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Teaching Geometrical Principles to Design Students

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Abstract

We propose a new method of teaching the principles of geometry to design students. The students focus on a field of design in which geometry is the design: tessellation. We review different approaches to geometry and the field of tessellation before we discuss the setup of the course. Instead of employing 2D drawing tools, such as *Adobe Illustrator*, the students define their tessellation in mathematical formulas, using the *Mathematica* software. This procedure enables them to understand the mathematical principles on which graphical tools, such as Illustrator are built upon. But we do not stop at a digital representation of their tessellation design we continue to cut their tessellations in *Perspex*. It moves the abstract concepts of math into the real world, so that the students can experience them directly, which provides a tremendous reward to the students.

Keywords:

Design, math, tessellation, Escher, geometry

Introduction

One of design's main goals is to give form to products and communication. However, most design students approach form intuitively, neglecting the understanding of the underlying geometry. Adobe Illustrator, for example, gives powerful tools to students to design geometrical shapes, such as Bézier curves (see Figure 1).

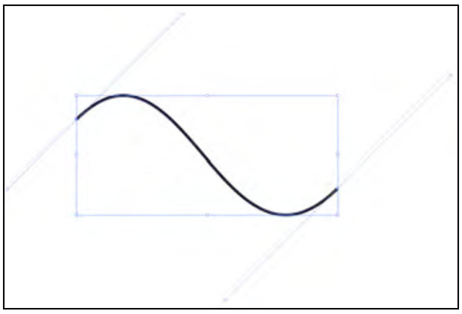

Figure 1: Bézier curve in Adobe Illustrator

The mathematical principles of Bézier curves and other geometrical functions remain hidden. A quadratic Bézier curve, for example, is the path traced by the function B(t), given the points P_0, P_1, and P_2:

$$B(t) = (1 - t)^2 P_0 + 2 (1 - t) tP_1 + t^2 P_2, t \in [0, 1]$$

Teaching such abstract and technical topics to design students is challenging and we experimented with new approaches for industrial design students (Vlist et al., 2008). In this study, we describe a new method of teaching geometry to industrial design students, but we believe that it can be used for other design disciplines as well, such as graphic design or fashion design.

Geometry is of course a topic characteristic of most high school maths education and therefore we focus on slightly more advanced topics than, for example, the Pythagorean theorem. When it comes to geometrical principles of design, the golden ratio is frequently brought forward. It is surprising that a book entitled "*Geometry of Design*" (Elam, 2001) is limited to the Golden Ratio, root squares and the human proportion. Moreover, the authors approach to directly apply these classic proportioning systems to design classics indicates how much design is concerned with the concretization and how little it is concerned with abstraction (Bartneck & Rauterberg, 2007). We capitalized on this move towards the concrete by focusing on a field of design in which geometry *is* the design: tessellation.

A tessellation is a collection of plane figures that fills the plane with no overlaps and no gaps. Tessellations are used in industrial design, textile design and interior design. Industrial design applies tessellations, amongst others, to tiles for bathrooms and kitchens and to paving stones. The principle of tessellation has also found great attention in manufacturing, since it minimizes the amount of wasted material when punching pieces from sheet metal (Bigalke & Wippermann, 1994). These application areas focus on the outer shape of the tiles. Sometimes tiles of different color are combined to introduce more variations.

Textile design and interior design focus even more on the inside of the tiles. They add decorative patterns onto the tiles, leaving the principle of the tessellation in the background. Throughout the centuries, fabrics and wallpapers utilized tessellation to create patterns. Maurice Cornelis Escher (1898-1972) took tessellation to the next level by adding meaning to the tiles. The graphical elements on the tiles are no longer just decoration, but they describe, for example, animals (see the fish in Figure 2). Schattschneider (1997) explored the different combinations in depth.

