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# A Power System Equivalent Impedance Based Voltage Control

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***Abstract*--Voltage control is traditionally achieved through reactive power control, mainly because of the inductive nature of the transmission grids. However, when considering distribution networks, reactive power may not be an effective mean for voltage control. This is because of the X/R ratio which determines the controllability of voltage through active and/or reactive power. Therefore, in this paper a voltage control scheme is proposed based on the estimation of the X/R ratio of the grid. The estimated X/R will then be used to determine the active and reactive power to be injected into the grid in order to keep the voltage between permissible limits.**

***Index Terms*-- Distributed generation, FACTS, equivalent circuit, Recursive Least Squares, voltage control,**

## I. Introduction

The expected change in the use and design of the electric power system is urging for new control and operation strategies of the networks. The main cause for this is the massive introduction of renewable energy sources leading to large amounts of distributed generation in the grid. It has been shown that distributed generation (DG) has some challenging impacts on the grid. Among these are voltage variations, power quality and protection issues [1], [2]. Voltage control is traditionally done via reactive power control, using synchronous generators, capacitors and recently FACTS devices. However, in distribution networks, reactive power control is not so effective because the network impedance is not predominantly inductive. Voltage control strategies in distribution networks are based on changing substation transformer tap changers, besides using reactive power compensation devices [3], [4], and possibly, using active and reactive power from DGs [5]. In order to have an efficient voltage control scheme, the X/R ratio should be determined so that the right amount of active and reactive power is injected [6]. However, this ratio is dependant on the network's structure, which in turn is prone to changes regularly. In this paper, a voltage control scheme is proposed, based on the estimation of the network's equivalent impedance as seen from the connection point of the power electronics devices.

This work is part of the research project: Electrical Infrastructure of the Future (Elektrische Infrastructuur van de toekomst in Dutch), sponsored by the Ministry of Economic Affairs of the Netherlands.

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In chapter II, current and voltage phasors are identified through Kalman filtering using the measurements of voltages and currents injected by the power electronics devices. In chapter III, the identified voltage and current phasors are used to estimate the grid's equivalent impedance. The estimation algorithm is based on a classical Recursive Least Squares (RLS) algorithm [7], [8]. This algorithm is extended to include constraints imposed by the physical nature of the identified parameters. In chapter IV, the quantity X/R is deduced from the identified equivalent impedance, and used to calculate the required active and reactive powers for keeping the voltage into permissible limits. Finally, simulation results are presented to prove the performance of the proposed method.

## II. Current and Voltage Phasors Identification

The voltage and current harmonics are identified by using a classical discrete Kalman filtering technique [9]. Other techniques such as Fast Fourier Transform (FFT), or Discrete Fourier Transform (DFT) could also be used. However, these techniques show a poor performance because of the fact that voltage and currents are time-varying [10]. Before applying the Kalman filter, a state space model of the system has to be established.
The sampled current and voltage measurements are used by the Kalman filter to calculate the real and imaginary voltage and current at different frequencies:
$V_{1,r} \quad V_{1,i} \quad \ldots \quad V_{n,r} \quad V_{n,i}\,,\; I_{1,r} \quad I_{1,i} \quad \ldots \quad I_{n,r} \quad I_{n,i}\,.$
The continuous signal (voltage or current) s (t) composed of *n* harmonics is given by:

$$s(t) = \sum_{h=1}^{h=n} \left[ A_{h,r}(t)\cos(h\omega t) - A_{h,i}\sin(h\omega t) \right] \qquad (1)$$

Where $f = \dfrac{\omega}{2\pi}$ represents the fundamental frequency.

The discrete state space model of the system is in general given by :

$$\begin{cases} x(t_{k+1}) = x(t_k) + \chi(t_k) \\ y(t_{k+1}) = C_k^T x(t_k) + \nu(t_k) \end{cases} \qquad (2)$$

Where $t_k = kT_{sf}$, $k \geq 1$, $T_{sf}$ is the Kalman filter sampling period, $\chi(t_k) \in \Re^{2n}$ is a random variable vector representing the process noise, with zero mean value, no time correlation and covariance matrix.

