Stiffness of planar tensegrity beam topologies

Stiffness of Planar Tensegrity Beam Topologies

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ABSTRACT

This paper demonstrates a symbolic procedure to compute the stiffness of structures. Geometrical design parameters enter in this computation. A set of equations linear in the degrees-of-freedom, but nonlinear in the design parameters, is solved symbolically. The resulting expressions reveal the values of these parameters which yield desirable properties for the stiffness or stiffness-to-mass ratio. By enumerating a set of topologies, stiffness properties are optimized over this set of topologies. This procedure is applied to a planar tensegrity beam. The results make it possible to optimize the structure with respect to stiffness properties, not only by appropriately selecting (continuous) design parameters like dimensions, but also by selecting an appropriate topology for the structure (a discrete design decision).

1 INTRODUCTION

As a first step in integrating system design in system theory, we consider a powerful class of mechanical systems, namely tensegrity structures of class 1 or 2, that are relatively easy to analyze, and so permit analytical solutions.

Tensegrity structures are web-like mechanical structures that consist of two types of members: tensile ones (tendons) and compressive ones (bars). This class of systems has been studied for a long time, see, *e.g.,* Maxwell (Forew. 1890), whose terminology consisted of ties and struts instead of tendons and bars. In a class 1 tensegrity structure (Skelton *et al.,* 2001; Skelton and Chan, n.d.) the bar endpoints, *i.e.,* the nodal points, are only connected to tendons, not to other bars. Tendons are exclusively loaded in tension, otherwise they would buckle because they are very slender. Bars are normally loaded in compression only and not in tension.

The main advantage of tensegrity systems is that an equilibrium is possible in different configurations or shapes. Tendons can function as sensor or as actuator or both (Skelton and Adhikari, 1998). Alternatively, bars can be used as actuators or sensors. By controlling the length of all tendons simultaneously, a tensegrity structure can be made very stiff for any static load acting on the nodal points. It is not always feasible to equip all tendons as actuators, so the

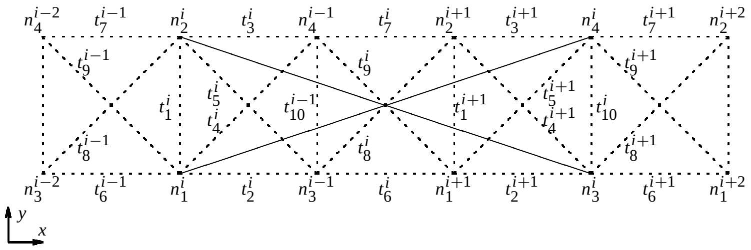

Figure 1 Single stage of planar tensegrity structure. Bars: —, tendons: - -

stiffness properties inherent to the structure are important. Because those properties depend on topology and geometry it is of interest to study this influence.

Our main goal is to obtain guidelines in the design of planar tensegrity structures of class 1 or 2. Structural aspects studied are changes in stiffness and stiffness-to-mass ratio due to variations in topology and geometry of a planar tensegrity structure. This work is an extension to other topologies of the work reported in (De Jager and Skelton, 2001).

In the rest of this paper we first discuss several aspects of modeling tensegrity systems of class 1 and 2. Then some computational results are given and discussed, and guidelines for the design of tensegrities are formulated.

2 PLANAR TENSEGRITY STRUCTURES

A tensegrity structure consists of bars and tendons, arranged in such a way that the structure has integrity and is not a mechanism. This is achieved by pre-stressing the tendons by a tensile force. A planar tensegrity structure is one that only extends in the plane. A tensegrity structure can be of class 1, where bars are only connected by tendons, and do not connect directly, or of class 2, where in a nodal point up to two bars and a number of tendons are connected. This can be generalized to a class k definition. Often a tensegrity structure is made up of nested tensegrity structures, giving it a fractal character. This is beneficial for analysis and design, because only a limited number of structures needs to be investigated. Those structures can then be used to build up a more complex structure.

2.1 Description of Planar Tensegrity Structures

An elementary stage, numbered i, of a planar tensegrity structure of class 1 is given in Fig. 1. This stage can be repeated indefinitely, by replicating it, shifted some distance of the horizontal dimension, to build up a planar structure in x-direction. It could also be replicated in y-direction or both.

Without diagonal tendons the stiffness is derived from second order effects (*i.e.,* it is zero in the linear approximation, except for pre-stress), so diagonal tendons are included.

