Price-based control of electrical power systems

Chapter 5
Price-based Control of Electrical Power Systems

A. Jokić, M. Lazar, and P.P.J. van den Bosch

Abstract In this chapter we present the price-based control as a suitable approach to solve some of the challenging problems facing future, market-based power systems. On the example of economically optimal power balance and transmission network congestion control, we present how global objectives and constraints can in real-time be translated into time-varying prices which adequately reflect the current state of the physical system. Furthermore, we show how the price signals can be efficiently used for control purposes. Becoming the crucial control signals, the time-varying prices are employed to optimally shape, coordinate and synchronize local, profit-driven behaviors of producers/consumers to mutually reinforce and guarantee global objectives and constraints. The main focus in the chapter is on exploiting specific structural properties of the global system constraints in the synthesis of price-based controllers. The global constraints arise from sparse and highly structured power flow equations. Preserving this structure in the controller synthesis implies that the devised solutions can be implemented in a distributed fashion.

A. Jokić, M. Lazar, P.P.J. van den Bosch
Eindhoven University of Technology, Department of Electrical Engineering, Eindhoven, The Netherlands, e-mail: {a.jokic,m.lazar,p.p.j.v.d.bosch}@tue.nl

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5.1 Introduction

The aim of this chapter is to present, illustrate and discuss the role of prices in devising certain control solutions for electrical power systems. In particular we focus on the problem of *capacity management* in the sense of optimal utilization of scarce transmission network capacity. The term *price-based control*, as we use it in this chapter, can be considered as equivalent to *market-based* or *incentives-based* operation and control. In general terms, the main idea of the chapter is to present how the price signals can be used as the main control signals for *coordination* of many *local* behaviors (subsystems) to achieve some crucial *global* objectives.

As an introduction to the control problem considered in this chapter, we continue with pointing out to some of the changes that are taking place in the operation of today's power systems, and to some specific features of these systems which make their control an extremely challenging task.

5.1.1 Power systems restructuring

In spite of their immense complexity and inevitable lack of our full comprehension of all dynamic phenomena that are taking place in electrical power systems, to the present days these systems have shown an impressive level of performance and robustness. To a certain extent, this can be attributed to the long persistence of a traditional, regulated industry, which had a practice of rather conservative engineering, control and system operation. Another reason for their success is that traditional power systems are characterized by highly repetitive daily patterns of power flows, with a relatively small amount of suddenly occurring, uncertain fluctuations on the aggregated power demand side, and with well-controllable, large-scale power plants on the power production side. As a consequence, in traditional power systems, a large portion of power production could be efficiently scheduled in an open-loop manner, while the classical automatic generation control (AGC) scheme [17] sufficed for efficient real-time power balancing of uncertain demands.

Market-based operation

The most significant change that is taking place in power systems over the past decade is a liberalization and a policy shift towards competitive market mechanisms for their operation. From a monopolistic, one utility controlled operation, the system is being restructured to include many parties competing for energy production and consumption, and for provision of many of the *ancillary services* necessary for the system operation, e.g., provision of various classes of capacity reserves [30]. The main operational goal has shifted from centralized, utility cost minimization objective to decentralized profit maximization objectives of individual parties, e.g., of producers, consumers, retailers, energy brokers, etc. Fulfillment of crucial system constraints, such as global power balance and transmission network limits, has become a responsibility of market and system operators. The challenging task in

designing the control and decision algorithms for these "global" operators is to ensure that the autonomous, profit driven behaviors of local subsystems will not act in a way that the system is driven to a highly unreliable and fragile state (or even instability), but will rather mutually reinforce on ensuring its integrity. The physical properties of electrical power systems play a prominent role in designing these markets and control architectures, and they are responsible for a very tight coupling in between economical and physical/technical layers of an electrical power system. They are also a reason why a straightforward transfer of knowledge and experience from deregulation, restructuring, operation and control of other sectors to the electric power system sector is often hampered or, even more often, is simply impossible.

Distributed generation and renewable energy sources

Another major change that is taking place in today's power systems is large-scale integration of distributed power generators (DG), many of which are based on intermittent renewable sources like wind and sun. Non-dependence on fossil fuels of many DG technologies, together with environmental issues, are the main driving forces for this change, and many countries have posed high targets concerning deployment of renewable sources over ten years horizons.

As a consequence, future power system will face a significant increase of uncertainties in any future system state prediction. Large and relatively fast fluctuations in production are likely to become normal operating conditions, standing in contrast to today's operating conditions characterized by highly repetitive, and therefore highly predictable, daily patterns. Note that the success of the present power systems heavily relies on this high predictability, while in the future, the need for fast acting, power balancing control loops will increase significantly.

5.1.2 Some specific features of power systems

Electrical power systems are one of the largest and most complex engineering systems ever created. They consist of thousands of generators and substations, and hundreds of millions of consumers all interconnected across circuits of continental scale. A distinguishing feature of electrical power systems, when compared for example to telecommunication networks, internet or road traffic networks, is that the subsystems in a electrical power system are all *physically* interconnected, i.e., dynamics of subsystems in the network are directly coupled.[1,2]

[1] Direct dynamical coupling is expressed through a set of equality constraints relating certain physical quantities among subsystems, e.g., coupling power flow equations in the electrical power system.

[2] Note that the networks in other infrastructures, e.g., telecommunication networks, internet or road traffic networks, could also be considered as *physically interconnected* in the sense that the subsystems in a network are related through certain physically realized links (which are possibly further characterized by some constraints), e.g., a highway in a road traffic network. However, the distinguishing feature of these systems, when compared to an electrical power system, is that

Large scale, physical interconnections, and the following specific features of electrical power systems makes mastering their complexity and devising an efficient operational and control solutions an extremely challenging task:

- *Heterogeneity and autonomy.* There is an enormous variety of physical devices interconnected to the network, with huge spectra of possible dynamical (physical) characteristics. All these *local*, almost exclusively nonlinear, dynamical characteristics of the subsystems are taking part in shaping the *global* dynamic behavior of the system, as they are all physically interconnected. In the economical layer of liberalized power systems, power producers and consumers (prosumers) are autonomous decision makers which are driven by their *local*, profit-driven objectives. As they are sharing the same power transmission network, which has a limited capacity, care has to be taken that the *local* and *autonomous* behaviors do not overload or destabilize the system. This generic *global* goal is not the natural aim of autonomously acting prosumers.
- *No free routing.* Unlike other transportation systems, which assume a free choice among alternative paths between source and destination, the flow of power in electrical energy transmission networks is governed by physical laws and is characterized by complex dependencies on nodal power injections (nodal productions and consumptions). Due to the complex relations, creating efficient congestion management schemes to cope with the transmission constraints is one of the toughest problems in design of market, operational and control architectures for power systems.
- *No buffering of commodities.* Electrical energy cannot be efficiently stored in large quantities, which implies that production has to meet rapidly changing demands immediately in *real-time*. This characteristic makes electricity a commodity with fast changing production costs, and is responsible for a tight coupling between economical and physical/techical layers of a power system.