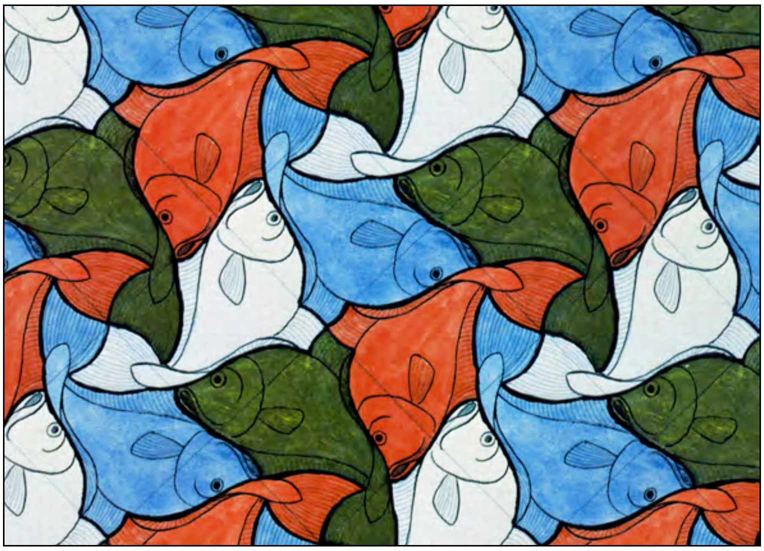

Figure 2: Regular division of the plane drawing #20 (Escher, 1938)

Before describing the process of the course, we will first give a short introduction to the different types of geometry, including group theory, matrices, and topology.

Geometry

Group theory: the mathematics of symmetry

Before going into the particularities of the mathematics used, we discuss a number of distinct views on geometry. Roughly the views were developed historically from concrete to abstract and from solutions of specific problems to powerful tools. In the context of design and education it is also important to pay attention to the mediums used. Each distinct view upon geometry naturally comes with its own preferred tools for representing and manipulating the constructions. We distinguish the following views upon geometry:

- Euclidian geometry,
- analytical geometry,
- transformational geometry.

If we are to assign one mathematician as the main or most famous person behind each of these three views, then we have to name Euclid, Descartes and Klein. To simplify matters we call them the inventors of Euclidian geometry, analytical geometry and transformational geometry, respectively. Euclid (Greek: Εὐκλείδης) lived around 300BC and his main work is known as "*The Elements*" (Euclid, Heath, & Densmore, 2002). René Descartes (1596-1650) published his main filosophical work "*Discours de la méthode*" (Descartes & Clarke, 1999) and the mathematical "*La Géométrie*" (Descartes et al., 1659) both in 1637. Felix Klein (1849-1925) presented his influential "Erlangen Program" in 1872 (Sharpe, 1997).

Euclidian geometry studies points, lines, circles, angles, lengths, areas, etc. It is the heart of the geometry taught at secondary schools with nowadays aspects of the other views blended in. Euclidian geometry can be studied on a purely axiomatic basis, and although the mathematics community honors Euclid for this, modern primary and secondary school teaching of geometry relies heavily on links to practical applications and examples from optics, physics and everyday live. Particularly in the Netherlands, this is the case (Feijs, 2005). Figure 3 shows one of the oldest fragments of a textbook on geometry and indeed the text is about lines, segments, rectangles, sections etc.[1]

Figure 3: Fragment of Euclid's "*The Elements*" (Euclid, 75-125 A.D.)

The ideal media for this type of geometry are books and classrooms using pencil and paper, with tools such as straightedge (ruler) and compass. It also goes very well with traditional blackboard and chalk or modern white board and marker. Although things

[1] translation: *If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half",* [5]

can be done using computers, usually it means that someone has to program most of it first, using some form of analytical geometry. A typical example is The Geometry Applet by Joyce (1996). Plane geometry fits well with today's flat computer screens and ink jet printers. Important results include Pythagoras' law, functions such as sine, cosine, tangent and their inverses. These results are indispensable when designing tessellations.

Now we turn to the *second view*, analytical geometry. Descartes connected geometry and algebra by coding the points of geometry as number pairs. Each point is given an x and a y coordinate. In Figure 4 this approach is shown in action.

