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Abstract Traffic network disruptions lead to a significant increase in travel time and hence transportation costs. An efficient routing model requires a good description of the congestion dynamics. One of these dynamics is the spillback effect, i.e. congestion propagation to upstream links. This leads to a secondary delay in the upstream road where the speed of the vehicles in the upstream road decreases while increasing the travel time. In this paper, we analyze the influence of considering or ignoring the spillback effect in dynamic routing decisions. For this, we model the stochastic dynamic routing problem with travel-time-dependent stochastic disruptions as a discrete time finite horizon Markov Decision Process (MDP). Then, we incorporate the spillback effect into the MDP formulation. To analyze the effect of the spillback, we also formulate the MDP ignoring the spillback effect. Comparing both results shows how much delay can be avoided by considering the spillback information. To reduce computational time for large network sizes, we consider a stochastic dynamic programming algorithm considering the dynamic and stochastic traffic and spillback information for a limited part of the network. We also provide online and static algorithms mostly used in practice for the routing decisions. We use a test bed of different network structures with different levels of disruption rates and spillback rates. Then, we compare the solution quality and the computational time of different strategies in case of the spillback effect in the network.

Keywords Dynamic shortest path problem · dynamic programming · spillback effect

1 Introduction

In traffic networks, travelers experience disruptions due to accidents, road closures and road bottlenecks. These dynamic disruptions in traffic networks cause a drastic increase in travel times and decrease the probability of being on-time at the destination. Dynamic and stochastic routing algorithms are emerging to reduce the impact of these disruptions by taking into account the stochastic and dynamic nature of the travel times using real-time and probabilistic information. However, in case of a disruption, an efficient routing model also requires a good description of the congestion dynamics. One of these dynamics is the spillback effect, a congestion propagation to an upstream road. When there is a congestion at a route, due to the limited capacity, the queue at this route spills back to the upstream roads in time. In this phenomenon, a queue on a downstream road affects the possible output rate of the upstream roads.

In the literature, several studies have been done to analyze the impact of spillback effect on travel time variability (Gentile et al 2007, Knoop et al 2007, Knoop 2009 and Osorio et al 2011). These studies show that the travel time distributions and the patterns obtained with and without modeling the spillback phenomenon are considerably different, especially when the congestion level is high. For instance, Knoop (2009) shows that when spillback effects are not considered, the delay increases significantly. The high impact of the spillback phenomenon in highly disrupted networks provides an opportunity to analyze the effect of the spillback in dynamic routing decisions.

Furthermore, several studies focus on the effect of spillback modeling in adaptive routing. Knoop et al (2008) investigates the value of considering the spillback information in fixed route choice and adaptive route choice models via a simulation model. They show that the effect of not considering the spillback information is higher in fixed route choice models. In adaptive routing as the drivers choose alternative routes, the spillback effect decreases due to lower densities at downstream links. In Huang and Gao (2012) and Huang (2012) the effect of spillback is considered implicitly by modeling travel times as a multivariate normal distribution with random coefficient of correlations. In these studies it is shown that the correlations between link travel times decrease with temporal and spatial distances. When link dependency is not taken into account, travelers underestimate the risk of vulnerable links and hence delays increase.

In this paper, we analyze the influence of considering or ignoring the spillback effect on the quality of routing decisions for dynamic shortest path problems. For this, we model the dynamic shortest path problem with travel-time-dependent stochastic disruptions as a discrete time finite horizon Markov Decision Process (MDP). We assume that the disruption status of the vulnerable links in the network changes as we travel along the current road and depends on the travel time of the current road. We denote this as the travel-time-dependency. We incorporate the spillback effect into the MDP formulation. To analyze the effect of the spillback, we also formulate the MDP ignoring the spillback effect. Comparing both results shows how much delay can be avoided by using the spillback information.

We model the spillback effect by using the simplified Kinematic Wave Model (KWM) (Lighthill and Whitham 1955, Newell 1993). We find an approximate value for the shockwave speed which is the rate of decrease in speed of the vehicles in the upstream roads. Then, we integrate the rate of decrease in speed to the state transition probability functions in the MDP formulation. We choose to use KWM as the queue dynamics are realistic and the computation is efficient(Knoop 2009).

This paper analyzes the effect of considering or ignoring the spillback information also for different routing algorithms. First, we provide the optimal algorithms considering the spillback model or ignoring it. However, for increasing network size and disruption levels, the computational complexity increases causing the MDP to become computationally intractable. Therefore, to reduce the state-space, we use a hybrid dynamic programming algorithm using the detailed disruption and spillback information for a limited part of the network only (Sever et al 2013). We also provide online and static routing algorithms mostly used in practice. We use a test bed of different network structures with different levels of disruption rates and spillback rates. Then, we compare the solution quality and the computational time of different algorithms in case of the spillback effect in the network.

The main contributions of this paper are as follows:

1. We model the dynamic shortest path problem as a discrete time finite horizon Markov Decision Process. We also model the spillback effect by using the well known Kinematic Wave Model. We explicitly integrate the spillback effect into the MDP formulation by modifying the state transition function. Using this framework, we model a stochastic dynamic programming approach using dynamic disruption and spillback information for a limited part of the network. We also model an online and an offline routing algorithm that are mostly used in practice.
2. We analyze the value of considering the spillback information on the dynamic routing decisions by comparing the algorithms considering the spillback effect and ignoring the spillback effect. Numerical results show that considering the spillback effect in the dynamic routing decisions significantly improves the solution quality for the networks with higher number of vulnerable links. Moreover, the hybrid dynamic programming approach with the disruption and the spillback information for the limited part of the network, significantly reduces the computational time while providing on average significantly higher solution quality than the full information model that ignores the spillback effect.

The structure of the paper is as follows. Section 2 provides the details of the spillback model and Section 3 describes our modeling framework based on dynamic programming. Section 4 discusses the offline, online and dynamic programming algorithms within this framework in detail. In Section 5, the experimental design, the numerical results and the important insights are discussed. In Section 6, we conclude the paper by providing an overview of the results and future directions.

2 Modeling the Spillback Effect

In disrupted networks with limited capacities, spillbacks occur when the growing queues at the downstream links block the departures from the upstream link. Spillback is the phenomenon that a queue on a downstream link affects the possible output volume of the upstream link or links connected to it. In the traffic theory literature, several analytical models are used to analyze the spillback dynamics: finite capacity queueing models (Jain and Smith 1997, Van Woensel and Vandaele 2007, Osorio and Bierlaire 2009, Osorio et al 2011) and kinematic wave model (Ziliaskopoulos 2000, Skabardonis and Geroliminis 2005, Gentile et al 2007, Knoop et al 2007, Hoogendoorn et al 2008, Knoop et al 2008, Knoop 2009).

