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# Experimental and Numerical Analysis of Water Hammer in a Large-Scale PVC Pipeline Apparatus

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## Abstract

This paper investigates the effects of the pipe-wall viscoelasticity on water-hammer pressures. A large-scale pipeline apparatus made of polyvinyl chloride (PVC) at Deltares, Delft, The Netherlands, has been used to carry out water-hammer experiments. Tests have been conducted in a reservoir-pipe-valve system with a 275.2 m long DN250 PVC pipeline. Rapid closure of a manually operated ball valve at the downstream end generated water hammer. Computed results are compared with experimental runs. Calibrated creep functions have been obtained using optimization in conjunction with an inverse hydraulic transient solver and these are used in the simulations. It is shown that the incorporation of both unsteady skin friction and viscoelastic pipe wall mechanical behaviour in the hydraulic transient model contributes to a favourable fitting between numerical results and observed data.

**Keywords:** Water hammer, Unsteady Skin Friction, Viscoelasticity, Pipeline Apparatus, PVC Pipe.

## 1. Introduction

The increased application of pipes made of viscoelastic materials in recent years has made it necessary to do more theoretical and experimental researches on their mechanical behaviour under transient loads. The paper focuses on the mathematical modelling of hydraulic transients in PVC pipes by adding the retarded strain in the transient elastic pipe flow equations and by taking into account unsteady friction losses. Here, a large-scale pipeline apparatus at Deltares, Delft, The Netherlands has been used to obtain experimental data for the validation of mathematical and numerical models that predict water-hammer responses in viscoelastic pipes. The main part of the system under consideration was a horizontal 235.4 mm inner diameter PVC pipe of 261.2 m length with control valves at the downstream and upstream ends, and with nine locations to collect transient pressures along the system including the supply steel pipeline and short outlet steel pipe. Numerical and experimental data were analysed and the analysis shows a considerable damping partly due to unsteady skin friction and partly due to viscoelastic behaviour of the pipe wall.

The role of unsteady skin friction in one-dimensional pipe flow depends on the system under analysis. The assumption of steady viscous losses may be satisfactory for slow pressure transients where the wall shear stress has a quasi-steady behaviour. Previous investigations on the behaviour of steady friction models for rapid transients in elastic-pipelines (Golia [1], Bergant *et al.* [2]) showed large discrepancies in attenuation, shape and timing of pressure traces when computational results were compared with measurements. Unsteady friction arises due to the extra losses from the two-dimensional nature of the unsteady velocity profile. If turbulence is considered unsteady friction is a three-dimensional problem. However, modelling both the two-dimensional and three-dimensional cases is complicated and computationally intensive. Numerous unsteady skin friction models have been proposed to date including one- (1D) and two-dimensional (2D) models (Bergant *et al.* [2], Pezzinga and Brunone [3]). The 1D models approximate the actual cross-sectional velocity profile and corresponding viscous losses in different ways and the 2D models compute the actual cross-sectional profile continuously during the transient event. The drawbacks of the 2D models in comparison with 1D models are complex modelling and larger CPU times. For engineering practice it is desirable to have a model that takes into account higher dimensional velocity profile behaviour, but still can be efficiently implemented in the one-dimensional analysis. This paper deals with the convolution-based unsteady friction model (Zielke [4], Vardy and Brown [5]). The friction term is dependent on instantaneous mean flow velocity and weights for past velocity changes.

The major property of viscoelastic materials is that the strain response lags the applied stress. This behaviour influences the pressure response during transient events by attenuating the pressure fluctuations and by increasing the dispersion of the pressure wave front (Pezzinga and Brunone [3], Covas *et al.* [6], [7]). Viscoelastic energy dissipation is due to viscous resistance of pipe-wall material during the creep process (Duan *et al.* [8]). The viscoelastic pipe-wall property can be dealt with by a mechanical model which eventually provides an appropriate relation between stress and strain. The strain is decomposed into instantaneous-elastic strain and retarded-viscoelastic strain. The elastic strain is included in the wave speed, whereas the retarded strain is an additional term included in the mass-balance equation (Rieutord and Blanchard [9]). In this paper, a method of characteristics (MOC) based numerical scheme is implemented including unsteady skin friction and viscoelastic behaviour of the pipe-wall. The rate of the retarded circumferential strain is a linear function of pressure, allowing for the application of MOC with a constant wave speed. Incorporation of the retarded strains in the mathematical model introduces a significant damping in the results which is in accordance with the observations (Covas *et al.* [6, 7]). Large-scale experimental and numerical results are compared and the possible causes of existing discrepancies are discussed.

