

# The effect of interstage buffer storage on the output of two unreliable production units in series, with different production rates

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The effect of interstage buffer storage on  
the output of two unreliable production  
units in series, with different production  
rates

by

J. Wijngaard

Eindhoven, November 1977

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J. Wijngaard

## 1. Introduction

The effect of interstage buffer storage on the output of a production line is discussed very frequently. Two classes of models are considered. In the first place models where the processing time of the production units is stochastic (see for instance [2], [3], [4], [6], [8], [10]). In the second place models where the processing time is constant but where each production unit is subject to failures (see for instance [1], [5], [7], [9], [11]).

For the case of a two stage production line there are some exact analytical results, for three or more stages simulation is used or only approximated solutions are given.

For the two-stage case analytical results are found by considering the equations for the stationary probabilities. At least for the model with machine failures this is probably not the easiest way to solve the problem (Buzacott [1] gives only exact formula's for the case with two identical production units with geometrically distributed time to failure and repair time, Okamura and Yamashina [9] give only numerical results).

In the failure model it is easy to distinguish regeneration points, for instance the points of time where the buffer becomes empty. The time between two subsequent regenerations is called a cycle. The output rate of the production line can be written then as the quotient of the expected production per cycle and the expected cycle time. This is worked out in this paper.

We use a continuous time model. This makes it possible to consider also cases where the production rates of the two stages differ.

If  $v$  is the production rate of a (working) machine,  $\lambda$  is the failure rate and  $\mu$  the repair rate, then the net production rate is  $v \cdot \frac{\mu}{\lambda + \mu}$ . The effect of a buffer storage decreases if the difference between the net production rates of the two units increases. Some numerical results are given for the cases:  $v_1 = v_2$ ,  $\mu_1 = \mu_2$ ,  $\lambda_1 \neq \lambda_2$ ;  $v_1 = v_2$ ,  $\mu_1 \neq \mu_2$ ,  $\lambda_1 = \lambda_2$ ;  $v_1 \neq v_2$ ,  $\mu_1 = \mu_2$ ,  $\lambda_1 = \lambda_2$ .

## 2. The model

The system considered in this paper is a two stage production line with an interstage buffer storage.



The production is intended to be continuous but there are unexpected machine failures. For both production units the time until the first failure is assumed to be negative exponentially distributed with parameters  $\lambda_1$  and  $\lambda_2$  respectively. The duration of the down-time of a production unit is also assumed to be negative exponentially distributed (parameters  $\mu_1$  and  $\mu_2$ ). The unit of time is chosen such that the production rate of  $P_1$  is 1. The production rate of  $P_2$  is called  $v$ .

If the buffer is completely occupied and production unit  $P_2$  is down, then production unit  $P_1$  has to stop also (or the products which are made are wasted), if  $P_2$  is running but  $v < 1$  then  $P_1$  has to slow down to rate  $v$  ( $P_1$  is blocked). If the buffer is empty and production unit  $P_1$  is down then production unit  $P_2$  has to stop also, if  $P_1$  is running but  $v > 1$  then  $P_2$  has to slow down to rate 1 ( $P_2$  is starved).

We are interested in the question how the average production rate of the whole line depends on the capacity of the buffer (this capacity is called  $K$ ). To answer this question we introduce regeneration points.

The points of time where  $P_2$  is running,  $P_1$  is down and the buffer becomes empty are defined as regeneration points. The time between two regeneration points is called a cycle. Let  $P_T$  be the expected production of the line per cycle and  $T$  the expected cycle length, then the net production rate of the line is  $P_T/T$ . The quantities  $P_T$  and  $T$  can be calculated in the same way, both can be seen as "costs" per cycle. The rate at which the costs grow during the cycle depends on the state of the system. The state of the system can be described by the triple  $(a, b, x)$ , where  $a$  is the state of production unit  $P_1$  ( $a = 0$  means down,  $a = 1$  means running),  $b$  is the state of production unit  $P_2$  and  $x$  is the inventory level in the buffer. (The regeneration points are therefore the entrances in state  $(0, 1, 0)$ ).

