

# Distributed, price-based control approach to market-based operation of future power systems

**Citation for published version (APA):**

Jokic, A., Bosch, van den, P. P. J., & Hermans, R. M. (2009). Distributed, price-based control approach to market-based operation of future power systems. In *Proceedings of the 6th International Conference on the European Energy Market (EEM 2009) 27-29 May 2009, Leuven, Belgium* (pp. 5207170-1/6). Institute of Electrical and Electronics Engineers. <https://doi.org/10.1109/EEM.2009.5207170>

**DOI:**

[10.1109/EEM.2009.5207170](https://doi.org/10.1109/EEM.2009.5207170)

**Document status and date:**

Published: 01/01/2009

**Document Version:**

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

**Please check the document version of this publication:**

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

**General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

[www.tue.nl/taverne](http://www.tue.nl/taverne)

**Take down policy**

If you believe that this document breaches copyright please contact us at:

[openaccess@tue.nl](mailto:openaccess@tue.nl)

providing details and we will investigate your claim.

# Distributed, Price-based Control Approach to Market-based Operation of Future Power Systems

A. Jokic,

P.P.J. van den Bosch,

R.M. Hermans

**Abstract**—In this paper we present, discuss and illustrate on examples the price-based control paradigm as a suitable approach to solve some of the challenging problems facing future, market-based power systems. It is illustrated how global objectives and constraints are optimally translated into time-varying prices. The real-time varying price signals are guaranteed to adequately reflect the state of the physical system and present the signals that optimally shape, coordinate and synchronize local, profit driven behaviors of producers/consumers to mutually reinforce and guarantee global objectives and constraints. As an illustrative example, the real-time price-based power balance control with congestion management is presented.

**Index Terms**—Power System Markets; Distributed control; Network congestion management; Optimization

## I. INTRODUCTION

Electrical power systems are going through some major restructuring processes and it has been widely recognized that the feasibility, reliability and efficiency of the future European power grid cannot be achieved by a simple extrapolation of the current state. Regarding market, operational and control structures, national power grids in Europe currently exhibit rather significant diversity of employed solutions, many of which are posed on ad-hoc basis, are characterized by insufficient coordination among involved parties, have inconsistencies, non-optimality and lack of stability and robustness proofs. In spite of vast research efforts, see e.g. [1], [2], [3], [4], [5], and commonly shared vision for the future, we can say that there is yet no unifying and fundamental scientific theory to shape the optimal management and control structure of the future EU power grid.

In this paper, our main goal is to present distributed control systems theory as a scientific framework which is suitable to adequately formulate and cope with some of the major challenges facing future power systems. The *leitmotiv* and the core idea of the presented approaches is in utilization of real or near real-time varying price signals to shape the behaviors of local subsystems so that the desired overall system objectives and constraints are met. Depending on the considered time-scale and global objectives of interest, the term *local subsystem* can mean either producer or consumer as an active player (market agent) in economical layers, or, in other cases, generator or some other physical device as local dynamical system in the physical layer. It is a nice

observation that different problems and on different time-scales can be treated with the same mathematical tools, where prices, which in some cases have purely mathematical interpretations, act as crucial control signals.

Indeed, the idea of price-based control is long present in power system community, dating back to work of Fred Schweppe and his co-workers, see e.g. [1], [6], [7], [8]. In this paper we further explore this idea by taking a control systems view on the problems and suitable emerging solutions. Although analogy between market operation and dual formulation of optimization problems has been often noted in the past, a novel approach and one of the main contributions of this paper is to present how specific structure of power flow equations can be preserved in devising efficient and flexible *distributed* control solutions, as alternative to centralized solutions. In addition to addressing some general theoretical notions and results, we will present several concrete examples to illustrate their potential for real-life applications.

