi_d = \omega t$, with $\omega = \frac{\pi}{2}$ [rad/s], is the desired angular position, and $a = 0.8$ [m], $b = 0.8$ [m] specify the center of the working area of the manipulator, see Fig. 6.3. The constant angle ψ_0 is used to select the trajectory: if $\psi_0 = \frac{\pi}{2}$ the trajectory is a circle, if ψ_0 has another value the circle is deformed to an ellipse or even a straight (diagonal) line.

The periodic nature of the task makes it easy to compute accurate and repeatable tracking error statistics, without influence of initial transients.

REMARK 6.2. Compared with the simulations in Section 5.4 the desired trajectory is more than twice as slow. Here, we track the circle in 4 [s] instead of in 3.5/2 [s]. Also the radius of the circle is 0.15 [m] instead of 0.2 [m]. This makes it possible to have a large margin before the motors saturate. A phenomenon that is present in practice, but is not accounted for in the

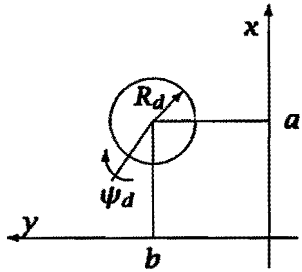

FIGURE 6.3. Desired trajectory

model and simulations, and is to be avoided, to prevent a hodge-podge of unmodeled dynamics.

6.4.2. Results for adaptive controllers. The control schemes of Slotine/Li and Kelly are compared by establishing the tracking errors of the XY-table with a stiff link for a circular trajectory. The controller tuning resulted in the controller parameters in Table 6.3.

Parameters	PD	Slotine/Li	Kelly
ω_0	$4 \cdot 2\pi$	$4 \cdot 2\pi$	$4 \cdot 2\pi$
β	0.7	0.7	0.7
Γ^{-1}	-	$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 10^{3} & \\ & & & 10^{2} \end{bmatrix}$	$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 10^{3} & \\ & & & 10^{2} \end{bmatrix}$
λ	-	-	$\begin{bmatrix} 5 & \\ & 5 \end{bmatrix}$

TABLE 6.3. Controller parameters for the stiff XY-table

For $\omega_0 = 5 \cdot 2\pi$ chattering occurred. The results presented below were obtained with initial values for the parameter estimates equal to those in Table 6.1.

Figure 6.4 presents the tracking error in x-direction. The largest errors occur for $t \approx 0$ and are due to stiction effects and incorrect initial conditions of the controllers. Neglecting these effects, the largest error for both schemes is ≈ 0.7 [mm]. Figure 6.5 gives the tracking error in y-direction. This error is much larger, due to larger inaccuracies in the model, especially in the Coulomb friction model. The largest error for Slotine/Li is ≈ 7 [mm] and for Kelly ≈ 10 [mm]. When we neglect the first 0.5 [s] of the responses

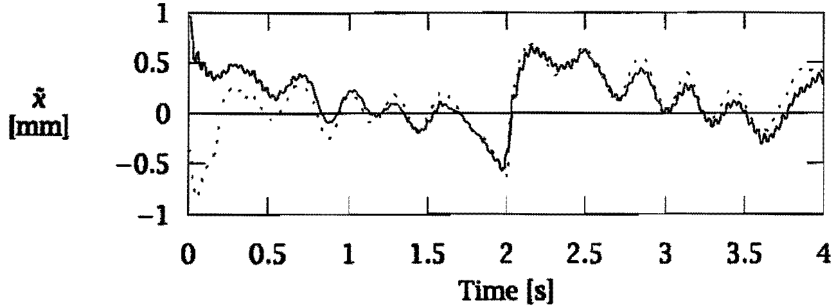

FIGURE 6.4. Error in x-direction, (–) Slotine/Li, (· · ·) Kelly

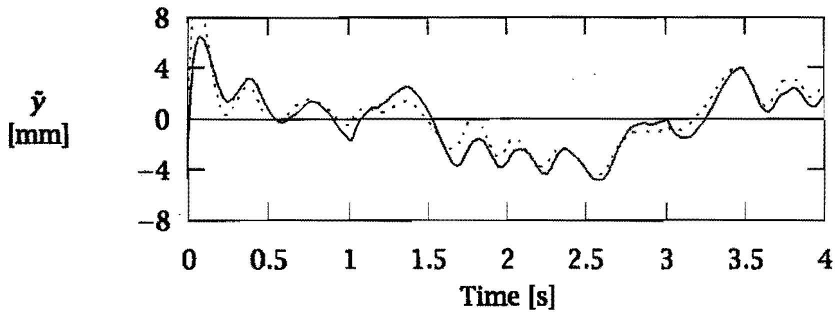

FIGURE 6.5. Error in y-direction, (–) Slotine/Li, (· · ·) Kelly

the controller of Kelly performs better: 4 [mm] maximum tracking error against 4.5 [mm] for Slotine/Li.

Hence, for the stiff system the controller of Kelly performs somewhat better than the controller of Slotine/Li. This can be attributed to a better parameter estimate (results not presented). The results for the PD controller are also not presented, but are much worse. The tracking errors are several times as large. By using friction compensation the difference in the tracking errors can be reduced by approximately a factor of 2.

REMARK 6.3. The measurements of the motor positions are used to compute a corresponding end-effector position. All results are presented in terms of this "fictive" computed end-effector position. Due to flexibility in the links and joints, the actual end-effector position does not coincide with the computed one. This discrepancy will be larger for the flexible system. For the control system the use of motor positions is OK, because they are readily available and do not endanger the stability, but for the evaluation the end-effector positions are needed, especially for the flexible system. Therefore only a limited number of those results are presented.

6.4.3. Results for adaptive controller with VSS. Due to model errors the adaptive controllers cannot be expected to give zero tracking errors. Also the parameters are unlikely to converge. Especially the Coulomb friction cannot be accurately represented by the model (5.3). Furthermore, there is some viscous friction, not incorporated in the design model used in the adaptive controllers. Therefore the tracking error cannot be made very small, especially in y-direction. To remedy this, a sliding mode component is added to the controller. It should correct the effects of the imperfect cancellation of the nonlinearities in the system.

Only the original version of the sliding mode controller (5.12) and the linear interpolation in a parallel boundary layer modification (4.22) are implemented. The control parameters for the adaptive control part of the VSS controller are the parameters used in the experiments of the adaptive controller of Slotine/Li, see Table 6.3. A value of λ corresponding with 20% of the allowable torque was used. Larger values for λ forced a larger σ to avoid chattering, so the corresponding gain in the boundary layer would hardly change. A value of $\sigma = 40$ [mm/s] was necessary to avoid chattering. This is twice as large as the value used in the simulations in Section 5.4.3, see Table 5.6, an indication of the lack of verisimilitude of the simulation model. For the experiment without interpolation a value of $\sigma = 0$ was used. Chattering in this case was clearly present, see Fig. 6.6.

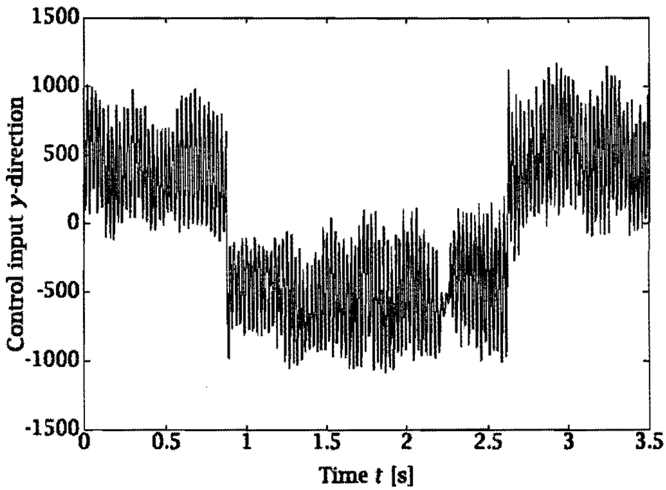

FIGURE 6.6. Experimental results, input signal u with VSS

No integral action was required, because persistent disturbances or biased Coulomb friction coefficients were not present. For the tracking error in

y-direction results see Fig. 6.7.

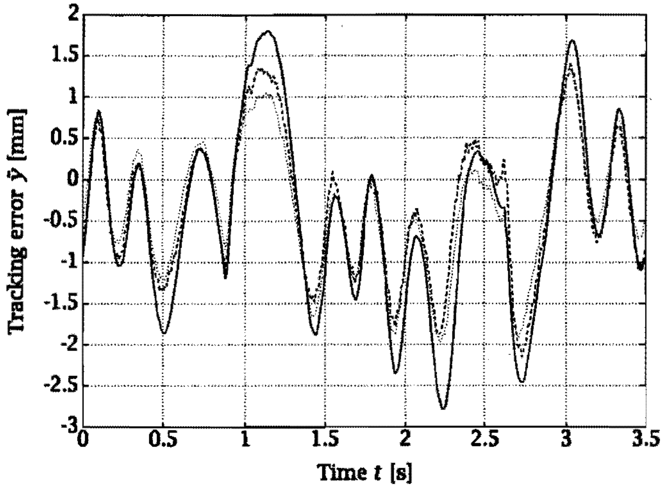

FIGURE 6.7. Experimental results, y-direction, (—) no VSS, (- -) VSS, (· · ·) VSS with interpolation

From these results it is clear that an additional sliding mode component improves the tracking accuracy. The use of boundary layer interpolation has negligible influence on the tracking error, but attenuates chattering. It is therefore desirable and profitable to include smoothing of the control input in applications.

The results in Fig 6.7 cannot be compared with the results in the previous section, *e.g.*, Fig. 6.5, because the XY-table has undergone extensive modifications between the experimental runs. The reference trajectory is also different. Here, two complete circles are traversed within 3.5 [s].

6.4.4. Results for acceleration feedback based controller. To verify our findings with the simulation of the acceleration feedback controller, see Section 5.4.3, some experiments have been performed. The acceleration feedback was appended to the adaptive controller of Slotine and Li, but with parameter adaptation disabled, to avoid eliminating the parameter error. The controller parameters were chosen to obtain a bandwidth of the controlled system of 3 [Hz] in x-direction, and of 4 [Hz] in y-direction. In both cases a damping factor $\beta = 0.7$ was chosen. As gain for the acceleration feedback $\alpha = 0.4$ was selected. This values is approximately optimal, it gives the largest reduction of the tracking error. The acceleration signal was processed by 2 presampling low pass filters, the first one is a first order filter with a cut-off frequency of 40 [Hz], the second one a Butterworth filter with 10 [Hz] bandwidth. For the results see Figs. 6.8–6.9.

