Multi-relaxation time viscoplastic modeling of thermorheological simple materials

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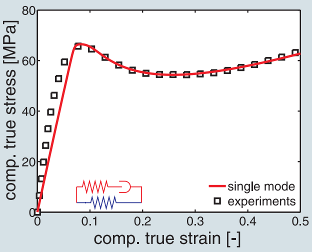
introduction

Over several decades, an in-house constitutive model for glassy polymers was developed [1, 2], that accurately captures the yield and post-yield response of glassy polymers. Despite these capabilities, several problems exist due to the nature of the single-mode modeling:

- inaccurate description of the pre-yield regime.
- incapability to qualitatively predict elastic recovery.

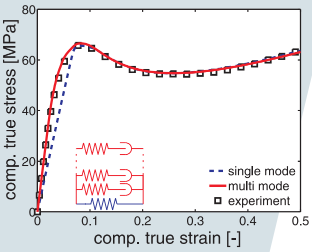


$$\boldsymbol{\sigma} = \kappa(J-1)\boldsymbol{I} + \ldots$$
$$G\tilde{\boldsymbol{B}}_e^d + G_r\tilde{\boldsymbol{B}}^d$$
$$\boldsymbol{D}_p = \frac{\boldsymbol{\sigma}_s^d}{2\eta(\overline{\tau}, p, T, S)}$$
$$\eta = \eta_0 \cdot a_\sigma \cdot a_p \cdot a_{Sa}$$

Figure 1 *Experiment and simulation using the single-mode model for uni-axial compression (left); governing constitutive relations (right).*

solution

An elegant solution to this problem was proposed by [1], hence a multi-mode approach is employed.



$$\boldsymbol{\sigma} = \kappa(J-1)\boldsymbol{I} + \ldots$$
$$\sum_{i=1}^{n}(G_i\tilde{\boldsymbol{B}}_{e_i}^d) + G_r\tilde{\boldsymbol{B}}^d$$
$$\boldsymbol{D}_{p_i} = \frac{\boldsymbol{\sigma}_{s_i}^d}{2\eta_i(\overline{\tau}, p, T, S)}$$

Figure 2 *Experiment and simulation using the multi-mode model for uni-axial compression (left); modified constitutive relations (right).*

This is a straightforward extension to multi-relaxation times, with the special feature that the non-linearity is coupled to the total stress.

material characterization

As model material, polycarbonate was selected, and values for material parameters were taken from literature [2]. The moduli and initial viscosities of the modes were obtained from an intrinsic deformation curve as depicted in figures 1 and 2. The stress up to yield can be described with the well known Boltzmann integral.

By taking into account the non-linearity of the stress in the viscosities, the moduli and corresponding viscosities can be calculated.

$$\sigma(t) = \sum_{i=1}^{n}\left[E_i\dot{\epsilon}\int_{-\infty}^{t}\exp\left(-\frac{\psi-\psi'}{\lambda_i}\right)dt'\right] \quad ; \quad \eta_i = \lambda_i \cdot E_i$$

where

$$\psi = \int_{-\infty}^{t}\frac{dt''}{a_\sigma[\sigma(t'')]} \qquad \psi' = \int_{-\infty}^{t'}\frac{dt''}{a_\sigma[\sigma(t'')]} \qquad a_\sigma = \frac{\overline{\tau}/\tau_0}{\sinh(\overline{\tau}/\tau_0)}$$

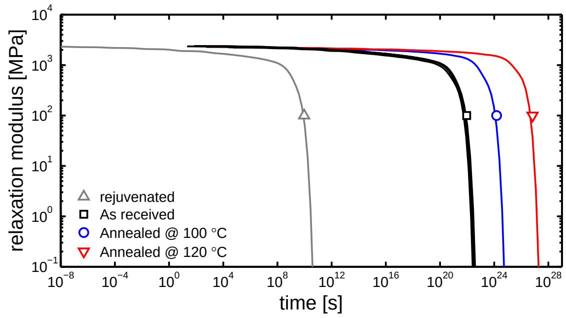


Figure 3 *Relaxation moduli versus time.*

Similar to the single-mode approximation [2], a unique rejuvenated relaxation time spectrum can be defined that accurately describes the pre-yield range in different strain rates and thermal history.

results

The multi-mode model accurately describes the pre-yield regime and is therefore capable of calculating loading and unloading curves for flat-tip indentation (Figure 4).

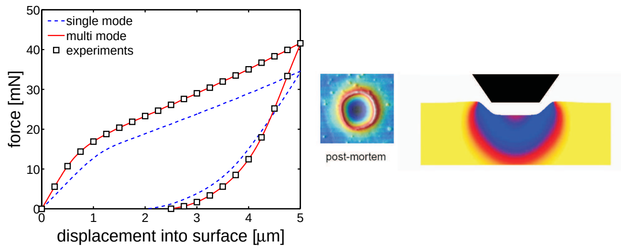


Figure 4 *Flat-tip indentation using the single and multi-mode model.*

conclusion

The new multi-mode implementation is capable of describing the non-linear viscoelastic pre-yield regime accurately, while the post-yield behavior of the single-mode model remains unaffected.

References:

[1] Tervoort, T.A.: *Constitutive modelling of polymer glasses. Finite, non-linear viscoelastic behaviour of polycarbonate* (Eindhoven University of Technology 1996)

[2] Klompen, E.T.J.: *Mechanical properties of solid polymers: Constitutive modelling of long and short term behaviour* (Eindhoven University of Technology 2005)

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