## 2. Water Hammer with Unsteady Friction in Viscoelastic Pipes

The water-hammer equations including viscoelastic behaviour of the pipe wall and unsteady skin friction can be expressed in terms of pressure head  $H$  and discharge  $Q$  as follows (adapted from Bergant *et al.* [10])

$$\frac{\partial H}{\partial t} + \frac{a^2}{g A} \frac{\partial Q}{\partial x} + \frac{2a^2}{g} \frac{\partial(\varepsilon_\varphi)_{ret}}{\partial t} = 0 \quad (1)$$

$$\frac{\partial H}{\partial x} + \frac{1}{g A} \frac{\partial Q}{\partial t} + h_f = 0 \quad (2)$$

with the wave speed  $a$  defined by

$$a = \sqrt{\frac{K/\rho}{1 + \alpha(K/E)(D/e)}} \quad (3)$$

The partial derivative of the retarded circumferential strain  $(\varepsilon_\varphi)_{ret}$  in the viscoelastic pipe wall is

$$\frac{\partial(\varepsilon_\varphi)_{ret}}{\partial t}(x, t) = \frac{\alpha D}{2e} \rho g \frac{\partial}{\partial t} \int_0^t [H(x, t-t^*) - H(x, 0)] \frac{\partial J(t^*)}{\partial t} dt^* \quad (4)$$

and the unsteady skin frictional head loss  $h_f$  per unit length is given by

$$h_f(t) = \frac{fQ|Q|}{2gDA^2} + \frac{16\nu}{gD^2A} \int_0^t \frac{\partial Q(t^*)}{\partial t} W(t-t^*) dt^* \quad (5)$$

where  $A$  = cross-sectional flow area,  $D$  = internal pipe diameter,  $E$  = Young's modulus of elasticity of pipe-wall material,  $e$  = pipe-wall thickness,  $f$  = friction coefficient according to Darcy-Weisbach,  $g$  = gravitational acceleration,  $J$  = creep-compliance function,  $K$  = bulk modulus of elasticity of liquid,  $t$  and  $t^*$  = time,  $W$  = weighting function,  $x$  = axial distance,  $\alpha$  = axial pipe-constraint parameter,  $\nu$  = kinematic viscosity, and  $\rho$  = mass density of the liquid. Eqs. (1) and (2) describe the acoustic behaviour of weakly compressible low-Mach-number flows in prismatic viscoelastic pipes of circular cross-section. The pipe-wall is assumed to behave in a linearly viscoelastic manner.

The linear viscoelastic behaviour of the pipe wall for small strains and no dynamic fluid-structure interaction (FSI) effects can be expressed by the hoop strain of the pipe-wall  $\varepsilon_\varphi$  as a function of internal pressure (Güney [11]). The creep-compliance function  $J(t)$ , which describes the viscoelastic behaviour of the pipe material, can be determined experimentally using a mechanical test or calibrated (tuned) on collected transient data (Covas *et al.* [6], [7]; method used in this paper). A mechanical model of the viscoelastic solid can be used to describe the creep function. The generalised Kelvin-Voigt model consisting of  $N$  parallel spring and dashpot elements in series is represented by

$$J(t) = J_0 + \sum_{k=1}^N J_k (1 - e^{-t/\tau_k}) \quad (6)$$

where the stiffness of each spring is  $E_k = 1/J_k$ , the viscosity of each dashpot is  $\eta_k$ , and the associated retardation time is  $\tau_k = \eta_k/E_k$ . The influence of the elastic strain is included in the liquid wave speed  $a$  (Eq. (3)).

The friction term in Eq. (5) comprises a steady part and an unsteady part (first and second term on the right-hand side of the equation, respectively). The unsteady component follows from the convolution of a weighting function ( $W$ ) with past temporal discharge variations ( $\partial Q/\partial t$ ). Zielke [4] derived the weighting function based on analytical solutions obtained for *laminar* flow. Vardy and Brown [5] used the *frozen viscosity* assumption to derive a weighting function for *smooth-pipe turbulent* flow (used in this paper). Their approximate weighting function is

$$W_{app}(\tau) = \frac{A^* e^{-B^* \tau}}{\sqrt{\tau}} \quad (7)$$

where  $A^*$  and  $B^*$  depend on the Reynolds number ( $\mathbf{Re}_0 = Q_0 D / (\nu A)$ ) of the pre-transient flow, and  $\tau = 4 \nu t / D^2 =$  dimensionless time. Vardy and Brown [5] defined the coefficients  $A^*$  and  $B^*$  for smooth-pipe turbulent flow as

$$A^* = \frac{1}{2\sqrt{\pi}} \quad \text{and} \quad B^* = \frac{\mathbf{Re}_0^\kappa}{12.86} \quad \text{with} \quad \kappa = \log_{10}(15.29 \mathbf{Re}_0^{-0.0567}) \quad (8)$$

These coefficients are accurate in the range  $2 \cdot 10^3 < \mathbf{Re}_0 < 10^8$  (Vardy and Brown [5]). The *frozen viscosity* assumption is satisfactory for flow situations with a large Ghidaoui *et al.* ratio of the diffusion time scale to the wave time scale  $P$  expressed by the following relation ( $P \gg 1$ , Ghidaoui *et al.* [12])

$$P = \frac{2D / (\sqrt{f} Q / A)}{L/a} \quad (9)$$

where  $L =$  pipe length.