In this paper, we choose to use the kinematic wave model (KWM) as it is widely accepted in the literature for modeling the spillback effect where the evolution of queues are modeled in a realistic manner and it is computationally efficient and simple (Knoop et al 2007, Knoop 2009, Qian et al 2012). Furthermore, we formulate the problem in terms of a Markov Decision Process (MDP) where we use average travel times as inputs to the model. So, KWM is more suitable for our model as it is a first-order-model using average traffic variables such as average flow and average speed on a road.

KWM is based on a wave phenomenon where it is applied to highway traffic flows (Lighthill and Whitham 1955, Newell 1993). Daganzo (1994) and Daganzo (1995) also explain the kinematic wave theory for traffic flow as a discretized cell transmission model which can be used to predict the evolution of traffic over time and space. The theory states that the traffic states are separated by boundaries. The speed at which these boundaries propagate can be computed using the shock wave speed. The shock wave speed is the rate of change in the speed of vehicles at the upstream links. Figure 1 illustrates that at the downstream of the road ("part b") there is a higher density of vehicles due to a disruption. Due to the limited capacity of the road and the outflow of the upstream road ("part a"), the density will move to the upstream road creating a shockwave between the boundary of the roads. This shockwave speed moves in the opposite direction of the traffic slowing down the vehicles at the upstream. We consider this type of spillback effect where the shockwave occurs between two links.

To use the shock wave speed theory, it is assumed that the number of vehicles is conserved and the flow of vehicles depends on the density in a link. The rate of change is formulated by the ratio of the difference between the flows of vehicles at these links to the difference between the densities of the two links given

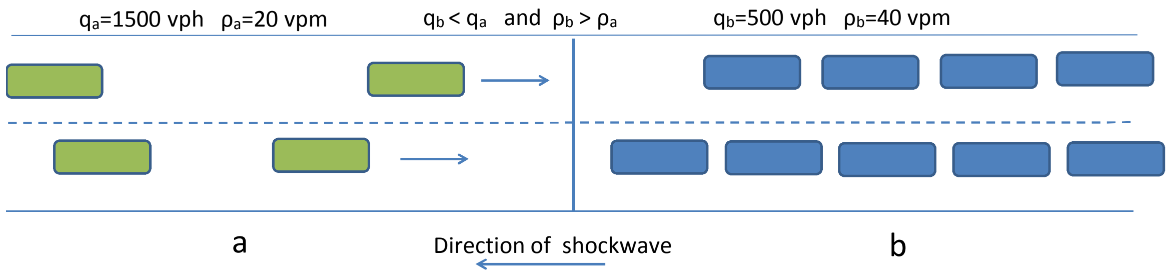

Fig. 1: Shockwave in Traffic

in Equation (1).

$$\omega = (q_d - q_u)/(\rho_d - \rho_u) \tag{1}$$

q_i is the vehicle flow at link i (where u is the upstream link and d is the downstream link) and ρ_i is the density of link i. We only make routing decisions decisions at the nodes and we approximate this equation using only the travel time and the distance, which are the inputs to the MDP formulation. For simplicity, we use first-order-models based on average values and assume that the flow of vehicles at non-congested state is the same for all links. Considering the fundamentals of traffic theory (Daganzo 2006), we use the equation below: (v_{ik}: speed at link i at stage k, v_{fi}: free flow speed at link i, L_i: length of the link i, t_{ik}: the observed travel time to travel link i at stage k and t_{fi}: travel time during the free flow speed at link i)

$$\rho_i = \frac{q_i}{v_{ik}} \tag{2}$$

When we replace variable ρ in Equation (1) with the Equation (2), we obtain the rate of change in the speed of the upstream link as:

$$\omega = \frac{(q_d - q_u)}{(\frac{q_d}{v_{dk}} - \frac{q_u}{v_{uk}})} \tag{3}$$

The above formulation can be easily calculated with traffic data. However, in this paper the input to the model is the average travel time or average speeds for each link. Thus, we investigate relating the traffic flow to the average speed and to the average travel time. It is intuitive that increasing congestion on a link results in a decrease in the average speed on that link. When we consider a congestion model using a discrete Markov process with a link capacity, the rate of change in the traffic flow on a link is defined as the ratio between the speed of the link at that stage and the free flow speed: $\frac{v_{ik}}{v_{fi}}$ (Jain and Smith 1997 and Wang et al 2010).

By using the fundamentals of the traffic theory (Daganzo 1994, Rakha and Zhang 2005), the average travel time observed at stage k for a link i during the congestion can be computed as the ratio of the length of the link, L_i and the average speed during traveling on the link, v_{ik}:

$$v_{ik} = \frac{L_i}{t_{ik}} \tag{4}$$

Using these equations, Equation (3) is approximated to: (t_{fd} and t_{fu}: travel time during the free flow speed at the downstream and the upstream link respectively; L_d: length of the downstream link)

$$\omega \approxeq \frac{(\frac{t_{fd}}{t_{dk}} - \frac{t_{fu}}{t_{uk}}) * aL_d}{(a * t_{fd} - t_{fu})} \tag{5}$$

"a" is the ratio of the distance of the upstream to the downstream link ($\frac{L_u}{L_d}$). So when $a \geq 1$, the spillback effect is higher at the upstream link because the queue at the shorter and congested downstream link spill to the upstream link at a higher rate.

The rate of change in travel time for the upstream link is then the ratio of the travel time observed at stage k to the new travel time with modified speed due to the spillback effect becomes:

$$\gamma_r \approxeq \frac{t_{dk} * (a * t_{fd} - t_{fu})}{t_{dk} * (a * t_{fd} - t_{fu}) + (t_{fd} * t_{uk} - t_{fu} * t_{dk})} \tag{6}$$

Osorio et al (2011) argues that KWM captures average deterministic traffic conditions. Our problem is stochastic and transient. So, in our model, we use KWM to determine the shock wave speed for each disruption state. Then, we incorporate the rate of change in travel time due to the spillback to our probability transition matrix in the MDP formulation. In this way, we modify the spillback model such that it is state dependent and transient.

To model the link dependency and the approximate effect of propagation from downstream links, we use a linear relationship between the one-step-transition function for the upstream links at stage k. We consider spillback phenomenon as an effect that increases the probability of having disruptions. Because of this, we incorporate the spillback effect in the probability transition function.

We assume that the downstream links, affecting the travel time of an upstream link, i.e. r, are limited to the links that are two-links ahead of link r which is denoted by the neighborhood vector: $Zone(r)$. Note that in Huang (2012), it is shown that the correlations between link travel times decrease with temporal and spatial distances.

