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# ADVANCED SPINDLE RUNOUT-ROUNDNESS SEPARATION METHOD

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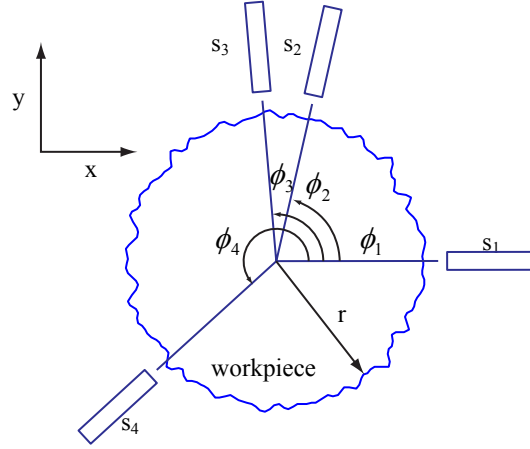
Abstract: a flexible and accurate method for separating spindle error motion and workpiece roundness is presented. The method makes use of three or more displacement probes. Angle measuring probes can also be used. The angular positions of the probes as well as errors in sensor amplification are determined directly from the measurement data and require no extra measurements. The method can be used for real-time runout measurements with nanometer accuracy.

## 1 Introduction

Philips CFT Mechatronics Eindhoven produces among many other things high precision turning lathes that are equipped with hydrostatic bearings. The roundness of turned products is in a large degree determined by the spindle runout at the chisel position. For measuring the performance of the lathe the spindle runout and the achieved workpiece roundness had to be measured at high rotational speed with nanometer accuracy. Traditional measuring techniques only allow cumulative measurement of the spindle runout and of the workpiece geometry. To overcome this problem a new multipoint separation method has been developed.

## 2 Method

There are three unknown parameters: the x-displacement  $x(\phi)$ , y-displacement  $y(\phi)$  and the workpiece roundness deviation  $r(\phi)$ . Therefore a minimum of three probes is required. The use of an extra sensor will show to provide some significant benefits. First we will deduce how spindle displacement and roundness affect the probe reading.



The probe signal to expect can be constructed from the workpiece displacement and the workpiece radius. For the workpiece rotated over angle  $\phi$  the following relation can be derived for a position probe at angle  $\phi_k$ :

$$s_k(\phi) = x(\phi) \cdot \cos(\phi_k) + y(\phi) \cdot \sin(\phi_k) + r(\phi - \phi_k) \quad (\text{eq. 2.1})$$

Eq. 2.1 can be expanded into a Fourier series:

$$s_k(\phi) = \text{Re} \left( \sum_{n=1}^{\infty} s_{kn} e^{in\phi} \right) \quad (\text{eq. 2.2})$$

The  $n^{\text{th}}$  complex Fourier coefficient  $s_{kn}$  of the position probe signal can be written as a linear function of the complex Fourier coefficients of the spindle runout  $x_n, y_n$  and of the workpiece radius  $r_n$ :

$$s_{kn} = x_n \cdot \cos(\phi_k) + y_n \cdot \sin(\phi_k) + r_n \cdot e^{-i(n\phi_k)} \quad (\text{eq. 2.3})$$

In our experiments four identical capacitive position probes were used. However, any type of probe can be used. For a sensitive angle probe for instance we can write:

$$s_k(\phi) = \frac{1}{r_0} \frac{dr}{d\phi} \quad (\text{eq. 2.4})$$

After expanding into a Fourier series (eq. 2.2) the  $n^{\text{th}}$  complex Fourier coefficient of the angle probe signal can be derived:

$$s_{kn} = x_n \cdot 0 + y_n \cdot 0 + r_n \cdot \left( -i \frac{n}{r_0} \cdot e^{-i(n\phi_k)} \right) \quad (\text{eq. 2.5})$$

$N_k$	= angular position of the probe $k$ (defined in the direction of rotation)
$N$	= angle of rotation of the workpiece
$s_k$	= distance from the $k^{\text{th}}$ probe to the workpiece
$r$	= workpiece radius
$r_0$	= nominal radius of the workpiece
$x$	= $x$ -position of the workpiece
$y$	= $y$ -position of the workpiece
$s_{kn}$	= $n^{\text{th}}$ complex Fourier-coefficient of the $k^{\text{th}}$ sensor data
$r_n$	= $n^{\text{th}}$ complex Fourier-coefficient of the workpiece radius
$x_n$	= $n^{\text{th}}$ complex Fourier-coefficient of the spindle displacement in $x$ -direction
$y_n$	= $n^{\text{th}}$ complex Fourier-coefficient of the spindle displacement in $y$ -direction
$n$	= harmonic number
$i$	= complex operator