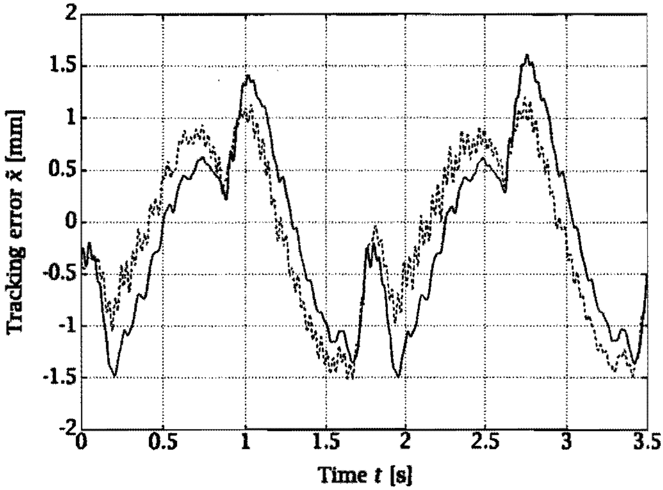

FIGURE 6.8. Experimental results, x-direction, (—) no AF, (- -) AF

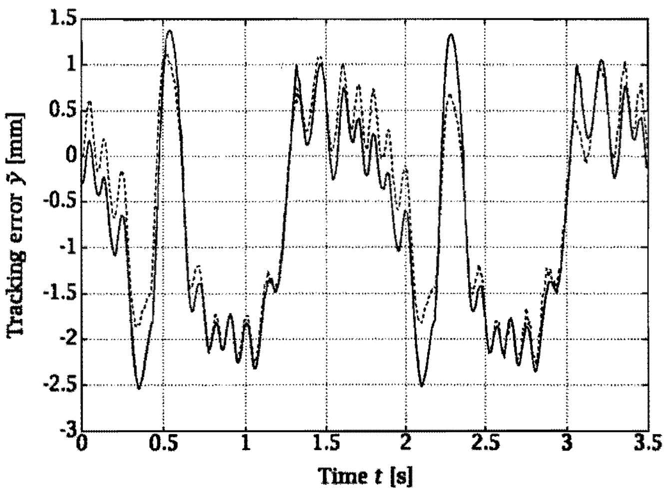

FIGURE 6.9. Experimental results, y-direction, (—) no AF, (- -) AF

The results show that a slight improvement has been obtained. The difference in performance is certainly not spectacular, so the use of acceleration feedback is, because it requires extensive signal processing to obtain a noise free signal, although the sensor is relatively cheap, questionable for applications. As noted before, perhaps one of the other approaches for acceleration feedback can improve the results.

6.4.5. Robustness for additional dynamics. We asses the robustness of the adaptive control schemes by using them to control the flexible XY-table and base our conclusions on

- the degree of detuning the controllers used for the stiff XY-table, to avoid chattering,
- the tracking errors of the controllers.

Due to the fact that the end-effector positions are not available, the results presented are in term of motor positions errors. They can only be used for comparisons and are of limited value for other purposes, *e.g.*, to assess the ultimate tracking accuracy of the end-effector, if significant unmodeled dynamics is present.

Only the adaptive controller results are presented, because the simulations showed that they are more sensitive to unmodeled dynamics than the other controllers, especially the adaptation component. Again, a PD controller is used as reference.

To avoid chattering the design parameter ω_0 for the x-direction had to be changed. The other parameters remain the same. See Table 6.4 for the parameters.

Parameter	PD	Slotine/Li	Kelly
ω_{0_x}	$3.5 \cdot 2\pi$	$2.5 \cdot 2\pi$	$3.5 \cdot 2\pi$
ω_{0_y}	$4 \cdot 2\pi$	$4 \cdot 2\pi$	$4 \cdot 2\pi$
β	0.7	0.7	0.7
Γ^{-1}	-	$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 10^{3} & \\ & & & 10^{2} \end{bmatrix}$	$\begin{bmatrix} 10^{-3} & & & \\ & 10^{-4} & & \\ & & 10^{3} & \\ & & & 10^{2} \end{bmatrix}$
λ	-	-	$\begin{bmatrix} 5 & \\ & 5 \end{bmatrix}$

TABLE 6.4. Controller parameters for the flexible XY-table

The Slotine/Li controller had to be detuned more than the Kelly and PD controller. This will result in larger tracking errors because the control is less tight. Also, this control scheme can be called less robust then the Kelly and PD scheme, because it is influenced more by the additional dynamics.

The tracking errors for the Slotine/Li and Kelly controllers are presented in Figs. 6.10 and 6.11, and the tracking errors for the Slotine/Li and PD controllers are in Figs. 6.12 and 6.13.

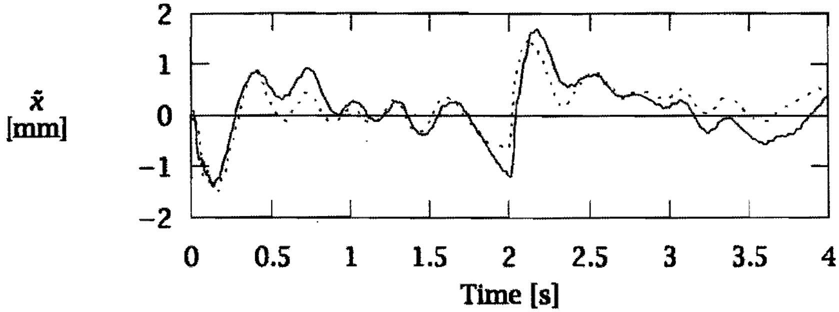

FIGURE 6.10. Error in x-direction, (–) Slotine/Li, (· · ·) Kelly

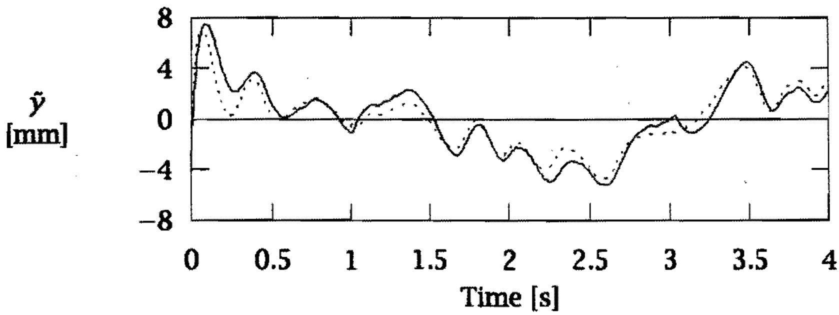

FIGURE 6.11. Error in y-direction, (–) Slotine/Li, (· · ·) Kelly

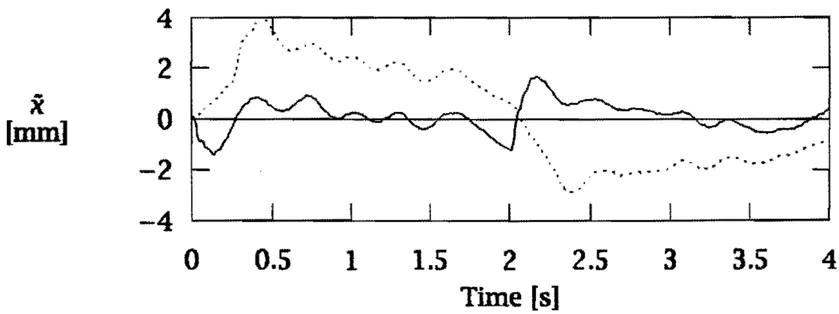

FIGURE 6.12. Error in x-direction, (–) Slotine/Li, (· · ·) PD

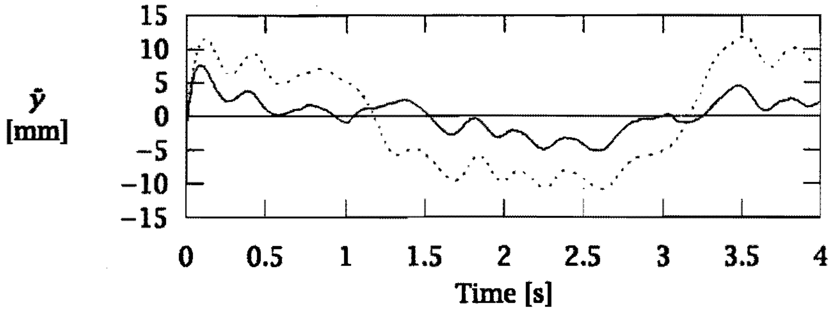

FIGURE 6.13. Error in y-direction, (–) Slotine/Li, (· · ·) PD

The maximum error in x-direction for Slotine/Li is ≈ 1.7 [mm] and is larger than the maximal error for Kelly, ≈ 1.5 [mm]. Both errors are more than 2 times as large as for the stiff XY-table. The error in y-direction is only marginally larger. It is evident from the tracking error for the PD controller (without friction compensation) that the performance of the PD controller is worse compared with the adaptive controllers. However, the PD controller performance changes only by a factor 1.3 (30% increase) between the stiff and flexible system. The change in absolute values is slightly larger.

Using both criteria mentioned above, we conclude that the controller of Slotine/Li is not as robust as the controller of Kelly or a PD controller. The robustness of the last two controllers is comparable. The deterioration of the performance of the Slotine/Li controller is due to the parameter estimation, presented in Figs. 6.14–6.17.