5.1.3 Related work

The publications of Fred Schweppe and his co-workers can be considered as the first studies that systematically investigated the topic of price-based operation of electrical power systems. Many of the results from that period are summarized in [8, 26–28]. Ever since, there has been a tremendous amount of research devoted to a market-oriented approach for the electrical power system sector. For a detailed introduction and an overview, the interested reader is to referred to many books on the subject, e.g., [14, 18, 27, 29, 30]. In particular, for a detailed overview and some recent results concerning different approaches to price-based power balancing and congestion management of transmission systems we refer to [3, 6, 10, 13, 23, 31] and the references therein.

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the dynamics of subsystems (e.g., cars on a highway) are not *directly coupled*, but possibly only indirectly, e.g., through some common performance objectives (common tasks) and/or inequality constraints (e.g., collision avoidance).

Probably the most closely related to some of the results presented in this chapter is the work of Alvarado and his co-workers [1–4, 11]. In [11], the authors have investigated how an independent system operator (ISO) could use electricity prices for congestion management without having an *a priori* knowledge about cost functions of the generators in the system. There, the authors illustrated how, in principle, a sequence of market observations could be used to estimate the parameters in the cost function of each generator. Based on these estimates, and by solving a suitably defined optimization problem, an ISO could issue the nodal prices causing congestion relieve. Although dealing with an intrinsically dynamical problem, the paper considered all the processes in a static framework. In [2, 3] the results of [11] have been extended by addressing possible issues of concern when price-based congestion management is treated as a dynamical process. The usage of price as a *dynamic feedback control* signal for power balance control has been investigated in [4]. There, the effects of interactions of price update dynamics and the dynamics of an underlying physical system (e.g., generators) on the stability of the overall system have been investigated. However, no congestion constraints have been considered and therefore only one, scalar valued, price signal was used to balance the power system.

In this chapter we present how nodal prices can be efficiently used for real-time power balance control and congestion management of a transmission network. These results, which treat the considered problem in a dynamical framework, present an extension of the above mentioned contributions. In particular, the emphasis in this chapter is on devising efficient control structures that exploit the specific structure of global power flow equations and constraints related to the transmission network. We show how preserving this structure in the proposed solutions results in a price-based control structure with an advantageous property that it can be readily implemented in a *distributed* fashion.

5.1.4 Nomenclature

The field of real numbers is denoted by $\mathbb{R}$, while $\mathbb{R}^{m\times n}$ denotes m by n matrices with elements in $\mathbb{R}$. For a matrix $A \in \mathbb{R}^{m\times n}$, $[A]_{ij}$ denotes the element in the i-th row and j-th column of A. For a vector $x \in \mathbb{R}^n$, $[x]_i$ denotes the i-th element of x. The transpose of a matrix A is denoted by $A^\top$. A vector $x \in \mathbb{R}^n$ is said to be nonnegative (nonpositive) if $[x]_i \geq 0$ ($[x]_i \leq 0$) for all $i \in \{1,\dots n\}$, and in that case we write $x \geq 0$ ($x \leq 0$). For $u, v \in \mathbb{R}^k$ we write $u \perp v$ if $u^\top v = 0$. We use the compact notational form $0 \leq u \perp v \geq 0$ to denote the complementarity conditions $u \geq 0$, $v \geq 0$, $u \perp v$. Ker$A$ and ImA denote the kernel and the image space of A, respectively. We use I_n and $\mathbf{1}_n$ to denote an identity matrix of dimension $n \times n$ and a column vector with n elements all being equal to 1, respectively. The operator $\mathrm{col}(\cdot,\dots,\cdot)$ stacks its operands into a column vector, and $\mathrm{diag}(\cdot,\dots,\cdot)$ denotes a square matrix with its operands on the main diagonal and zeros elsewhere. The matrix inequalities $A \succ B$ and $A \succeq B$ mean A and B are Hermitian and $A - B$ is positive definite and positive semi-definite, respectively. For a scalar-valued differentiable function $f : \mathbb{R}^n \to \mathbb{R}$,

$\nabla f(x)$ denotes its gradient at $x = \text{col}(x_1, \ldots, x_n)$ and is defined as a *column vector. With a slight abuse of notation we will often use the same symbol to denote a signal, i.e., a function of time, as well as possible values that the signal may take at any time instant.*

5.2 Optimization decomposition: Price-based control

It is fair to say that the modern control systems theory is grounded on the following remarkable fact: virtually all control problems can be casted as optimization problems. It is insightful to realize that the same, far reaching statement, holds as well for the power systems: virtually all global operational goals of a power system can be formulated as constrained, time-varying optimization problems. Similarly as modern control theory accounts for efficiently solving these optimization problems (which is in most cases a far from trivial task), the same mathematical framework provides a systematic and rigorous scientific approach to shape operational and control architectures of power systems[3]. For illustration, in mathematical terms, a shift from monopolistic, one utility controlled system, to the market-based system is seen as a shift from explicitly solving a *primal problem* (e.g., economic dispatch at the control center) to solving its *dual problem* (e.g., operating real-time energy market). The former case can be called the cost-based operation, while the latter can be called the price-based operation. Before continuing with consideration of some specific problems in power systems, and their price-based solutions, we will first recall some basic notions from optimization theory. For the general introduction, closely related subjects and the state-of-the-art results on the distributed optimization, the interested reader is refereed to [5, 7, 9, 19, 22] and the references therein.

Consider the following structured, time varying[4], optimization problem

$$\min_{x_1,\ldots,x_N} \quad \sum_{i=1}^{N} J_i(x_i), \tag{5.1a}$$

subject to

$$x_i \in \mathcal{X}_i, \qquad i = 1,\ldots,N, \tag{5.1b}$$

$$G(x_1,\ldots,x_N) \leq 0, \tag{5.1c}$$

$$H(x_1,\ldots,x_N) = 0, \tag{5.1d}$$

where $x_i \in \mathbb{R}^{n_i}$, $i = 1,\ldots,N$ are the local decision variables, the functions $J_i : \mathbb{R}^{n_i} \to \mathbb{R}$, $i = 1,\ldots,N$, denote the local objective functions, while each set $\mathcal{X}_i \subseteq \mathbb{R}^{n_i}$ is defined through a set of *local* constraints on the corresponding local variable x_i as follows

[3] The interested reader is referred to the excellent paper [9] where the role of alternative ways for solving optimization problems is reflected in devising alternative operational structures for communication networks.

[4] For notational convenience, we have omitted the explicit reference to the time dependence.

$$\mathcal{X}_i = \{x_i \in \mathbb{R}^{n_i} \mid g_i(x_i) \leq 0,\ h_i(x_i) = 0\}, \tag{5.2}$$

where $g_i(\cdot)$ and $h_i(\cdot)$ are suitably defined vector valued functions. The functions G and H, which respectively take values in $\mathbb{R}^k$ and $\mathbb{R}^l$, define *global* inequality and equality constraints.

Note that the optimization problem (5.1) is defined on the overall, global system level, where global objective function is sum of local objectives as indicated in (5.1a). Furthermore, note that if the global constraints (5.1c) and (5.1d) are omitted, the optimization problem (5.1) becomes separable in a sense that it is composed of N *independent* local problems which can be solved separately. For such a completely separable case, we say that the optimization problem can be solved in a *decentralized* way. For the future reference, we will call the problem (5.1) the *primal problem*.