The left side of the structure has to be modified for the boundary condition. The left side removes the three DOF of the rigid body, in effect, it restricts movement of the upper left node in both x and y-coordinate direction, *i.e.,* the node is translationally fixed, and of the lower left node in the x-direction.

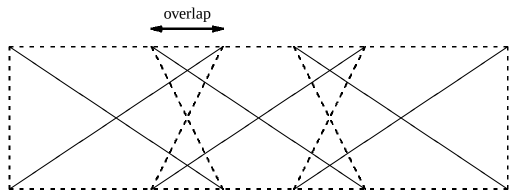

Figure 2 Tensegrity with minimal number of tendons. Bars: —, tendons: - -

There are no restrictions specified at the right side. Only differences in geometry are taken into account, the connection of some tendons is to different nodes than in Fig. 1.

2.2 Model of Planar Tensegrities

The basic assumptions in setting up the model are the same as in (De Jager and Skelton, 2001).

The symbolic model is derived for a planar tensegrity system, as seen in Fig. 2 for a 3-stage structure, with a minimal number of tendons, so compared to Fig. 1 the "inner" vertical tendons and the "uneven" pairs of diagonal tendons are removed. Only the horizontal and the left and right vertical tendons and the diagonal tendons that cross the overlap are included in the model.

The equilibrium conditions for small perturbations of the DOF are used to derive a set of equations that is linear in the perturbations of the DOF. Loading the structure and computing the deflection will then give insight in the stiffness and stiffness-to-mass properties of the structure.

The goal is to optimize the geometry, characterized by the overlap between the stages of a multi-stage tensegrity structure and the angles of the bars, for several topologies. The optimum depends on the assumption on the stiffness of the tendons, on the number of stages, and on the slenderness of the structure. The number of stages, overlap and slenderness together determine the bar angle, so not all factors are independent. To get an "easy" parameterization, overlap and slenderness are used as geometric parameters.

The set of equilibrium equations can be solved symbolically because the equations are linear in the unknown DOF, and the parameter dependency can be parameterized polynomially when over-parameterization is used, *i.e.*, three instead of two parameters. Given the analytical solution for the displacements of the DOF, as functions of the design parameters, the superfluous design parameter can be eliminated, and it is straightforward to obtain values for the overlap and slenderness that minimize the displacement of a specific point of the structure for a given load at the nodal points, or to obtain values that optimize the stiffness or stiffness-to-mass ratio. This can be done by differentiation, establishing stationary points, and solving the resulting equation for the design parameters.

3 STIFFNESS AND STIFFNESS-TO-MASS RATIO

To characterize the geometry the two parameters are nondimensionalized, so the slenderness ratio $l = l_x/l_y$, with l_x and l_y the horizontal and vertical dimensions of the structure, and the overlap factor s in the overlap $h = sl_x$ between stages are used. The overlap h is the distance

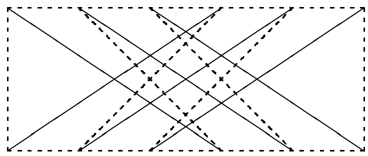

Figure 3 Super tensegrity cross. Bars: —, tendons: - -

between the right nodes of stage i and the left nodes of stage $i+1$. The parameters s and l will be varied in characterizing the solutions. The external force w and the stiffness factor k of the tendons always appear in the combination w/k in the deflection. In the stiffness the force w drops out and k appears affinely.

For the stiffness-to-mass ratio the mass is computed assuming a constant cross-sectional area A_b and the same specific mass ρ for all bars. The mass of the bars is then proportional to their lengths, which can be expressed as a function of l_y, l and s, where l_y appears affinely.

The tendon lengths depend on s. Only for $0 < s < 1/(n+1)$ the topology given before is a valid one, except for $n = 2$ where $0 < s < 1$ holds. For $n = 3$ and $1/4 < s < 1$ we can change the topology and get a super tensegrity cross, see Fig. 3.

For $n = 3$ there are three other topologies that are of interest, namely for $s = 0$, $s = 1/4$, and $s = 1$, where some nodal points can be “pinned” together to get a class 2 or higher tensegrity. In the sequel we report results for the 5 topologies of $n = 3$.