## II. OPTIMIZATION DECOMPOSITION: A MATHEMATICAL THEORY OF FUTURE POWER SYSTEM CONTROL

Modern control systems theory is grounded on the following remarkable fact: virtually all control problems can be casted as optimization problems. It is insightful to realize that the same, far reaching statement, holds as well for the power systems: virtually all global operational goals of a power system can be formulated as constrained, time-varying optimization problems. Similarly as modern control theory accounts for efficiently solving these optimization problems (which is in most cases a far from trivial task), the same mathematical framework provides a systematic and rigorous scientific approach to shape operational and control architectures of power systems<sup>1</sup>. For illustration, in mathematical terms, a shift from monopolistic, one utility controlled system, to the market-based system is seen as a shift from explicitly solving *primal problem* (e.g. economic dispatch at the control center) to solving its *dual problem* (e.g. operating real-time energy market). The former case can be called the cost-based operation, while the latter can be called the price-based operation. Before continuing with consideration of some specific problems in power systems, and their price-based solutions, we will first recall some basic notions from optimization theory. For closely related subjects and the state-of-the-art results on the distributed optimization

A. Jokic, P.P.J. van den Bosch and R.M. Hermans are with the Department of Electrical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands, E-mails: a.jokic@tue.nl, p.p.j.v.d.bosch@tue.nl, r.hermans@tue.nl.

<sup>1</sup>The interested reader is referred to the excellent paper [9] where the role of alternative ways for solving optimization problems is reflected in devising alternative operational structures for communication networks.

and control, the interested reader is referred to [10], [11], [9], [12], [13] and the references therein.

Consider the following structured, time varying<sup>2</sup>, optimization problem

$$\min_{x_1, \dots, x_N} \sum_{i=1}^N J_i(x_i), \quad (1a)$$

$$\text{subject to } x_i \in \mathcal{X}_i, \quad i = 1, \dots, N, \quad (1b)$$

$$G(x_1, \dots, x_N) \leq 0, \quad (1c)$$

$$H(x_1, \dots, x_N) = 0, \quad (1d)$$

where  $x_i \in \mathbb{R}^{n_i}$ ,  $i = 1, \dots, N$  are the local decision variables, the functions  $J_i : \mathbb{R}^{n_i} \rightarrow \mathbb{R}$ ,  $i = 1, \dots, N$ , denote the local objective functions, while each set  $\mathcal{X}_i \subseteq \mathbb{R}^{n_i}$  defines local constraints on the corresponding local variable  $x_i$ . The functions  $G$  and  $H$ , which respectively take values in  $\mathbb{R}^k$  and  $\mathbb{R}^l$ , define global inequality and equality constraints. The vector valued inequality (1c) should be interpreted elementwise.

Note that the optimization problem (1) is defined on the overall, global system level, where global objective function is sum of local objectives as indicated in (1a). Furthermore, note that if the global constraints (1c) and (1d) are omitted, the optimization problem (1) becomes separable in a sense that it is composed of  $N$  independent local problems which can be solved separately. For such a completely separable case, we say that the optimization problem can be solved in a *decentralized* way. For the future reference, we will call the problem (1) the *primal problem*.

Next, from (1) we formulate the *dual problem* as follows

$$\max_{\lambda, \mu} \quad l(\lambda, \mu) \quad (2a)$$

$$\text{subject to } \quad \mu \geq 0, \quad (2b)$$

where

$$l(\lambda, \mu) := \min_{x_1 \in \mathcal{X}_1, \dots, x_N \in \mathcal{X}_N} \left( \sum_{i=1}^N J_i(x_i) - \lambda^\top H(x_1, \dots, x_N) + \mu^\top G(x_1, \dots, x_N) \right). \quad (3)$$

In (2) and (3)  $\lambda \in \mathbb{R}^k$  and  $\mu \in \mathbb{R}^l$  are the dual variables (Lagrange multipliers) and have an interpretation of *prices* for satisfying the global constraints (1c) and (1d). If (1) is a convex optimization problem, it can be shown that the solutions of the primal and the dual problem coincide<sup>3</sup>.

**Remark II.1** Suppose that the functions  $H$  and  $G$  have an additive structure in local decision variables  $x_i$ , meaning that  $H(x_1, \dots, x_N) = \sum_{i=1}^N \tilde{h}_i(x_i)$  and  $G(x_1, \dots, x_N) = \sum_{i=1}^N \tilde{g}_i(x_i)$  with some given functions  $\tilde{h}_i, \tilde{g}_i$ ,  $i = 1, \dots, N$ .