We also define a constant α to control the rate of the propagation which can change from network to network. We calculate the ratio of the increase in unit-time-transition probability for the vulnerable link r, (β_r) by rate of the change in the travel time of upstream link calculated using Equation (7)(if there is a decrease in speed, otherwise $\beta_r = 0$):

$$\beta_r = max(0, \gamma_r) \tag{7}$$

Let $p^r_{u,u'}(k)$ denote the one-stage disruption transition probability between any two disruption levels for vulnerable arc r, $p^r_{u,u'}(k) = P\{\hat{D}_{k+1}(r) = u' | \hat{D}_k(r) = u\}$.

Transition rate to higher disruption scenarios is modeled as:
$\lambda^r_{u,u'}(k) = p^r_{u,u'}(k)(1 + \alpha * \sum_{r' \in Zone(r)} \beta_{r'})$
Repair rate to lower disruption scenarios is modeled as:
$\mu^r_{u,u'}(k) = p^r_{u,u'}(k)/(1 + \alpha * \sum_{r' \in Zone(r)} \beta_{r'})$

The Markov Process is uniformizable if there exists a constant, δ, such that for disruption scenario u given the time at stage k, we have $(\sum_{u';u'>u} \lambda^r_{u,u'}(k) + \sum_{u';u'<u} \mu^r_{u,u'}(k)) \leq \delta$. Then we can define the related discrete-time process with transition probabilities for going into disruption: $p'^r_{u,u'}(k)$:

$$p'^r_{u,u'}(k) = \begin{cases} \lambda^r_{u,u'}(k)/\delta, & \text{if } u' > u, \\ \mu^r_{u,u'}(k)/\delta, & \text{if } u' < u, \\ 1 - (\sum_{u';u'>u} \lambda^r_{u,u'}(k) + \sum_{u';u'<u} \mu^r_{u,u'}(k))/\delta, & \text{otherwise }, \end{cases} \tag{8}$$

3 Model Formulation- Markov Decision Process

We model the Dynamic Shortest Path Problem with the disruptions in the network as a discrete time finite horizon Markov Decision Process (MDP). Consider a traffic network represented by the directed graph $G(N, A, A_v)$ where the set N represents nodes (or intersection of roads), A the set of directed links (arcs) and A_v the set of vulnerable links, potentially in disruption ($A_v \subseteq A$). The number of vulnerable links is R: $R = |A_v|$. Each of these vulnerable links has a known probability of going into a disruption and a known probability of recovery from the disruption.

In this paper, we assume that the travel times are affected by a spillback phenomena for the travelers who are already travelling on the link. The spillback effect is caused by the backward propagation of the congestion at the downstream links towards the upstream links due to limited capacity of the links. We model the spillback effect by the Kinematic Wave Theory and we integrate the spillback effect into the transition function of the MDP.

The travel time on an link follows a discrete distribution (predictable from the historical data) given the disruption level at each link. We assume that the actual travel time for the current link can change due the spillback effect from the downstream links. We only know after traveling which actual travel time has been realized. We learn the information about the disruption status for all the vulnerable links when we arrive at a node. At each node, we make a decision to which node to travel next. Our objective is to travel from the initial node to the destination node with the minimum total expected travel time. We derive the optimal routing decision by the MDP formulation with finite number of stages where stage k represents the number of nodes that have been visited so far from the start node. The end of horizon, i.e. K, is reached by arriving to the destination node.

In this section, we formulate the problem as an MDP.

States

The state of the system at stage k, S_k, is represented by two components:

- The current node at stage k, $i_k \in N$.
- The disruption status information at stage k is denoted by $\hat{D}_k$ which gives the disruption status for all vulnerable links. Each vulnerable link can take any value from the disruption scenario vector U^r. For each link there can be M_r different types of disruption scenarios: $\hat{D}_k(r) \in U^r$: $U^r = \{u^1, u^2, ...u^{M_r}\}$. Note that at each stage, we use a realization of the disruption vector, $\hat{D}_k$.

The state of the system at stage k is then: $S_k = (i_k, \hat{D}_k)$. The final state is reached by arriving to the destination node.

Actions

Our action, x_k, is the next node to travel given the state S_k. We note that each action x_k is an element of the set of all possible actions $\mathcal{X}(S_k)$ which is the neighbor set of the current node. So, the node in the next stage is actually the action decided at the previous state: $i_{k+1} = x_k$.

The Exogenous Information

The disruption status of the vulnerable links change as we proceed to the next stage. The exogenous information consists of the realization of the disruption status of all the vulnerable links. Let $\hat{D}_{k+1}$ denote the disruption status realization that becomes known when stage $k+1$ is reached:

$W_{k+1} = \hat{D}_{k+1}$

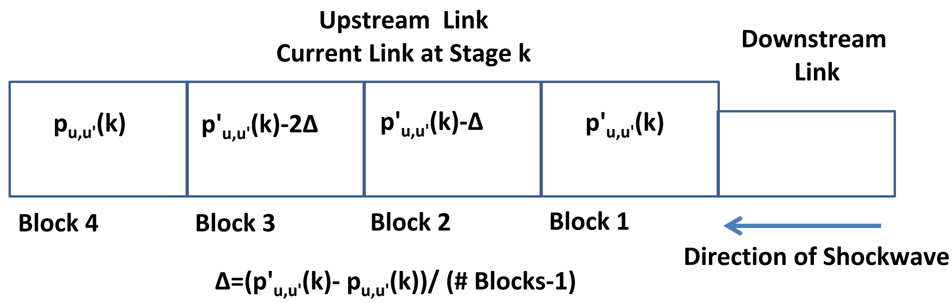

Fig. 2: The Cost Function Considering the Spillback Effect on the Current Link

The State Transition Function

At stage k, if the system is in state S_k, we make a decision x_k and then observe the new exogenous information W_{k+1}. The system transition occurs to a new state S_{k+1} according to the transition function: $S_{k+1}= S^M(S_k, x_k, W_{k+1})$

The state transition involves the following transition functions:

$$i_{k+1} = x_k \tag{9}$$

$$\hat{D}_{k+1} = \hat{D}_{k+1} \tag{10}$$

The Cost Function

The cost of traveling from current node at stage k, i.e. i_k, to the next node, x_k, given $\hat{D}_k$ is denoted by $C(S_k, x_k) = t_{i_k,x_k}(\hat{D}_k)$. In real-life, the travel time on the current link changes due to the spillback effect. The sections of the current link located at different distances from the downstream link are affected from the spillback at a different rate. To model this we divide the current link in blocks of time units. So, that for each block, we compute the relative spillback effect and related increase in travel time. In our model, changes in travel time depend on how the disruption at the vulnerable links in $Zone(r)$ spills back to the current link (Figure 2). We model this behavior as follows: we divide the current link into B blocks. Then, we compute the rate of increase in unit transition probabilities after considering the spillback effect. For each block, we add the weighted effect of the spillback such that the spillback effect increases as the block is located nearer to the congested downstream road.