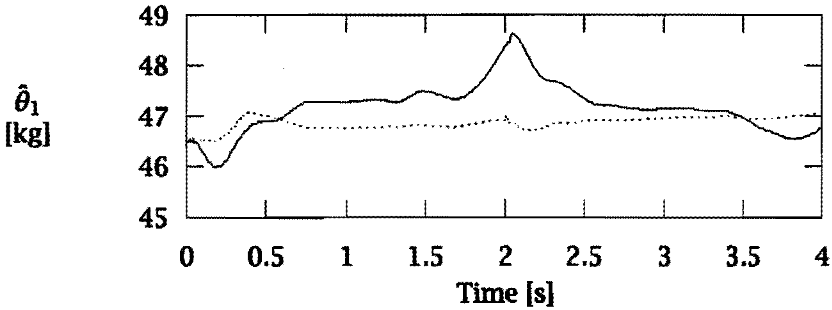

FIGURE 6.14. Estimate of θ_1, (–) Slotine/Li, (· · ·) Kelly

The dynamics of the parameter estimates is quite different for both controllers. The estimates of Slotine/Li are more fluctuating than those of Kelly. Although it appears in Fig. 6.17 that the parameter $\hat{\theta}_4$, estimated by the controller of Slotine/Li, is drifting, this figure only shows a short time frame. As remarked in the presentation of the control schemes, drift-

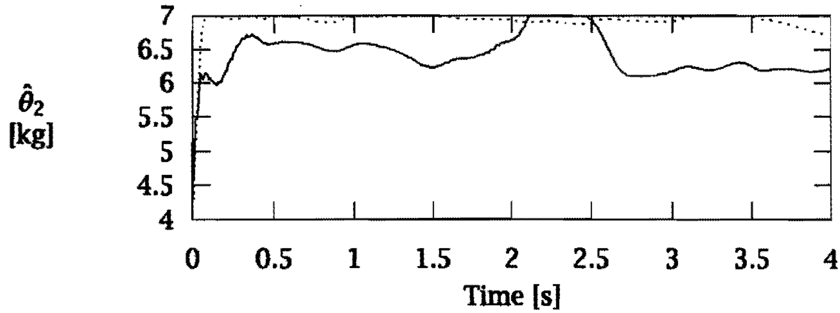

FIGURE 6.15. Estimate of θ_2, (–) Slotine/Li, (· · ·) Kelly

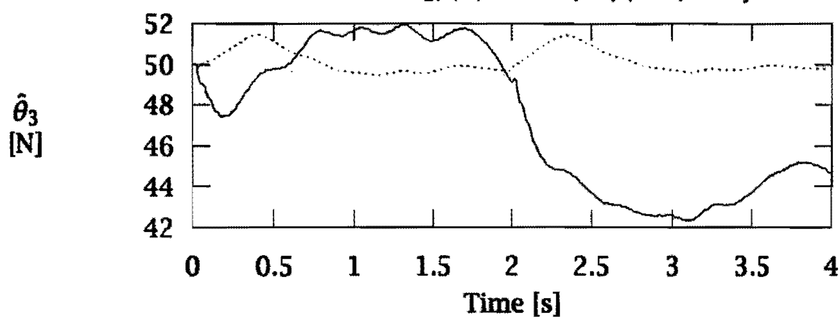

FIGURE 6.16. Estimate of θ_3, (–) Slotine/Li, (· · ·) Kelly

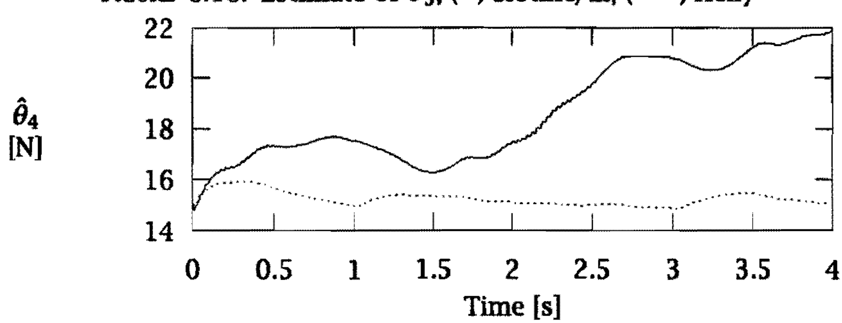

FIGURE 6.17. Estimate of θ_4, (–) Slotine/Li, (· · ·) Kelly

ing has been observed when the reference trajectory is not persistently exciting, *e.g.*, for a regulator task. This is not so in our case. Also, the same phenomenon is not always observed in other experiments. Anyway, even when the parameter is drifting, it will hit the upper bound allowed for this parameter at ≈150% of the nominal value.

REMARK 6.4. The estimated parameters bear no relation to the physical parameters. The adaptation tries to compensate for the incorrect model structure. The parameters are given a value depending on s or v, according to (4.17) and (4.20). Because s and v do not become zero, the parameter estimates do not converge. As can be seen in Fig. 6.15, the parameter estimates are also limited to $\approx$150% of the nominal parameter values. This was one of the ad-hoc remedies, mentioned in Section 2.3.3.3, to improve the robustness of adaptive controllers.

REMARK 6.5. The estimates of the Coulomb friction parameters also change with time and the state of maintenance of the XY-table, so they may differ considerably from the values in Table 6.1.

6.5. Discussion of results

In this discussion we compare the control schemes implemented on the XY-table, *i.e.*, the adaptive computed torque like, the VSS, and the AF controllers.

The main lines are that the results of the experiments coincide with the results of the simulation study.

The adaptive controllers can reduce the tracking error considerably compared with a PD controller, but are sensitive to unmodeled dynamics. The VSS controller combined with the adaptive controller can further reduce the tracking error. The AF controller can also improve the tracking accuracy, but has some implementation disadvantages, and should perhaps only be used when an acceleration signal is available for other reasons.

Our final qualification is that our findings indicate a slight preference for an adaptive controller combined with VSS control. Of both adaptive controllers, the controller of Slotine and Li is preferred because it is easier to design and tune, and a possible disadvantage with respect to sensitivity for unmodeled dynamics is compensated by the VSS component.

Of course, we have to refuse to commit a stroke in a generalization of this qualification to other mechanical systems. The class of stiff mechanical manipulators is, however, nicely covered by our recommended controllers, although the results are obtained with a system of relatively low dimension.

CHAPTER 7

Measures to improve the control quality

We discuss two aspects of the controller implementation that can help to improve the performance of the control system. First, the influence of friction compensation. Second, the relation between sampling rate or prediction type of the Kalman filter and performance.

7.1. Friction compensation

The main cause of nonlinearity and model errors for the XY-table, with a stiff spring in the x-transmission, is friction. It is therefore necessary to accurately compensate friction to improve the tracking performance. In this section we discuss the friction phenomenon, give an extended friction model and present simulation and experimental results that show how extended friction compensation can improve the performance, see also [201]. All results are obtained for the XY-table.

7.1.1. Introduction. The presence of friction in mechanical systems, where material parts move relative to each other and contact is necessary due to a guiding or bearing function of the parts, is unavoidable. It is not always possible to eliminate friction by using advanced tribological measures. When traditional techniques to eliminate backlash are used, the problem of friction becomes even more pronounced. In general, friction is a limiting factor for the tracking performance of mechanical control systems.

There are several ways to overcome the effects of friction

- the use of high gain feedback, but this has disadvantages, such as large input signals and no robust performance due to excitation of high frequency unmodeled dynamics,
- the use of additional dither signals, that prevent the system from stiction,
- compensation of friction by the controller; the accuracy of the compensation largely depends on the correctness of the structure of the friction model used for the compensation and on an accurate knowledge of the friction model parameters.

The main problem is the formulation of accurate friction models. These models are difficult to obtain, due to the complexity of friction phenomena. Even the physical causes of friction are not well understood [202]. One approach is to perform some measurements on the system in question and deduce an indication of the structure of the equations describing the effects of friction. Some experiments in this direction are performed [203], but the conclusions with respect to the structure of the friction model are closely related to the system investigated and can hardly be generalized.

Another approach, chosen in this work, is to use an elaborate friction model, and to adapt the parameters of the model. When some terms in the model are not significant, the corresponding parameters will be small. After an initial period of use, the structure of the friction model can be simplified by deleting terms that are related with small parameters (*i.e.*, insignificant terms) or have parameters of equal value, *e.g.*, for direction dependent parameters. It is necessary to use a sufficiently rich model to encompass all effects that can appear and are related to friction. Yet, the number of parameters should not be too large, to avoid problems with the adaptation (overparametrization) and to avoid modeling of disturbances that are not related to friction.

Adaptive friction compensation has been used by [127, 128, 204–206], but they use relatively simple friction models.

7.1.2. Friction model. It can be deduced from the characteristics of the tracking error in y-direction, that there is an harmonic disturbance force, which is the cause of the lower tracking accuracy for the y-direction. This difference in accuracy is completely in contradiction with common expectations. The transmission of the torque from the motor to the end-effector in y-direction is much simpler than in x-direction and therefore the model in y-direction could be assumed to be much more accurate than in x-direction.

The reason for this discrepancy between reality and expectation can be deduced from the harmonic nature of the disturbance force. The period of the force fluctuation is equivalent to the time needed for one complete revolution of the y-motor, and so of its shaft, bearings, and belt wheel. Therefore, it seems logical to assume that the disturbance force stems from some imperfections and friction in the shaft and bearings. Another possible explanation could be the presence of imperfections in the electro-magnetic fields in the motor due to, *e.g.*, a lack of rotational uniformness or symmetry, leading to an inhomogeneous magnetical or electrical field. If the motor is brush-less, the motor constant (relating motor current to torque) is position dependent [207], so current control cannot be identified with torque control and the result is a periodic torque ripple for constant current. In this case we are not modeling friction, but state dependent disturbances.

The use of a reduction in the transmission for the x-motor alleviates these effects for the x-direction.

A solution for the periodic friction would be to eliminate it by replacing the shaft and bearings, but, incidentally, it provides a source of model error, which does not endanger the stability, but significantly reduces the performance. None of the control schemes used can cope directly with this type of disturbance, except by using larger gains in the PD part of the schemes, but those large gains do endanger the stability and can therefore not be applied in practice.

Another solution is canceling the disturbance force by compensation. This can be regarded as an extension of standard Coulomb friction compensation, it just requires an extended friction model.

The appearance of periodic or position dependent friction components has been observed previously and is reported by, *e.g.*, [203], but for their system the harmonic friction component was small, in the order of 7% of the Coulomb friction. In our case this is not true, so we should explicitly consider periodic friction.