Let:

$\alpha(x)$  be the rate at which the "costs" grow in state  $(1, 0, x)$   
 $\beta(x)$  be the rate at which the "costs" grow in state  $(0, 1, x)$   
 $\gamma(x)$  be the rate at which the "costs" grow in state  $(0, 0, x)$   
 $\sigma(x)$  be the rate at which the "costs" grow in state  $(1, 1, x)$

If  $\alpha(x) = \beta(x) = \gamma(x) = \sigma(x) = 1$  for all  $x, 0 \leq x \leq K$ , then the expected "costs" per cycle are equal to the expected cycle length,  $T$ .

If  $\alpha(x) = 0, \beta(x) = v, \gamma(x) = 0, \sigma(x) = v$  for  $x, 0 < x \leq K$  and  $\alpha(0) = 0, \beta(0) = 0, \gamma(0) = 0, \sigma(0) = \min(1, v)$ , then the expected "costs" per cycle are equal to the expected production per cycle,  $P_T$ .

### 3. The expected costs per cycle

In this section it is shown how in general the expected costs per cycle,  $C_T$ , can be determined. Therefore we have to introduce the functions  $f(\cdot), g(\cdot), h(\cdot), \ell(\cdot)$ .

$f(x)$  are the expected costs until the end of the cycle if the system is now in state  $(1, 0, x), 0 \leq x \leq K$ .

The functions  $g(\cdot), h(\cdot), \ell(\cdot)$  are defined analogously for the states  $(0, 1, \cdot), (0, 0, \cdot), (1, 1, \cdot)$ . The definition of  $g(0)$  is somewhat ambiguous, but it is easy to see that  $\lim_{x \rightarrow 0} g(x) = 0$  and to make  $g(x)$  continuous in  $x = 0$  we define  $g(0) = 0$ .

For the expected costs per cycle we can write now

$$C_T = \frac{\beta(0)}{\lambda_2 + \mu_1} + \frac{\lambda_2}{\lambda_2 + \mu_1} h(0) + \frac{\mu_1}{\lambda_2 + \mu_1} \ell(0) \quad (1)$$

Notice that it is assumed here that  $P_2$  has the same failure rate when it is starved as when it is running. In the same way it is assumed that  $P_1$  blocked has the same failure rate as  $P_1$  running.

Other assumptions about the failure rates can be handled in the same way.

Now it will be shown how the functions  $f$ ,  $g$ ,  $h$ ,  $\ell$  can be determined.

The costs  $f(x)$  can be divided in the costs during the first small time interval  $\Delta$  and the rest of the costs until the end of the cycle.

For  $0 \leq x < K$  we get (deleting some terms of order  $\Delta^2$ ).

$$f(x) = \alpha(x) \cdot \Delta + (1 - \lambda_1 \Delta)(1 - \mu_2 \Delta)f(x + \Delta) + \lambda_1 \Delta(1 - \mu_2 \Delta)h(x) + \mu_2 \Delta(1 - \lambda_1 \Delta)\ell(x)$$

Hence (deleting again terms in  $\Delta^2$ ).

$$\frac{f(x) - f(x + \Delta)}{\Delta} = \alpha(x) - (\lambda_1 + \mu_2)f(x) + \lambda_1 h(x) + \mu_2 \ell(x)$$

Taking the limit for  $\Delta \rightarrow 0$  this yields

$$-f'(x) = \alpha(x) - (\lambda_1 + \mu_2)f(x) + \lambda_1 h(x) + \mu_2 \ell(x), \quad 0 \leq x < K \quad (2)$$

In the same way we can derive

$$vg'(x) = \beta(x) - (\lambda_2 + \mu_1)g(x) + \lambda_2 h(x) + \mu_1 \ell(x), \quad 0 < x \leq K \quad (3)$$

$$0 = \gamma(x) - (\mu_1 + \mu_2)h(x) + \mu_1 f(x) + \mu_2 g(x), \quad 0 \leq x \leq K \quad (4)$$

$$(v-1)\ell'(x) = \delta(x) - (\lambda_1 + \lambda_2)\ell(x) + \lambda_1 g(x) + \lambda_2 f(x), \quad 0 < x < K \quad (5)$$

For  $x = K$ , instead of (2) we find

$$0 = \alpha(K) - (\lambda_1 + \mu_2)f(K) + \lambda_1 h(K) + \mu_2 \ell(K) \quad (2a)$$

If  $v \leq 1$  then equation (5) also holds for  $x = 0$ , for  $x = K$  we get

$$0 = \delta(K) - (\lambda_1 + \lambda_2)\ell(K) + \lambda_1 g(K) + \lambda_2 f(K) \quad (5a)$$