Let $p'^{r}_{u,u'}(k)$ denote the unit-time-transition probability between any two disruption levels for vulnerable arc r after considering the spillback effect from the downstream links at its zone. The immediate traveling cost is also dependent on the disruption status of its zone due to the spillback effect: $C(S_k, x_k) = C(S_k, x_k|\hat{D}_k(Zone(r))) = t_{i_k,x_k}(\hat{D}_k|\hat{D}_k(Zone(r))$

The cost function becomes:

$$C(S_k, x_k|\hat{D}_k(Zone(r))) = \sum_{j=1}^{B} \sum_{u'=1}^{u'=M_r} (p'^r_{u,u'}(k) - (j-1) * \Delta) * t_{i_k,x_k}(\hat{D}_{k+1}(r) = u') \tag{11}$$

$$\Delta = (p'^r_{u,u'}(k) - p^r_{u,u'}(k))/(B-1) \tag{12}$$

For simplification, the rate of travel time increase in the blocks are only calculated depending on the disruption state of the downstream at stage k. Here, we do not calculate what will happen to the downstream links when we reach the next block in the current link.

The Transition Probability for Travel-Time Dependency

The disruption status vector transits from $\hat{D}_k$ to $\hat{D}_{k+1}$ according to a Markovian transition matrix. We define the transition matrix for a vulnerable link r from stage k to $k+1$ as $\Theta^r(k|S_k, x_k, \hat{D}_k(Zone(r)))$. This transition matrix is travel-time-dependent: the probability of being in the disruption status of the next stage D_{k+1}, depends on the travel time between the current node at stage k and the next node at stage $k+1$ given the disruption status at stage k and the ones in the relevant zone. As its is discussed in the cost function, we include the effect of spillback on the current link. The travel time for the current link after calculating the effect of spillback is denoted as $C(S_k, x_k|\hat{D}_k(Zone(r)))$.

$p'^r_{u,u'}(k)$ denotes the one-stage disruption transition probability between any two disruption levels for vulnerable arc r after considering the spillback effect:
$p'^r_{u,u'}(k) = P\{\hat{D}_{k+1}(r) = u^{'}|\hat{D}_k(r) = u\}$.

$\Theta^r(k|S_k, x_k, \hat{D}_k(Zone(r)))$ is the transition vector for a vulnerable arc r from a disruption status realization, $\hat{D}_k(r) = u$, to a disruption status at stage $t+1$, $\hat{D}_{k+1}(r) = u^{'}$:

$$\Theta^r(k|S_k, x_k, \hat{D}_k(Zone(r))) = \sum_{t=1}^{t=t_{max}} P(t_{i_k,x_k}(\hat{D}_k|\hat{D}_k(Zone(r)) = t) \left[p'^r_{u,u'1}(k) \; ... \; p'^r_{u,u'M^r}(k)\right]^t \tag{13}$$

Here t_{max} is determined by the maximum possible travel time of a link given the disruption state and the spillback effect from its zone.

The probability of having the new state S_{k+1} given S_k is then calculated as:

$$P(S_{k+1}|S_k) = \prod_{r=1}^{R} \Theta^r_{u,u'}(k|S_k, x_k, \hat{D}_k(Zone(r)) \tag{14}$$

The Objective Function

The objective is to minimize the expected total travel time from the initial state until the final state:

$$\min_{\pi \in \Pi} \mathbb{E} \quad \sum_{k=1}^{K} C(S_k, \mathcal{X}^{\pi}(S_k)) \tag{15}$$

where π denotes a policy or decision rule and Π denotes the set of all policies.

Bellman Equations

The optimization problem in Equation (15) can be solved using the Bellman Equations:

$$V_k(S_k) = \min_{x_k \in \mathcal{X}_k} C(S_k, x_k | \hat{D}_k(Zone(r))) + \sum_{S_{k+1}} P(S_{k+1}|S_k) V_{k+1}(S_{k+1}) \tag{16}$$

The solution to the MDP formulation gives the optimal solution (Opt_S). We refer the reader the optimal algorithm used in Sever et al (2013).

To investigate the effect of not using spillback in our routing decisions, we also solve the MDP without including the spillback effect. The optimal solution without considering the effect of the spillback is denoted as Opt_{NS}. The cost function becomes: $C(S_k, x_k) = t_{i,x_k}(\hat{D}_k)$. Furthermore, we replace the transition matrix for the vulnerable link r in Equation (13), as:

$$\Theta^r(k|S_k, x_k) = \left[p^r_{u,u'^1} \; p^r_{u,u'^2} \; ... \; p^r_{u,u'^{M^r}} \right]^{C(S_k, x_k)} \tag{17}$$

4 Routing Algorithms

In this section, we describe the algorithms used for the routing decisions with or without considering the spillback effect.

4.1 Dynamic Programming with Two-links Ahead Look Policy with Spillback Effect ($DP(2, H)$)

The number of state variables increases exponentially with the number of disruption levels. Because of this, the MDP faces the curses of dimensionality. To solve large scale problems with many disruption levels, the dynamic programming approach for solving the Bellman's equations becomes computationally intractable. To reduce the computational time, we use the travel-time-dependent probability distributions for a limited part of the network. For the vulnerable links that are far from the current node, it is intuitive to assume that we experience the long run averages without the spillback effect. We use a Dynamic Programming Approach with Two-Links Ahead Policy, $(DP(2, H))$ developed in Sever et al (2013).

$(DP(2, H))$ considers limited online information for each stage. The intuition behind this is that due to the structure of the disruptions (where the probability of experiencing the disruption decreases within time), by the time we arrive to the further links, we will experience the steady-state probabilities. Therefore, we eliminate the computational burden to calculate the transition probabilities for all vulnerable links.

In this policy, we have online information for two-links ahead from the decision node. Therefore, the state space of the hybrid policy for any current node i_k is modified as $S_k = (i_k, \hat{D}_k^{i_k})$ where $\hat{D}_k^{i_k} = \{u_k^{r_{i_k}1}, u_k^{r_{i_k}2}, .., u_k^{r_{i_k}R_{i_k}}\}$ with r_{i_k} which is the vector of the vulnerable links that are two-links-ahead neighborhood of the node i_k and $R_{i_k} = |r_{i_k}|$, $R_{i_k} \leq R$. For the rest of the links, we calculate the memoryless distributions.