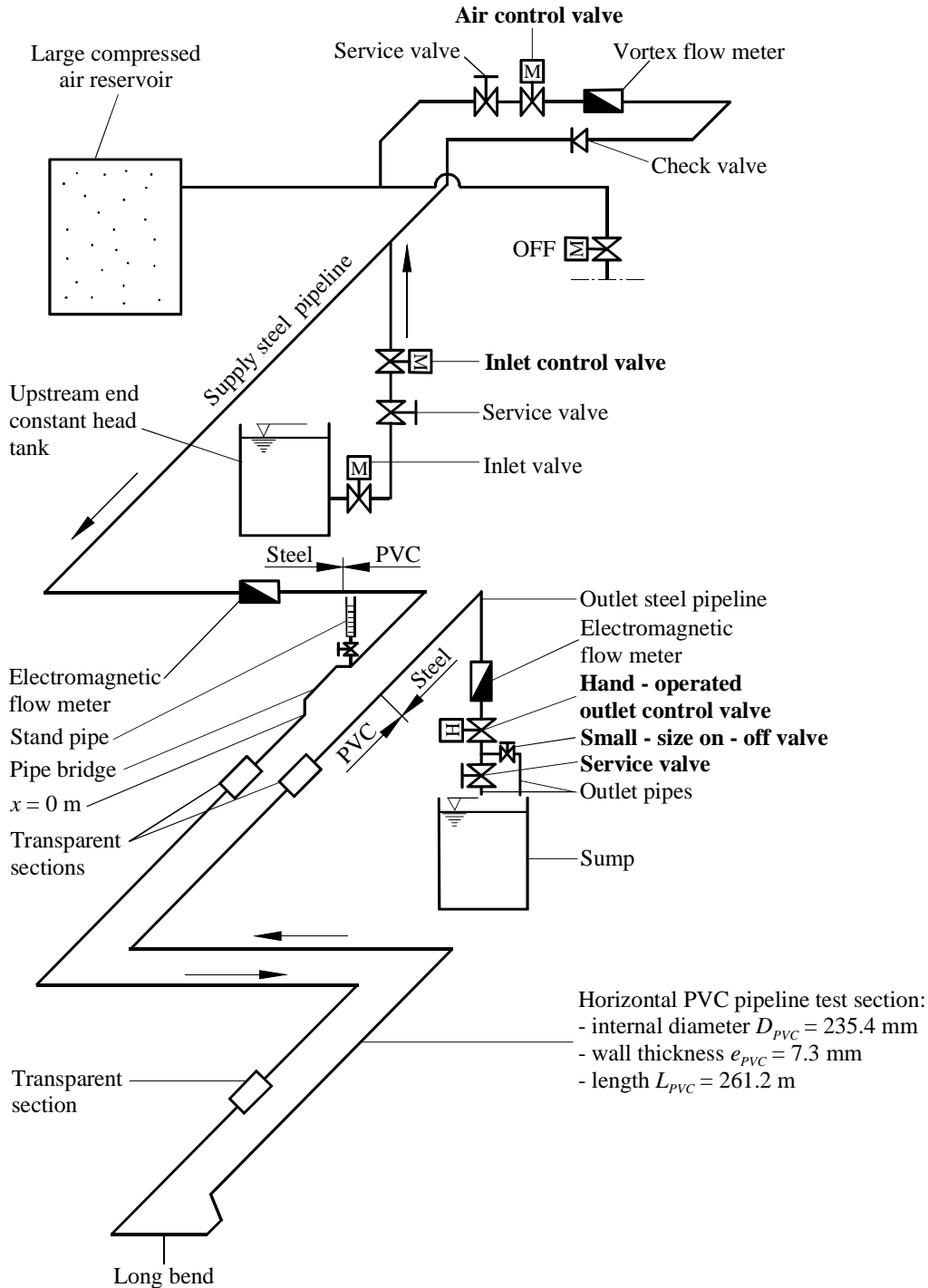
The MOC transformation of the water-hammer equations (1) and (2) gives the following set of compatibility equations

$$\frac{dH}{dt} \pm \frac{a}{gA} \frac{dQ}{dt} + \frac{2a^2}{g} \frac{\partial(\varepsilon_\varphi)_{ret}}{\partial t} \pm ah_f = 0 \quad \text{along} \quad \frac{dx}{dt} = \pm a \quad (10)$$

The numerical solution of the compatibility equations (10) is standard and can be found elsewhere (Covas *et al.* [6], [7], Bergant *et al.* [10]).