If  $v \geq 1$  equation (5) holds also for  $x = K$ , for  $x = 0$  we get

$$0 = \delta(0) - (\lambda_1 + \lambda_2)\ell(0) + \lambda_1 g(0) + \lambda_2 f(0) \quad (5b)$$

For  $x = 0$  instead of (3) we have

$$g(0) = 0 \quad (3a)$$

Using equation (4) the function  $h(\cdot)$  can be substituted into the functions  $f(\cdot)$ ,  $g(\cdot)$ ,  $l(\cdot)$ . That gives a system of three first order linear differential equations in the functions  $f(\cdot)$ ,  $g(\cdot)$ ,  $l(\cdot)$  with boundary conditions (2a), (3a) and (5a) or (5b).

In the solution of these equations we have to distinguish between the cases  $v = 1$  and  $v \neq 1$ .

#### 4. The case $v = 1$

In this case the left hand side of equation (5) reduces to 0. Substitution of (4) and (5) into (2) and (3) yields

$$-f'(x) = \epsilon(x) + r_f(g(x)-f(x)) \quad , \quad 0 \leq x < K \quad (6)$$

$$g'(x) = \eta(x) + r_g(f(x)-g(x)) \quad , \quad 0 < x \leq K \quad (7)$$

$$, \text{ where } r_f = \frac{\lambda_1 \mu_2}{\mu_1 + \mu_2} + \frac{\lambda_1 \mu_2}{\lambda_1 + \lambda_2} \quad , \quad r_g = \frac{\lambda_2 \mu_1}{\mu_1 + \mu_2} + \frac{\lambda_2 \mu_1}{\lambda_1 + \lambda_2}$$

$$\text{and } \epsilon(x) = \alpha(x) + \frac{\lambda_1}{\mu_1 + \mu_2} \gamma(x) + \frac{\mu_2}{\lambda_1 + \lambda_2} \delta(x) \quad , \quad 0 \leq x \leq K$$

$$\eta(x) = \beta(x) + \frac{\lambda_2}{\mu_1 + \mu_2} \gamma(x) + \frac{\mu_1}{\lambda_1 + \lambda_2} \delta(x) \quad , \quad 0 \leq x \leq K$$

Addition of (6) and (7) yields

$$g'(x) - f'(x) = \epsilon(x) + \eta(x) + (r_f - r_g)(g(x) - f(x)), \quad 0 < x < K \quad (8)$$

By substitution of equation (4) for  $x = K$  and equation (5a) into equation (2a) we get the boundary condition

$$0 = \epsilon(K) + r_f(g(K) - f(K)) \quad (9)$$

Let the function  $w(\cdot)$  be defined by  $w(x) = g(x) - f(x)$ . This function  $w(\cdot)$  is determined by (8) and (9). Substitution of (4) for  $x = 0$  and (5b) in equation (1) yields

$$C_T = \frac{1}{\lambda_2 + \mu_1} (\eta(0) + r_g f(0)) = \frac{1}{\lambda_2 + \mu_1} (\eta(0) - r_g w(0)) \quad (10)$$

Hence the expected costs per cycle,  $C_T$ , depend only on the function  $w(\cdot)$ .

In order to find the expected cycle length we have to substitute  $\alpha(x) = \beta(x) = \gamma(x) = \delta(x) = 1$  for  $x, 0 \leq x \leq K$ . For  $r_f \neq r_g$  this yields

$$T = \frac{1}{\lambda_2 + \mu_1} \left\{ \eta^* - (\epsilon^* + \eta^*) \frac{\lambda_2 \mu_1}{\lambda_2 \mu_1 - \lambda_1 \mu_2} + \left( \frac{\lambda_2 \mu_1}{\lambda_1 \mu_2} \epsilon^* + \frac{\lambda_2 \mu_1}{\lambda_2 \mu_1 - \lambda_1 \mu_2} (\epsilon^* + \eta^*) \right) e^{(r_g - r_f)K} \right\}$$

where  $\epsilon^*, \eta^*$  are equal to  $\epsilon(\cdot)$  and  $\eta(\cdot)$  with  $\alpha(x) = \beta(x) = \gamma(x) = \delta(x) = 1$ . For  $r_f = r_g$  we get

$$T = \frac{1}{\lambda_2 + \mu_1} \{ \eta^* + \epsilon^* + r_g (\eta^* + \epsilon^*) K \}$$

The expected production per cycle is found by substituting  $\alpha(x) = \gamma(x) = 0$ ,  $\beta(x) = \delta(x) = 1$  for  $0 < x \leq K$  and  $\alpha(0) = \beta(0) = \gamma(0) = 0$ ,  $\delta(0) = 1$ .