When the compensation is based on the angular position $\omega_p q$ of the shaft, only the amplitude b_p and phase ϕ_p of the sinusoidal compensation force has to be determined. When adaptive controllers are used, one could try to use adaptive friction compensation [128, 204, 206] by estimating amplitude and phase. However, when the compensating force is of the form

$$f_p = b_p \sin(\omega_p q + \phi_p)$$

the parameter ϕ_p does not appear linear in the control force, which is required for the adaptation part of the controller. The angular frequency ω_p is assumed to be known to avoid the same problem. Fiddling with the phase to get a small error is possible, but tedious and should be repeated for each arrangement of belt wheels and belt, and must be repeated every time the connection between motor, belt wheel and end-effector is changed, *e.g.*, a belt change or even re-attachment of the belt, and in general as a result of maintenance works. So, a much better solution is to incorporate the adaptation of the phase in the control scheme. For this purpose, write the previous expression for f_p as

$$f_p = a_{p_1} \sin(\omega_p q) + a_{p_2} \cos(\omega_p q)$$

and now the two amplitudes $a_{p_1} = b_p \cos(\phi_p)$ and $a_{p_2} = b_p \sin(\phi_p)$ and no phase has to be adapted. Both parameters appear linear in the control force. A disadvantage of this method is that both sine and cosine have to be computed, resulting in a slightly longer computation time.

In this section we will use a slightly extended model of the XY-table instead of (6.1)

$$
\begin{aligned}
\theta_1 \ddot{x} + g_x(x, \dot{x}) &= f_x \\
\theta_2 \ddot{y} + g_y(y, \dot{y}) &= f_y
\end{aligned} \tag{7.1}
$$

where g_x and g_y are disturbance forces due to Coulomb, viscous, and other types of friction or due to other state dependent disturbances. The absence of Coriolis, centripetal, and gravitational forces makes the XY-table an ideal object for the study of the merits of friction compensation.

Including Coulomb, viscous, and periodic friction in the model, we obtain for the friction force g

$$
\begin{aligned}
g(q, \dot{q}) &= a_c^+ \operatorname{sgn} \dot{q} + a_v^+ \dot{q} + a_{p_1}^+ \sin(\omega_p q) + a_{p_2}^+ \cos(\omega_p q) \quad \text{for } \dot{q} \ge 0 \\
g(q, \dot{q}) &= a_c^- \operatorname{sgn} \dot{q} + a_v^- \dot{q} + a_{p_1}^- \sin(\omega_p q) + a_{p_2}^- \cos(\omega_p q) \quad \text{for } \dot{q} < 0
\end{aligned} \tag{7.2}
$$

where we assume that all parameters in the friction model are direction dependent.

7.1.3. Controller. We use the controller given by (4.16), *i.e.*, the adaptive controller proposed by Slotine and Li and recommended in Chapter 6. We now apply this control scheme to the extended model of the XY-table (7.1). The expressions for M, C, and f are the same as in Section 5.1.2. Only g is changed

$$
g(q, \dot{q}, \theta) = \begin{cases} g_x(q_1, \dot{q}_1, \theta_i) & \text{for } i = 3, \ldots, 3 + n_{g_x} \\ g_y(q_2, \dot{q}_2, \theta_i) & \text{for } i = 4 + n_{g_x}, \ldots, 4 + n_{g_x} + n_{g_y}. \end{cases}
$$

Here, the parameters $\theta_i, i > 2$, correspond in an obvious way to the parameters a^+, a^- in (7.2).

This results in expressions for Y in (4.17) as follows

$$
Y^+ = \begin{bmatrix} \ddot{x}_r & 0 & \operatorname{sgn} \dot{x} & \dot{x} & \sin \omega_p x & \cos \omega_p x & 0 & 0 & 0 & 0 \\ 0 & \ddot{y}_r & 0 & 0 & 0 & 0 & \operatorname{sgn} \dot{y} & \dot{y} & \sin \omega_p y & \cos \omega_p y \end{bmatrix}
$$

for positive velocities, used for adaptation of the θ^+ parameters and an equivalent expression Y^- for negative velocities to adapt the θ^- parameters. The parameters $\theta^\pm$ are ordered as $\theta^+ = \theta_1, \theta_2, a_x^+, a_y^+$ and similar for $\theta^- = \theta_1, \theta_2, a_x^-, a_y^-$, but, of course, the velocities for x and y direction change sign independent of each other.

The design of the control parameters K_v and Λ is performed by choosing a favorable dynamics of the tracking error, characterized by the undamped characteristic frequency f_c and damping coefficient β_c of a second order system. The goal was to get a small tracking error without exciting high frequency dynamics that could endanger stability. The selection of Γ^{-1} was guided by the rule given in [128], but the gains had to be detuned to avoid stability problems.

Some preliminary simulations were performed to verify the viability of using an extended friction model and adapting the parameters. For the nominal parameter values used in the model of the XY-table and used for the controller design, see Table 7.1. Parameter a_n is the maximum amplitude of a band limited pseudo white noise disturbance force used to model torque ripple and other random disturbances. The value of a_v is by accident much higher than in practice.

Parameter	Value x	Value y	Unit
θ_1, θ_2	46.5	4.3	kg
$a_c^+ = a_c^-$	45.0	12.5	N
$a_v^+ = a_v^-$	6.0	10.0	N s m^{-1}
$b_p^+ = b_p^-$	12.5	3.5	N
ω_p	1/9.7	1/10.5	rad mm^{-1}
ϕ_p^+/ω_p	−815	−790	mm
ϕ_p^-/ω_p	−835	−820	mm
a_n	6.25	2:5	N
f_c	4.0	4.0	Hz
β_c	0.7	0.7	-

TABLE 7.1. Nominal parameters of the simulation model and controller

7.1.4. Simulation results. An overview of the simulation results for extended friction compensation is given. The control task is again the tracking of a circle, with the same specifications as in Section 5.4.1.

Five sets of results are presented, all for the second of two cycli of 3.5 [s] duration each. The results can be divided in two groups. Consideration of Figs. 7.1–7.3 indicates the effect of using more elaborate friction models. Figures 7.3–7.5 give an opportunity to assess the effect of using adaptation of parameters instead of fixed parameters in the scheme of Slotine and Li. We now present the five sets of results in more detail, starting with an assessment of the effects of extended friction compensation.

First, the results without extended friction compensation in y-direction are shown, where only the standard Coulomb friction is present in the computed torque part. See Fig. 7.1. Results without and with adaptation of the parameters are shown, both starting with the nominal parameters.

The tracking error is mainly due to the lack of viscous friction compensation. The tracking error is reduced by the adaptation, *i.e.*, the inertia and Coulomb friction parameters are given values, that change in time, to compensate somehow the effects of the viscous and the periodic friction.

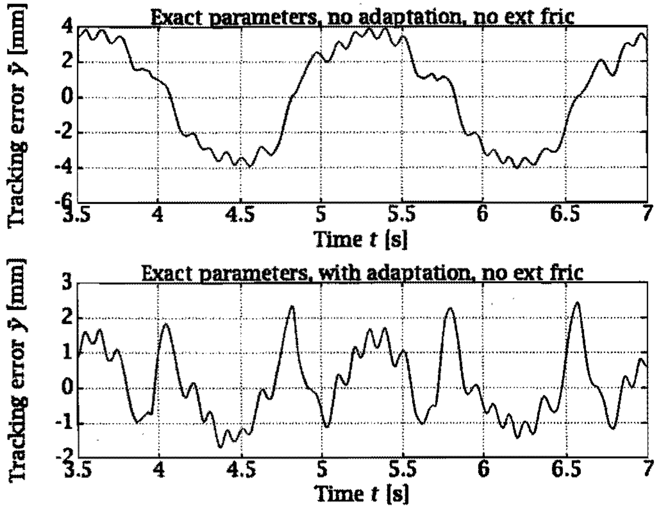

FIGURE 7.1. Simulation results without extended friction compensation

Second, the results without periodic friction compensation in y-direction are shown, where only Coulomb and viscous friction are compensated. Both the results without and with adaptation of the parameters are presented in Fig. 7.2, starting with the nominal parameters. The tracking error is smaller by a factor of 2, due to the compensation of the viscous friction. Again, the use of adaptation can partly compensate for the unmodeled periodic friction.

Third, the results with extended friction compensation in y-direction, including Coulomb, viscous, and periodic friction compensation are considered. The almost ideal tracking error is given in Fig. 7.3. The results without and with adaptation of the parameters are presented, both starting with the nominal parameters. The remaining tracking error is almost completely caused by the torque ripple. When the torque ripple is absent the error is much smaller, but not equal to 0 due to

- the quantization error in the position measurement,
- the prediction error in position and velocity of the one step ahead Kalman filter,
- inexact cancellation of the Coulomb friction, because the compensation can detect the instance of a change of sign of the velocity with an

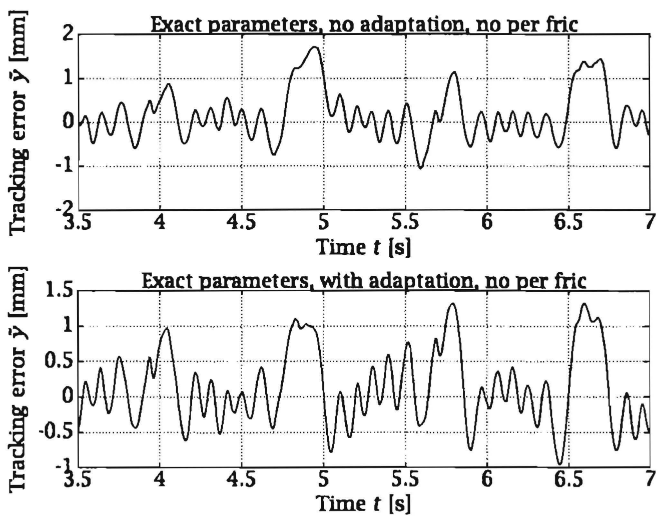

FIGURE 7.2. Simulation results without periodic friction compensation

accuracy of 1 sample only, due to the discrete time implementation of the controller.

See also Fig. 7.11 for the influence of the first two causes.