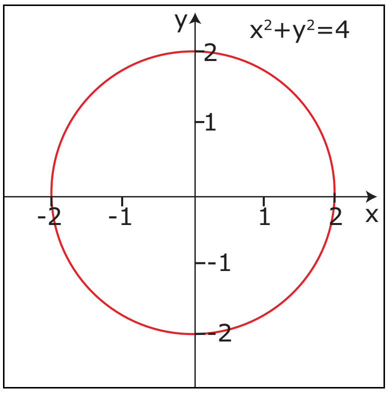

Figure 4: Circle described by equation in Cartesian coordinate system.

The circle with radius 2 around the origin is described by an equation $x^2 + y^2 = 4$. The circle can be shifted, for example, the substituting $x \rightarrow x{-}2$ and $y \rightarrow y{-}2$ and a bit of algebra give the equation of the circle around {2,2}, which is $x^2 + y^2 - 4x - 4y + 4 = 0$. Straight lines, ellipses, parabolas etc. each have their own type of equation. Geometry is replaced by algebra. Deep geometric results can be obtained by unleashing the power of algebra, for example to prove that one cannot construct a square whose area is equal to that of a given circle using straight ruler and compass needs Galois theory (Galois, 1830a, 1830b), which is algebraic in nature. The ideal medium for analytical geometry is anything that helps to solve equations. First pen and paper, later log tables, calculators and now of course computers. When using computers, numerical procedures are useful, symbolic procedures are useful and their combination is best.

Finally we address the *third view* on geometry, transformational geometry. As we shall see, this is about symmetries, including the symmetries encountered in art, from ancient times to Escher and beyond. It also is useful for modern physics, for example quantum physics, but that is outside the scope of our teaching. In order to explain the basic idea, we refer to Figure 5 (snapshot of *Microsoft PowerPoint*).

Consider geometric transformations such as rotation, translation and reflection, for example those shown in the menu of Figure 5. The two-arrowed block shape can be rotated left over 90° or right over 90° but in both cases it becomes a different shape. When flipped horizontally or vertically it turns into the same shape again. The same applies rotation over 180° (twice 90°). Therefore we can say that the two-arrowed block shape is a symmetric shape. More precisely, each of the transformations which leave the figure the same, is a *symmetry* of this figure. Formulated somewhat more abstractly, the underlying mathematics is called group theory. Two operations can be composed, doing one and then the other, which is a transformation again. For example rotate left over

90° and then (again) left over 90° has the combined effect of the rotate 180° transformation[2].

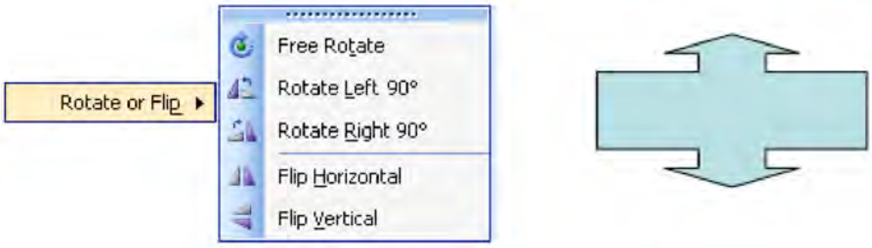

Figure 5: Symmetry operations and symmetric figure (Microsoft, 2007)

In two dimensions one can create patterns filling the entire plane such that the pattern remains the same under certain transformations. For a specific pattern, the set of transformations forms a so-called group. Next to translations, reflections and translations one needs an extra transformation called glide-reflection. Group theory is in the heart of studying regular patterns. There are three possibilities:

- If there are no translations, the patterns are so-called rosettes. There exist two sets of rosette groups, either with or without reflections. They have applications in art and architecture, most notably the rose windows in Gothic cathedrals. A number of rosettes of logos, each with different symmetries, are in Figure 6. The logos are Opel (180° rotation), Crosiers (180° rotation and reflections), Mitsubishi (120° rotation and reflection), Yamaha (idem), and NATO (90° rotation, no reflection).
- If there are only translations in one direction, the patterns are so-called friezes. There exist seven distinct frieze groups. They have numerous applications in art and architecture such as friezes of ancient temples and court buildings.
- If there are translations in two or more directions, the patterns are so-called wall-paper patterns. There exist seventeen distinct wallpaper groups. They appear on wallpaper indeed, in brickwork, in floor tessellations, in Escher's work and so on.