Next, from (5.1) we formulate the *dual problem* as follows

$$\max_{\lambda,\mu} \quad \ell(\lambda,\mu) \tag{5.3a}$$

$$\text{subject to} \quad \mu \geq 0, \tag{5.3b}$$

where

$$\ell(\lambda,\mu) := \min_{x_1\in\mathcal{X}_1,\ldots,x_N\in\mathcal{X}_N} \left(\sum_{i=1}^{N} J_i(x_i) - \lambda^\top H(x_1,\ldots,x_N) + \mu^\top G(x_1,\ldots,x_N) \right). \tag{5.4}$$

In (5.3) and (5.4) $\lambda \in \mathbb{R}^k$ and $\mu \in \mathbb{R}^l$ are the dual variables (Lagrange multipliers) and have an interpretation of *prices* for satisfying the global constraints (5.1c) and (5.1d). If (5.1) is a convex optimization problem, it can be shown that the solutions of the primal and the dual problem coincide[5].

Remark 5.1. ***Decomposition and local optimization.*** If the functions H and G have an additive structure in local decision variables x_i, meaning that $H(x_1,\ldots,x_N) = \sum_{i=1}^N \tilde{h}_i(x_i)$ and $G(x_1,\ldots,x_N) = \sum_{i=1}^N \tilde{g}_i(x_i)$ with some given functions $\tilde{h}_i(\cdot)$, $\tilde{g}_i(\cdot)$, $i = 1,\ldots,N$, then for a fixed λ and μ the optimization problem in (5.4) is separable and can be solved in a decentralized way. In that case the i-th local optimization problem is given by

$$\min_{x_i\in\mathcal{X}_i} \quad J_i(x_i) - \lambda^\top \tilde{h}_i(x_i) + \mu^\top \tilde{g}_i(x_i). \tag{5.5}$$

□

[5] In fact, an additional mild condition, the so-called Slater's constraints qualification, is required for the solutions to coincide, see e.g., [7] for more details.

Remark 5.2. ***Coordination via global price determination.*** Updating the the dual variables (prices) λ, μ to solve the maximization problem in (5.3) can be achieved in a centralized way on a global level, e.g., at the central market operator which calculates the market clearing price. In some important cases, as it will be presented in the following section, the optimal prices (λ, μ) can also be efficiently calculated in a *distributed* way. This means that they can be calculated even if there is no one central unit that gathers information and communicates with all the subsystems (local optimization problems) in the network, but the optimal price calculations are based only on the locally available information and require only limited communication among neighboring systems. □

Example 5.1. Loosely speaking, the market-based power system can be seen as solving the dual optimization problem (5.3). When the power limits in transmission network are not considered, this can be more precisely described as follows. Suppose that for each i, local decision variable x_i is a scalar and represents the power production ($x_i > 0$) of a power plant i, or a power consumption ($x_i < 0$) if the subsystem i is a consumer. Furthermore, let $J_i(x_i)$ denote power production costs when the i-th subsystem is a producer, and its negated benefit function when the i-th subsystem is a consumer. Since we do not consider the transmission network limits, the only global constraint is the power balance constraint $\sum_{i=1}^{N} x_i = 0$, i.e., in (5.1) and (5.3) we have that $H(x_1,\ldots,x_N) := \sum_{i=1}^{N} x_i$. Obviously, the *primal problem* (5.1) now corresponds to minimization of total production costs and maximization of total consumers benefit, while the power balance constraint is *explicitly* taken into account via (5.1d). Let us now consider the *price-based solution* through the corresponding *dual problem*. First note that minimization problem in (5.4) is in this case given by

$$\ell(\lambda) := \min_{x_1,\ldots,x_N} \sum_{i=1}^{n} \left(J_i(x_i) - \lambda x_i\right). \tag{5.6}$$

In (5.6) each term in the summation, i.e., $J_i(x_i) - \lambda x_i$, denotes the benefit of a subsystem i where λ denotes the price for electricity. Obviously, the dual problem (5.3) then amounts to maximizing the total benefit of the system. Note that in solving the dual problem, the power balance constraint is not explicitly taken into account. However, the corresponding maximum in (5.3) is attained precisely when the price λ is such that for the solution to the corresponding minimization problem in (5.6) it holds that $\sum_{i=1}^{N} x_i = 0$. In other words, the price λ which maximizes the total benefit of the system is precisely the price for which the system is in balance.

To summarize, while in primal solution the global constraints were *explicitly taken into account*, in the dual solution they are *enforced implicitly through the price* λ. □

The observation from the above presented example can be generalized to the core idea of the price price-based control approach: In the price-based control, a price (Lagrange multiplier) is assigned to each crucial global constraint (i.e., each row in

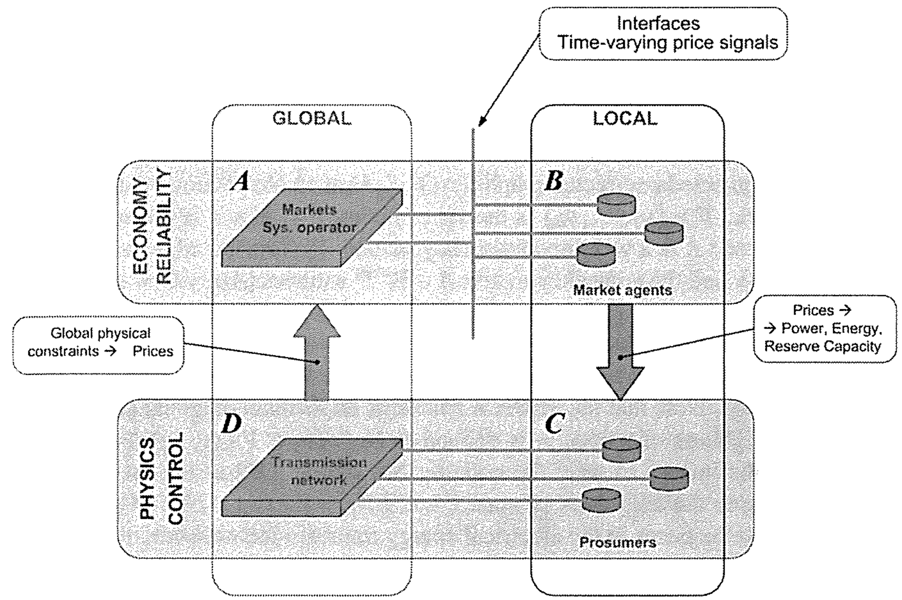

Figure 5.1: The price-based control loop.

(5.1c) and (5.1d), see (5.4)) and is used to implicitly enforce this constraint via local optimization problems (see e.g., (5.5)).

In a rather general sense, the price-based control loop can be illustrated as shown in Figure 5.1, and is encompassing the interplays between:
i.) *physical layer* of a power system (C and D in Figure 5.1), with time varying power flows as prominent signals; and *economical layer* (A and B in Figure 5.1) with time varying price signals as the prominent information carriers;
ii.) *local objectives* of producers/consumers (prosumers) (B and C in Figure 5.1, corresponding to (5.5)) and *global constraints*, e.g., power balance, transmission network limits and reliability constraints (A and D in Figure 5.1, corresponding to (5.1c) and (5.1d)).