<sup>2</sup>For notational convenience, we have omitted the explicit reference to the time dependence.

<sup>3</sup>In fact, an additional mild condition, the so-called Slater's constraints qualification, is required for the solutions to coincide, see e.g. [14] for more details.

Then for a fixed  $\lambda$  and  $\mu$  the optimization problem in (3) is separable and can be solved in a decentralized way.  $\square$

**Remark II.2** Updating the the dual variables (prices)  $\lambda$ ,  $\mu$  to solve the maximization problem in (2) can be achieved in a centralized way on a global level, e.g. at the central market operator which calculates the market clearing price. In some important cases, as it will be presented in Section III, the optimal prices  $(\lambda, \mu)$  can also be efficiently calculated in a *distributed* way. This means that they can be calculated even if there is no one central unit that gathers information and communicates with all the subsystems in the network, but the optimal price calculations are based only on the locally available information and require only limited communication among neighboring systems.  $\square$

#### A. Example 1.

Loosely speaking, and as already mentioned in the introduction, the market-based power system can be seen as solving the dual optimization problem (2). When the power limits in transmission network are not considered, this can be more precisely described as follows.

Suppose that for each  $i$ , local decision variables  $x_i$  is a scalar and represents the power production ( $x_i > 0$ ) of a power plant  $i$ , or a power consumption ( $x_i < 0$ ) if the subsystem  $i$  is a consumer. Furthermore, let  $J_i(x_i)$  denote power production costs when the  $i$ -th subsystem is a producer, and its negated benefit function when the  $i$ -th subsystem is a consumer. Since we do not consider the transmission network limits, the only global constraint is the power balance constraint  $\sum_{i=1}^N x_i = 0$ , i.e. in (1) and (2) we have that  $H(x_1, \dots, x_N) := \sum_{i=1}^N x_i$ .

Obviously, the *primal problem* (1) now corresponds to minimization of total production costs and maximization of total consumers benefit, while the power balance constraint is *explicitly* taken into account via (1d).

Let us now consider the *price-based solution* through the corresponding *dual problem*. First note that minimization problem in (3) is in this case given by

$$l(\lambda) := \min_{x_1, \dots, x_N} \sum_{i=1}^n (J_i(x_i) - \lambda x_i). \quad (4)$$

In the above equation each term in the summation, i.e.  $J_i(x_i) - \lambda x_i$ , denotes the benefit of a subsystem  $i$  where  $\lambda$  denotes the price for electricity. Obviously, the dual problem (2) then accounts to maximizing the total benefit of the system. Note that in solving the dual problem, the power balance constant is not explicitly taken into account. However, mathematics tells us that the corresponding maximum in (2) is attained precisely when the price  $\lambda$  is such that for the solution to the corresponding minimization problem in (4) it holds that  $\sum_{i=1}^N x_i = 0$ . In other words, the price  $\lambda$  which maximizes the total benefit of the system is precisely the price for which the system is in balance.

To summarize, while in primal solution the global constraints were *explicitly taken into account*, in the dual solution they are *enforced implicitly through the price*  $\lambda$ .

**Remark II.3** The observation from the above presented example can be generalized to the core idea of the price-based control paradigm:

*In the price-based control, a price (Lagrange multiplier) is assigned to each crucial global constraint (i.e. each row in (1c) and (1d), see (3)) and is used to implicitly enforce this constraint.*

With this interpretation in mind, the main objective of the market operator in a power system is to adequately translate *global constraints* into price signals so that the reaction of *local subsystems* to this price (via minimization problem in (3)) will result in a situation when these global constraints are necessarily satisfied.