In order to determine the Fourier components  $x_n$ ,  $y_n$  and  $r_n$  a solution matrix containing the equations for all probes (eq. 2.3 and eq. 2.5) can be formulated for every harmonic. The equation to be solved can be written in the form  $\mathbf{s} = \mathbf{H} \cdot \mathbf{x}$ :

$$\begin{bmatrix} s_{1n} \\ s_{2n} \\ \vdots \\ s_{5n} \\ \vdots \end{bmatrix} = \begin{bmatrix} \cos(\phi_1) & \sin(\phi_1) & e^{-i(n\phi_1)} \\ \cos(\phi_2) & \sin(\phi_2) & e^{-i(n\phi_2)} \\ \vdots & \vdots & \vdots \\ 0 & 0 & -i \frac{n}{r_0} \cdot e^{-i(n\phi_s)} \\ \vdots & \vdots & \vdots \end{bmatrix} \cdot \begin{bmatrix} x_n \\ y_n \\ r_n \end{bmatrix} \quad (\text{eq. 2.6})$$

In the example above (eq. 2.6) the upper half of matrix  $\mathbf{H}$  represents position probes, while the lower half represents angle probes. The solution matrix  $\mathbf{H}$  can be extended for any number of probes.

$\mathbf{H}$ : matrix containing information about positions and characteristics of the probes.

$\mathbf{x}$ : vector containing the unknown complex Fourier coefficients of the runout error in  $x$ - and  $y$ -direction and of the workpiece radius.

$\mathbf{s}$ : vector containing the complex Fourier coefficients of the probedata.

In case four or more probes are used a least-square-estimate  $\hat{\mathbf{x}}$  can be made for every harmonic:

$$\hat{\mathbf{x}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \cdot \mathbf{s} \quad (\text{eq. 2.7})$$

$$\mathbf{e} = (\mathbf{H} \hat{\mathbf{x}} - \mathbf{s})$$

$\hat{\mathbf{x}}$ : vector containing the estimated complex Fourier-coefficients of the spindle runout in  $x$ - and  $y$ -direction and of the workpiece radius.

$\mathbf{e}$ : error residue

After solving equation 2.7 for all relevant harmonics, a complete solution of  $x(\phi)$ ,  $y(\phi)$  and  $r(\phi)$  can be found through inverse Fourier transformation.

### 3 Selecting probe angles

The probe positions cannot be chosen arbitrarily. To avoid harmonic suppression the matrix  $\mathbf{H}$  must be well conditioned: the condition number  $\kappa(\mathbf{H})$  should be sufficiently small.

$$\kappa(\mathbf{H}) = \|\mathbf{H}\| \|\mathbf{H}^{-1}\|$$

$$\frac{\|\delta \mathbf{x}\|}{\|\mathbf{x}\|} = \kappa(\mathbf{H}) \cdot \frac{\|\delta \mathbf{H}\|}{\|\mathbf{H}\|}, \quad \frac{\|\delta \mathbf{x}\|}{\|\mathbf{x}\|} = \kappa(\mathbf{H}) \cdot \frac{\|\delta \mathbf{s}\|}{\|\mathbf{s}\|} \quad (\text{eq. 3.1})$$

*\*When over 3 probes are used,  $\kappa(\mathbf{H})$  should be replaced by  $\kappa(\mathbf{H}^T \mathbf{H})$ .*

A large condition number (e.g.  $\kappa(\mathbf{H}) = 10$ ) means that relatively small errors of  $\mathbf{H}$  (probe angle estimation) or  $\mathbf{s}$  (probe reading) may result in a large error of  $\mathbf{x}$ . In the example of figure 3.1 a small condition number (e.g.  $\kappa(\mathbf{H}) = 3$ ) is obtained when probe 1 and probe 2 are not separated close to a multiple of one wavelength.