For  $r_f \neq r_g$  this yields

$$P_T = \frac{1}{\lambda_2 + \mu_1} \left\{ \frac{\mu_1}{\lambda_1 + \mu_2} - (\epsilon' + \eta') \frac{\lambda_2 \mu_1}{\lambda_2 \mu_1 - \lambda_1 \mu_2} + \left( \frac{\lambda_2 \mu_1}{\lambda_1 \mu_2} \epsilon' + \frac{\lambda_2 \mu_1}{\lambda_2 \mu_1 - \lambda_1 \mu_2} (\epsilon' + \eta') \right) e^{(r_g - r_f)K} \right\}$$

where  $\epsilon', \eta'$  are equal to  $\epsilon(\cdot)$  and  $\eta(\cdot)$  with  $\alpha(x) = \gamma(x) = 0$  and  $\beta(x) = \delta(x) = 1$ .

For  $r_f = r_g$  we get

$$P_T = \frac{1}{\lambda_2 + \mu_1} \left\{ \frac{\mu_1}{\lambda_1 + \mu_2} + \epsilon' + r_g (\eta' + \epsilon') K \right\}$$

The average production rate of the line is equal to  $P_T/T$ .

This can also be written as  $\mu_2/(\lambda_2 + \mu_2) - (1/(\lambda_2 + \mu_1))/T$ , where  $\mu_2/(\lambda_2 + \mu_2)$  is the net production rate of  $P_2$  and  $(1/(\lambda_2 + \mu_1))/T$  is the average loss of production per unit of time due to an empty buffer ( $P_2$  starved).

##### 5. The case $v \neq 1$

Substitution of the equation (4) into the equations (2), (3), (5) yields

$$\underline{f}'(x) = \underline{c}(x) + A \underline{f}(x)$$



where  $\underline{f}(x)$  is the vector  $\begin{pmatrix} f(x) \\ g(x) \\ \ell(x) \end{pmatrix}$ ,  $\underline{f}'(x)$  is the vector  $\begin{pmatrix} f'(x) \\ g'(x) \\ \ell'(x) \end{pmatrix}$

, A is the matrix  $\begin{pmatrix} \mu_2 + \frac{\lambda_1 \mu_2}{\mu_1 + \mu_2} & -\frac{\lambda_1 \mu_2}{\mu_1 + \mu_2} & -\mu_2 \\ \frac{1}{v} \frac{\lambda_2 \mu_1}{\mu_1 + \mu_2} & -\frac{1}{v} \left( \mu_1 + \frac{\lambda_2 \mu_1}{\mu_1 + \mu_2} \right) & \frac{\mu_1}{v} \\ \frac{\lambda_2}{v-1} & \frac{\lambda_1}{v-1} & -\frac{(\lambda_1 + \lambda_2)}{v-1} \end{pmatrix}$

and  $\underline{c}(x)$  is the vector  $\begin{pmatrix} -\alpha(x) - \frac{\lambda_1}{\mu_1 + \mu_2} \gamma(x) \\ \frac{1}{v} \left( \beta(x) + \frac{\lambda_2}{\mu_1 + \mu_2} \gamma(x) \right) \\ \frac{1}{v-1} \end{pmatrix}$

If  $\frac{\mu_1}{\lambda_1 + \mu_1} \neq v \cdot \frac{\mu_2}{\lambda_2 + \mu_2}$  (the units have different net production rates) then A has three different eigenvalues,  $\lambda_1, \lambda_2, \lambda_3$ .

One of these eigenvalues (say  $\lambda_1$ ) is always 0 and has eigenvector  $\underline{e}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

The eigenvectors of the other two eigenvalues are called  $\underline{e}_2$  and  $\underline{e}_3$ .

The general solution of the homogeneous equation is

$$c_1 \underline{e}_1 + c_2 \underline{e}_2 e^{\lambda_2 x} + c_3 \underline{e}_3 e^{\lambda_3 x}$$

, where  $c_1, c_2, c_3$  are arbitrary constants.