For the stiffness of the tendons three cases are explored

1. $k_i = k$ is constant and the same for all tendons.
2. $k_i = EA/l_i$, with l_i the tendon length, E the modulus of elasticity, and A the cross-sectional area of a tendon.
3. $k_i = EV/l_i^2$. This relation is relevant when it is assumed that in the previous relation for k_i the cross-sectional area A varies inversely proportional to the length, due to a constant volume restriction.

For optimizing the geometry we use a single load condition and two criteria to optimize. The load w is a vertical force F at the top/right node. The first criterion is the stiffness of the structure regarded as a beam, w/y, the ratio of force and displacement of the top/right node. The second criterion is the stiffness-to-mass ratio, $w/y/M$, with $M = 2nm$ the total mass and m the mass of a single beam, equal to $\rho A_b l_b$.

The resulting relations are proportional to k, EA, or EV and inversely proportional to various powers of l_y, depending on the stiffness model, and further have a “polynomial” denominator (including powers of square roots) in s and l. These relations are differentiated with respect to s, to obtain stationary points by equating them to zero. The resulting algebraic relation can be solved for s analytically, but only when the degree of the polynomial is not too large. In general this relation is quite complex for polynomials of degree 3 and larger, and a graphical solution is preferred.

4 RESULTS FOR STRUCTURAL ANALYSIS

We now present and discuss the results for several cases:

- three tendon stiffness models,
- stiffness and stiffness-to-mass ratio.

The results in Fig. 4 are for the three different constitutive relations. In all three cases class 2 topologies give better stiffness than class 1. The disadvantage of class 2 is that there are more restrictions when the shape of the structure needs to be changed. The results for stiffness-to-mass show the same tendencies, but are tilted w.r.t. to the results above, to favor smaller s.

5 DISCUSSION

5.1 Conclusion

We can conclude the following

- the presented approach is useful
- it provides detailed analytical information
- but for a stylized problem formulation
- while class 1 beams do not excel at stiffness.

5.2 Recommendations

The following can be recommended

- expand this approach to more elaborate set-ups (less assumptions, larger scale)
- exploit other approaches to defy the shortcomings of this approach (free material design, elimination techniques)
- extend to include (parametric) control.

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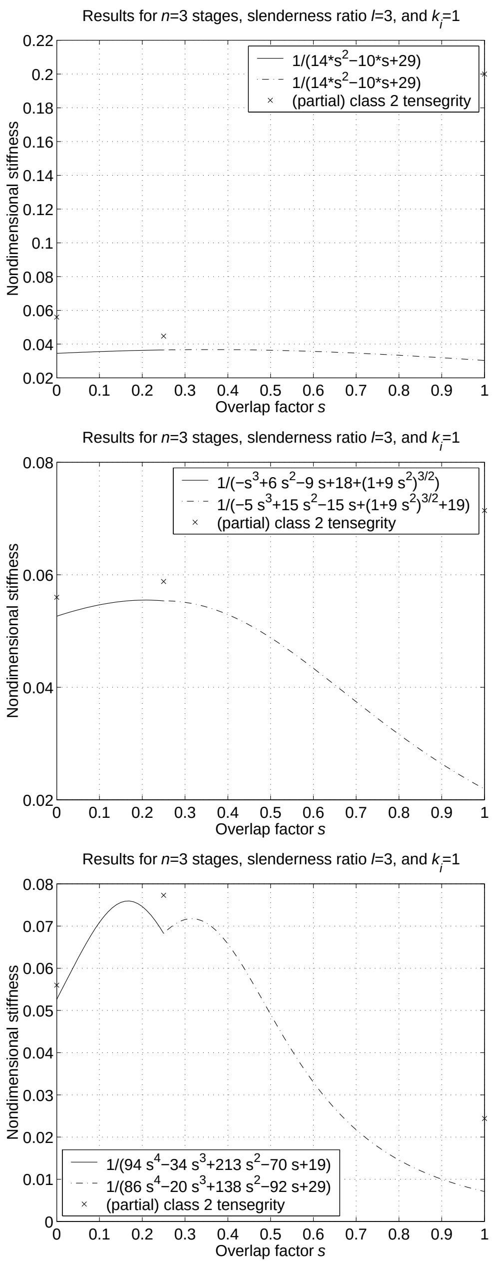

Figure 4 Results for stiffness, upper: equal stiffness, middle: equal cross-sectional area, lower: equal volume