Given the state is S_k, a transition is made to the next decision state, $S_{k+1} = (x_k, \hat{D}_{k+1}^{x_k})$. For the $(DP(2, H))$ policy, we use the travel-time-dependent state transition matrix for the vulnerable links for which we have online information. The probability distribution of being in state S_{k+1} from the state S_k given travel-time-dependent transition matrix is the same as the one given in Equation (14) except we only consider the limited part of the transition matrix where only the vulnerable links that are at most two-links ahead from i_k are included. Note that we incorporate the spillback effect into the model by considering the transition probabilities given in (13).

For the links that are beyond the two-links ahead neighborhood of i_k, we use the probability distributions with travel-time-independent memoryless probability. The one-step transition probability for the r^{th} vulnerable link is denoted as $p^r_{u,u'} = P\{\hat{D}_{k+1}(r) = u^{'} | \hat{D}_k(r) = u\}$. Please note that r^{th} vulnerable link is an element of the vector r_i. For the formulation, we use memoryless transition probabilities for the vulnerable links at each node i as:

$$\Theta^r(k|S_k, x_k) = \left[p^r_{u,u'^1} \; p^r_{u,u'^2} \; ... \; p^r_{u,u'^{M^r}}\right] \tag{18}$$

Then, the probability distribution of being in state S_{k-1} with memoryless probabilities is:

$$P(S_{k+1}|S_k) = \prod_{r=1}^{R} \Theta^r_{u,u'}(k|S_k, x_k) \tag{19}$$

4.2 Expected Shortest Path(ESP)

In practice, if there is no real-time information available, only historical information is used to determine the expected shortest path from the start node to the destination node. As we use steady-state probabilities for the calculation of the expected travel time, we do not involve the effect of spillback in our routing decisions. The output of this strategy is a single offline route from the start node to the destination node.

The steady-state probability to be in state u^r for any vulnerable link r is denoted by $P(u^r)$. Please note that $u^r \in U_r$. The expected value of the link r for traveling from node i_k to node x_k is denoted by $\bar{t}_{i_k,x_k}$:

$$\bar{t}_{i_k,x_k} = \sum_{u^r} P(u^r) t_{i_k,x_k}(u^r) \tag{20}$$

For finding the minimum expected shortest path from x_k to the destination node n, $\tilde{V}_{x_k}$, we apply a backward recursion to the following equations:

$$\tilde{V}_{i_k} = \min_{x_k \in N, x_k \in Neighbor(i_k)} \bar{t}_{i_k,x_k} + \tilde{V}_{x_k} \quad \text{where} \quad \tilde{V}_n = 0 \tag{21}$$

4.3 Online Routing Policy ($Online$)

In practice, routing algorithms mostly use real-time information and historical information. For representing these algorithms, we develop an online policy for the dynamic shortest path problems. At each node, we have the complete online information available for the neighbor links of the current node i_k. For the links that are not in this zone, no online information is considered and therefore, the expected values are used based on historical information.

The steady-state probability to be in state u^r for any vulnerable link r is denoted by $P(u^r)$, with $u^r \in U_r$. The expected value of the link r for traveling from node i_k to node x_k is denoted by $\bar{t}_{i_k,x_k}$ and computed by using the Equation (20).

At each decision node i, online information of the one-link-ahead links of the current node plus the expected values of the links until the destination node that are located further from the one-link-ahead-neighborhood are considered to determine x_t. The online route can be found by solving the following equations with a backward recursion:

$$V(S_k, x_k) = \min_{x_k \in N, x_k \in Neighbor(i_k)} C(S_k, x_k) + \tilde{V}_{x_k} \tag{22}$$

For finding the minimum expected shortest path from x_k to the destination node n, $\tilde{V}_{x_k}$, we apply a backward recursion to the following equations:

$$\tilde{V}_{x_k} = \min_{x_{k+1} \in N, x_{k+1} \in Neighbor(x_k)} \bar{t}_{x_k, x_{k+1}} + \tilde{V}_{x_{k+1}} \quad \text{where} \quad \tilde{V}_n = 0 \tag{23}$$

5 Experimental Design

We generate different network instances using a number of dimensions to characterize and construct these instances. The dimensions considered are as follows:

- Network Size: The network size consists of small, medium and large networks with 16, 36 and 64 nodes respectively. The network is designed such that the origin and destination nodes are situated in the top-left corner and bottom-right corner respectively. With this structure, we prevent evaluating unnecessary nodes far from the shortest path. Clearly, this does not limit the applicability of our results, but merely reduces the number of unnecessary calculations to be evaluated.
- Spillback rate: The parameter, α that changes the effect of spillback based on the network type. We use low and high spillback rates for each network. After preliminary experiments for the generated networks, the low spillback rate is set to $\alpha = 1$ and high spillback rate is set to $\alpha = 15$ to reflect a significant difference between low and high spillback rate.
- Disruption Level: A vulnerable link can have 2 and 3 different levels of disruptions. We set the expected travel time for each vulnerable link the same regardless of the disruption level. For this, we adjust the steady state probabilities accordingly.
- Number of Vulnerable links: Vulnerable links are randomly assigned to the network with an iterative process. At each iteration, a shortest-path is found by the current vulnerable links list and a new vulnerable link is assigned on the path of the current shortest path. This continues until we reach the total number of vulnerable links. We have instances with low and high numbers of vulnerable links. Specifically, 20% (low vulnerability) or 80% (high vulnerability) of the least-cost links in the network are labeled to be vulnerable.
- Travel Times: Each link has a randomly selected travel time taken from a uniform distribution $U[1, 10]$. The travel time follows a discrete distribution depending on the number of disruption levels.
- Probability of having disruption in the link: Define a low probability of having disruptions to be between $[0.2 - 0.5)$,and a high probability in the range $[0.5 - 0.8)$.

We define "network type" as the network that has specific network dimensions. For instance, small size network with low number of vulnerable arcs with low disruption probability is a unique network type.

In this paper, we generate 24 different network types with the relevant properties as seen in Table 1. For each of the network type, we randomly generate 25 instances with random travel times and random locations for vulnerable arcs along the actual shortest path.

Table 1: Network Dimensions

Number of Nodes	Number of vulnerable arcs		Probability of having a disruption		Number of disruption levels
	Low	High	Low	High	
16	3	5, 7	[0.2- 0.5)	[0.5- 0.8)	2, 3
36	3	5, 7	[0.2- 0.5)	[0.5- 0.8)	2, 3
64	3	5, 7	[0.2- 0.5)	[0.5- 0.8)	2, 3

For the evaluation of the algorithms, we use an exact evaluation. In the exact evaluation, for each algorithm, we obtain routing policies considering each possible state and with/without the spillback effect. Then, with these pre-determined policies for each instance, we compute the exact value function using the Equation (16) considering that there is a spillback effect in the network . This value gives the expected cost of the algorithm considering all possible states.