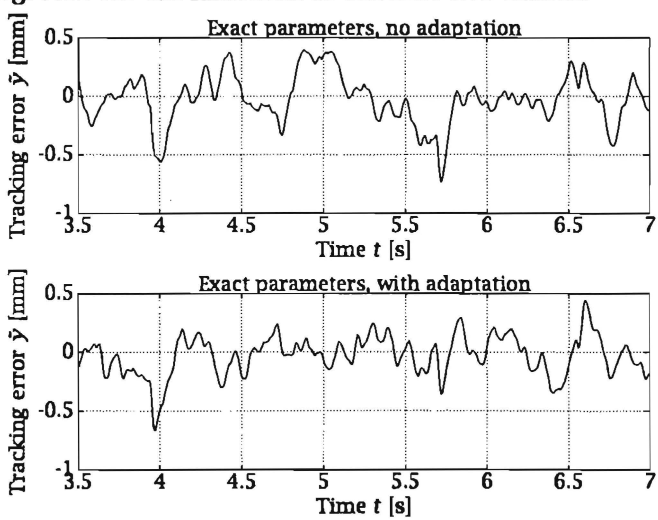

FIGURE 7.3. Simulation results with extended friction compensation

Comparison with the previous figure shows that the addition of periodic friction compensation results in a small, but noticeable, improvement in the performance. In relative terms, it is again a factor of 2. With adaptation the tracking error is only slightly smaller then without, which means that the parameter adaptation somehow cancels the effects of the three causes for the remaining tracking error mentioned above, although the first cause is mainly of a random nature. Further improvement is hardly possible, due to the lack of structure in the pseudo white noise signal used to model the torque ripple.

To show the influence of the initial parameters estimates and the rate of convergence of the adapted parameters, or better: the rate of convergence of the tracking error due to the adaptation of the parameters, the results starting from an initial parameter estimate of 80% and 0% of all nominal parameters are presented in Figs. 7.4 and 7.5. So, all parameters, including the inertia and Coulomb friction parameters, are assumed to be approximately known or even completely unknown.

Figure 7.4 shows the advantage of using adaptation. The tracking error is reduced by a factor of 2. The adaptation is fast, so an error comparable with the result given in Fig. 7.3 for exactly known parameters can be obtained after approximately 1 control cycle.

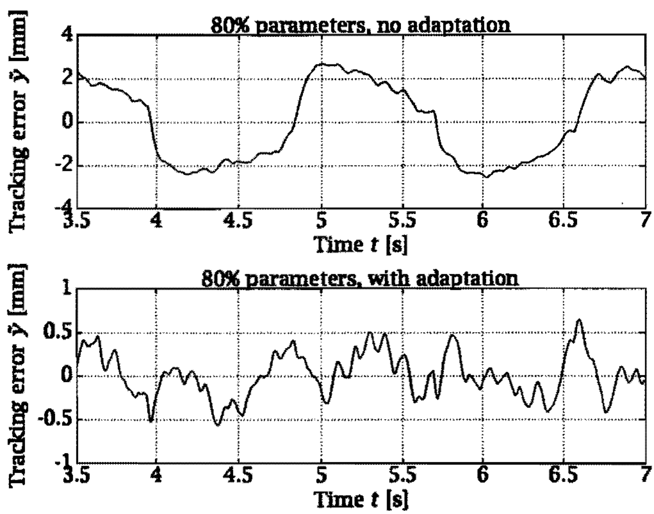

FIGURE 7.4. Simulation results with extended friction compensation but approximate parameters

Finally, the results starting from a zero initial estimate for all parameters show clearly the advantage of using a computed-torque-like control scheme. With a zero initial estimate for the parameters the control scheme of Slotine and Li degenerates to a pure PD feedback of the tracking error.

The tracking error is an order of magnitude larger than the error obtainable with more advanced control schemes. Figure 7.5 also clearly shows that the parameters obtain values that reduce the tracking error significantly after 2 cycli when adaptation is used. In the long run, the error will be as small as in Fig. 7.3.

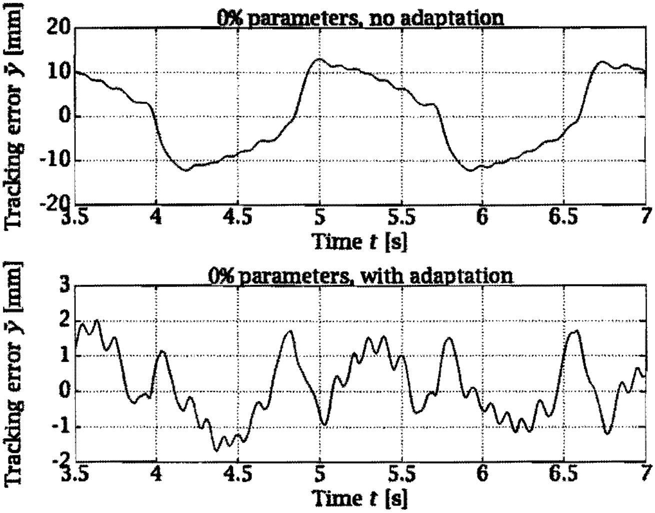

FIGURE 7.5. Simulation results with extended friction compensation but unknown parameters

7.1.5. Experimental results. We present five experimental results, also for the second of two cycli of 3.5 [s] duration each, except for the last result. The results for PD feedback are obtained by using the controller with zero values of the parameters and no adaptation. From all other experiments only results with adaptation are shown and the initial values of the parameters are assumed to be completely unknown. Results with fixed parameters are not obtained, because, in contrary to the simulation, exact model parameters are not defined. The results are presented in order of increasing tracking performance.

A reference result for the tracking error in y-direction, presented in Fig. 7.6, is obtained with a PD controller. Compare this with the first plot in Fig. 7.5 to see the similarity of simulation and experiment.

Figure 7.7 gives the result when only inertial forces and Coulomb friction, without direction dependent parameters, are compensated. The error is already small after two cycli.

In Fig. 7.8 the influence of the directional dependency of the Coulomb friction gives the largest improvement of the tracking error.

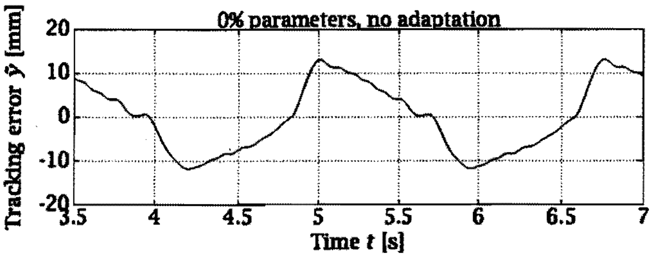

FIGURE 7.6. Experimental result without feedforward

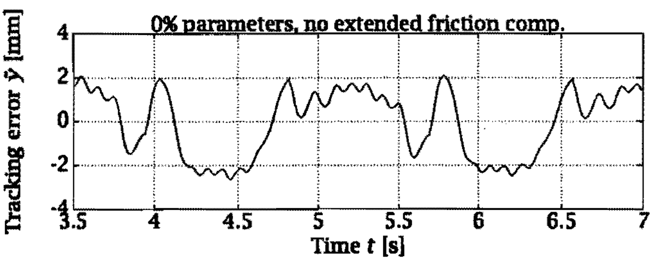

FIGURE 7.7. Experimental result without extended friction compensation, unknown parameters

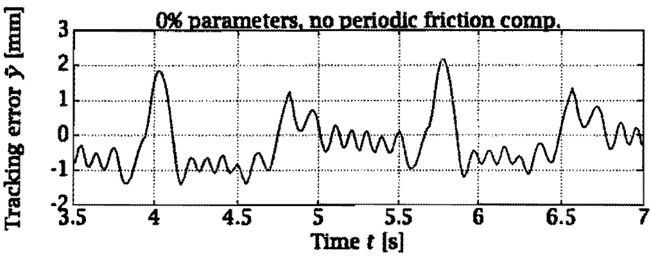

FIGURE 7.8. Experimental result without periodic friction compensation, unknown parameters

Good results are obtained with the full friction model, as shown in Fig. 7.9, although the improvement is not as large as suggested by the simulation results. See the second plot of Fig. 7.5 for comparison with the simulation.

To unfold the potential of extended friction compensation: the result of Fig. 7.10, where a longer period to obtain appropriate values $\hat{\theta}$ for the parameters was allowed (5 cycli), is the best that could be obtained experimentally.

We stress that to obtain this result both adaptation and an extended friction model are necessary. The periodic components in the tracking error

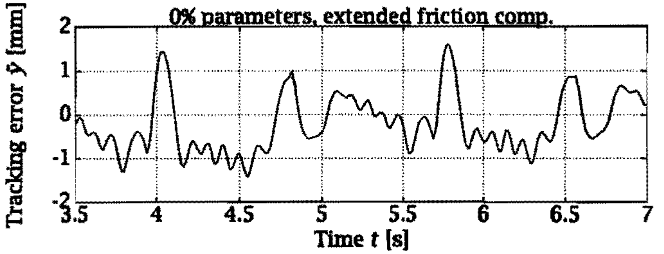

FIGURE 7.9. Experimental result with extended friction compensation but unknown parameters

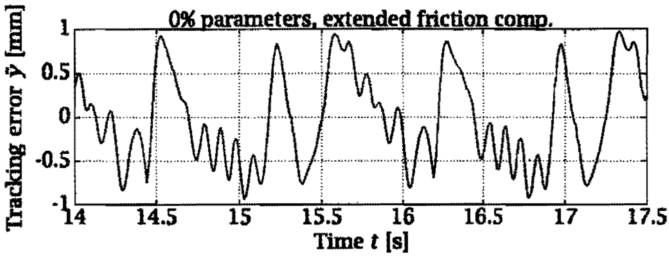

FIGURE 7.10. Experimental result with extended friction compensation but unknown parameters, prolonged adaptation time

are remnants of the periodic friction in the system that is not completely compensated or is over compensated. This result is comparable with the second plot of Fig. 7.3. A faster adaptation, by choosing larger gains in Γ^{-1}, was not possible due to stability problems, but the assumption that the parameters are initially completely unknown is also not very realistic. In general the parameters will "converge" within ≈ 7 [s].

7.1.6. Discussion of results. Both simulations and experiments show a marked performance improvement using the adaptive computed torque scheme with extended friction compensation instead of PD feedback. More extensions of the friction model lead to better performance. When only model based compensation with fixed, but inaccurate, parameters is used, the performance is worse, so adaptation is profitable. Due to stability problems during the experiments, that shows up in oscillations of the control signal, the adaptation could not be tuned to guarantee a "converged" tracking error within 3.5 [s]. This was caused by the unmodeled dynamics, that also limit the PD gains.