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$$Q_{\chi,k} = E[\chi(t_k)\chi_k^T(t_k)] \tag{3}$$

$x(t_k) \in \Re^{2n}$ is the state space vector of the identified real and imaginary components of the fundamental and harmonic phasors:

$$x(t_k) = [A_{1,r} \quad A_{1,i} \quad \ldots \quad A_{n,r} \quad A_{n,i}]^T \tag{4}$$

$y(t_k)$ is the $k^{\text{th}}$ sampled measurement of $s(t)$, $C_k$ is the measurement vector given by:

$$C_k = [\cos(\omega t_k) \quad -\sin(\omega t_k) \quad \ldots \quad \cos(\omega t_k) \quad -\sin(\omega t_k)]^T \tag{5}$$

$\nu(t_k)$ is a random variable representing the measurement noise, which is assumed to be white noise, with covariance $Q_{\nu,k}$ and having no correlation with the process noise, $\chi(t_k)$.

The state estimation given by the Kalman filter at the $k^{\text{th}}$ iteration is given by:

$$\hat{x}(t_k) = \hat{x}(t_{k-1}) + L_k\left[y(t_k) - C_k^T\hat{x}(t_{k-1})\right] \tag{6}$$

$L_k$ is the Kalman gain matrix and is given by:

$$L_k = \frac{\tilde{R}_k C_k}{C_k^T \tilde{R}_k C + Q_{\nu,k}} \tag{7}$$

$\tilde{R}_k$ is the estimated (before measurement) covariance error matrix, and is given by :

$$\tilde{R}_k = E\left[(x(t_k) - x(t_{k-1}))(x(t_k) - x(t_{k-1}))^T\right] \tag{8}$$

The actual (after measurement) covariance error matrix $\hat{R}_k$ is:

$$\hat{R}_k = \left(I - L_k C_k^T\right)\tilde{R}_k \tag{9}$$

$I$ being the identity matrix.

The convergence of such an algorithm is given by the following condition [10]

$$\omega < \frac{\pi}{2n^2 T_{sf}} \tag{10}$$

## III. EQUIVALENT CIRCUIT IDENTIFICATION

The equivalent circuit identification procedure is carried out using a classical Recursive Least Squares (RLS) algorithm [7]. Voltage and current phasors identified by the Kalman filter are used as inputs to the RLS algorithm, as shown in figure 1. This algorithm is based on the solution of an unconstrained optimization problem.

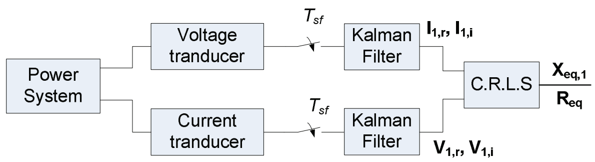


Fig. 1. Equivalent impedance identification

The equivalent circuit model is given by:

$$y_h(kT_e) = \phi_h(kT_e)\Theta_h \tag{11}$$

Where $y_h(kT_e)$ represents the voltage phasor measurements at each sampling period, and is given by:

$$y_h(kT_e) = \left(\hat{V}_{h,r}(kT_e) \quad \hat{V}_{h,i}(kT_e)\right) \tag{12}$$

$\phi_h(kT_e)$ represents the current phasor measurements.

$$\phi_h(kT_e) = \left(\hat{I}_{h,r}(kT_e) \quad \hat{I}_{h,i}(kT_e) \quad c_h\right) \tag{13}$$

Current dynamics could also be included to form higher order models, but this would result only in minor improvements in the estimation algorithm [8].