Furthermore, to evaluate the reliability of each algorithm, we calculate the variance of travel times among different disruption scenarios. The variance is calculated as follows (Here, S_0 is the disruption state at time 0 at the origin):

$$\begin{aligned}\sigma^2 = &\sum_{S_0} \text{probability}(S_0) * (\text{Cost Alternative Policy}(S_0))^2 \\ &- [\sum_{S_0}(\text{Cost Alternative Policy}(S_0) * probability(S_0))]^2\end{aligned} \tag{24}$$

For each instance, the percentage cost difference(gap) relative to the optimal policy for each routing policy is calculated as follows:

$$\Delta(\%) = \frac{(E[\text{Alternative Policy}]) - E[\text{Optimal Policy}]}{E[\text{Optimal Policy}]} * 100 \tag{25}$$

The algorithms presented in this paper are programmed in Java. The computational results in this section are obtained by using IntelCore Duo 2.8 Ghz Processor with 3.46 GB RAM.

6 Numerical Results

For each network type, we compute the average of the 25 instances. Then, we aggregate these results according to these network dimensions: the disruption rate, the network size, the vulnerability and the spillback rate. In the numerical results, the aggregated results for the expected travel time, the variance, the percentage gap and the computational time are shown for the network size, the vulnerability and the spillback rate.

Effect of the Network Size

To investigate the effect of considering the spillback information on different network sizes, we compare the expected travel time and the variance of the routing algorithms for small, medium and large networks with 16, 36 and 64 nodes respectively. On small size networks, as the alternatives are limited, the effect of ignoring the spillback effect and using the limited information is higher. For instance, Table 3 shows that the percentage gap of the optimal algorithm without the spillback effect (Opt_{NS}) is on average 7.035% worse than the optimal algorithm with the spillback effect (Opt_S) under low disruption rate for small networks. This difference shows that the delay caused by not considering the spillback information in our routing decisions is higher for the small networks.

The performance of $DP(2, H)$ is better than the Opt_{NS}, but still 2.236% worse than Opt_S. This gap is the value of considering only a limited part of the network. Figure 3 shows that as the network size increases, the gap and the variance decrease. Figure 3 shows that the performance difference between the $DP(2, H)$ and the Opt_{NS} and the Expected Shortest Path algorithms also decreases with the increase in the network size. This shows that as the vulnerable links become scattered, the effect of spillback rate in routing decisions decreases compared to the smaller networks. The variance of total travel time decreases for all algorithms as there are more alternatives in large networks. This way, in case of a high effect disruption an alternative lower cost decision is made.

When we compare the Opt_{NS} with the Online and the Expected Shortest Path algorithm, we observe that using the full information by ignoring the spillback effect gives similar performances as static and online algorithms both in small and large size networks. This indicates that considering the spillback information has on average higher impact of the quality of the routing decisions than considering detailed information about the disruption for the whole network.

Table 2: The Performance of the algorithms evaluated with the spillback effect for different network sizes

Network Type	Performance	Low Disruption Rate					High Disruption Rate				
		Opt_S	Opt_NS	DP(2,H)	Online	ESP	Opt_S	Opt_NS	DP(2,H)	Online	ESP
Small Network	Expected Value	26.990	28.514	27.479	29.052	32.022	29.158	30.270	29.207	30.910	32.156
	Variance	2.395	4.125	2.878	4.771	0.111	2.101	6.689	3.389	6.427	0.861
	Δ (%)	n.a.	7.035	2.236	9.456	13.376	n.a.	7.346	1.438	9.271	8.091
	Cpu(sec)	296.819	141.630	0.119	0.001	0.001	297.593	145.830	0.118	0.000	0.000
Medium Network	Expected Value	39.635	40.772	40.185	41.871	43.269	41.595	42.035	41.803	42.211	43.580
	Variance	0.233	0.928	0.646	1.085	0.034	0.144	0.692	0.574	0.868	0.007
	Δ (%)	n.a.	4.029	2.169	6.964	8.310	n.a.	2.127	1.397	2.783	4.028
	Cpu(sec)	1551.290	775.658	0.225	0.003	0.001	1579.452	775.503	0.201	0.001	0.001
Large Network	Expected Value	53.066	54.360	53.507	55.780	55.852	54.546	55.334	54.714	55.203	56.106
	Variance	0.199	1.011	0.551	0.796	0.004	0.180	2.122	0.670	0.642	0.002
	Δ (%)	n.a.	3.462	1.377	5.905	4.493	n.a.	3.310	1.023	1.880	2.137
	Cpu(sec)	4541.214	1453.135	0.523	0.008	0.005	4481.012	1181.773	0.752	0.005	0.004

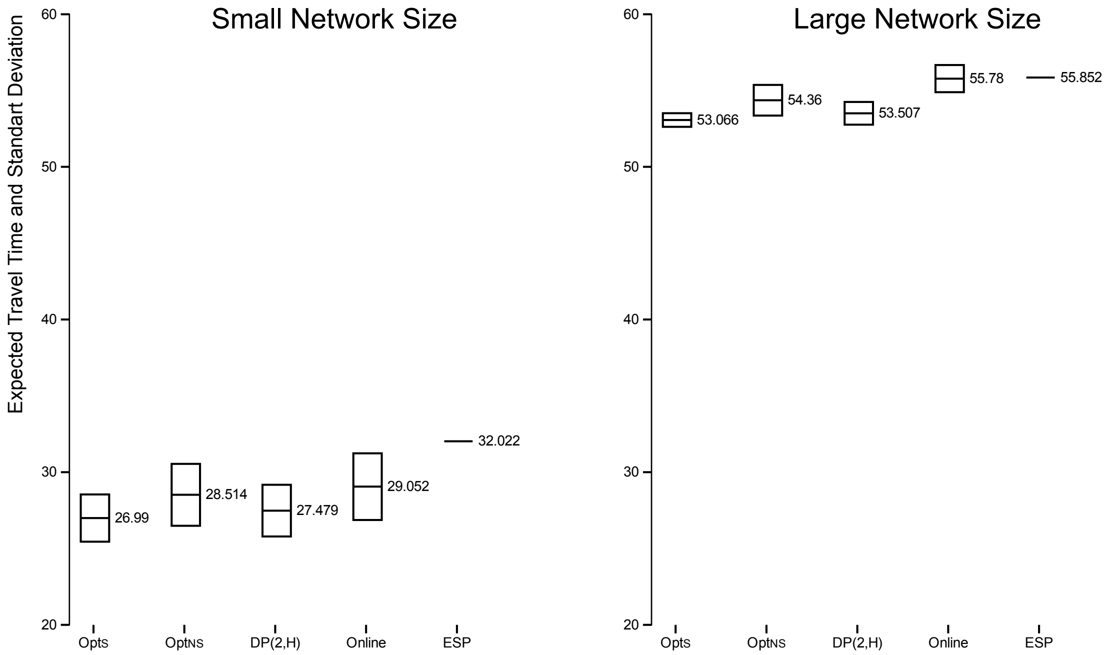

Fig. 3: The expected value and the variance of the algorithms evaluated with the spillback effect for different network sizes