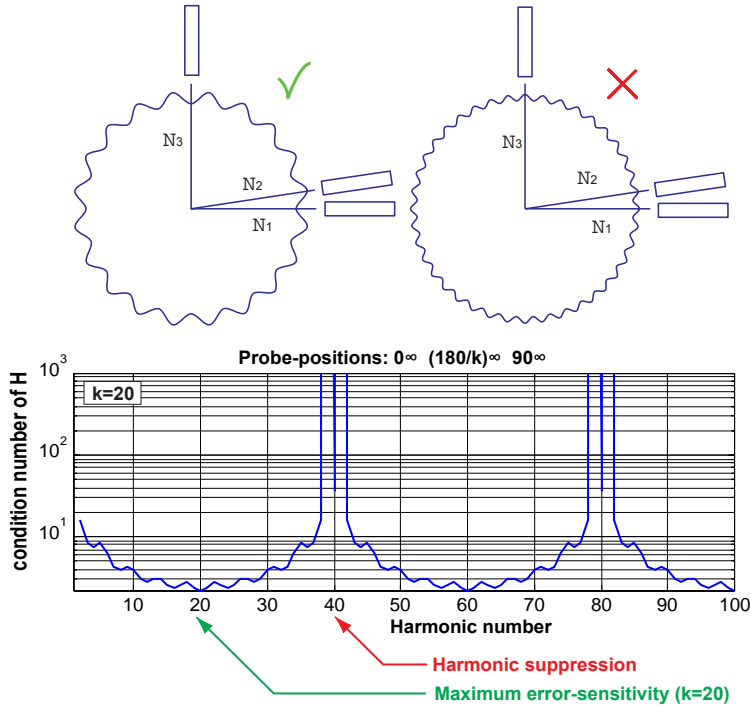


Fig 3.1: Condition number for the 2<sup>nd</sup> until the 100<sup>th</sup> harmonic. Probe angles: 0°, (180/20)°, 90°

The workpiece roundness and the spindle runout can of course contain a broad range of harmonics. By checking the condition numbers for relevant harmonics the probe angles can be selected to closely match our region of interest.

When using three probes, the probe angles can only be optimized for a very limited range of harmonics. Carefully positioning an extra probe allows us to optimize the condition numbers for a wider range of harmonics. This has the added advantage of the possibility of automatic probe angle detection (see section 4). In our test-setup the probes were estimated to be  $0^\circ$ ,  $35.5^\circ$ ,  $121.3^\circ$  and  $229.6^\circ$ .

#### 4 Estimating the probe angles and probe amplification factors

The angular position of the probes as well as differences in sensor amplification can be determined automatically without the need for extra measurements, provided that four or more probes are used. For reliable processing of the sensor data the probe positions need to be known accurately. This is especially important for the processing of high harmonics, because for the  $n^{\text{th}}$  harmonic an angle error of  $d\theta$  results in a phase error of  $n \cdot d\theta$ .

The probe amplification factors can be estimated by minimizing the error residues for preferably low harmonics that have large amplitude. The first harmonic will do perfectly when the workpiece is positioned slightly eccentric to the axis of rotation. Wrongly estimated probe angles have only little effect on the calculated amplification factors when using lower harmonic range.

Next, the error residues have to be minimized for optimum probe position estimates. This could be done for a single harmonic, however extra accuracy can be obtained when minimizing the error residues of several harmonics simultaneously. In every optimization step the optimum amplification factors are determined.

Because of the complexity of this optimization problem, the calculation is best performed numerically. To minimize the influence of sensor noise or non-repetitive runout it is recommended to average data over several revolutions for estimating the probe angles.

#### 5 Evaluating the results

Because we make use of more than three probes while we have only three unknown parameters we can use the error residue as a suitable check. A small error residue accompanied with a small condition number guarantees an accurate calculation of  $x_n$ ,  $y_n$  and  $r_n$ . Phase- and amplitude should be consistent among all probes, provided that a certain harmonic has a sufficiently high signal to noise ratio. This can be observed by comparing the real measurements to simulated measurement data.

Checking  $\angle(\mathbf{H} \cdot \hat{\mathbf{x}}) - \angle \mathbf{s}$  and  $\|\mathbf{H} \cdot \hat{\mathbf{x}} - \mathbf{s}\|$  for all evaluated harmonics.

In our measurements we used an optical flat diamond turned brass workpiece with a nominal radius  $r_0$  of 25 mm. It was turned and measured on the same lathe. Measurements were performed using four ADE Model 2102/2036K non-contact capacitive probes. The data acquisition board was equipped with a sample-and-hold accessory to guarantee a simultaneous readout of all probes.

At low speeds measurements were performed with up to 2500 samples per revolution. This way a total error residue for the 1<sup>st</sup> until 100<sup>th</sup> harmonic in the order of only 1 nm has been obtained. At a rotational speed of 2200 rpm the sample rate was limited by the acquisition board at 250 samples per revolution.

Because the workpiece geometry is fixed, we should expect the calculated workpiece geometry not to change for measurements performed at different rotational speed. At the same time the spindle runout is expected to change due to dynamic lubricating effects in the hydrostatic bearing.

