

## Correction to 'On the error probability for a class of binary recursive feedback strategies'

**Citation for published version (APA):**

Schalkwijk, J. P. M., & Post, K. A. (1974). Correction to 'On the error probability for a class of binary recursive feedback strategies'. *IEEE Transactions on Information Theory*, 20(2), 284-284.  
<https://doi.org/10.1109/TIT.1974.1055175>

**DOI:**

[10.1109/TIT.1974.1055175](https://doi.org/10.1109/TIT.1974.1055175)

**Document status and date:**

Published: 01/01/1974

**Document Version:**

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

**Please check the document version of this publication:**

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

**General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

[www.tue.nl/taverne](http://www.tue.nl/taverne)

**Take down policy**

If you believe that this document breaches copyright please contact us at:

[openaccess@tue.nl](mailto:openaccess@tue.nl)

providing details and we will investigate your claim.

We can recover  $j$  from  $g(j)$  as follows:

$$j_n = g_n^j$$

$$j_k = j_{k+1} + g_k^j \bmod 2$$

which gives

$$j_{n-1} = g_n^j + g_{n-1}^j$$

$$j_{n-2} = g_n^j + g_{n-1}^j + g_{n-2}^j, \dots$$

Thus

$$j_k = \sum_{m=k}^n g_m^j \bmod 2.$$

We know that the integers  $i$  and  $i'$ , where  $0 \leq i < 2^{n-1}$  and  $i' = i + 2^{n-1}$ , differ in only one digit, i.e.,  $i_n = 0, i'_n = 1, i_k = i'_k, k = 1, 2, \dots, n-1$ . Hence

$$g_k^i = i_k + i_{k+1} = g_k^{i'}, \quad \text{if } k \leq n-2. \quad (2)$$

Furthermore,

$$g_n^i + g_{n-1}^i = i_{n-1}$$

while

$$g_n^{i'} + g_{n-1}^{i'} = i'_{n-1} = g_n^i + g_{n-1}^i. \quad (3)$$

We have thus shown that, if we add together the first two columns of a  $2^n$ -level Gray code and copy the remaining  $n-2$  columns, the resulting  $n-1$  columns contain two identical parts. It remains to be proved that each half is a  $2^{n-1}$ -level Gray code. We denote the latter by  $G(i), 0 \leq i \leq 2^{n-1} - 1$ . Then

$$G_{n-1} = i_{n-1}, \quad G_k = i_k + i_{k+1}, \quad 0 \leq k \leq n-2.$$

The previous are identical to the expressions (2) and (3). Thus the  $n-1$  columns do consist of two repetitions of  $2^{n-1}$ -level Gray code. Now if we combine the first two columns again, we reduce each  $2^{n-1}$ -level Gray code into two  $2^{n-2}$ -level Gray codes, or, the complete array into four  $2^{n-2}$ -level Gray codes. This can continue until we have only  $m$  columns, which would be  $2^{n-m}$  repetitions of  $2^m$ -level Gray code. We have thus derived the lemma.

#### REFERENCES

- [1] M. Gardner, "Mathematical games," *Sci. Amer.*, vol. 227, pp. 106-109, Aug. 1972.

### Correction to "On the Error Probability for a Class of Binary Recursive Feedback Strategies"

J. PIETER M. SCHALKWIJK AND KAREL A. POST

In the above paper<sup>1</sup>, p. 499, (2) should have read

$$p_{n+1}(\theta) = \begin{cases} \frac{(1 - y_{n+1})p + y_{n+1}q}{(1 - y_{n+1})[aq + (1 - a)p] + y_{n+1}[(1 - a)q + ap]} p_n(\theta), & \text{for } \theta > a_n \\ \frac{(1 - y_{n+1})q + y_{n+1}p}{(1 - y_{n+1})[aq + (1 - a)p] + y_{n+1}[(1 - a)q + ap]} p_n(\theta), & \text{for } \theta < a_n. \end{cases} \quad (2)$$

Manuscript received September 14, 1973.

The authors are with the Technological University, Eindhoven, The Netherlands.

<sup>1</sup> J. P. M. Schalkwijk and K. A. Post, *IEEE Trans. Inform. Theory*, vol. IT-19, pp. 498-511, July 1973.

On the right side of p. 505, the fifth and sixth line from the bottom, the lower error exponent  $\bar{E}^-(R)$  is valid for the 1 output and the upper error exponent  $\bar{E}^+(R)$  for the 0 output.

#### ACKNOWLEDGMENT

The authors want to thank Dr. J. C. Tiernan for pointing out the mistake in (2).

### Optimal Decoding of Linear Codes for Minimizing Symbol Error Rate

L. R. BAHL, J. COCKE, F. JELINEK, AND J. RAVIV

**Abstract**—The general problem of estimating the *a posteriori* probabilities of the states and transitions of a Markov source observed through a discrete memoryless channel is considered. The decoding of linear block and convolutional codes to minimize symbol error probability is shown to be a special case of this problem. An optimal decoding algorithm is derived.

#### I. INTRODUCTION

The Viterbi algorithm is a maximum-likelihood decoding method which minimizes the probability of word error for convolutional codes [1], [2]. The algorithm does not, however, necessarily minimize the probability of symbol (or bit) error. In this correspondence we derive an optimal decoding method for linear codes which minimizes the symbol error probability.

We first tackle the more general problem of estimating the *a posteriori* probabilities (APP) of the states and transitions of a Markov source observed through a noisy discrete memoryless channel (DMC). The decoding algorithm for linear codes is then shown to be a special case of this problem.

The algorithm we derive is similar in concept to the method of Chang and Hancock [3] for removal of intersymbol interference. Some work by Baum and Petrie [4] is also relevant to this problem. An algorithm similar to the one described in this correspondence was also developed independently by McAdam *et al.* [5].

#### II. THE GENERAL PROBLEM

Consider the transmission situation of Fig. 1. The source is assumed to be a discrete-time finite-state Markov process. The  $M$  distinct states of the Markov source are indexed by the integer  $m, m = 0, 1, \dots, M-1$ . The state of the source at time  $t$  is denoted by  $S_t$  and its output by  $X_t$ . A state sequence of the source extending from time  $t$  to  $t'$  is denoted by  $S_t^{t'} = S_t, S_{t+1}, \dots, S_{t'}$ , and the corresponding output sequence is  $X_t^{t'} = X_t, X_{t+1}, \dots, X_{t'}$ .

The state transitions of the Markov source are governed by the transition probabilities

$$p_t(m | m') = \Pr \{S_t = m | S_{t-1} = m'\}$$

and the output by the probabilities

$$q_t(X | m', m) = \Pr \{X_t = X | S_{t-1} = m'; S_t = m\}$$

where  $X$  belongs to some finite discrete alphabet.

Manuscript received July 27, 1972; revised July 23, 1973. This paper was presented at the 1972 International Symposium on Information Theory, Asilomar, Calif. January 1972.

L. R. Bahl and J. Cocke are with the IBM Thomas J. Watson Research Center, Yorktown Heights, N.Y.

F. Jelinek is with the IBM Thomas J. Watson Research Center, Yorktown Heights, N.Y., on leave from Cornell University, Ithaca, N.Y.

J. Raviv is with the IBM Thomas J. Watson Research Center, Yorktown Heights, N.Y. He is now at the IBM Scientific Center, Haifa, Israel.