Furthermore, prices are the signals used for *coordination* and *time synchronization* of actions from decentralized decision makers, so that the global system objectives are necessarily achieved in such that way that the total social welfare of the system is maximized, i.e., that they are achieved in the economically optimal way. It is also insightful to interpret the price-based solutions as *incentives-based* solutions, as prices λ are used to give incentives to the local subsystems so that their local objectives will make them behave in a way which serves global needs.

5.3 Preserving the structure: Distributed price-based control

5.3.1 Problem definition

Consider a connected undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ as an abstraction of an electrical power network. $\mathcal{V} = \{\nu_1, \ldots, \nu_n\}$ is the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of undirected edges, and A is a weighted adjacency matrix. Undirected edges are denoted as $\varepsilon_{ij} = (\nu_i, \nu_j)$, and the adjacency matrix $A \in \mathbb{R}^{n \times n}$ satisfies $[A]_{ij} \neq 0 \Leftrightarrow \varepsilon_{ij} \in \mathcal{E}$ and $[A]_{ij} = 0 \Leftrightarrow \varepsilon_{ij} \notin \mathcal{E}$. No self-connecting edges are allowed, i.e., $\varepsilon_{ii} \notin \mathcal{E}$. We associate the edges with the power lines of the electrical network and, for convenience, we set the weights in the adjacency matrix as follows: $[A]_{ij} = -b_{ij}$, where b_{ij} is the line susceptance. Note that the matrix A has zeros on its main diagonal and $A = A^\top$. The set of neighbors of a node ν_i is defined as $N_i \triangleq \{\nu_j \in \mathcal{V} \mid (\nu_i, \nu_j) \in \mathcal{E}\}$. Often we will use the index i to refer the node ν_i. We define $I(N_i)$ as the set of indices corresponding to the neighbors of node i, i.e., $I(N_i) \triangleq \{j \mid \nu_j \in N_i\}$. We associate the nodes with the buses in the electrical energy transmission network.

5.3.1.1 Primal: Optimal power flow problem

To define the optimal power flow problem as a primal optimization problem (5.1) on the global level, with each node ν_i we associate a set of local decision variables $\{p_i, \delta_i\}$, i.e., in (5.1) $x_i := \mathrm{col}(p_i, \delta_i)$, a singlet $\hat{p}_i$ and a triplet $(\underline{p}_i, \overline{p}_i, J_i)$. Here $p_i, \delta_i, \underline{p}_i, \overline{p}_i, \hat{p}_i \in \mathbb{R}$, $\underline{p}_i < \overline{p}_i$ and $J_i : \mathbb{R} \to \mathbb{R}$ is a strictly convex, continuously differentiable function. The values p_i and $\hat{p}_i$ denote the reference values for node power injections into the network, while δ_i denotes a voltage phase angle at the node ν_i. Positive values of p_i and $\hat{p}_i$ correspond to a flow of power into the network (production), while negative values denote power extracted from the network (consumption). Both p_i and $\hat{p}_i$ can take positive as well as negative values, and the only difference is that, in contrast to $\hat{p}_i$, the value p_i has an associated objective function J_i and a constraint $\underline{p}_i \leq p_i \leq \overline{p}_i$. In the case of a positive p_i, the function J_i represents the variable costs of production, while for negative values of p_i, it denotes the negated benefit function of a consumer. We will refer to p_i as the power from a price-elastic producer/consumer (or simply, power from a price-elastic unit), and to $\hat{p}_i$ as the power from a price-inelastic producer/consumer (price-inelastic unit).

Note that the assumption that one price-elastic unit and one price-inelastic unit are associated with each node is made to simplify the presentation and it does not result in any loss of generality of the presented results.

We use a "DC power flow" model[6] to determine the power flows in the network for given values of node power injections. The power flow in a line $\varepsilon_{ij} \in E$ is given by $p_{ij} = b_{ij}(\delta_i - \delta_j) = -p_{ji}$. If $p_{ij} > 0$, power in the line ε_{ij} flows from node ν_i to node ν_j. The power balance in a node yields $p_i + \hat{p}_i = \sum_{j \in I(N_i)} p_{ij}$. With the

[6] The DC power flow model is a linear approximation of a complex AC power flow model and is often used in practice. For a study comparing the AC and DC power flow models, and in particular the impact of the linear approximation on the nodal prices, the interested reader is referred to [20].

abbreviations $p = \mathrm{col}(p_1, \ldots, p_n)$, $\hat{p} = \mathrm{col}(\hat{p}_1, \ldots, \hat{p}_n)$, $\delta = \mathrm{col}(\delta_1, \ldots, \delta_n)$ the overall network balance condition is $p + \hat{p} = B\delta$, where the matrix B is given by $B = A - \mathrm{diag}(A\mathbf{1}_n)$.

Problem 5.1. *Optimal Power Flow (OPF) problem.*
For any constant value of $\hat{p}$,

$$\min_{p,\delta} J(p) \triangleq \min_{p,\delta} \sum_{i=1}^{n} J_i(p_i) \tag{5.7a}$$

subject to

$$p - B\delta + \hat{p} = 0, \tag{5.7b}$$

$$\underline{p} \leq p \leq \overline{p}, \tag{5.7c}$$

$$b_{ij}(\delta_i - \delta_j) \leq \overline{p}_{ij}, \ \forall (i, j \in I(N_i)), \tag{5.7d}$$

where $\underline{p} = \mathrm{col}(\underline{p}_1, \ldots, \underline{p}_n)$, $\overline{p} = \mathrm{col}(\overline{p}_1, \ldots, \overline{p}_n)$, and $\overline{p}_{ij} = \overline{p}_{ji}$ is the maximal allowed power flow in the line ε_{ij}. □

We will refer to a vector p that solves the OPF problem as a *vector of optimal power injections*.

For an appropriately defined matrix L and a suitably defined vector of power line limits $\overline{p}_L$, the set of constraints in (5.11c) can be written in a more compact form as follows:

$$L\delta \leq \overline{p}_L. \tag{5.8}$$

Note that the constraints (5.7b) and (5.7d) (or equivalently (5.8)) represent global equality and inequality constraints (5.1d) and (5.1c), respectively. Furthermore, for each node i, the corresponding constraint (row) in (5.7c) represent the local inequality constraint in (5.2).

Remark 5.3. In Problem 5.1 we have included δ explicitly as a decision variable, which will be crucial in the price-based control design. Another possibility, common in the literature, is to introduce a "slack bus" with zero voltage phase angle and to solve the equations for the line flows, completely eliminating δ from the problem formulation. However, in that case a specific structure, i.e., sparsity, of the power flow equations is lost. As we will see later in this chapter, preserving this sparsity will show to be beneficial for *distributed* controller implementation. □

Remark 5.4. The matrix B is a singular matrix with rank deficiency one and with the kernel space spanned by the vector $\mathbf{1}_n$. Physically, this reflects the fact that only the relative voltage phase angles determine the power flow. □

In traditional power system structures, where the production units are owned by one utility and there are little or no price elastic consumers, adjusting the production according to the solution of the OPF problem is one of the major operational goals of a utility. In such a system, the OPF problem is directly (explicitly) solved at a utility dispatch center, and the optimal reference values p are sent to the production units.