### 3. Large-Scale PVC Pipeline Apparatus

The dynamic two-phase flow large-scale PVC pipeline apparatus at Deltares, Delft, The Netherlands has been primarily used for pipeline filling and emptying experiments (Laanearu *et al.* [13]). The test rig consists of a water-supply tower of 25 m head, a pressurized air tank (volume of about 70 m<sup>3</sup>), a supply steel pipeline (20.6m long), PVC inlet pipe and pipe-bridge (14 m long), a horizontal 261.2 m long PVC pipeline (measured from  $x = 0$  m), an outlet steel pipeline (10.4 m long) and a basement reservoir (see test rig operation scheme in Fig. 1). The inner diameter of the steel pipes was 206 mm and of the PVC pipeline it was 235.4 mm. The PVC and steel pipe-wall thickness were 7.3 mm and 5.9 mm, respectively. The total PVC pipe length including inlet supply section and pipe bridge was  $L = 275.2$  m. The water supply into the PVC pipe system was regulated by an inlet control valve (automatically operated butterfly valve) at position  $x = -29.9$  m in the steel supply pipeline (Fig. 2). The PVC pipeline outlet was connected to the outlet steel pipeline which had a hand-operated control valve, service valve (closed during water hammer tests) and a small-size DN25 ball valve (used only for water hammer tests) at its downstream end. The PVC pipeline consisted of six straight pipes that were connected by four 90 degree elbows and one horizontal turning bend (Fig. 2). The PVC pipeline was fixed to the concrete floor by metal anchors and supported with wooden bricks to reduce sagging. In the upstream part of the PVC pipeline a pipe bridge elevated 1.3 m above the pipeline axis was supported by a tube-frame; the rest of the PVC pipeline was horizontal. It was attempted to structurally restrain the pipe system as much as possible to suppress FSI effects. However it was hard to fix the most downstream free bend; at this point a very heavy mass was attached using a rope to decrease vertical movements (axial movement was restrained by anchors and supports (Bergant *et al.* [14]).



**Fig. 1** Layout of the large-scale PVC pipeline apparatus at Deltares, Delft, The Netherlands

The pressure along the pipeline was measured by ten pressure transducers located at distances, measured relative to the inflow ( $x = 0$  m) into the horizontal PVC pipe test section, of  $-27.7$  m ( $p_{uv}$ ),  $-14.0$  m ( $p_u$ ),  $1.6$  m ( $p_1$ ),  $46.6$  m ( $p_3$ ; two pressure transducers (top and bottom)),  $111.7$  m ( $p_5$ ),  $183.7$  m ( $p_7$ ),  $206.8$  m ( $p_8$ ),  $252.7$  m ( $p_9$ ), and  $269.5$  m ( $p_{dv}$ ) (see Fig. 2). The valve generating the water-hammer event was placed  $271.3$  m downstream from the horizontal PVC pipe section inlet. All pressure transducers were of the strain-gauge type with a resonance frequency of  $10$  kHz. The accuracy of all transducers was  $0.1\%$  of their maximum range. They were all installed flush-mounted as good as possible. Water flow rates were measured using two electromagnetic flow meters at in- and outlet of the PVC pipe test section. This paper focuses on water-hammer tests that have been performed in the PVC pipeline by rapid closing of the small-size ball valve at the outlet (Fig. 1). The objective of these tests was to investigate the dynamic behaviour of the viscoelastic PVC pipeline. In each test, the sampling rate of each measured quantity was  $f_s = 1000$  Hz. All valve closure experiments were performed with the same initial steady discharge of  $Q_0 = 0.007$  m<sup>3</sup>/s.