Figure 6: Five rosette logos with different symmetries (Opel, Canons Regular of the Order of the Holy Cross, Mitsubishi, Yamaha, NATO)

There exist also non-periodic tessellations such as invented by Penrose (1974), which have only been noted a few decades ago (folk wisdom of crystallography assumed such patterns would not be possible). In our teaching we used the rosette and the frieze

[2] A collection of such transformations is said to form a *group* if the following properties hold: Closure: if two transformations are in the group, then so is their composition; Associativity: composing T_1 with "T_2 composed with T_3" is the same as "T_1 composed with T_2" composed with T_3; Identity: there is a special transformation called the identity (doing nothing) which can be composed with any T with the effect of T; Inverses: for any transformation T in the group there is a transformation denoted as T^{-1} such that T composed with T^{-1} equals the identity and also T^{-1} composed with T equals the identity transformation.

patterns as easy forerunners of the more complex wallpaper patterns. We did so both in the lectures and in the assignments given to the students. As literature we provided Chapter I of the book *Regular Figures* by L. Fejes Toth (1964). The pedagogic idea behind this choice is that the book is mathematically quite rigorous[3] and that it is a good exercise for the students to develop their skill of getting information from such a book directly, instead of low-threshold and colorful Internet resources such as the Wikipedia Wallpaper group article [http://en.wikipedia.org/wiki/Wallpaper_group]. Another reason is that the idea of one transformation being transformed by another turns out useful later, when implementing transformations. It is very practical being able to read formulas like $S_2^{-1}S_1S_2$ and interpret them as action sequences to be programmed.

What is the ideal medium for studying and using this type of geometry? Again, pen with paper and computer algebra are useful, but both the development of intuition and the practical applications call for ways to make the transformations happen in physical reality and before one's eyes, or even better, in one's hands. Shifting pieces of paper one over another, transparent sheets, sliding and rotating images and semi-transparent images in picture editors, physical tiles, paper and scissors are all very useful. For certain Heesch tilings (see next sections for a detailed discussion of the Heesch tilings) there exist a trick called "The Envelop Method" based on slicing 3D objects (envelopes) into tile-able ground forms (Feijs, 2008). It is important to realize that the student can not "see" the transformations in the same way he or she can see the points and lines of Euclidean geometry.

Matrices: implementing transformations

In this section we briefly outline a technique for programming and executing transformations in practice. The same technique is used in computer graphics, computer games, etc. The technique was explained to our students, examples being coded in *Mathematica* (Wolfram Research, 2009, Version 7) and in this form it was also used to make the tessellations. Stephen Wolfram conceived *Mathematica* and is a standard software package used in scientific, engineering, and mathematical fields and other areas of technical computing. It is also the foundation of Wolfram's work on cellular automata (Wolfram, 2002). Points, lines and other geometric objects are moved around the plane by using Descartes' trick: manipulating their coordinates. To move a point with coordinates $\{x,y\}$ into a new position one needs four parameters: how the old x contributes to the new x, how the old x contributes to the new y and so on. It is customary to group those four numbers in a two by two block called a matrix. There are calculation rules how to apply a matrix to a coordinate pair. There are also rules how to multiply two matrices to yield a new one, representing the composed transformation. In Figure 7 four example matrices are shown. They represent the identity transformation, flipping along a diagonal, counter clockwise rotation over 90° and 45°, respectively.