In contrast to this, in deregulated and liberalized power systems, the OPF problem is only indirectly (implicitly) solved by utilization of nodal prices. Next we define the optimal nodal prices problem as a central problem of liberalized, price-based operated systems.

5.3.1.2 Dual: Optimal nodal prices problem

According to price-based control approach we assign prices (Lagrange multipliers) to the global coupling constraints (5.7b) and (5.7d) (i.e., (5.8)) in Problem 5.1 to obtain the corresponding dual problem:

$$\max_{\mu \geq 0, \lambda} \quad \ell(\lambda, \mu) \tag{5.9}$$

where

$$\ell(\lambda, \mu) := \min_{\delta, p \in \{\tilde{p} | \underline{p} \leq \tilde{p} \leq \overline{p}\}} \quad \sum_{i=1}^{N} J_i(p_i) - \lambda^\top (p - B\delta + \hat{p}) + \mu^\top (L\delta - \overline{p}_L). \tag{5.10}$$

In (5.9) and (5.10) λ and μ are (vector) Lagrange multipliers.

Remark 5.5. The global coupling constraints (5.7b) and (5.8) do not have an additive structure in the decision variables p and δ, see Remark 5.1. Therefore for some fixed λ and μ the optimization problem in (5.10) is not separable (see Remark 5.1). However, when only the prices λ and the decision variables p are considered, the problem (5.10) becomes separable, i.e., it can readility be decomposed into n local problems, each assigned to one price-elastic unit. □

With respect to Remark 5.5, it is insightful the reformulate the dual problem (5.9) to the following equivalent problem.

Problem 5.2. *Optimal nodal prices (ONP) problem.* For a constant value of $\hat{p}$,

$$\min_{\lambda, \delta} \sum_{i=1}^{n} J_i(\Upsilon_i(\lambda_i)) \tag{5.11a}$$

$$\text{subject to} \quad \Upsilon(\lambda) - B\delta + \hat{p} = 0, \tag{5.11b}$$

$$L\delta \leq \overline{p}_L, \tag{5.11c}$$

where $\lambda = \text{col}(\lambda_1, \ldots, \lambda_n)$ is a vector of nodal prices,

$$\Upsilon(\lambda) \triangleq \text{col}(\Upsilon_1(\lambda_1), \ldots, \Upsilon_n(\lambda_n))$$

and

$$\Upsilon_i(\lambda_i) \triangleq \arg\min_{\tilde{p}_i} \{J_i(\tilde{p}_i) - \lambda_i \tilde{p}_i \mid \underline{p}_i \leq \tilde{p}_i \leq \overline{p}_i\}. \tag{5.12}$$

□

Although equivalent to (5.9), the problem formulation via (5.11) and (5.12) is insightful as it clearly indicates on one hand the role and *global* objectives of a system operator and on the other hand the *local* objectives of price elastic units. The letter is described by (5.12) and has the following interpretation: when a price elastic unit at node i receives a price λ_i for electricity at that particular node, it will adjust its production p_i to maximize its own benefit $J_i(p_i) - \lambda_i p_i$. The role of a system operator is to determine and issue a vector of nodal prices λ such that the overall system benefit is maximized (5.11a) while the system is in balance (5.11b) and while no line in the transmission system is congested (5.11c). A vector λ that solves the ONP problem is the *vector of optimal nodal prices.*

5.3.1.3 Price-based control problem

Consider a power network where each unit, i.e., producer/consumer, is a dynamical system, and assign to each such unit an appropriate model of its dynamics. Let G_i and $\hat{G}_i$ denote respectively a dynamical model of price-elastic and price-inelastic unit at node i as follows:

$$G_i : \dot{x}_i = f_i(x_i, p_i^A, p_i) = f_i(x_i, p_i^A, \Upsilon_i(\lambda_i)), \ \forall i, \tag{5.13a}$$

$$\hat{G}_i : \dot{z}_i = \hat{f}_i(z_i, \hat{p}_i^A, \hat{p}_i), \ \forall i, \tag{5.13b}$$

where x_i and z_i are the state vectors, p_i^A and $\hat{p}_i^A$ denote the *actual* node power injection from the system G_i and $\hat{G}_i$, respectively, into the network. As already mentioned, the input $p_i = \Upsilon_i(\lambda_i)$ denotes a price-dependent *reference* signal for power injection, i.e., $p_i = \Upsilon_i(\lambda_i)$ represents desired production/consumption of a price-elastic unit, while the input $\hat{p}_i$ denotes a *reference* value for the power injection of a price-inelastic unit. The desired production/consumption $\hat{p}_i$ of a price-inelastic unit does not depend on the electricity price λ_i, neither on any other signal from the power system.

Note that (5.11b) is always fulfilled when $\Upsilon(\lambda)$ and $\hat{p}$ are replaced respectively with $p^A = \text{col}(p_1^A, \ldots, p_n^A)$ and $\hat{p}^A = \text{col}(\hat{p}_1^A, \ldots, \hat{p}_n^A)$, since in this case (5.11b) represents the conservation law, i.e.,

$$p^A - B\delta + \hat{p}^A = 0. \tag{5.14}$$

To summarize, the complete dynamical model of a power system is described with the set of differential algebraic equations (5.13) and (5.14), with λ and $\hat{p}$ as inputs.

As opposed to the *actual* power injections, which are always in balance (5.14), keeping the balance in *reference* values (5.11b), i.e., balance in *desired* production and consumption, is a control problem. For future reference, we will always use the term *power balance* to refer to the power balance in sense of (5.11b), and not to the physical law (5.14).

To solve the power balance control problem, a measure of imbalance has to be available. The network frequency serves that purpose. Let $\Delta f \triangleq \text{col}(\Delta f_1, \ldots, \Delta f_n)$ denote the vector of nodal frequency deviations. In steady-state the network fre-

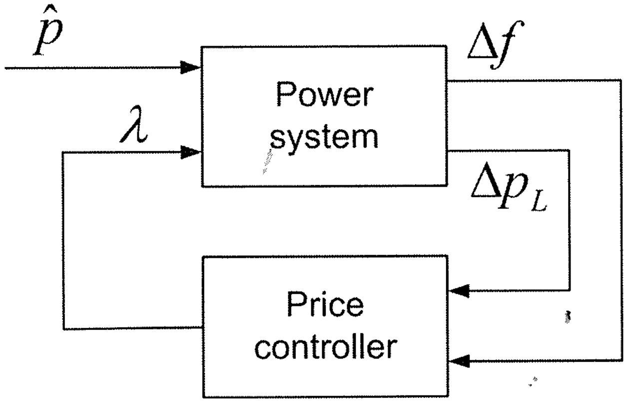

Figure 5.2: Price-based control scheme for real-time power balancing and congestion management.

quency is equal for all nodes in the system and the system is in balance if the network frequency is equal to its reference value, i.e., if $\Delta f = 0$. More precisely, if a system is in a steady-state with $\Delta f = 0$, then for each node (5.13a) implies $p_i = \Upsilon_i(\lambda_i) = p_i^A$, while (5.13b) implies $\hat{p}_i = \hat{p}_i^A$, and therefore (5.14) implies (5.11b).

In addition to controlling the power balance, nodal prices are used for congestion control, i.e., for fulfilment of the inequality constraints (5.8). For convenience we will define the vector of line overflows as $\Delta p_L \triangleq L\delta - \overline{p}_L$.