Effect of the Disruption Rate

Tables 2, 3 and 4 show that as the disruption rate increases, the gap between the algorithms in terms of the expected travel time decreases as they choose the similar routes which are generally the risk aversive ones. As the spillback rate increases, the transition probabilities to a higher disruption level increases at a higher rate. Therefore, the algorithms considering the spillback effect in their routing decisions choose the routes with lower expected travel times. Similarly, as the disruption probabilities are higher and vulnerable links are consecutive each other, the algorithms ignoring the spillback effect also choose alternative routes with lower expected travel times.

Though, when we consider the variance, however, due to higher transition probabilities the variance of the routes for each algorithm increases compared to the lower disruption rate case.

Table 3: The Performance of the algorithms evaluated with the spillback effect for high and low vulnerable networks

Vulnerability	Performance	Low Disruption Rate					High Disruption Rate				
		Opt_S	Opt_NS	DP(2,H)	Online	ESP	Opt_S	Opt_NS	DP(2,H)	Online	ESP
Low Vulnerability	Expected Value	38.364	39.503	38.745	39.994	41.281	39.744	40.296	39.883	40.376	41.546
	Variance	0.582	1.327	0.808	1.132	0.011	0.693	2.564	1.172	2.021	0.002
	Δ (%)	n.a.	3.905	1.322	4.936	5.771	n.a.	3.255	0.958	3.011	2.499
	Cpu(sec)	22.785	3.338	0.089	0.000	0.001	22.091	3.454	0.082	0.000	0.001
High Vulnerability	Expected Value	40.919	42.850	41.543	43.727	45.337	43.115	44.046	43.258	44.373	45.549
	Variance	1.182	2.484	1.725	2.941	0.075	0.885	3.570	1.793	3.062	0.482
	Δ (%)	n.a.	5.759	2.022	8.178	8.581	n.a.	4.268	1.230	4.694	4.965
	Cpu(sec)	3534.434	2981.343	0.811	0.006	0.003	3517.527	2166.090	0.923	0.003	0.002

Table 4: The Performance of the algorithms evaluated with the different rates of spillback effect

Spillback Rate	Performance	Low Disruption Rate					High Disruption Rate				
		Opt_S	Opt_NS	DP(2,H)	Online	ESP	Opt_S	Opt_NS	DP(2,H)	Online	ESP
Low Rate	Expected Value	39.615	40.967	40.164	41.972	43.639	41.569	42.332	41.744	42.623	43.860
	Variance	0.893	2.049	1.391	2.196	0.099	0.814	3.168	1.577	2.930	0.059
	Δ (%)	n.a.	4.532	1.933	7.136	8.367	n.a.	3.862	1.244	4.386	3.841
	Cpu(sec)	2125.778	722.740	0.410	0.004	0.002	2084.807	724.695	0.397	0.001	0.001
High Rate	Expected Value	40.179	41.464	40.683	42.496	43.790	41.963	42.761	42.072	42.926	43.927
	Variance	0.992	1.993	1.326	2.238	0.000	0.803	3.167	1.511	2.362	0.521
	Δ (%)	n.a	4.129	1.600	6.841	6.364	n.a.	3.922	1.032	3.742	4.175
	Cpu(sec)	2133.770	857.542	0.434	0.004	0.003	2153.898	677.376	0.422	0.003	0.002

Effect of the Number of Vulnerable Links

The expected travel time and the variance increases as there are more vulnerable links in the network. The impact of taking into account the spillback effect also increases with the number of vulnerable links. For instance, Table 3 shows that the performance gap between the *Opt_S* and the Opt_{NS} increases from 3.905% to 5.759% from low vulnerable to highly vulnerable networks with low disruption rate. This result is intuitive because as there are more vulnerable links consecutive to each other, the transition rates and so the travel time increase due to the spillback effect. Furthermore, the realized travel times on the current link at stage $k+1$ is more different than the observed travel times at stage k due to the higher impact of the spillback effect. Therefore, the algorithms using the spillback information choose risk aversive routes. Figure 4 shows that the online algorithms performance on highly vulnerable networks is much worse than the performance on the low vulnerable networks. The intuition behind this is that the online algorithm uses the observed travel times for the current link and expected travel times for the rest of the vulnerable

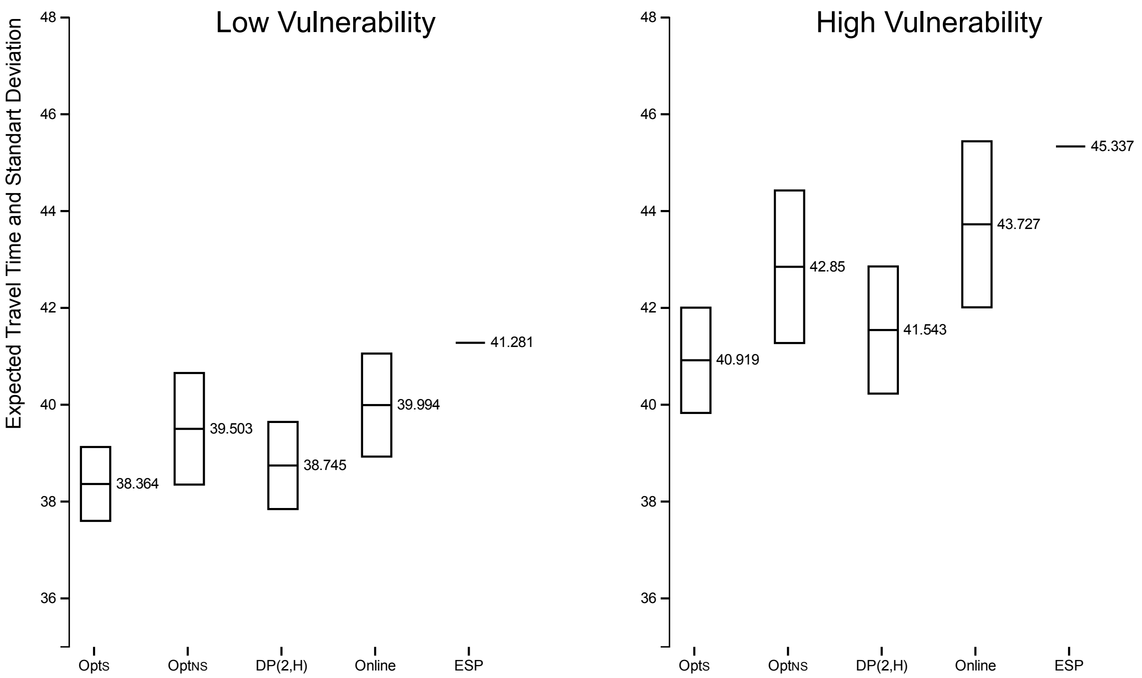

Fig. 4: The expected value and the variance of the algorithms evaluated with the spillback effect for different network sizes

links. When the observed value is much more different from the realized value due to the spillback effect, the solution quality decreases significantly.