Mathematica has very useful built-in notations and algorithms to work with matrices. It takes another trick to move from the level of points to the level of lines and curves (sets of coordinate pairs) and to the higher levels of complex shapes and complete figures (for example birds, fish, etc. as appearing in Esher's work). The trick

[3] To give an impression what we mean with "quite rigorous" we give a short quotation from Fejes Toth (pages 11 and 12), the paragraph where it is explained why a rosette group can only have rotations around a single centre: *Such a group contains only rotations about a single centre. For, if there were two rotations S_1, and S_2 with distinct centres O_1 and O_2, the transformation $S_1^{-1}S_2^{-1}S_1S_2$ of the group would be a degenerate rotation, by the additivity theorem for angles of rotation. It cannot be the identity, since it displaces O_1, into the image of O_1, under $S_2^{-1}S_1S_2$, i.e. under S_1 transformed by S_2. But the centre of this transformed rotation is the image of O_1 under S_2 i.e. a point different from O_1 and therefore $S_2^{-1}S_1S_2$ effects a change in O_1 . Hence $S_1^{-1}S_2^{-1}S_1S_2$ would be a non-degenerate translation, contrary to our assumption.*

deploys "mapping", using the *Mathematica* operator /@. This operator is well known to programmers in special languages such as LISP, ML or Clean but no such thing exists in C, C++ or Java.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$$

Figure 7: Matrices for identity, flipping, 90° rotation and 45° rotation.

Topology: from patterns to tilling.
It is tempting to assume that each of the 17 regular wallpaper patterns gives rise to precisely one schema of regular tiling. However, the relation is not one-to-one. What is still missing from the wallpaper patterns is: where are the cutting lines between the tiles? This question has been explored by Escher, aiming at his beautiful art. It also has been explored very thoroughly by Heesch and Kienzle (1963) aiming at industrial application and standardization. Heesch and Kienzle (1963) were well aware of group theory, but they added an analysis of the networks to be formed by the cutting lines between the tiles. Essentially this is about the edges (cutting line segments) and the vertices (nodes where three or more edges come together). For example, consider the structure of a honeybee comb where each tile has $n = 6$ edges and each vertex is joining three edges, see Figure 8.

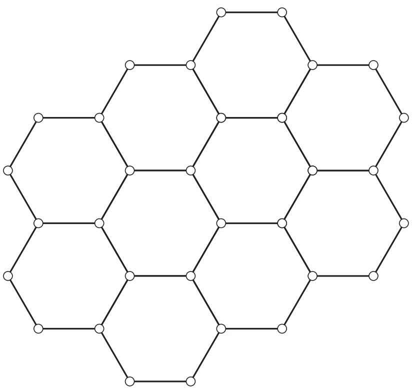

Figure 8: Network of tile edges.

This is a network of tile edges. This network has 32 vertices, 41 edges and 11 faces (including the outer space). Now there exists a mathematical result about such networks named after Leonhard Euler (1707-1783). It belongs to topology, that is the geometry of properties which are preserved by "rubber" transformations. Euler found that $V - E + F = 2$, where V is the number of vertices, E the number of edges and F the number of faces. Indeed, $32 - 41 + 11 = 2$. Heesch (1963) characterizes this network as type 333333, meaning that the six consecutive vertices when going round one tile, each connect 3 edges. The above network has eight tiles, so a first very rough guess could be that is has 10 x n vertices but of course most vertices are then counted three times instead of once. And the first guess of 10 x n edges has to be corrected because edges are counted twice. Going from 10 tiles to N tiles and inserting the corrections we find that $N.(\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}) - N.\frac{1}{2}.n + N = 2$. For very large N, taking the limit, we find $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1}{2}.n - 1$. This is for the given honeybee network, but the same reasoning can be applied to find all networks of triangles (n = 3). Let the three vertices have i, j and k edges. Now $1/i + 1/j + 1/k = \frac{1}{2}.3 - 1$. This is only possible for certain numbers of i, j and k. For example $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$, so $i = 4$, $j = k = 8$ is a possibility. Thus the network type 488 is found. Working further in this way only 11 networks are possible. Finally Heesch (1963) integrates this theory with wallpaper theory to get 28 tiling types. He describes them in a prescriptive style (in German). This is what we gave to the students (actually we gave access to all of Heesch's book, but we have no indication that they read anything except the prescriptions). An example is Figure 9. The network type is 43433. The tile itself is given a type code $CC_4C_4C_4C_4$ where C refers to 180° rotational symmetry and C_4 to 90° symmetry. These prescriptions are very practical to use. They could be used without any group theory or network theory, but we did present the essentials of those theories nevertheless.