Finally, we are ready to define the control problem.

Problem 5.3. *Optimal price-based control problem.* For a power system (5.13) - (5.14), design a feedback controller that has the network frequency deviation vector Δf and the vector of line overflows Δp_L as inputs, and the nodal prices λ as output (see Figure 5.2), such that the following objective is met: for any constant value of $\hat{p}$ such that the ONP problem is feasible, the state of the closed-loop system converges to an equilibrium point where the nodal prices are the *optimal nodal prices* as defined in Problem 5.2. □

5.3.2 Distributed price-based controller

In this subsection we first present an algebraic characterization (the Karush-Kuhn-Tucker optimality conditions) of optimal nodal prices in Problem 5.2 and study the structure of the matrices B and L which define the global coupling constraints (5.7b) and (5.7d) (i.e., (5.8)). As the main point, we show that this structure is preserved in the algebraic characterization of optimal nodal prices. Secondly, we show how an appropriate dynamic extension of these algebraic optimality conditions can be used as a solution to Problem 5.3.

5.3.2.1 Algebraic characterization of optimal nodal prices: the KKT conditions

The optimal power flow problem (5.7) is a convex problem which satisfies Slater's constraint qualification [7]. Therefore, for this problem the strong duality holds and the first-order Karush-Kuhn-Tucker (KKT) conditions [7] are *necessary and sufficient* conditions for optimality, and present us with the following characterization of optimal nodal prices.

Consider some constant value $\hat{p}$ such that the ONP problem (and therefore the optimal power flow problem (5.7)) is feasible. The KKT conditions for the optimal power flow problem (5.7) are given by:

$$p - B\delta + \hat{p} = 0, \tag{5.15a}$$
$$B\lambda + L^\top \mu = 0, \tag{5.15b}$$
$$\nabla J(p) - \lambda + \nu^+ - \nu^- = 0, \tag{5.15c}$$
$$0 \leq \ (-L\delta + \overline{p}_L) \ \perp \ \mu \ \geq 0, \tag{5.15d}$$
$$0 \leq \ (-p + \overline{p}) \ \perp \ \gamma^+ \ \geq 0, \tag{5.15e}$$
$$0 \leq \ (p + \underline{p}) \ \perp \ \gamma^- \ \geq 0, \tag{5.15f}$$

where λ and μ are Lagrange multipliers associated to global constraints (5.7b) and (5.7d) (i.e., (5.8)) as before in (5.9),(5.10), while γ^+ and γ^- the Lagrange multipliers associated with the local inequality constraints $p \leq \overline{p}$ and $p \geq \underline{p}$, respectively. Recall that the for $a, b \in R^n$, the expression $0 \leq a \perp b \geq 0$ means $a \geq 0$, $b \geq 0$ and $a^\top b = 0$.

Notice that if no line is congested in the system, then the Lagrange multiplier μ in (5.15) is equal to zero and (5.15b) yields $B\lambda = 0$. This implies $\lambda \in \operatorname{Ker} B$ or $\lambda = \mathbf{1}_n \lambda^*$, $\lambda^* \in \mathbb{R}$ (see Remark 5.4), i.e., at the optimum, there is one price in the network for all nodes. In the case that at least one line in the system is congested, it follows that the optimal nodal prices will in general be different for each node in the system.

Remark 5.6. The only "direct" coupling of the elements in of optimal Lagrange multipliers λ and μ is present in equality (5.15b) and is completely determined by matrices B and L, while the "indirect" coupling beween elements of λ and μ is via p and δ and through (5.15a), (5.15c) and (5.15d). □

Example 5.2. Consider a simple network depicted in Figure 5.3 and let $\overline{p}_{12}$ and $\overline{p}_{13}$ denote the line flow limits in the lines ε_{12} and ε_{13}, respectively. With μ_{12} and μ_{13} denoting the corresponding Lagrange multipliers from (5.15d), the optimality condition (5.15b) relates the optimal nodal prices with the following equality:

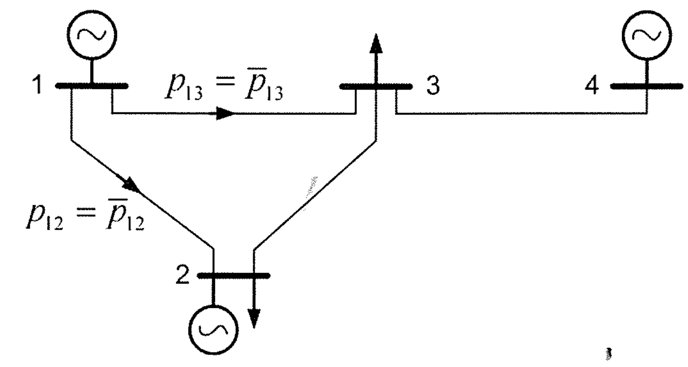

Figure 5.3: An example of a simple congested network.

$$\begin{bmatrix} b_{12,13} & -b_{12} & -b_{13} & 0 & b_{12} & b_{13} \\ -b_{12} & b_{12,23} & -b_{23} & 0 & -b_{12} & 0 \\ -b_{13} & -b_{23} & b_{13,23,34} & -b_{34} & 0 & -b_{13} \\ 0 & 0 & -b_{34} & b_{34} & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \overline{\mu_{12}} \\ \mu_{13} \end{bmatrix} = 0, \qquad (5.16)$$

where $b_{12,13} = b_{12} + b_{13}$ and so on. Each row in (5.16) represents an equality related to the corresponding node in the network, i.e., the first row is related to the first node etc. Note that the i-th row directly relates the nodal price λ_i only with the nodal prices of its neighboring nodes, i.e., with λ_j, $j \in I(N_i)$, and that only the nodal prices in the nodes corresponding to the congested line ε_{ij} are directly related to the corresponding Lagrange multiplier μ_{ij}. □

5.3.2.2 Price-based controller

Next, we present the price-based controller that solves Problem 5.3. Let K_λ, K_f, K_p and K_o be positive definite diagonal matrices, such that $K_f = \alpha K_\lambda$, $\alpha \in \mathbb{R}$ and $\alpha > 0$. Consider the following dynamic linear complementarity[7] controller:

$$\begin{bmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{bmatrix} = \begin{bmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_\lambda \\ x_\mu \end{bmatrix} + \begin{bmatrix} -K_f & 0 \\ 0 & K_p \end{bmatrix} \begin{bmatrix} \Delta f \\ \Delta p_L + w \end{bmatrix}, \qquad (5.17a)$$

$$0 \leq w \perp K_o x_\mu + \Delta p_L + w \geq 0, \qquad (5.17b)$$

$$\lambda = \begin{bmatrix} I_n & 0 \end{bmatrix} \begin{bmatrix} x_\lambda \\ x_\mu \end{bmatrix}, \qquad (5.17c)$$

[7] For an introduction to complementarity systems, the interested reader is referred to e.g., [12, 25] and the references therein.

where x_λ and x_μ denote the controller states, $\mathrm{col}(\Delta f, \Delta p_L)$ and w denote inputs to the controller, while λ denotes the output. The matrices K_λ, K_f, K_p and K_o represent the controller gains. The input $\mathrm{col}(\Delta f, \Delta p_L)$, which collects the nodal frequency and line overflow vectors, is an exogenous input to the controller, while the input w is required to be a solution to the finite dimensional complementarity problem (5.17b). The output λ is a vector of nodal prices.