It is also insightful to interpret the price-based solutions as *incentives-based* solutions, as prices  $\lambda$  are used to give incentives to the local subsystems so that their local objectives will make them behave in a way which serves global needs.  $\square$

**Remark II.4** When the transmission network constraints are completely neglected, as it was done in the above example, the global equality constraint

$$H(x_1, \dots, x_N) := \sum_{i=1}^N x_i = 0 \quad (5)$$

has an additive structure as described in Remark II.1. Therefore, for a fixed  $\lambda$  the solution to the corresponding optimization problem in (3) is separable and it can be *decomposed* into a set of  $N$  optimization independent problems. The prices can then be interpreted as the variables that are used to *coordinate* these independent problems with the goal that their solution coincide with the solution of the “original” primal problem. In such a cases, we say that the optimization problem is solved through the *dual decomposition*.  $\square$

The simplicity of the problem in the above presented example was instrumental to illustrate the main “philosophy” of using prices as the curtail signals in control of power systems. The examples presented in the following section are based on the same general idea as outlined in Remark II.3, however, there the underlying problems and solutions are much more complex and involved. Therefore, we will restrict our presentation to only some of their basic and the most illustrative features, while for detailed treatment and all the proofs the interested reader is referred to [15], [16].

### III. PRESERVING THE STRUCTURE: DISTRIBUTED CONTROL FOR TRANSMISSION SYSTEM MANAGEMENT

When the optimal power balance problem from the previous section (Example 1) is complemented to include the security limits on the power flows in tie-lines of the transmission network, the solution becomes much more complex, see e.g. [17], [4], [15]. In fact, devising efficient operational and control schemes to optimally cope with the transmission network limits in competitive environment of restructured power systems is considered to be one of the toughest problems in market structure design [3]. In this section we illustrate,

by using appropriate examples, how this problem can be solved in a *distributed* way. The crucial points on the path towards the solution are, firstly, adequate formulation of the problem as structured optimization problem, and secondly, preserving the structure of the problem in the solution. The latter is crucial as it allows for a far more flexible, scalable and robust distributed solutions, as opposed to centralized solutions which depend on extensive communications among distant subsystems in the network, are less robust and become increasingly complex as the size of the system increases.

Let us complement the optimal power balance problem discussed in Example 1 by adding the global inequality constraints (1c) which represents the security limits imposed on the power flows in tie-lines of the transmission network. In this case, the local variables  $x_i$  from Example 1 have to be extended to include at least the voltage phase angles at each bus in the system, see e.g. [17]. The voltage phase angles are necessary variables to describe the power flows in the transmission network. For simplicity, suppose that  $i$  is an index of a bus in the transmission network, and that at each bus there is connected either one producer or consumer with the local objective function  $J_i$ . As the transmission system is taken in the account, it is now not sufficient any more to represent the global power balance as a single scalar equality constraint, as it was done before in equation (5). The power balance constraint is now a vector valued equality constraint

$$H(x_1, \dots, x_N) = 0, \quad (6)$$

where  $H(x_1, \dots, x_N) \in \mathbb{R}^N$  and the  $i$ -th row in (6) represents the power balance of the  $i$ -th bus in the network. Furthermore, suppose that the network has a set of  $M$  lines for which, and due to security and reliability issues, there are determined the upper limits on the allowable power flows. In other words, we have to add  $M$  inequality constraints of the form

$$G(x_1, \dots, x_N) \leq 0, \quad (7)$$

where  $G(x_1, \dots, x_N) \in \mathbb{R}^M$  and the  $j$ -th row in (6) states that the power flow limit violation in the  $j$ -th line has to be less than zero, i.e. that the line must not be congested.

With the above definitions and interpretations of the global constraints, the optimal power balance problem with congestion management can be formulated as a primal problem (1), or the corresponding dual problem (2). Note that in the dual problem  $\lambda$  is now a vector, i.e.  $\lambda \in \mathbb{R}^N$ , where  $i$ -th entry in  $\lambda$  corresponds to the price of power balance constraint at the  $i$ -th bus. Similarly, the  $j$ -th entry in  $\mu$  (note that  $\mu \in \mathbb{R}^M$ ) is interpreted as a price for not overloading the  $j$ -th line.

According to the price-based control paradigm, see Remark II.3, the goal of the system operator is to calculate the prices  $\lambda$  and  $\mu$ , which will result in satisfaction of the constraints (6) and (7). The challenge of solving this problem originates from the complicating fact that the functions  $H$  and  $G$  have no longer an additive structure, as it was the case in Example 1. However, and as it is illustrated in the following example, the functions  $H$  and  $G$  are highly

structured and this structure can be further exploited in the solution.