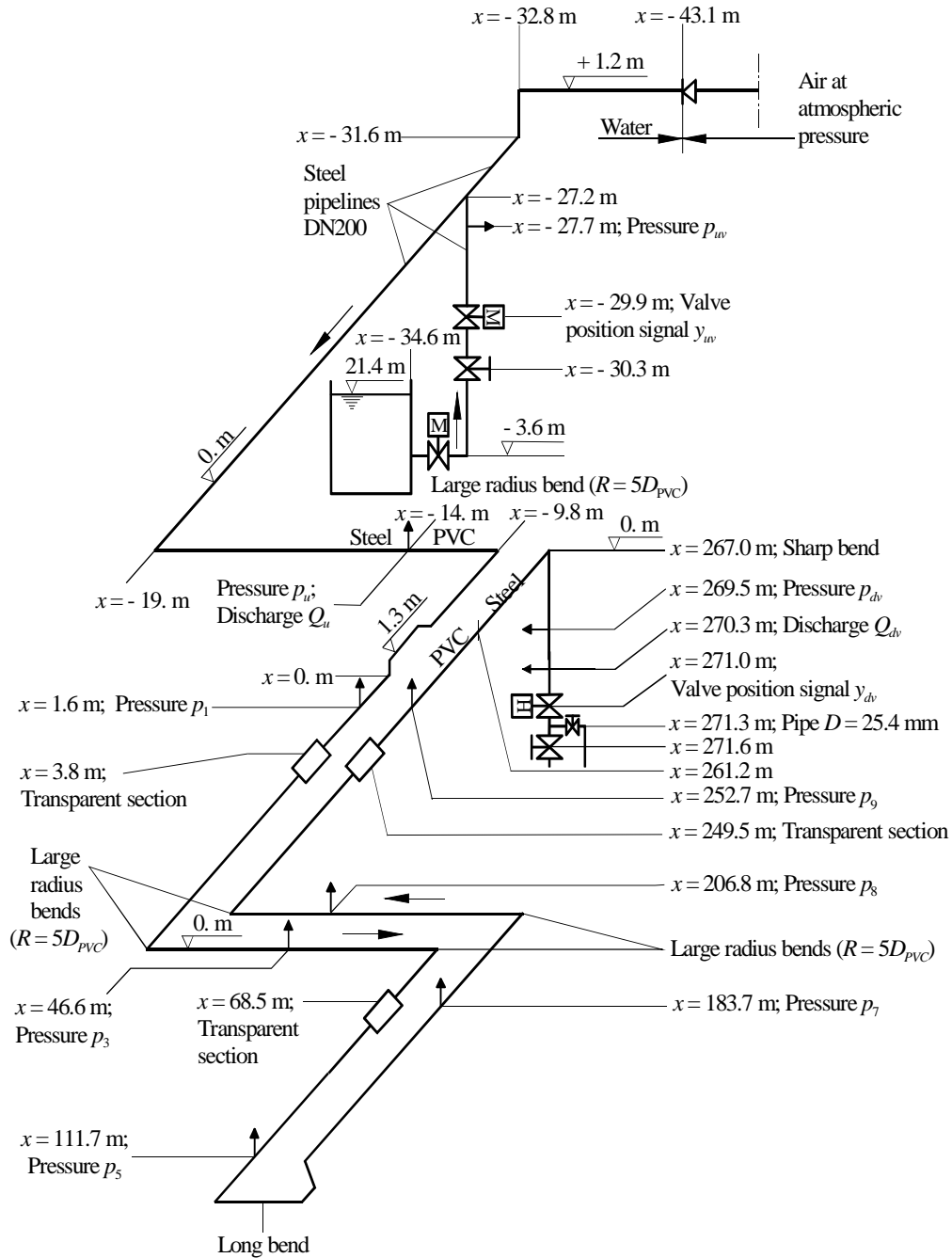


Fig. 2 Layout of instruments in the large-scale PVC pipeline apparatus

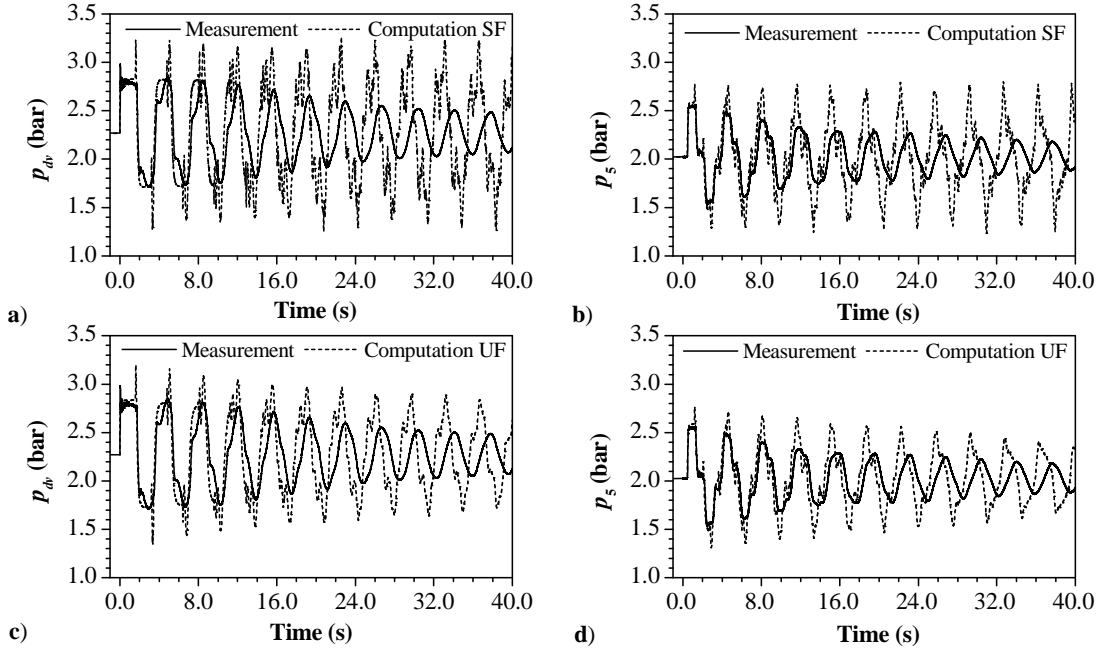
#### 4. Case Study

The effect of unsteady skin friction and viscoelastic behaviour of the pipe-wall on the transient wave form is presented for the case of rapid closure of the small-size ball valve positioned at the downstream end of the PVC pipeline apparatus as shown in Figs. 1 and 2. The horizontal PVC pipe axis at elevation  $z = 0$  m was taken as the horizontal datum level (Fig. 2). The static head in the supply tank was  $H_T = 21.4$  m, the initial flow rate in pipe was  $Q_0 = 0.007$  m<sup>3</sup>/s and the measured water temperature was  $T_w = 18.6$  °C. In numerical simulation the pipeline system included inlet steel pipe (length 20.6 m) with dead end steel pipe (length 15.9 m) at T-junction ( $x = -27.2$  m, Fig. 2), PVC pipeline with pipe-bridge (full length 275.2 m) and outlet steel pipe (length 10.4 m). The Darcy-Weisbach coefficient in the pipeline system was  $f = 0.015$  (measured), the wave speed in the steel pipe was  $a = 1239$  m/s (calculated using Eq. (3)) and in the PVC pipeline  $a_{PVC} = 348$  m/s (measured). The Young's modulus of elasticity for the steel pipe was  $E = 210$  GPa and for the PVC pipe  $E_{PVC} = 2.9$  GPa. The creep-compliance function of the PVC pipe material was not available and therefore was calibrated on measured transient pressure traces.