Assumption 5.1. The closed-loop system resulting from the interconnection of the controller (5.17) with the power system (5.13) - (5.14) is globally asymptotically stable for any constant value of $\hat{p}$ (i.e., with respect to the corresponding steady-state) such that the ONP problem is feasible. □

Theorem 5.1. *Suppose that* Assumption 5.1 *holds. Then the dynamic controller* (5.17) *solves the optimal power balance and congestion control problem, as defined in Problem 5.3.*

The proof of Theorem 5.1 follows from straightforward algebraic manipulations on the steady-state relations of the closed-loop system, i.e., of the power system (5.13),(5.14) in the closed-loop with the controller (5.17), where it can be shown that these steady-state relations necessarily include the KKT optimality conditions (5.15). The complete proof is omitted here for brevity, and for all the details, as well as for an approach how to verify Assumption 5.1, the interested reader is referred to [15] and [16].

Note that the controller (5.17) is in fact nothing else than a suitable dynamic extension of the optimality condition (5.15b), which is further appropriately updated by input signals $\mathrm{col}(\Delta f, \Delta p_L)$. With a reference to Remark 5.6, we have the following insightful interpretation of (5.17): the controller (5.17) explicitly includes the "direct" coupling among the elements in λ and μ, while the "indirect" coupling is obtained by adjustment of λ and μ to the inputs Δf and Δp_L which respectively carry the information if the constraints (optimality conditions) (5.15a) and (5.15d) are satisfied or not. The remaining optimality conditions (5.15c), (5.15e) and (5.15f), from (5.15) are satisfied on the local level through profit maximization behavior of price-elastic units as defined by (5.12).

Remark 5.7. The *only* system parameters that are explicitly included in the controller (5.17) are the transmission network parameters, i.e., the network topology and line impedances, which define the matrices B and L. To provide the correct nodal prices, the controller requires no knowledge of cost/benefit functions J_i and of power injection limits $(\underline{p}_i, \overline{p}_i)$ of producers/consumers in the system (neither is it based on their estimates). Furthermore, note that in practice often only a relatively small subset of all lines is critical concerning congestion, and for the controller (5.17) it suffices to include only these critical lines by appropriately choosing Δp_L and L. □

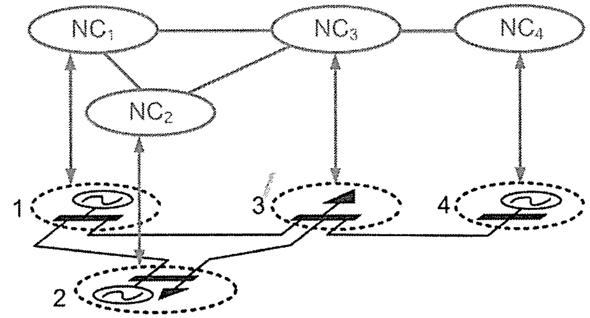

Figure 5.4: Distributed control scheme for power balance and congestion control.

Example 5.3. Consider again a simple network depicted in Figure 5.3 and described in Example 5.2. The highly structured relations from the optimality condition (5.15b) are as well present in the proposed controller (5.17), allowing for its *distributed* implementation. This means that the control law (5.17) can be implemented through a set of *nodal controllers*, where a nodal controller (*NC*) is assigned to each node in the network, and each *NC* communicates only with the *NC*'s of the neighboring nodes. From (5.17) and (5.16) it is easy to derive that the *NC* corresponding to node 1 in the network depicted in Figure 5.3 is given by:

$$\begin{bmatrix}\dot{x}_{\lambda_1}\\ \dot{x}_{\mu_{12}}\\ \dot{x}_{\mu_{13}}\end{bmatrix} = \begin{bmatrix}-k_{\lambda_1}b_{12,13} & k_{\lambda_1}b_{12} & k_{\lambda_1}b_{13}\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_{\lambda_1}\\ x_{\mu_{12}}\\ x_{\mu_{13}}\end{bmatrix}$$

$$+\begin{bmatrix}k_{\lambda_1}b_{12} & k_{\lambda_1}b_{13} & -k_{f_1} & 0 & 0\\ 0 & 0 & 0 & k_{p_{12}} & 0\\ 0 & 0 & 0 & 0 & k_{p_{13}}\end{bmatrix}\begin{bmatrix}x_{\lambda_2}\\ x_{\lambda_3}\\ \Delta f_1\\ \Delta p_{12}+w_{12}\\ \Delta p_{13}+w_{13}\end{bmatrix}, \tag{5.18a}$$

$$0 \leq \begin{bmatrix}w_{12}\\ w_{13}\end{bmatrix} \perp \begin{bmatrix}k_{o_{12}}x_{\mu_{12}}\\ k_{o_{13}}x_{\mu_{13}}\end{bmatrix}+\begin{bmatrix}\Delta p_{12}\\ \Delta p_{13}\end{bmatrix}+\begin{bmatrix}w_{12}\\ w_{13}\end{bmatrix} \geq 0, \tag{5.18b}$$

$$\lambda_1 = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_{\lambda_1}\\ x_{\mu_{12}}\\ x_{\mu_{13}}\end{bmatrix}, \tag{5.18c}$$

where $k_{\lambda_1} = [K_\lambda]_{11}$, $k_{f_1} = [K_f]_{11}$, and $k_{p_{12}}$, $k_{p_{13}}$, $k_{o_{12}}$, $k_{o_{13}}$ are the corresponding elements from the gain matrix K_p and K_o in (5.17c). Note that the state $x_{\mu_{ij}}$ is present only in one of the adjacent nodal controllers, i.e., in node i or in node j, and is communicated to the *NC* in the other node. □

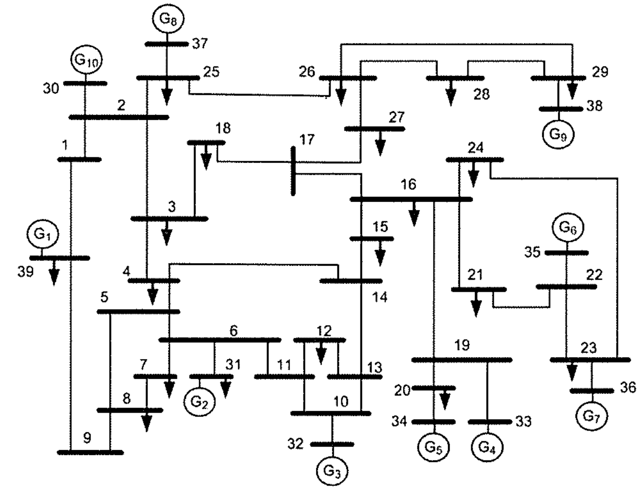

Figure 5.5: IEEE 39-bus New England test system.

The distributed implementation of the developed controller is graphically illustrated in Figure 5.4. The communication network graph among NC's is the same as the graph of the underlying physical network. Any change in the network topology requires only simple adjustments in NC's that are in close proximity to the location of the change. A distributed control structure is specially advantageous taking into account the large-scale of electrical power systems. Since in practice B is usually sparse, the number of neighbors for most of the nodes is small, e.g., two to four.