#### A. Example 2.

Consider a simple network depicted in Figure 1 and suppose that the line connecting bus 1 and bus 2 has an upper power flow limit set to  $\bar{p}_{12}$ , i.e. with  $p_{12}$  denoting the power flow in the corresponding line we have that  $p_{12} - \bar{p}_{12} \leq 0$ . With  $\mu_{12}$  denoting the corresponding the price (Lagrange multiplier) for this inequality constraint, and with  $\lambda_i$  denoting the price (Lagrange multiplier) for the power balance at bus  $i$  ( $i \in \{1, 2, 3, 4\}$ ), it can be shown that when the optimum of the corresponding dual problem (2), the prices necessarily satisfy the following equation:

$$\left( \begin{array}{cccc|c} b_{12,13} & -b_{12} & -b_{13} & 0 & b_{12} \\ -b_{12} & b_{12,23} & -b_{23} & 0 & -b_{12} \\ -b_{13} & -b_{23} & b_{13,23,34} & -b_{34} & 0 \\ 0 & 0 & -b_{34} & b_{34} & 0 \end{array} \right) \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \mu_{12} \end{pmatrix} = 0, \quad (8)$$

where  $b_{12,13} = b_{12} + b_{13}$  (and so on) and where  $b_{pq}$  is a susceptance of the line connecting busses  $p$  and  $q$ . The above relations among the prices are derived from the so-called Karush-Kuhn-Tucker (KKT) optimality conditions for the corresponding optimization problems (1) and (2), see e.g. [14] for the KKT conditions in general, and [15] for detailed derivation for this particular problem. What is important is that the optimality condition (8) preserves the structure of the power flow equations present in  $H$  and  $G$ . This structure is more precisely described as follows.

Each row in (8) represents an equality related to the corresponding bus in the network, i.e. the first row is related to the first bus etc. Note that the  $i$ -th row directly relates the price  $\lambda_i$  only with the prices of its neighboring busses, i.e. in the  $i$ -th row of (8) there explicitly appears  $\lambda_j$  if and only if there is a line connecting bus  $i$  with the bus  $j$ . Similarly, only the prices  $\lambda_i$  and  $\lambda_j$  in the busses corresponding to the congested line  $i - j$  are directly related to the corresponding Lagrange multiplier  $\mu_{ij}$ .

Note that in practice transmission network graph is usually sparse in a sense that the number of neighboring busses for most of the busses is small, e.g. two to four.

**Remark III.1** The highly structured relations among nodal prices, as illustrated above in the equation (8), are a consequence of an appropriate problem formulation, i.e. special care was taken already at the initial phase of formulating the global power system objectives as an optimization problems (1) and (2). More precisely, and for all the details we refer the interested reader to [15], to obtain the highly structured relations (8) it was necessary to induce the voltage phase angles explicitly in (1) in (2) as decision variables. Another possibility, common in the literature, is to introduce a “slack bus” with zero voltage phase angle and to solve the equations for the line flows, completely eliminating voltage phase angles from the problem formulation [17], [1]. However, in that case a specific structure, i.e. sparsity, of the power flow equations is lost.  $\square$

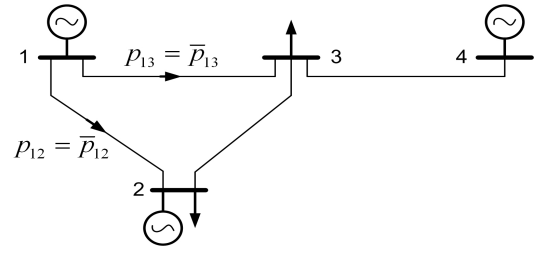


Fig. 1. An example of a simple congested network.

These highly structured relations from the optimality conditions can be used in devising *distributed* real-time price-based power balance control and congestion management solution as shortly described in the following subsection and further illustrated in the example in Subsection III-C.