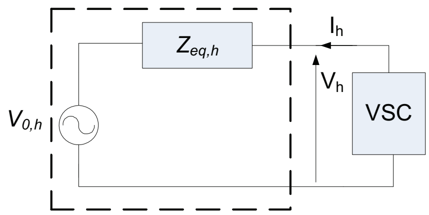


Fig. 2. Thevenin equivalent circuit

$\Theta_h \in \Re^{3\times2}$ is a matrix representing the power system parameters to be estimated:

$$\Theta = \begin{bmatrix} R_h & X_h \\ -X_h & R_h \\ V_{0,h,r}/c_h & V_{0,h,i}/c_h \end{bmatrix} = [\Theta_1 \quad \Theta_2] \tag{14}$$

$c_h$ is a scaling factor chosen in such a way that the $h^{\text{th}}$ order current phasor variations have the same order of magnitude as the term $\overline{V}_{0,h}/c_h$ [7].

For the estimation algorithm, when N couples (real and imaginary) of voltage and current phasor are available, the estimation algorithm consists of solving the following optimization problem:

$$\min_{\theta} f(\Theta) \tag{15}$$

Subject to

$$\begin{aligned} \theta_{11} - \theta_{22} &= 0 \\ \theta_{12} - \theta_{21} &= 0 \end{aligned} \tag{16}$$

Where

$$f(\Theta) = \varepsilon_1^T\varepsilon_1 + \varepsilon_2^T\varepsilon_2 \tag{17}$$

$$[\varepsilon_1 \quad \varepsilon_2] = Y - \Phi\Theta = [Y_1 - \Phi\Theta_1 \quad Y_2 - \Phi\Theta_2] \tag{18}$$

$$Y = [y^T(kT_e) \quad ... \quad y^T(NT_e)] = [Y_1 \quad Y_2] \qquad (19)$$

$$\Phi = [\phi^T(kT_e) \quad ... \quad \phi^T(NT_e)] \qquad (20)$$

Constraints are added to account for the fact that the identified parameters have a physical meaning.

If enough input vectors $\phi_h(T_k)$ are available, then the problem (15), subject to (16) is a convex quadratic programming problem, and therefore admits an analytical solution. This solution is determined by first finding the Lagrangian function, $L$, of (15) subject to (16), and then solving the system $\frac{\partial L}{\partial \Theta} = 0$ for $\Theta$. This solution is then given by:

$$\Theta = \Theta^* - \frac{P}{p_{11} + p_{22}} \begin{bmatrix} \theta_{11}^* - \theta_{22}^* & \theta_{12}^* - \theta_{21}^* \\ \theta_{12}^* - \theta_{21}^* & \theta_{22}^* - \theta_{22}^* \\ 0 & 0 \end{bmatrix} \qquad (22)$$

Where,

$$\Theta^* = \left(\Phi^T \Phi\right)^{-1} \Phi^T Y \qquad (23)$$

is the solution to the unconstrained problem (15).

## IV. Voltage Control

Due to the fact that in MV voltage grids, and especially in cable networks, the inductive part of the network impedance is not as predominant as in HV grids, voltage control in these networks would require huge amounts of reactive power to be injected. In this case, it is preferable to include active power control for voltage control as well. This is achieved by modifying the active and reactive power as follows:

$$\begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} X/Z & -R/Z \\ R/Z & X/Z \end{bmatrix} \begin{bmatrix} P' \\ Q' \end{bmatrix} \qquad (24)$$

Where $Q'$ is the modified reactive power supplied by the voltage droop controller given by:

$$V - V_0 = k_V (Q_0 - Q') \qquad (25)$$

Where $k_V$ is the voltage droop constant, $V_0$ and $Q_0$ are the nominal values for voltage and reactive power respectively.

$P'$ is the modified active power supplied by the frequency droop controller. In this paper, only voltage control is considered ($P'=0$), then (24) becomes:

$$P = -\frac{R}{Z} Q' \qquad Q = \frac{X}{Z} Q' \qquad (26)$$

This means, that the amount of active and reactive power injected into the grid are proportional to the X/R ratio, which is identified online using the RLS algorithm (see previous section).