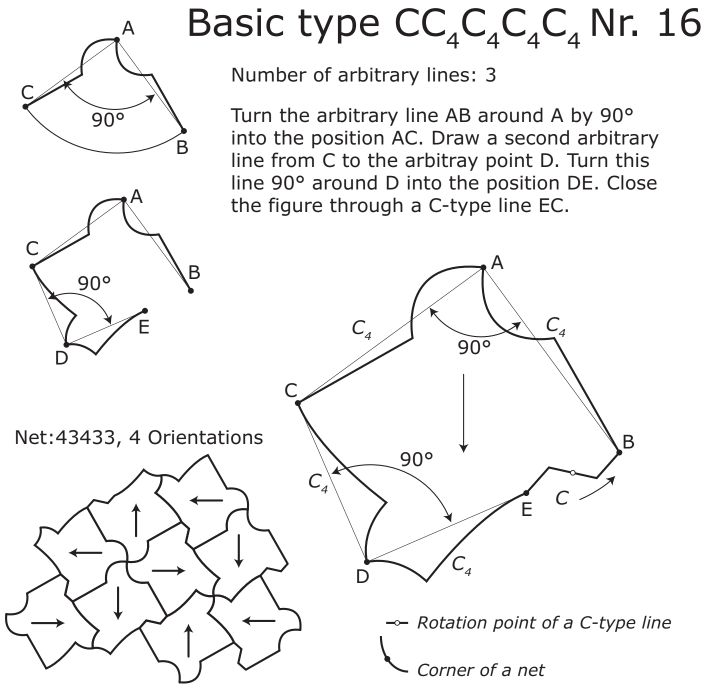

Figure 9: One of the 28 Heesch tile descriptions (Heesch & Kienzle, 1963), translated from German by the authors.

Execution of the course

Several steps need to be taken in advance of the course. First, the material needs to be ordered, ideally already cut to the dimensions of what the laser cutter can process. We use a Speedy300 laser cutter that can cut plates up to 40 x 70 cm. We used *Perspex* in at least three different colors to allow color variations. Dark colors are preferable since they bring out the engravings. Transparent *Perspex* is also available, but it requires additional attention during mounting.

Software licenses for *Mathematica*, *Illustrator* and *Corel Draw* must be available and the laser cutter needs to be booked. The duration of the cutting varies, in particular depending on the amount of engravings. In our experience, the works of six students can be cut in one day.

Workflow

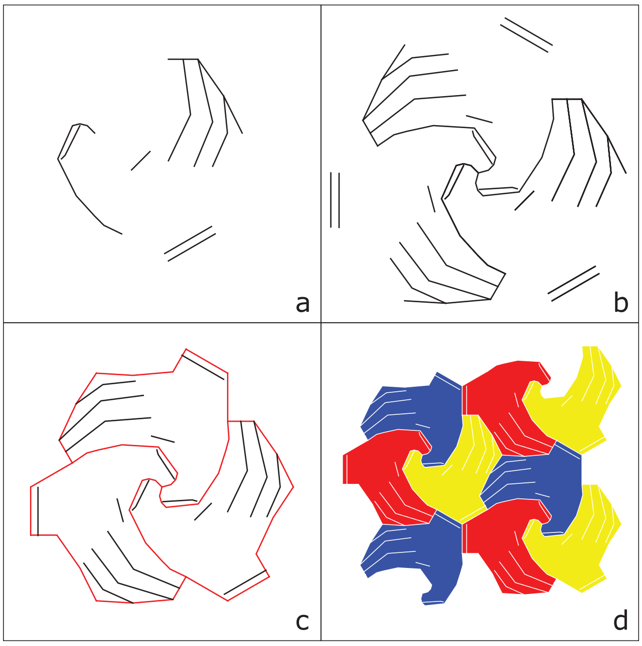

Figure 10: Workflow of tessellation. (design by Wouter Widdershoven)