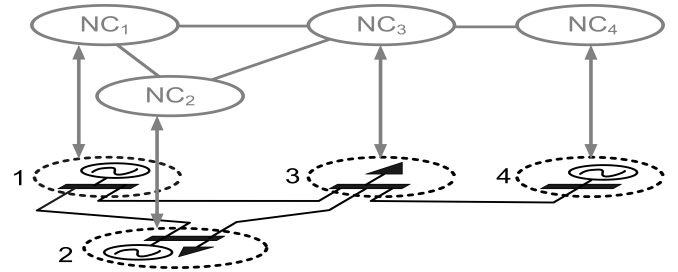


Fig. 2. Distributed control scheme for power balance and congestion control.

#### B. Distributed price-based control

When the violations of global system contracts (6) and (7) can be measured in real-time, in [16], [15] it was shown that a suitable dynamic extension of the optimality condition (8) can be used as a control law for real-time price update of the nodal prices  $\lambda_i$ . This dynamical extension of the optimality conditions was appropriately named the *KKT controller* [18], [15]. When the prices are updated according to the KKT control law, and when each producer or consumer is adjusting its power production/consumption according to its benefit maximization strategy, i.e.

$$p_i = \arg \min_{\tilde{p}_i} (J_i(\tilde{p}_i) - \lambda_i \tilde{p}_i)$$

with  $p_i$  denoting the power production/consumption of a subsystem at bus  $i$ , it was proven that the system will settle in the operating point where the total benefit of is maximized, i.e. that the operating point will coincide with the corresponding solution of (2).

Furthermore, in [15] it was shown that the KKT controller further preserves the highly structured relations present in the optimality conditions. The implication of this fact is that the price-based KKT control laws can be implemented through a set of *nodal controllers*, where a nodal controller (*NC*) is assigned to each node (bus) in the network, and each *NC* communicates only with the *NC*'s of the neighboring nodes. The distributed implementation of the propose price-based control structure is graphically illustrated in Figure 2.

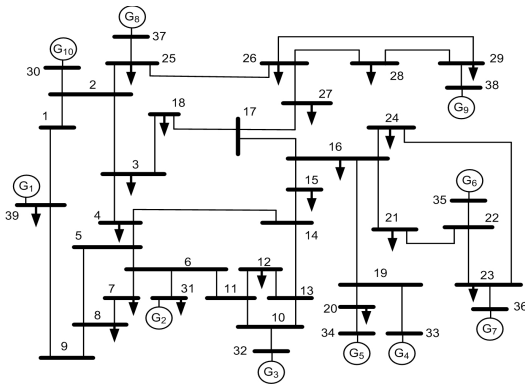


Fig. 3. IEEE 39-bus New England test system.

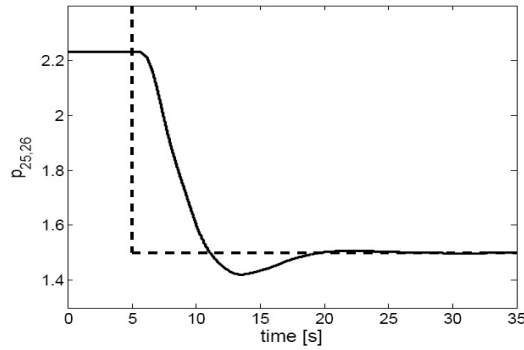


Fig. 4. Power flow in the line connecting buses 25 and 26.

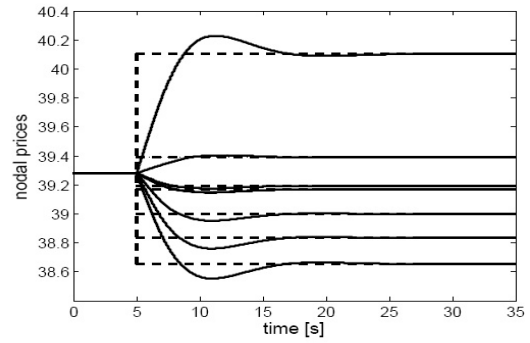


Fig. 5. Trajectories of nodal prices for generator buses 30-39.

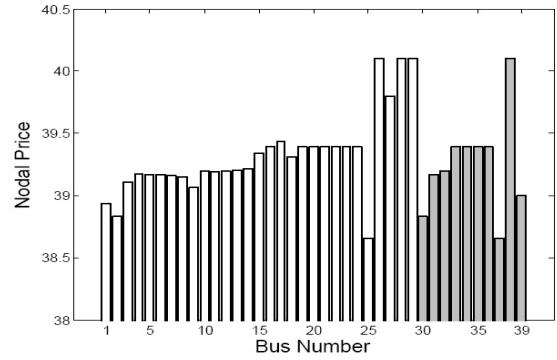


Fig. 6. Nodal prices in the case of congestion.