As shown in figure 3, the new values of P and Q, are fed to a control system of a compensation device, which is represented by a three phase PWM inverter, that is responsible for injection of the specified amounts of active and reactive power

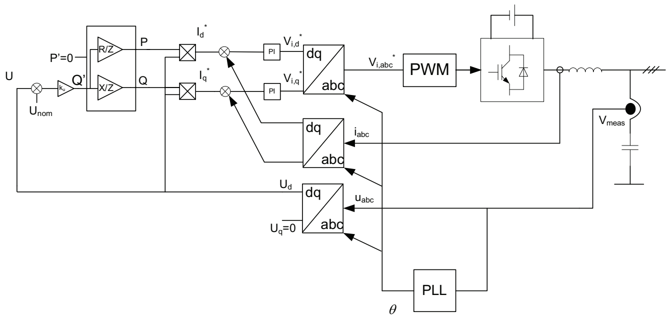


Fig. 3. Bus voltage control scheme.

## V. Simulation Results

The proposed approach is simulated under MATLAB/Simulink environment. The test system is a typical MV network in the Netherlands. The compensation device is a typical three phase inverter equipped with an active power source at the DC side. The rating of the device is 2 MVA/4 Mvar. The Recursive Least Squares algorithm is implemented with a sampling frequency $f_s$ = 50 kHz, to estimate the network equivalent impedance at the fundamental frequency. The droop constant used for this simulation is $k_V$=-6 kVar/V, $V_0$ = 1 pu, $Q_0$ = 0.

The test network is a cable grid, represented by 5 sections ($\Pi$ equivalent circuit); each section is 5 km long as seen in figure 4. In this simulation, only the inverter voltage control is considered, and the voltage at the substation is fixed at a constant value.

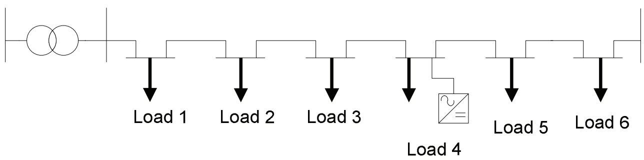


Fig. 4 The test network

### A. Case 1: voltage decrease

In this case, a load scenario is simulated in such a way to have a voltage decrease situation, i.e. a situation where voltage is below 0.95 pu. Table I gives the voltage levels at each bus, for three cases: No control, only reactive power control and active and reactive power control.

TABLE I
Voltage levels at different buses (voltage decrease)

| Bus # | No control | Reactive power | Active/Reactive Power |
|---|---|---|---|
| 1 | 0.974 | 0.981 | 0.982 |
| 2 | 0.956 | 0.971 | 0.972 |
| 3 | 0.945 | 0.967 | 0.969 |
| 4 | 0.937 | 0.968 | 0.970 |
| 5 | 0.931 | 0.969 | 0.971 |
| 6 | 0.926 | 0.964 | 0.967 |

Starting from bus 3, the voltages are below permissible limits $(1.05 < V < 0.95\,\text{pu})$, when no control is applied. The compensation device is now connected at bus 4. In that case, a voltage droop control scheme is implemented on the inverter; this means that only reactive power is used for voltage control. As seen, on figure 5, the maximum rating of 4 MVar is needed to achieve the desired voltage control. This is somehow a big amount that could be reduced by injecting a certain amount of active power as well. This is shown in figure 6, where the injected reactive power is being reduced by approximately a factor of two. In case the active power injection would not have been limited to 1.5 MW, the reactive power injection would have been even less, corresponding to $\frac{X}{Z}Q'$, which is nearly 30% out of the modified reactive power ($Q'$). Figure 7 gives the X/R ratio that is determined online adopting the algorithm mentioned in the previous section (RLS algorithm).