Before this calibration the role of unsteady friction was carefully investigated. The viscoelastic effects were ignored at this stage the simulation. Given the value of Ghidaoui *et al.* parameter  $P = 28$  (Eq. (9)), it was assumed that the convolution-based

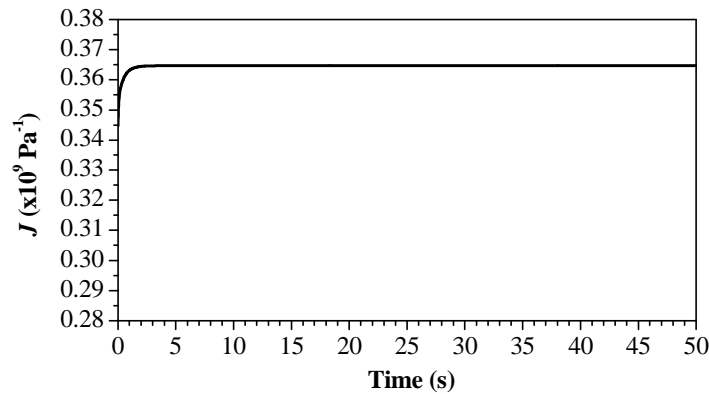


unsteady friction model would be sufficiently accurate to predict turbulent viscosity during the transient event considered in this case study. The smooth-pipe turbulent Vardy-Brown weighting function formulae were used (Eqs. (7) and (8),  $\mathbf{Re}_0 = 38,000$ ). The effect of unsteady friction is evident from the comparison of calculated and measured pressure heads at the closing valve ( $p_{dv}$ ;  $x = 269.5$  m;  $z = -2.5$  m) and close to the long PVC turning bend ( $p_5$ ;  $x = 111.7$  m;  $z = 0$  m) in Fig. 3. The simulation results clearly show that the steady friction model (SF) underestimates the damping observed in the experimental results. The unsteady friction model still underestimates the damping and dispersion but to a lesser extent than the viscoelastic behaviour of the pipe-wall.