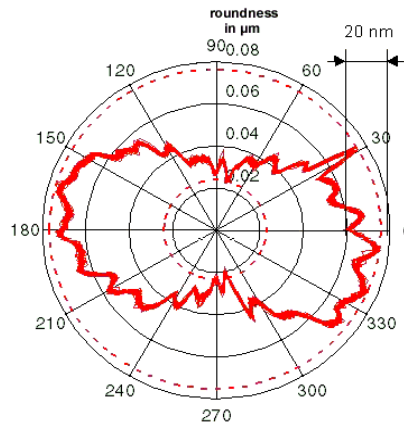
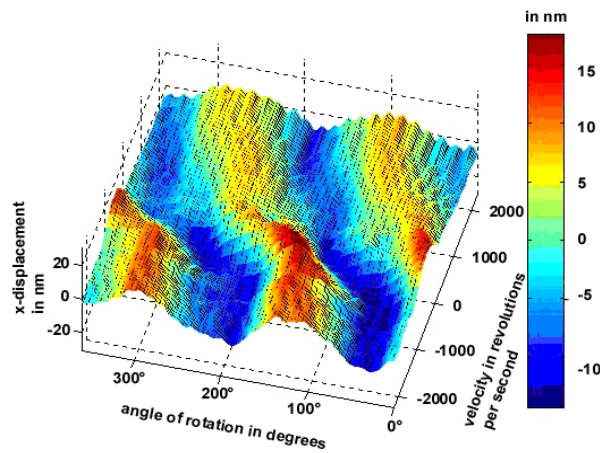


Fig 5.1



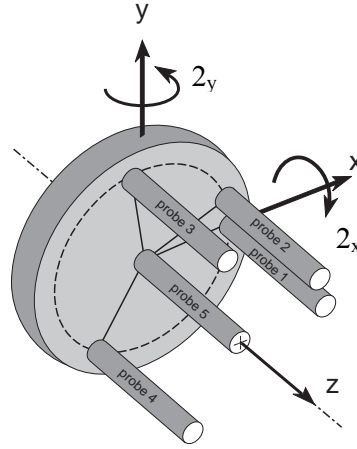
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Fig 5.2: Runout in x- direction from -2200 to 2200 rpm (2<sup>nd</sup> till 50<sup>th</sup> harmonic)

As figure 5.1 and 5.2 show, the calculated roundness and runout fully correspond to our expected results.

## 6 Measuring face error motion and surface flatness using 5 probes

Measuring surface flatness and face error motion took no part in our investigation. However, for determining the axial runout  $z$ , the  $x,y$ -tilt motion and the surface



flatness  $h$  of the spindle the same approach can be applied. The set of linear equations to be solved for the  $n^{\text{th}}$  harmonic can again be written in the form  $\mathbf{s} = \mathbf{H} \cdot \mathbf{x}$ :

$$\begin{bmatrix} s_{1n} \\ s_{2n} \\ s_{3n} \\ s_{4n} \\ s_{5n} \end{bmatrix} = \begin{bmatrix} -\cos(\phi_1) & \sin(\phi_1) & e^{-in\phi_1} & 1 \\ -\cos(\phi_2) & \sin(\phi_2) & e^{-in\phi_2} & 1 \\ -\cos(\phi_3) & \sin(\phi_3) & e^{-in\phi_3} & 1 \\ -\cos(\phi_4) & \sin(\phi_4) & e^{-in\phi_4} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_0 \theta_{yn} \\ r_0 \theta_{xn} \\ h_n \\ z_n \end{bmatrix} \quad (\text{eq. 6.1})$$

A least-square-estimate of the unknown Fourier components of the tilt, shape and  $z$ -position can be calculated with eq. 2.7. The tilt, flatness and  $z$ -translation can be obtained through inverse Fourier-transformation. The fifth probe does not need to be positioned in the centre, but it can also be positioned at the same radius as the other probes ( $r_0$ ). Optimization and determination of the probe positions and amplification factors can be done in the same way as described in section 4.



## 7 Method evaluation and recommendations

The presented method proves to be a flexible and accurate method for determining the spindle runout and the workpiece roundness. There are a few conditions that have to be taken into account. The quality of the workpiece is important and may be the limiting factor if one wants to measure up to nanometers. The workpiece must be as smooth as possible, because low harmonics in general offer the least difficulties.

An appropriate set of angle probes has to be selected to prevent harmonic suppression. The condition number of the condition number can be taken as an important guidance.

In case one is mainly interested in the workpiece geometry or in the repetitive part of the runout error it is recommendable to average the probe data. The effect of sensor noise can be reduced by a factor  $\sqrt{q}$  when averaging over  $q$  revolutions, or by increasing the number of data points by a factor  $q$ .

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