5.3.3 Illustrative example

To illustrate the potential of the developed, distributed price-based control methodology, we consider the widely used IEEE 39-bus New England test network. The network topology, generators and loads are depicted in Figure 5.5. The complete network data, including reactance of each line and load values can be found in [21]. All generators in the system are modeled using the standard third order model used in automatic generation control studies [17]. The parameter values, in per units, are taken to be in the ±20% interval from the values given in [24], pp. 545. Each generator is taken to be equipped with a proportional feedback controller for frequency control with the gain in the interval [18, 24].

We have used quadratic functions to represent the variable production costs, i.e., $J_i(p_i) = \frac{1}{2}c_{g,i}p_i^2 + b_{g,i}p_i$, where the values of parameters $c_{g,i}$, $b_{g,i}$, with $i = 30, 31, \ldots, 39$ denoting the indices of generator busses, are taken from [4] and are listed in Table 5.1. For simplicity, no saturation limits $\underline{p}$, $\overline{p}$ have been considered. All loads are taken to be price-inelastic, with the values from [21].

The proposed distributed controller (5.17) was implemented with the following values of the gain matrices: $K_\lambda = 3I_{39}$, $K_f = 8I_{39}$. For simplicity of exposition, the

Table 5.1: Production cost parameters for generator buses.

Bus number i	$c_{g,i}$	$b_{g,i}$
30	0.8	30.00
31	0.7	35.99
32	0.7	35.45
33	0.8	34.94
34	0.8	35.94
35	0.8	34.80
36	1.0	34.40
37	0.8	35.68
38	0.8	33.36
39	0.6	34.00

line power flow limit was assigned only for the line connecting nodes 25 and 26, and both corresponding gains K_p and K_o in the controller were set equal to 1. The simulation results are presented in Figure 5.6 and Figure 5.7. In the beginning of the simulation, the line flow limit $\overline{p}_{25,26}$ was set to infinity, and the corresponding steady-state operating point is characterized by the unique price of 39.16 for all nodes. At time instant 5s, the line limit constraint $\overline{p}_{25,26} = 1.5$ was imposed. The solid line in Figure 5.6 represents the simulated trajectory of the line power flow $p_{25,26}$. In the same figure, the dotted line indicates the limits on the power flow $\overline{p}_{25,26}$. The solid lines in Figure 5.7 are simulated trajectories of nodal prices for the generator buses, i.e., for buses 30 to 39, which is where the generators are connected. In the same figure, dotted lines indicate the off-line calculated values of the corresponding steady-state optimal nodal prices. For clarity, the trajectories of the remaining 29 nodal prices were not plotted. In the simulation, all these trajectories converge to the corresponding optimal values of nodal prices as well. The optimal nodal prices for all buses are presented in Figure 5.8. In this figure, the nodal prices corresponding to generator buses 30−39 are emphasized with the gray shaded bars. The obtained simulation results clearly illustrate the efficiency of the proposed distributed control scheme.

5.4 Conclusions and future research

In this chapter we have presented and illustrated on examples the price-based control paradigm as a suitable approach to solve some of the challenging problems facing future, market-based power systems. It was illustrated how global objectives and constraints, updated from the on-line measurements of the physical power system state, can be optimally translated into time-varying prices. The real-time varying price signals are guaranteed to adequately reflect the state of the physical system, present the signals that optimally shape, coordinate and in real or near real-time syn-

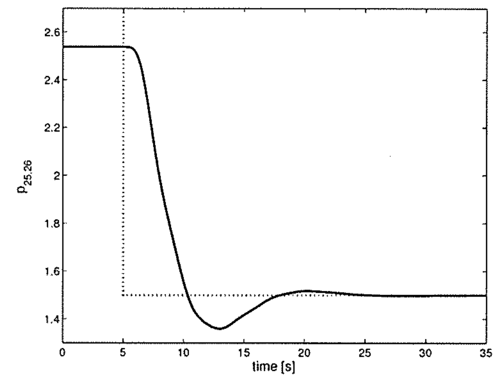

Figure 5.6: Power flow in the line connecting buses 25 and 26.

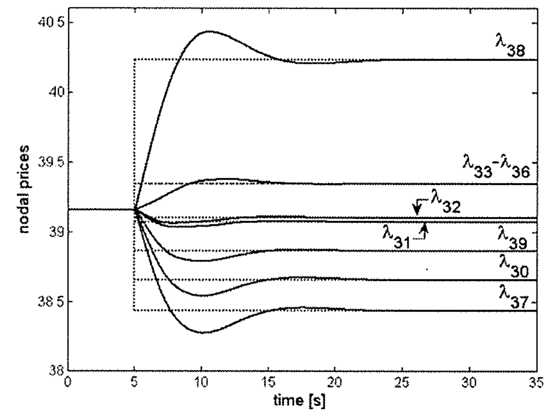

Figure 5.7: Trajectories of nodal prices for generator buses, i.e., for busses 30–39 where the generators are connected.

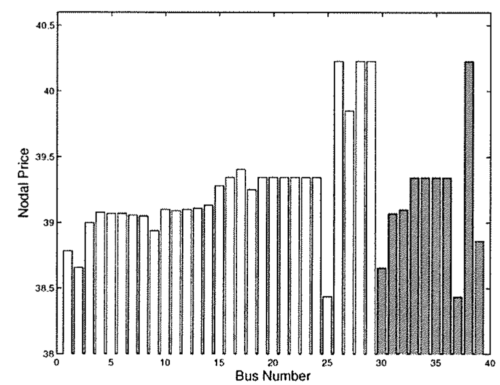

Figure 5.8: Optimal nodal prices in the case of congestion. The nodal prices corresponding to generator busses 30–39 are emphasized with the gray shaded bars.

chronize local, profit driven behaviors of producers/consumers to mutually reinforce and guarantee global objectives and constraints.

Future research will be devoted to modification of the devised price-based control scheme so that the prices are updated on the time scale of 5–15 minutes, rather than on the scale of seconds. Instead of using rapidly changing network frequency deviations as an indication of power imbalance in the system, one possibility is to use deviation of power production reference values to the generators which originate from (slightly modified) automatic generation control loops over the sampling period (i.e., over 5–15 minutes). These deviations can be used as a measure of imbalance in the system.

As a final remark, we would like to point out that in its core idea the price-based control approach presented in this chapter, which is based on a suitable dynamic extension of the Karush-Kuhn-Tucker (KKT) optimality conditions, is suitable for application in some other types of infrastructures as well. More precisely, when the system's objectives are characterized in terms of steady-state related constrained optimization problems, the time-varying price signals can be efficiently used for control purposes. In particular, the proposed approach is suitable for solving problems of economically optimal load sharing among various production units in a network. Examples of such systems include smart power grids in energy-aware buildings, industrial plants, large ships, islands, space stations or isolated geographical areas; water pumps, furnaces or boilers in parallel operation, etc. A distinguishing and advantageous feature of the presented approach is that the dynamic extension of the KKT optimality conditions preserves the structure of the underlying optimization problem, which implies that the corresponding price-based control structure can be implemented in a distributed fashion.

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