The students prepare their tessellation in *Mathematica*. The lines they draw define the paths that the laser cutter will use for the cutting and engraving. The colours of the lines define if a certain line is used for cutting or engraving. For the engraving, the line thickness defines the thickness of the engraving. For the cutting, an ideal value of 0.001 mm is used. It is important that the students do not simply create a tile and repeat it, since this would create overlapping lines. The laser cut would cut this line multiple times, which takes far too much time and can reduce the overall quality of the tiles. The students export the *Mathematica* graphic as an encapsulated post script (EPS) file and final adjustments in Illustrator. The illustrator file is then imported into *CorelDraw* and transferred to the *Laser Cutter* control software through a virtual printer driver. Figure 10 shows the workflow that the students go through. First, they create a tile (a) which is then repeated using rotations and translations in *Mathematica* (b) before the necessary colours and thicknesses are assigned (c). The graphic in (c) is then sent to the laser cutter. Last, the students create a coloured version of (d), which makes it easier to puzzle the pieces together (see Figure 11).

The tiles are then mounted to foam board or another plate of *Perspex*. Transparent Perspex is more difficult to mount, since the mounting plate and the glue shines through. Additional examples of the students' work are available at http://www.bartneck.de/2008/05/21/the-golden-ratio-course/.

Figure 11: *Perspex* tiles cut with a laser cutter. (Design by Wouter Widdershoven)

Conclusions

We described a new approach for bringing the mathematical foundations of geometry into the focus of design work. The approach is new to the best of our knowledge. One possible reason is that the tools used are relatively new and moreover the community of mathematicians and the community of industrial designers are usually very separated.

The students did not use common 2D design software packages, such as *Adobe Illustrator*, to create a tessellation, but we asked them to define the tessellation in the abstract language of math. This truly brought mathematical foundations of geometry into the focus of the course and *Mathematica* is one of the most accessible tools for design students to express their ideas in math. Of course the students could have stopped their effort after the production of a digital version of their tessellation, but going the extra mile of cutting real tiles with a laser cutter had a tremendous effect on the motivation of the students. The final tessellations have been hung at the walls of our department and it represents a great reward to our students. The *Perspex* tiles moved the abstract geometrical principles out of the computer and into the real world. The tessellations are physical objects to which the students can relate. They are not just an abstract ideas, but products that can be touched and experienced. The students responded that they appreciated the brushing-up of their math, but most of all it was fun. They truly enjoyed understanding the mathematical principles of vector graphics, as it is used by *Adobe Illustrator* and other design software. Moreover, it empowered them to create their own tessellations more easily. The patterns used in the currently fashionable handbags (e.g. Louis Vuitton) are now trivial to them and they can create advance tessellations for the design of textiles, bathroom tiles and visual design in general. Our approach facilitates design because of four reasons. First it empowers the design students because they become familiar with *Mathematica* and related tools, which allows them to address other modeling problems as well. Secondly it allows the students to design patterns whose aesthetic qualities can be added or embedded in other works of design. Thirdly it gives them additional knowledge about patterns as they appear in nature and in technical domains. Fourth it invites them to learn more about Escher, Japanese art and similar cultural topics relevant for design students.

We can also conclude that the technology for creating tessellations has developed rapidly. The math tools and graphic design tools allow the creation of tessellations within minutes and the remaining limiting factor is the creativity of the designer. The development of affordable laser cutters also has a tremendous effect on tessellation: the cutting line is negligible. It is no longer necessary to calculate in the width of the cutting line and true tessellations can be cut in one run. In the future we intend to extend the module to include aperiodic patterns, such as *Penrose* tilling. We are also further exploring the usage of semi-transparent *Perspex* that can then be glued against windows, creating an effect similar to windows in cathedrals.

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Biographical statement

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