The percentage gap beween the Opt_{NS} and the $DP(2, H)$ algorithm with the spillback effect increases as the number of vulnerable links increases (Table 3 and Figure 4). This shows that the value of using the spillback effect is more effective than the value of using full network information for highly vulnerable networks.

For all of the routing algorithms (especially for Opt_{NS} and Online) the variance increases with higher vulnerability. This shows that the route choice and its impacts differ significantly among different network states.

Effect of Spillback Rate

As the spillback rate increases, the gap between algorithms using and ignoring the spillback effect increases slightly (Figure 5). This is because the algorithms considering spillback choose risk aversive routes as the transition rates changes at a higher rate. This increases the expected travel time compared to low spillback rate. Note that the rate of increase is higher for the Online algorithm because it does not consider the spillback information.

Also, the gap between the optimal solution and the solution from the $DP(2, H)$ algorithm with the spillback decreases as both of the algorithms choose the risk aversive routes in case of higher spillback rate (Table 4).

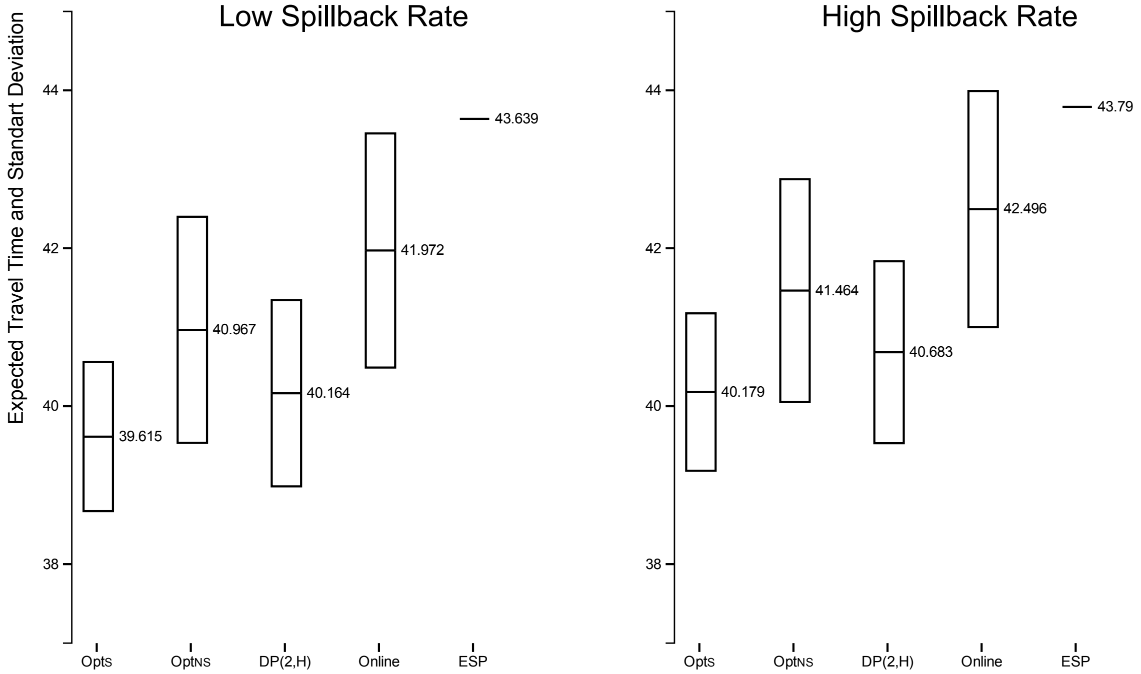

Fig. 5: The expected value and the variance of the algorithms evaluated with the Spillback effect for different network sizes

Computational Time

The computational time for the optimal algorithms via the MDP is increasing exponentially with the increase in the number of vulnerable links and the disruption levels (Table 3 and Table 5). The $DP(2, H)$, on the other hand, is less affected from the state space explosion as it considers limited part of the network while providing good quality of solutions. The expected shortest path and online algorithms are the fastest, but the performance of the routing decisions are much lower than the other algorithms.

7 Conclusions and Future Research

In this paper, we consider dynamic shortest path problems with travel-time-dependent network disruptions and spillback effect. The spillback effect is a congestion propagation to an upstream link. We analyze the effect of ignoring and considering the spillback effect in the dynamic routing decisions with network disruptions. We model the stochastic dynamic routing problem with travel-time-dependent stochastic disruptions as a discrete time finite horizon Markov Decision Process (MDP). We assume that the disruption status of the vulnerable links in the network changes as we travel along the current road. We denote this as the travel-time-dependency. We incorporate the spillback effect into the MDP formulation. The spillback effect is modeled with the kinematic wave model which is an accepted model in traffic theory. To analyze the effect of spillback, we also formulate the MDP ignoring the spillback effect. However, as the size of the network and the disruption levels increase, the computational complexity increases causing the MDP become computationally intractable. Therefore, to reduce the computational time, we use a hybrid dynamic programming algorithm using the detailed dynamic and stochastic traffic information for the limited part of the network. We also provide online and static algorithms mostly used in practice for the routing decisions. These algorithms do not include the spillback effect in their routing decisions.

We use a test bed of different network structures with different levels of disruption rates and spillback rates. Then, we compare the solution quality and the computational time of different strategies in case of the spillback effect in the network. The numerical results show that considering the spillback effect has the highest impact on the solution quality when the network has higher number of vulnerable links. Moreover, the hybrid dynamic programming approach with the disruption and the spillback information for the limited part of the network, significantly reduces the computational time while providing higher solution quality than the full information model that ignores the spillback effect.

Future research involves the analysis of the spillback effect by using other traffic models such as queueing theory to investigate the impact of the traffic models on the routing decisions. The extension of the spillback effect model by using time-dependent and second-order models will reflect the real dynamics even better.

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