To gain more than an order of magnitude in performance, compared with PD feedback, extended model based compensation is not sufficient. A further gain can only be achieved by modifying sensors, actuators or the controlled system itself. This should reduce the measurement error, eliminate the torque ripple or raise the frequency of the unmodeled dynamics. The performance implication of the controller implementation is discussed in Section 7.2.

A comparison of the simulation and experimental results shows a difference in performance. This can also be attributed to the erroneous model of the XY-table used. Especially the number of degrees-of-freedom of the model is too low, due to flexible connections, *e.g.*, the belts, that the model does not account for.

This discrepancy means that an evaluation of modifications of control schemes by simulations should always be checked by an implementation of the modification in the controller software and validation of the simulation results with experiments. This indispensable step is, however, often omitted in the development and presentation of control schemes.

7.2. Controller implementation

As discussed in Section 6.3.3, we can expect a marked influence on performance of the sample time and the predictive nature of the Kalman filter. Simulation results show the relation between these aspects. The simulation are performed for the XY-table model, discussed in Section 5.1.2. We first discuss the modification of the controller implementation, then we present the simulation results. A discussion summarizes the results.

7.2.1. Controller. We use the controller given by (4.16), *i.e.*, the adaptive controller proposed by Slotine and Li, but we disable adaptation, *i.e.*, we use a CT type controller. We use values for the parameter estimates of 100% of the nominal values, different from the XY-table simulations of the adaptive, VSS, and AF controllers where 80% was used. The following modification of the standard controller implementation are investigated

(1) the use of higher and lower sampling rates,
(2) the use of a predictive and a non predictive type of Kalman filter for the estimation of the states.

7.2.2. Simulation results. An overview of the simulation results is given. The control task is again the tracking of a circle, with the same specifications as in Section 5.4.1. We first present the results for several sampling rates under different conditions, then the results for the two Kalman filter implementations under the same conditions.

7.2.2.1. ***Sampling rate.*** For the sample time t_s the following values where used, $t_s = 0.014/n$ [s], $n = 1, 2, 4$. First, in Fig. 7.11, the tracking errors under ideal conditions are presented, *i.e.*, no friction and unmodeled dynamics are present in the evaluation model. Then, in Fig. 7.12, the tracking error with friction, followed by the results for an evaluation model with unmodeled dynamics in Fig. 7.13. The Kalman filters used are redesigned, based on the same noise covariances, for each sampling rate. In all three cases only the last part of the responses is shown, that cover exactly a complete circle.

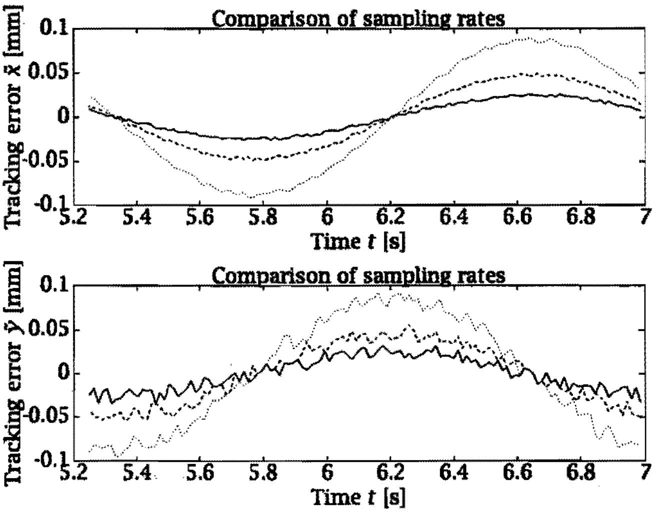

FIGURE 7.11. Tracking error, no friction, no unmodeled dynamics, 3 sampling rates, (—) $n = 4$, (- -) $n = 2$, (· · ·) $n = 1$

The tracking error in y-direction in Fig. 7.11 is much noisier than the one in x-direction. This is because there is no speed reduction in y-direction, so the quantization error in y-direction is more substantial. Under ideal conditions the relative increase in performance is the largest, but the absolute increase is largest for the case with unmodeled dynamics. In all cases the use of a higher sampling rate improves the tracking performance. Experiments, see [147], substantiate these results.

The sinusoidal component of the tracking error in Fig. 7.11, with a relatively small amplitude, is a remnant of the discrete time implementation of the controller. The effective form of the reference signal, a staircase, is the main reason for this error. Compared with the tracking error caused by model errors, it can be neglected.

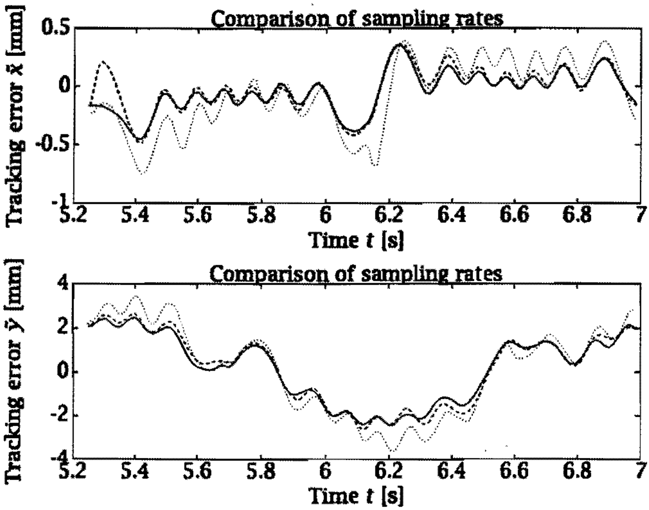

FIGURE 7.12. Tracking error, friction, no unmodeled dynamics, 3 sampling rates, (—) $n = 4$, (- -) $n = 2$, (· · ·) $n = 1$

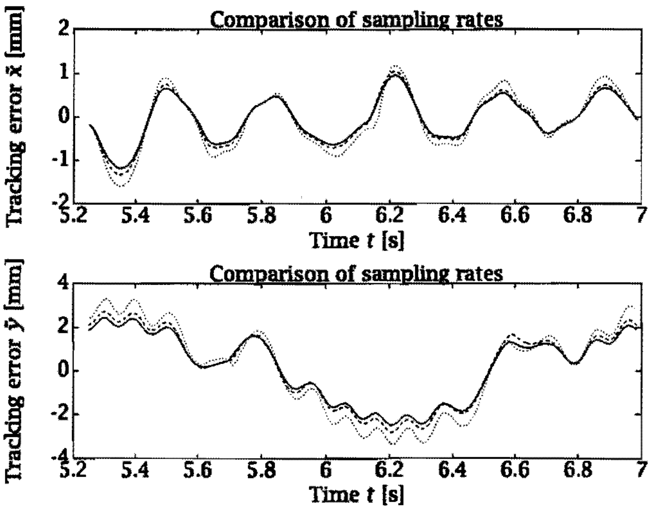

FIGURE 7.13. Tracking error, friction, unmodeled dynamics, 3 sampling rates, (—) $n = 4$, (- -) $n = 2$, (· · ·) $n = 1$

7.2.2.2. *Type of Kalman filter.* For the Kalman filter two implementations are used, the standard one in which the state is predicted one step ahead, and an alternative one without prediction, so the time delay in the controller implementation is not compensated for. The results are in Figs. 7.14–

7.16. Again, the results for three evaluation models are presented, with and without friction and with and without unmodeled dynamics.

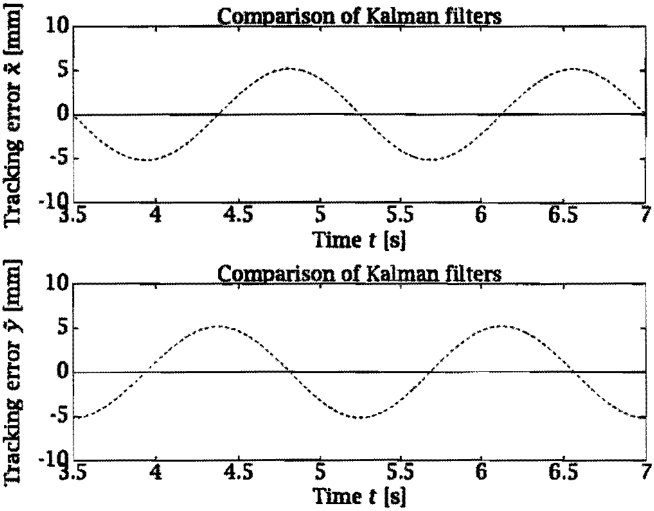

FIGURE 7.14. Tracking error, no friction, no unmodeled dynamics, 2 Kalman filters, (—) standard, (- -) alternative

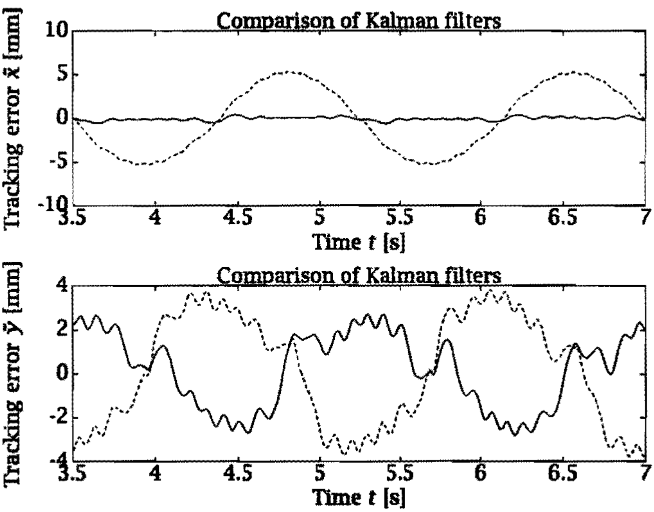

FIGURE 7.15. Tracking error, friction, no unmodeled dynamics, 2 Kalman filters, (—) standard, (- -) alternative