Note that the communication network graph among NC's is the same as the graph of the underlying physical network. Any change in the network topology requires only simple adjustments in NC's at the location of the change. A distributed control structure is specially advantageous taking into account the large-scale of electrical power systems.

Finally, it is worth to mention that the *only* system parameters that are explicitly included in the price update rules are the transmission network parameters, i.e. the network topology and line impedances. To provide the correct prices, the controller requires no knowledge of cost/benefit functions  $J_i(\cdot)$  of producers/consumers in the system (neither is it based on their estimates).

### C. Example 3.

To illustrate the potential of the price-based methodology for practical application we consider the widely used IEEE 39-bus New England test network. The network topology, generators and loads are depicted in Figure 3. The complete network data, including reactance of each line and load values can be found in [19]. All generators in the system are modeled using a third order model consisting of governor, turbine and rotor dynamics. This is a standard model used in "automatic generation control" studies [20]. The parameter values, in per units, are taken to be in the  $\pm 20\%$  interval from the values given in [21], pp. 545. Each generator is taken to be equipped with a proportional feedback controller

for frequency control with the gain in the interval [18, 24]. We have used quadratic functions to represent the variable production costs, i.e.

$$J_i(p_i) = \frac{1}{2}c_{g,i}p_i^2 + b_{g,i}p_i,$$

with the values of parameters  $c_{g,i}$ ,  $b_{g,i}$ , for  $i = 1, \dots, 10$  as listed in Table 5 in [22]. The lower saturation limit and the upper saturation limit for each generator was set to 0 and 10, respectively. All loads are taken to be price-inelastic, with the values from [19].

The distributed price-based controller was implemented in the simulation and for simplicity of exposition, the line power flow limit was assigned only for the line connecting nodes 25 and 26. The simulation results are presented in Figure 4 and Figure 5. In the beginning of the simulation, the line flow limit  $\bar{p}_{25,26}$  was set to infinity, and the corresponding steady-state operating point is characterized by the unique price of 39.28 for all nodes. At time instant 5s, the line limit constraint  $\bar{p}_{25,26} = 1.5$  was imposed. The solid line in Figure 4 represents the simulated trajectory of the line power flow  $p_{25,26}$ . In the same figure, the dashed line indicates the limits on the power flow  $\bar{p}_{25,26}$ . The solid lines in Figure 5 are simulated trajectories of nodal prices for the generator buses, i.e. for buses 30 to 39, which is where the generators are connected. In the same figure, dashed lines indicate the off-line calculated values of the corresponding steady-state optimal nodal prices. For clarity,

the trajectories of the remaining 29 nodal prices were not plotted. In the simulation, all these trajectories converge to the corresponding optimal values of nodal prices as well. The optimal nodal prices for all buses are presented in Figure 6. In this figure, the nodal prices corresponding to generator buses 30-39 are emphasized with the gray shaded bars. The obtained simulation results clearly illustrate the efficiency of the proposed distributed control scheme.

#### IV. CONCLUSION AND FUTURE WORK

In this paper we have presented and illustrated on examples the price-based control paradigm as a suitable approach to solve some of the challenging problems facing future, market-based power systems. It was illustrated how global objectives and constraints, updated from the on-line measurements of the physical power system state, can be optimally translated into time-varying prices. The real-time varying price signals are guaranteed to adequately reflect the state of the physical system, present the signals that optimally shape, coordinate and in real or near real-time synchronize local, profit driven behaviors of producers/consumers to mutually reinforce and guarantee global objectives and constraints.

Current research is devoted to modification of the devised price-based control scheme so that the prices are updated on the time scale of 5-15 minutes, rather than on unrealistically fast scale of seconds. In that approach, instead of using rapidly changing network frequency deviations as indication of power imbalance in the system, we use are using deviation of power production reference values to the generators which originate from (slightly modified) secondary control loops (automatic generation control) over the sampling period (i.e. over 5 - 15 minutes). These deviations are used as a measure of imbalance in the system.