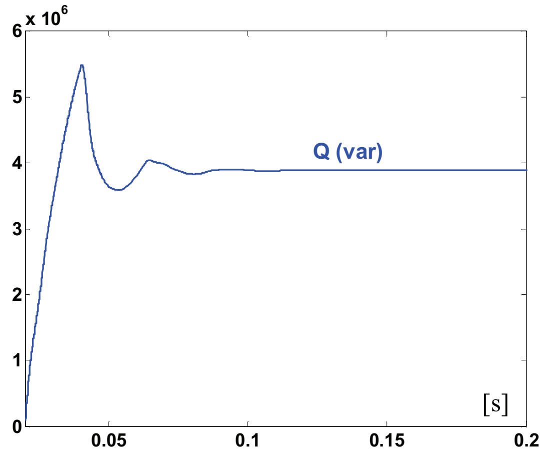


Fig. 5. Reactive power injection in case 1 (P = 0)

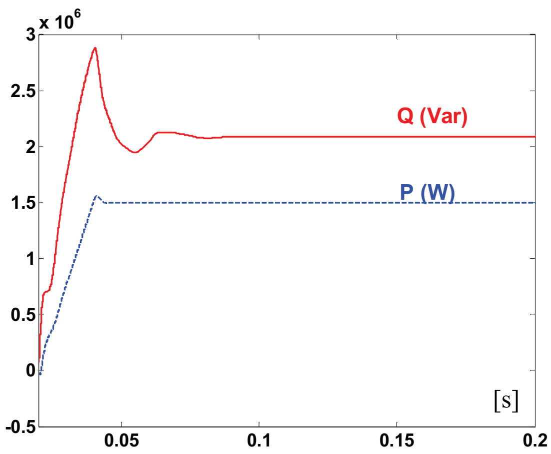


Fig. 6. Active and reactive power injection in case 1

### B. *Case 2: voltage rise*

In this scenario, a voltage rise situation is created by adding a generator injecting 4 MW at bus 6. Similarly as in the voltage decrease case, the inverter is used to control voltage at bus 4. Table II gives voltage levels for each control option.

TABLE II
VOLTAGE LEVELS AT DIFFERENT BUSES (VOLTAGE RISE)

| Bus # | No control | Reactive power | Active/Reactive Power |
|---|---|---|---|
| 1 | 1.052 | 1.045 | 1.045 |
| 2 | 1.046 | 1.032 | 1.032 |
| 3 | 1.045 | 1.025 | 1.025 |
| 4 | 1.048 | 1.021 | 1.021 |
| 5 | 1.051 | 1.024 | 1.024 |
| 6 | 1.057 | 1.031 | 1.031 |

Again using only reactive power for voltage control requires nearly 4 Mvar. Including the modified reactive power control scheme reduces this value to 2 Mvar, by absorbing 1.5 MW. This is shown Fig. 9.

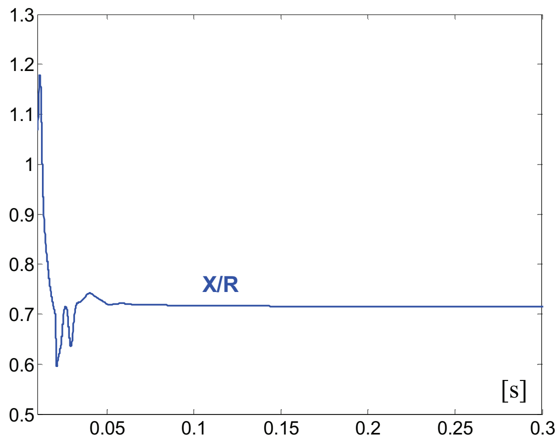


Fig. 7. The X/R ratio

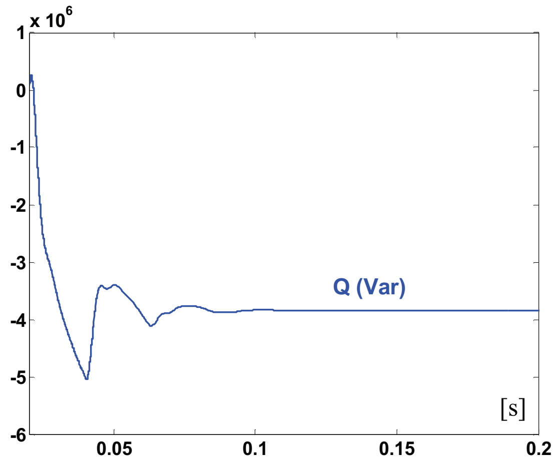


Fig. 8. Reactive power injection in case 2 (P = 0)