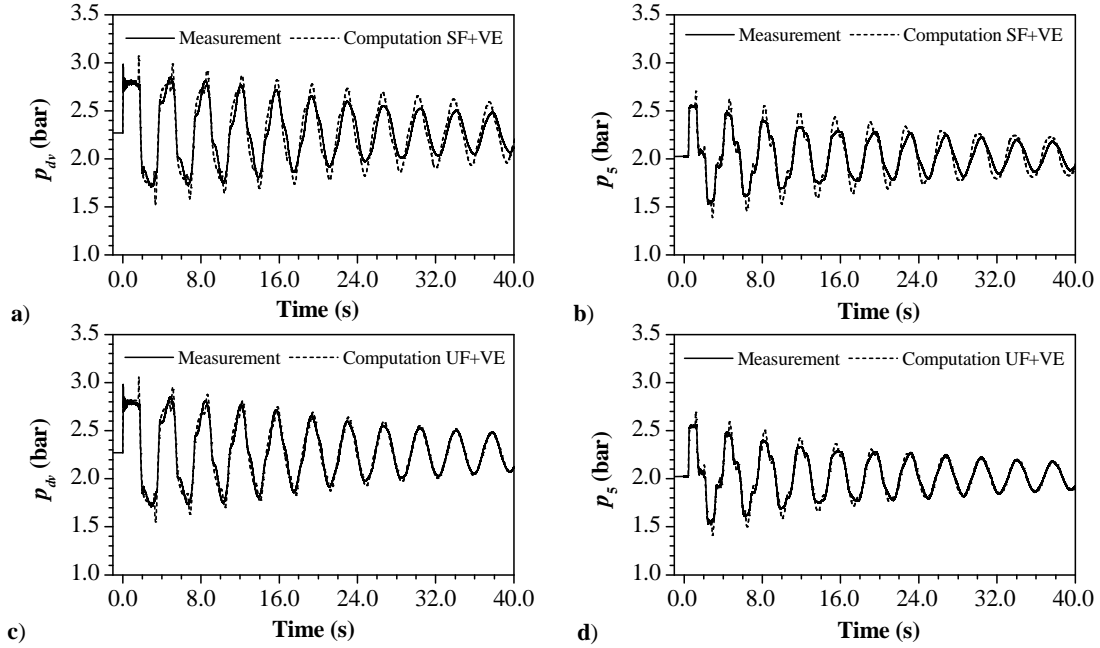
**Fig. 3** Steady and unsteady friction model computations in assumed elastic pipeline system (Fig. 2). Comparison of pressures at the valve ( $p_{dv}$ ;  $x = 269.5$  m;  $z = -2.5$  m) and at the long bend ( $p_5$ ;  $x = 111.7$  m;  $z = 0$  m).

The creep-function (Eq. (6)) was calibrated for the turbulent flow conditions ( $\mathbf{Re}_0 = 38,000$ ) considering unsteady frictional effects in the simulation. The parameters  $N$ ,  $J_k$  and  $\tau_k$  of the model were calibrated using an optimization algorithm that minimizes the difference between the measured and calculated heads at one pressure transducer (pressure at the valve  $p_{dv}$  (see Fig. 2) was used herein). The MATLAB function “lsqnonlin”, which solves nonlinear least-squares problems, was used to carry out the minimization. In this MATLAB tool, the objective function is a vector whose elements are the absolute differences between measurement and calculation at the valve. To avoid having numerous calibration runs and decrease the optimization time, an appropriate set of varied parameters (retardation time  $\tau_k$ ) was selected. The creep-compliance coefficients  $J_k$  were calibrated for the sample period of 50 s, thus obtaining the creep function as in Fig. 4. The creep function soon achieved the limit value (after around two seconds) implying that only two Kelvin-Voigt elements were enough to model viscoelastic property of the PVC pipe. The calibrated values were  $J_k = \{0.0848 \times 10^{-10}, 0.1136 \times 10^{-10}\}$  1/Pa and  $\tau_k = \{0.05, 0.5\}$  s;  $k = 1, 2$ . The elastic component of the creep-compliance function corresponds to an elastic Young’s modulus  $E_{PVC} = 2.9$  GPa i.e.  $J_0 = 1/E_{PVC} = 3.448 \times 10^{-10}$  1/Pa.



**Fig. 4** Creep compliance function ( $J$ ) for PVC pipeline system (Fig. 2).

Figures 5(a) and 5(b) show the viscoelastic effect combined with steady friction (SF+VE) on the pressure response at the valve and near the long bend. The lower phase velocity, larger dispersion and higher attenuation compared to Fig. 3 are clear but not yet sufficient to match the measured results perfectly. Figures 5(c) and 5(d) show the comparison of measured pressures with the viscoelastic simulation with unsteady friction. Again the smooth-pipe turbulent Vardy-Brown weighting function formulae were used (Eqs. (7) and (8),  $Re_0 = 38,000$ ). The numerical results in Figs. 5(c) and 5(d), when compared with the results in Figs. 5(a) and 5(b), show that the addition of unsteady friction (UF+VE) has positive effect on the pressure response. This effect is more pronounced for damping than for dispersion. Previous investigations of pressure transients in PVC pipes (Soares *et al.* [15]) showed that the effect of the viscoelastic behaviour of the pipe wall was dominant over unsteady friction. It may now be concluded that these effects are dependent on the type of transients, the pipe material and the system configuration. In addition, the two computed pressure spikes observed at the beginning and the end of first half-cycle of transient event refer to the downstream and upstream steel pipes, respectively. These high frequency spikes which quickly attenuate should have a minor effect on pressure transients. Meanwhile, the authors are currently investigating the existing small discrepancies and the first results indicate that it is due to FSI behaviour of the outlet steel pipe with sharp elbow.