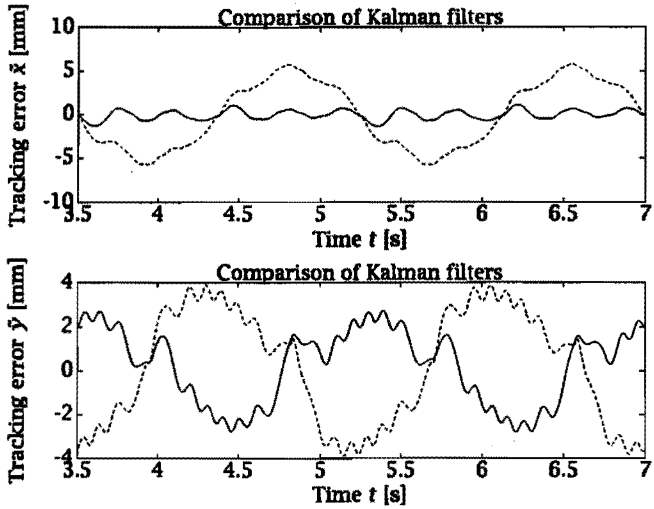

FIGURE 7.16. Tracking error, friction, unmodeled dynamics, 2 Kalman filters, (—) standard, (- -) alternative

Figure 7.13 shows that the tracking error for the non predictive type of Kalman filter is much larger. This could be expected, because the time delay in the controller implementation is not compensated, so the tracking error is almost the same as the difference between two version of the reference trajectory time series, shifted by one time sample relative to each other. When the tracking error is compensated for this difference, the remaining error is almost the same as in Fig. 7.11. In the case of model errors the predictive type of Kalman filter reduces the error in x-direction also, the behavior in y-direction is more complicated. At the moment, there is no real explanation for this behavior, except for the suspicion that the model error is somehow compensated by the computational delay and by the fact that the tracking error introduced by model errors dominates.

7.2.3. Discussion of results. It is evident from these responses that both the sampling rate and the type of Kalman filter used have a marked influence on the performance of the system.

The use of a predictive type of Kalman filter, or other measures to compensate the time delay in the controller, is mandatory to get low tracking errors.

When the sampling rate is high, the tracking error is dominated by the model errors in the controller, and any attempt to improve the control quality by using a better, *i.e.*, higher sampling rate, implementation is in vain. Therefore, when the sampling rate is high, most improvements of the tracking performance can be expected from more elaborate models in the controllers. In general this requires more computations, thereby limiting the maximum sampling rate if the computing power is fixed. On the other hand, the conclusions for a low sampling rate are the inverse of the conclusions for a high sampling rate, *i.e.*, some improvement can be expected from an increase of the sampling rate.

In general, there will be a trade off between model accuracy and sampling rate, or better, between the use of computing power to increase the sampling rate or to use more accurate models in the control algorithms. This conclusion is not new, nor original, but our results substantiate the view expressed before, *e.g.*, in [163].

CHAPTER 8

Discussion, conclusion and conjectures

In this chapter we will again, but this time from a more general point of view, discuss the results obtained in the previous chapters. Confronting the results with the goals set out in Chapter 3 will show the completeness with which these goals are attained, and where we did not succeed to resolve some open questions. We will try to delineate the domain of applicability of our results, which will indicate how broad the results are, and to which degree we can generalize them. The generalization of the results will, however, be limited by the fundamental problem that induction, which is needed for this generalization, has, as stated before, no logical validity [7]. Therefore, the main contributions of our research are not general statements of the form "to control a system of class X with goals of group Y, the best you can do is to apply a control scheme of type Z," but a set of new conjectures, in the spirit of the falsification paradigm [208] of Popper, that should be falsified in the next round of research activities.

8.1. Discussion

We discuss the following aspects of the results

- performance robustness for the three types of modeling errors,
- relation between extensive modeling, performance, and robustness,
- trade off between design model and error model,
- relation between area of applicability and performance,
- limitations of experimental evidence.

We distinguish three types of modeling errors, the parameter error, the unmodeled statics, and the unmodeled dynamics error. The adaptive controllers can compensate the parameter error. To guarantee low tracking errors, it is not necessary to estimate correctly the parameters. So, for this type of model error, we have relatively simple controllers that solve the problem of the parameter type of model error.

The unmodeled statics error, *e.g.*, neglected Coulomb friction, periodic torque ripple or persistent disturbances, cannot be compensated for by the adaptive controllers. This type of errors cannot be handled by high gains in the adaptive controller also, because that will inevitable excite unmodeled dynamics, leading to an unstable closed loop system. The use of VSS controllers has some advantage, especially in combination with an adaptive controller, but cannot fully compensate the error, due to chattering problems that are to be avoided. For this type of model error a fully satisfying robust controller has not been found, although the VSS controller has a definite advantage.

The unmodeled dynamics error, *e.g.*, neglected joint and link flexibility or actuator dynamics, cannot be handled well by the adaptive controllers, also VSS and AF based controllers are of limited value. Modifications to solve these problems for the specific case of joint flexibility are available, but require a more complete model, extended instrumentation, and accurate knowledge of some system parameters, because they are not adapted by these controllers. For less structured types of unmodeled dynamics little is known. So, also the unmodeled dynamics error cannot be handled satisfactory by the robust controllers studied.

The use of adaptive controllers is often advantageous, but there have been some cases where this is not true, see, *e.g.*, Figure 5.12. This case is quite artificial and has therefore little practical significance.

The results of Chapter 7 suggest that the use of extensive models, where the significant parts of the model equations are modeled more carefully, is profitable. These results also refute the naive assumption that the use of extended models makes the control system more vulnerable to modeling errors, because the control action is based more on feedforward, using an erroneous model, and less on feedback, the main robustifying control structure. The same is true for the results in Section 5.3 where controllers that implicitly use observes were as robust as a simple PD controller. In other cases, see [209], a performance deterioration was found for model based controllers, using observers, in the presence of parameter errors. A possible explanation for those contradictory observations is that the relative number of measurements, compared with the number of states to be estimated, was worse for the last case.

The fact that for high performance a model based controller, with a relatively complicated model, is still necessary suggests that the initial goal, to eliminate the modeling burden, is not always reached. It is true that the boundary or trade off between modeling and the use of advanced controllers can be shifted, in favor of fewer requirements on the accuracy of the model, but this shift will often not be sufficient. This depends on the specifications that the control system is required to fulfill, and a general statement of design directives can therefore only be given as a function of those requirements.

Chapter 7 also shows that the implementation of the controller can have substantial influence on the performance. The use of predicted states in a discrete implementation, to compensate for the computational delay, seems mandatory. Also the sampling rate is limited from below for performance reasons. The use of complicated models, that can improve the performance, should be balanced against the required sampling rate.

From Section 5.3 another observation can be made. The use of linear robust controllers did not always give the expected robust performance increase relative to a simple PD controller. Yet, the design by first mimicking a PD controller and then, by changing the weight functions, trying to improve the control system seems a viable way, as a first attack, for the design of the control system. It seems also plausible that the use of more detailed error models is necessary to improve the robust performance. Still, the question arises if the effort needed to develop an accurate error model cannot be used more fruitfully by developing a more accurate design model. This again implies a trade off between several modeling activities, that, in the opinion of the author, should be eliminated by developing and using design methods that make both modeling activities superfluous.

The fact that the controllers exploit the specific structure of the model, *e.g.*, in the case of the adaptive computed torque controllers, is the main disadvantage of the controllers used because it limits the area of applicability. It is also the main advantage because, on the other hand, it makes possible a performance level that probably could not be achieved with other controllers.

The experimental results obtained are hampered by two major factors. The end-effector position was not available for evaluation purposes. We also stress that not all aspect of the numerical experiments are verified by laboratory experiments, due to the choice of the experimental system. In this system Coulomb friction was dominant and, except for the inertia forces, other forces were virtually non existent.

8.2. Conclusion

The conclusion, that relates our results as discussed in the previous section with the goals set out in Chapter 3, will be given as a dialogue.[1] This dialogue will answer also some other questions that could be put forward.

[1] Inspired by "Dialogo sopra i due Massimi Sistemi del Mondo, Tolemaico e Copernicano," a.k.a. Dialogue of the Great World Systems, by Galileo, see [2,210]. The same players are around.

Simplicio: I have read the book, but forgot what your initial goals were. To which degree have you succeeded to reach these goals?

Sagredo: Just tumble back to Chapter 3, and you will see.

Salviati: No, I can tell you without looking it up. I tried to find out if robust controllers can be used for nonlinear systems, if they have certain advantages, and if they can be computed easily—without the need for detailed models. Succinctly, you can summarize the results as follows. For the models and experimental system used, those controllers exist, but for high performance an adequate extended model is still necessary, so my original aim to reduce the modeling burden is not always reached. Also, I have not found a robust controller that can eliminate the influence of modeling errors. The initial goals are therefore only partially realized.

Sagredo: So, your research hasn't worked out very much, isn't it?

Salviati: In a certain sense, yes. But you should realize that the aims you put forward are always higher than can be reached in a limited time.

Simplicio: Isn't that an ointment to lessen the pain?

Salviati: No. Or better, yes. It's always a pain to realize that the goals you set forward are not within easy reach. But the bitterness is sweetened by the knowledge that nobody could reach these goals.

Sagredo: But why didn't you realize that before you started your work?

Simplicio: His eyes where opened during the job, and he was blind before.

Salviati: No, it's more that my sight was out of focus. Now the picture is much clearer, although it's still not bright.

Simplicio: But why can't you say that robust controllers are always better, your work does give an indication in that direction, or better, in all cases you investigated their performance was not worse. Also, are not all systems like your systems?

Sagredo: That's because he used a few specific models to test his suppositions, and he isn't sure if the results are also valid for other systems. The last you said is pure nonsense.

Salviati: That's true, but it's not the whole picture. You should realize that it's sometimes possible to extend results from specific cases to an encompassing class of problems.

Simplicio: But why haven't you done that?

Salviati: For my problems that's only interesting if you are sure that the class of problems is broad enough to contain most practical problems. But, to delineate a class of problems, you need some assumptions that are never verified in practice. So, if I should extend those results, I implicitly assume that those assumptions *are* verified, and I can't do that because they aren't.

Sagredo: So, the assumptions that must be made to justify the use of an induction argument usually rest on conjecture and hope?

Salviati: Yes, indeed.

Sagredo: Your results are based on simulations and experiments. If you had instead developed a mathematical theory, should that not be better, in the sense that more general results could be expected?