Comparing Fig. 5 with Fig. 6, and Fig. 8 with Fig. 9, it can be clearly seen that the apparent power needed for voltage control is significantly reduced (4 MVA for only reactive power injection, and 2.5 MVA for active and reactive power injection). Thus the inverter rating would be significantly reduced.

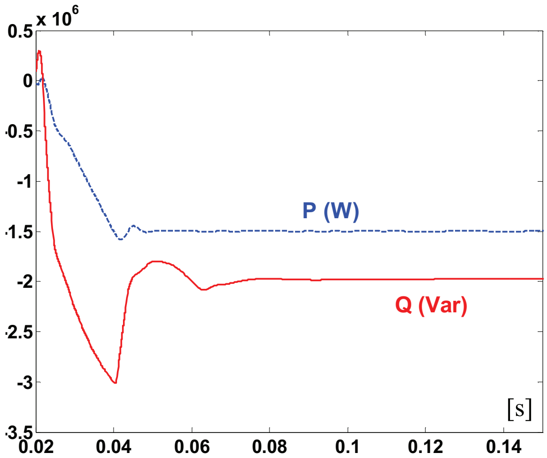


Fig. 9. Active and reactive power injection in case 2

## VI. Conclusion

Voltage control in distribution networks is becoming more and more an urgent matter, as the penetration level of distributed generation is increasing.

In the MV and LV grids reactive power injection is not an efficient way to keep voltage in a permissible range. It has been shown, that using an online identification of the network impedance, a power converter associated with an active power source, can effectively adapt to the active and reactive power needs of the system in order to achieve a good voltage profile. With this strategy, the inverter MVA rating would be significantly reduced. This approach could also be a promising tool for voltage control coordination without the need for an expensive and unreliable communication.

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## VIII. Biographies

**Abdelhamid Kechroud** Received his Electrical Engineering degree from University of Sciences and Technology Houari Boumediene, in 2005 and the M.Sc from Institut National Polytechnique de Grenoble in 2006. Since October 2006 he is with Electrical Power Systems Group at Eindhoven University of technology, working towards his PhD. His research interest are: decentralized control, voltage and frequency control and distributed generation integration

**Dr. Johanna M.A. Myrzik** was born in Darmstadt, Germany in 1966. She received her MSc. in Electrical Engineering from the Darmstadt University of Technology, Germany in 1992. From 1993 to 1995 she worked as a researcher at the Institute for Solar Energy Supply Technology (ISET e.V.) in Kassel, Germany. In 1995 Dr. Myrzik joined to the Kassel University, where she finished her PhD thesis in the field of solar inverter topologies in 2000. Since 2000, she is with the Eindhoven University of Technology, the Netherlands. In 2002, she became an assistant professor and in 2008 an associate professor in the field of residential electrical infrastructures. Her fields of interests are: power electronics, renewable energy, distributed generation, electrical power supply.

**Prof. W. L. Kling** received his M.Sc. degree in electrical engineering from the Technical University of Eindhoven, The Netherlands, in 1978. Since 1993, he is a part-time Professor at the Electric Power Systems Laboratory, Delft University of Technology, the Netherlands. And since 2000, also at the Eindhoven University of Technology, where he became full-time professor end 2008. Furthermore he was with TenneT, the Dutch Transmission System Operator, in the Asset Management Department responsible for network strategy. His experience is in the area of planning and operation of power systems. Prof. Kling is involved in scientific organizations such as Cigré and IEEE. He is the Dutch Representative in the Cigré Study Committee C6 Distribution Systems and Dispersed Generation and the Eurelectric/UCTE WG SYSTINT.