**Fig. 5** Steady and unsteady friction model computations in PVC pipeline system (Fig. 2). Comparison of pressures at the valve ( $p_{dv}$ ;  $x = 269.5$  m;  $z = -2.5$  m) and at the long bend ( $p_s$ ;  $x = 111.7$  m;  $z = 0$  m).

## 5. Conclusions

This paper investigates the effects of pipe-wall viscoelasticity on water-hammer pressures in a large-scale pipeline apparatus made of polyvinyl chloride (PVC). Tests have been conducted in a reservoir-pipe-valve system with a 275.2 m long DN250 PVC pipeline. Rapid closure of a manually operated ball valve at the downstream end generated water hammer. Computed results were compared with measured results. Calibrated creep functions based on optimization in conjunction with an inverse hydraulic transient solver with unsteady friction have been obtained and then used in the simulations. It was shown that the incorporation of both unsteady friction and viscoelastic pipe wall mechanical behaviour in the hydraulic transient model contributed to a favourable fitting between numerical results and observed data.

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## Nomenclature

$A$	Cross-sectional flow area [ $m^2$ ]	$y$	Valve opening position [deg]
$a$	Wave speed [m/s]	$z$	Elevation [m]
$A^*, B^*, \kappa$	Vardy-Brown weighting function coefficients [-]	$\alpha$	Parameter representing the axial pipe constraints [-]
$D$	Internal pipe diameter [m]	$\varepsilon_\varphi$	Total circumferential strain [-]

$E$	Modulus of elasticity of pipe-wall material [Pa]	$\eta_k$	Dashpot viscosity [Pa s]
$e$	Pipe-wall thickness [m]	$\nu$	Kinematic viscosity [ $\text{m}^2/\text{s}$ ]
$f$	Darcy-Weisbach friction coefficient [-]	$\rho$	Mass density of liquid [ $\text{m}^3/\text{s}$ ]
$f_s$	Sampling frequency [Hz]	$\tau$	Dimensionless time ( $\tau = 4vt/D^2$ ) [-]
$g$	Gravitational acceleration [ $\text{m}/\text{s}^2$ ]	$\tau_k$	Retardation time in Kelvin-Voigt model [s]
$H$	Piezometric head, head [m]	<i>Subscripts</i>	
$h_f$	Frictional head loss per unit length [-]	$dv$	Downstream valve
$J$	Creep-compliance function [1/Pa]	$e$	Elastic
$J_k$	Kelvin-Voigt model parameter [1/Pa]	$ret$	Viscoelastic retardation
$J_0$	Instantaneous creep-compliance [1/Pa]	$T$	Tank (reservoir)
$K$	Bulk modulus of elasticity of liquid [Pa]	$u, uv$	Upstream, upstream valve
$P$	Ghidaoui <i>et al.</i> ratio [-]	0	Steady-state (initial) conditions
$p$	Pressure [Pa]	<i>Abbrev.</i>	
$Q$	Discharge (flow rate) [ $\text{m}^3/\text{s}$ ]	FSI	Fluid-structure interaction
$R$	Inner pipe radius [m]	MOC	Method of characteristics
<b>Re</b>	Reynolds number ( <b>Re</b> = $(Q/A)D/\nu$ ) [-]	PVC	Polyvinyl chloride
$T_w$	Water temperature [ $^{\circ}\text{C}$ ]	SF	Steady skin friction
$t, t^*$	Time [s]	UF	Unsteady skin friction
$W$	Weighting function [-]	VE	Viscoelasticity
$x$	Axial distance [m]		

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**PREVIOUS PUBLICATIONS IN THIS SERIES:**

Number	Author(s)	Title	Month
II-47	M.H.A. van Geel C.G. Giannopapa B.J. van der Linden J.M.B. Kroot	Development of a blood flow model including hypergravity and validation against an analytical model	Sept. '11
II-48	M.H.A. van Geel C.G. Giannopapa B.J. van der Linden	Development of a blood flow model and validation against experiments and analytical models	Sept. '11
II-49	F.A. Radu A. Muntean I.S. Pop N. Suciu O. Kolditz	A mixed finite element discretization scheme for a concrete carbonation model with concentration-dependent porosity	Sept. '11
II-50	M.A.T. van Hinsberg J.H.M. ten Thijsse Boonkamp F. Toschi H.J.H. Clercx	On the efficiency and accuracy of interpolation methods for spectral codes	Sept. '11
II-51	A. Bergant Q. Hou A. Keramat A.S. Tijsseling	Experimental and numerical analysis of water hammer in a large-scale PVC pipeline apparatus	Oct. '11