Salviati: I admit that it may be possible to say more if the results obtained have a sound mathematical basis. Nevertheless, the only real profit theory would give is perhaps that the resulting new conjectures could be more effective and efficient, but even this limited advantage is speculative. In general one can say that the practical validity of a theory, using assumptions that are not verified in practice, can be even less than a practical evaluation, whose results can only be extended by using questionable induction and speculative "representative application" arguments. You should also keep in mind that the practical use of research results is inversely proportional with the theoretical level of the research, with some exceptions.

Simplicio: Well, now you claim that your findings are of some use in applications, but you never specified which practical problems can be solved that couldn't be solved before.

Sagredo: Yes, that's a nice topic, what is in essence the practical relevance of your research?

Salviati: You should keep in mind that my work has not been done in an application oriented setting, but I only smelled some practical problems, as sketched in the applications I mention in Chapter 1, and tried, not to solve a specific problem, but to develop a general line of thought or methodology, where emphasis is shifted from modeling to robust control, in the hope that a new balance could be obtained where the same or higher goals could be reached with less or equal exertion, respectively. I can't claim that a breakthrough has been made, and that it is much simpler now to design control systems that perform well in practical situations, where model errors are inevitable, but this field has been explored sufficiently to suspect that at the moment accurate modeling is still necessary, and that those models should be used in the design, to obtain control systems that achieve high marks.

8.3. Conjectures

Our final statements are a set of overlapping and even contradictory recommendations, put in the form of conjectures, that possibly can (and eventually should) be falsified. These conjectures can be regarded also as open questions that have to be resolved, to attain further the original goals of this research.

The experimental system used is an exponent of an interesting class of mechanical systems where the tracking error is mainly caused by forces that cannot be described by inertia, Coriolis, centrifugal, and potential (gravitational) forces. To cover a larger class of systems we state

CONJECTURE 8.1. The experimental results obtained are representative for the class of controllers studied, they do not depend on the choice of experimental system, and are therefore relevant for other systems.

The goal is, of course, to falsify this claim by extending the evaluation to other systems.

In the design of H_2, H_∞, and μ-synthesis controllers a global error model was not sufficient. An open question is if the situation can be improved by using an extensive error model.

CONJECTURE 8.2. The use of an extensive error model will not improve the norm based controllers when they are applied in practice.

The goal is to come up with an application where an extensive model does improve the performance, and then analyze why and when an extensive error model is profitable. Or, more generally, make up the contours of a table indicating, on one hand, the degree of complexity of a model of the system to be controlled, and on the other hand, the degree of complexity of an error model, needed for high performance and low expenditure.

Due to the way of designing VSS controllers, one can be certain that model errors that can be modeled as matched disturbance signals can be compensated for by the control input. If the accompanying problems like chattering can be solved, one can state the following.

CONJECTURE 8.3. Variable structure control is the method of choice for the control of systems with prominent unmodeled statics.

The question to be resolved is, of course, if there are no better methods.

The adaptive controllers studied in this thesis are perfectly suited to control models with unknown parameters. If unmodeled statics is present, the applicability becomes questionable. This is further amplified if unmodeled dynamics is present, so

CONJECTURE 8.4. Adaptive control is not suitable for robust control in the case of prominent unmodeled dynamics.

The question is if this disadvantage exist for all adaptive controllers, or only for the controllers evaluated in this study.

As remarked before, some modifications are possible that make the adaptive controllers more robust for unmodeled statics and dynamics errors. One can think of limiting the parameters values to reasonable values or stopping the parameters adaptation when the values of s are within a certain bound. So one can pose the following.

CONJECTURE 8.5. There are modifications of adaptive control that makes it suitable for robust control in the case of prominent unmodeled dynamics.

The question is if these modifications can assure good performance *for all possible types* of model errors.

If one does not believe that the answer to the previous question is affirmative, one can doubt if there are controllers at all that are able to work if the model is erroneous. This leads to

CONJECTURE 8.6. Robust controllers, applied in practice, cannot compensate for modeling errors completely. So a shift of the boundary between extensive modeling and the use of advanced control concepts, effectively eliminating a substantial part of the modeling activities, is a dream.

and the question is open.

A consequence of the last conjecture is that extensive models are necessary. But if the controller cannot optimally profit from the structure of the model equations, there is some remaining doubt if the goals can be reached. This is expressed by

CONJECTURE 8.7. Knowledge of the system to be controlled is better than knowledge of advanced control, during the design of controllers. In other words: Putting physics in control is always better than putting control into physics.

A possible consequence is that control schemes become more problem specific, *e.g.*, schemes for mechanical systems described by second order differential equations with a specific structure, and other control schemes for systems described by sets of equations with another structure.

Perhaps this can lead to a proliferation of control schemes, that cannot again be unified. Also, it is not alway easy to detect a structure in the model equations that can profitably be used in the control scheme. This makes the design and implementation of controllers much less standard and therefore more expensive. So finally

CONJECTURE 8.8. Always putting physics in control is economically not feasible.

With the recommendations implied by these conjectures we finish this dissertation and hope that the reader has found it of some value. For a further account of the research, with respect to the last questions, the reader is referred to the pertinent literature.

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- Rob Visser (acceleration feedback),
- Ruud Heikoop (μ-synthesis controller),
- André Blom (VSS controller).

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Eindhoven, 16 oktober 1992 Bram de Jager

Stellingen

behorende bij het proefschrift

Practical Evaluation of Robust Control for a Class of Nonlinear Mechanical Dynamic Systems

1. Het aantal, in de literatuur voorgestelde, regelkonsepten voor het robuust regelen van niet-lineaire systemen, dat nog een toepassingsgerichte analyse ontbeert, is groter dan voor een goede ontwikkeling van dit gebied wenselijk zou zijn.
   Dit proefschrift. Hoofdstuk 2.
2. De stelling "Every stable linear feedback system is robustly stable" [1] lijkt ook op te gaan voor niet-lineaire systemen.
   Dit proefschrift. Hoofdstukken 5 en 6.
3. Het begrip "computed torque" wordt door verschillende auteurs [2, 3] op eigenwijze ingevuld.
4. De mogelijkheden van modelvorming in de menswetenschappen zijn geringer dan in de techniek.
   Een voorbeeld is de invloed van model fouten op de te volgen regelstrategie, die, als fenomeen, pas de laatste dekade in de systeem- en regeltheorie expliciet wordt onderzocht. In de sociologie was dit fenomeen al eerder onderkend. Het komt bijvoorbeeld naar voren in het onderzoek "Decision making under Uncertainty", zie [4].
5. De belangrijkste eigenschap van een onderzoeker is zijn vermogen tot kreatieve zelfkritiek.
6. Indachtig het adagium "zachte heelmeesters maken stinkende wonden" zou het goed zijn als in de wetenschap, maar niet alleen daar, kollegiale kritiek minder als een persoonlijke aanval, maar meer als een waardevolle bijdrage zou worden beschouwd.
7. In de wetenschap is integriteit waardevoller dan opportunisme.

8. Het algemeen belang is meer gediend met niet extern gefinancierd onderzoek, dan met door bedrijven gefinancierd onderzoek.

> Toute société capitaliste fonctionne régulièrement grâce à des secteurs sociaux qui ne sont ni imprégnés ni animés de l'esprit de gain et de la recherche du plus grand gain. Lorsque le haut fonctionnaire, le soldat, le magistrat, le prêtre, l'artiste, le savant sont dominés par cet esprit, la société croule, et toute forme d'économie est menacée. ...

Citaat van François Perroux uit 1962, geciteerd in [5, p. 123].

9. Als de universiteit naar eer en geweten de haar opgelegde taak met betrekking tot het bijbrengen van maatschappelijk verantwoordelijkheidsbesef wil vervullen, zal ze het goede voorbeeld moeten geven en hoge eisen moeten stellen aan haar eigen ethisch gedrag. De huidige praktijk is daarmee soms in tegenspraak.

10. Ter vermindering van het papierverbruik verdient het aanbeveling om alleen proefschriften te verstrekken aan geïnteresseerden. Deze interesse komt naar voren in de geleverde terugkoppeling, bijvoorbeeld blijkend uit het gebruik van het recht om tijdens de verdediging van het proefschrift te opponeren vanuit de zaal.

11. Het vermijden van het gebruik van een metalen nietje bij theezakjes is een goede (maar geen grote) stap in de scheiding, en in de vermindering van de omvang, van het huisvuil. Beter is het om het zakje geheel te vermijden.

12. Ter vermindering van het beslag op primaire bronnen en de belasting van het ecosysteem is "lean consumption" minstens even belangrijk als "lean production".

13. Na de ontkoppeling van het pond sterling is de betrouwbaarheid van de Bank of England niet meer spreekwoordelijk.

14. De aan het gebruik van werktuigen gekoppelde immanente nadelen moeten leiden tot een bewuster gebruik, en tot een relativering van het primaire doel.
Een voorbeeld hiervan is het gebruik van motorvoertuigen.

> ... Auf den Straßen jagten Automobile, zum Teil gepanzerte, und machten Jagd auf die Fußgänger, überfuhren sie zu Brei, drückten sie an den Mauern der Haüser zuschanden. Ich begriff sofort: es war der Kampf zwischen Menschen und Maschinen, lang vorbereitet, lang erwartet, lang gefürchtet, nun endlich zum Ausbruch gekommen. ...

Citaat uit [6].

15. De recente strubbelingen in Oost-Europa wijzen erop dat een repressief regime niet alleen de goede, maar ook de slechte eigenschappen van mensen kan onderdrukken. Een onderdrukkend regime kan daarom *soms* nuttig zijn, maar blijft *altijd* ongewenst.

16. Het aan elkaar refereren door een "inner circle" is niet alleen bij wetenschappelijke publikaties, maar ook bij journalistieke, een gangbaar middel om de output te vergroten.

[redacted]

„Een mens is alleen onmisbaar bij zijn begrafenis."
De oma van Theo, in Cursor, weekblad TU Eindhoven.

[redacted]

Cursor wordt gelezen 2

Cursor's Theo heeft voor de tweede maal 'de Limburger' gehaald. Eerder drong zijn creatie 'Limbopad' al op hun voormalige achterpaginarubriek Klatsch door. Nu haalde hij de voorpagina van dinsdag 24 maart.

Afbeelding uit Cursor dd. 27/3/92.