#### REFERENCES

- [1] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R. E. Bohn, *Spot Pricing of Electricity*. Kluwer Academic Publishers, 1988.
- [2] M. Ilic, F. Geliana, and L. Fink, *Power Systems Restructuring: Engineering and Economics*. Kluwer Academic Publishers, 1988.
- [3] S. Stoft, *Power System Economics: Designing Markets for Electricity*. Kluwer Academic Publishers, 2002.
- [4] W. W. Hogan, "Contract networks for electric power transmission," *Journal of Regulatory Economics*, vol. 4, pp. 211–242, 1992.
- [5] F. F. Wu and P. Varaiya, "Coordinated multilateral trades for electric power networks: Theory and implementation," *International Journal of Electrical Power Energy Systems*, vol. 21, no. 2, pp. 75–102, 1999.
- [6] F. C. Schweppe, R. D. Tabors, J. L. Kirtley, H. R. Outhred, P. F. H., and C. A. J., "Homeostatic utility control," *IEEE Transactions on Power Apparatus and Systems*, vol. 99, no. 3, pp. 1151–1163, 1980.
- [7] M. C. Caramanis, R. E. Bohn, and F. C. Schweppe, "Optimal spot pricing, theory and practice," *IEEE Transactions on Power Apparatus and Systems*, vol. 109, no. 9, pp. 3234–3245, 1982.
- [8] F. C. Schweppe, M. C. Caramanis, and R. D. Tabors, "Evaluation of spot price based electricity rates," *IEEE Transactions on Power Apparatus and Systems*, vol. 104, no. 7, pp. 1644–1655, 1985.
- [9] M. Chiang, S. H. Low, A. R. Calderbank, and J. C. Doyle, "Layering as optimization decomposition: A mathematical theory of network architectures," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 255–213, 2007.
- [10] D. P. Palomar and M. Chiang, "Alternative distributed algorithms for network utility maximization: Framework and applications," *IEEE Transactions on Automatic Control*, vol. 52, no. 12, pp. 2254–2269, 2007.
- [11] A. Nedic and A. Ozdaglar, "Distributed subgradient methods for multi-agent optimization," *IEEE Transactions on Automatic Control*, vol. 54, no. 1, pp. 48–61, 2009.
- [12] M. Rotkowitz and S. Lall, "A characterization of convex problems in decentralized control," *IEEE Transactions on Automatic Control*, vol. 50, no. 12, pp. 1984–1996, 2005.
- [13] C. Langbort, R. S. Chandra, and R. D'Andrea, "Distributed control design for systems interconnected over an arbitrary graph," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1502–1519, 2004.
- [14] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge University Press, 2004.
- [15] A. Jokic, "Price-based optimal control of electrical power systems," Ph.D. dissertation, Eindhoven University of Technology, The Netherlands, 2007.
- [16] A. Jokic, M. Lazar, and P. P. J. van den Bosch, "Real-time control of power systems using nodal prices," in *16th Power Systems Computation Conference*, Glasgow, UK, 2008.
- [17] R. D. Christie, B. F. Wollenberg, and I. Wangenstein, "Transmission management in the deregulated environment," *Proceedings of the IEEE*, vol. 88, no. 2, pp. 170–195, 2000.
- [18] A. Jokic, M. Lazar, and P. P. J. van den Bosch, "On constrained steady-state regulation: Dynamic KKT controllers," *IEEE Transactions on Automatic Control*, vol. Accepted for publication, 2009.
- [19] M. A. Pai, *Energy Function Analysis for Power System Stability*. Kluwer Academic Publishers, 1989.
- [20] P. Kundur, *Power System Stability and Control*. McGraw-Hill, 1994.
- [21] H. Saadat, *Power System Analysis*. McGraw-Hill, 1999.
- [22] F. L. Alvarado, J. Meng, C. L. DeMarco, and W. S. Mota, "Stability analysis of interconnected power systems coupled with market dynamics," *IEEE Transactions on Power Systems*, vol. 16, no. 4, pp. 695–701, 2001.