If  $\frac{\mu_1}{\lambda_1 + \mu_1} = v \cdot \frac{\mu_2}{\lambda_2 + \mu_2}$  then A has only 2 different eigenvalues, 0 and  $\lambda$ .

Since  $\lambda$  has index 2 the general solution of the homogeneous equation in this case is

$$c_1 \underline{e}_1 + c_2 \underline{e}_2 e^{\lambda x} + c_3 (\underline{e}_2 x e^{\lambda x} + \underline{e}_2^* e^{\lambda x})$$

where  $c_1, c_2, c_3$  are arbitrary constants,  $\underline{e}_1$  is an eigenvector of eigenvalue 0,  $\underline{e}_2$  is an eigenvector of eigenvalue  $\lambda_2$  and  $\underline{e}_2^*$  satisfies  $(A-\lambda)\underline{e}_2^* = \underline{e}_2$ .

In case  $\alpha(x) = \beta(x) = \gamma(x) = \delta(x) = 1$  (to determine T) a solution of the inhomogeneous equation is given by

$$G \underline{e}_1^* x + \underline{d}^*$$

where  $G^*$  and  $\underline{d}^*$  have to satisfy  $G^* \underline{e}_1 = \underline{c}^* + A \underline{d}^*$  and  $\underline{c}^*$  is equal to  $\underline{c}(x)$  with  $\alpha(x) = \beta(x) = \gamma(x) = \delta(x) = 1$ .

In case  $\alpha(x) = \gamma(x) = 0, \beta(x) = \delta(x) = v$  (to determine  $P_T$ ) a solution of the inhomogeneous equation is given by

$$G' \underline{e}_1 x + \underline{d}'$$

where  $G'$  and  $\underline{d}'$  must satisfy  $G' \underline{e}_1 = \underline{c}' + A \underline{d}'$  and  $\underline{c}'$  is equal to  $\underline{c}(x)$  with  $\alpha(x) = \gamma(x) = 0, \beta(x) = \delta(x) = v$ .

For  $v < 1$  the boundary conditions are

$$\begin{aligned} \alpha(K) + \frac{\lambda_1}{\mu_1 + \mu_2} \gamma(K) - (\mu_2 + \frac{\lambda_1 \mu_2}{\mu_1 + \mu_2}) f(K) + \frac{\lambda_1 \mu_2}{\mu_1 + \mu_2} g(K) + \mu_2 \ell(K) &= 0 \\ g(0) &= 0 \\ \delta(K) + \lambda_2 f(K) + \lambda_1 g(K) - (\lambda_1 + \lambda_2) \ell(K) &= 0 \end{aligned}$$

For  $v > 1$  the latter boundary condition has to be replaced by

$$\delta(0) + \lambda_2 f(0) - (\lambda_1 + \lambda_2) \ell(0) = 0$$

In order to determine T we substitute into these equations  $\alpha(K) = \gamma(K) = \delta(K) = \delta(0) = 1$ .

To determine  $P_T$  we have to substitute  $\alpha(K) = \gamma(K) = 0, \delta(K) = v, \delta(0) = \min(1, v)$

Substitution of equation (4) for  $x = 0$  into (1) yields

$$C_T = \frac{1}{\lambda_2 + \mu_1} \left\{ \beta(0) + \frac{\lambda_2}{\mu_1 + \mu_2} \gamma(0) + \frac{\lambda_2 \mu_1}{\mu_1 + \mu_2} f(0) + \mu_1 \ell(0) \right\}$$

To determine  $T$  and  $P_T$  we can substitute into this expression the corresponding solutions of  $f(0)$  and  $\ell(0)$ .

#### 6. The effect of a buffer storage

First the case is considered where both production units are identical with respect to production rate, break down rate and repair rate

$$(\lambda_1 = \lambda_2 = \lambda, \mu_1 = \mu_2 = \mu, v_2 = 1).$$

In this case, according to section 4, the net production rate of the line is given by

$$\frac{\mu}{\lambda + \mu} - \frac{1}{2\eta^* (1 + r_g K)}$$

$$, \text{ where } \eta^* = \frac{(\lambda + \mu)^2}{2\lambda\mu} \text{ and } r_g = \frac{1}{2} (\lambda + \mu)$$

For  $\lambda = 0.01$ ,  $\mu = 0.09$  the net production rate of the line as function of  $K$ , the capacity of the buffer, is given in fig. 1.

For  $K \rightarrow \infty$  the line production rate goes to 0.9

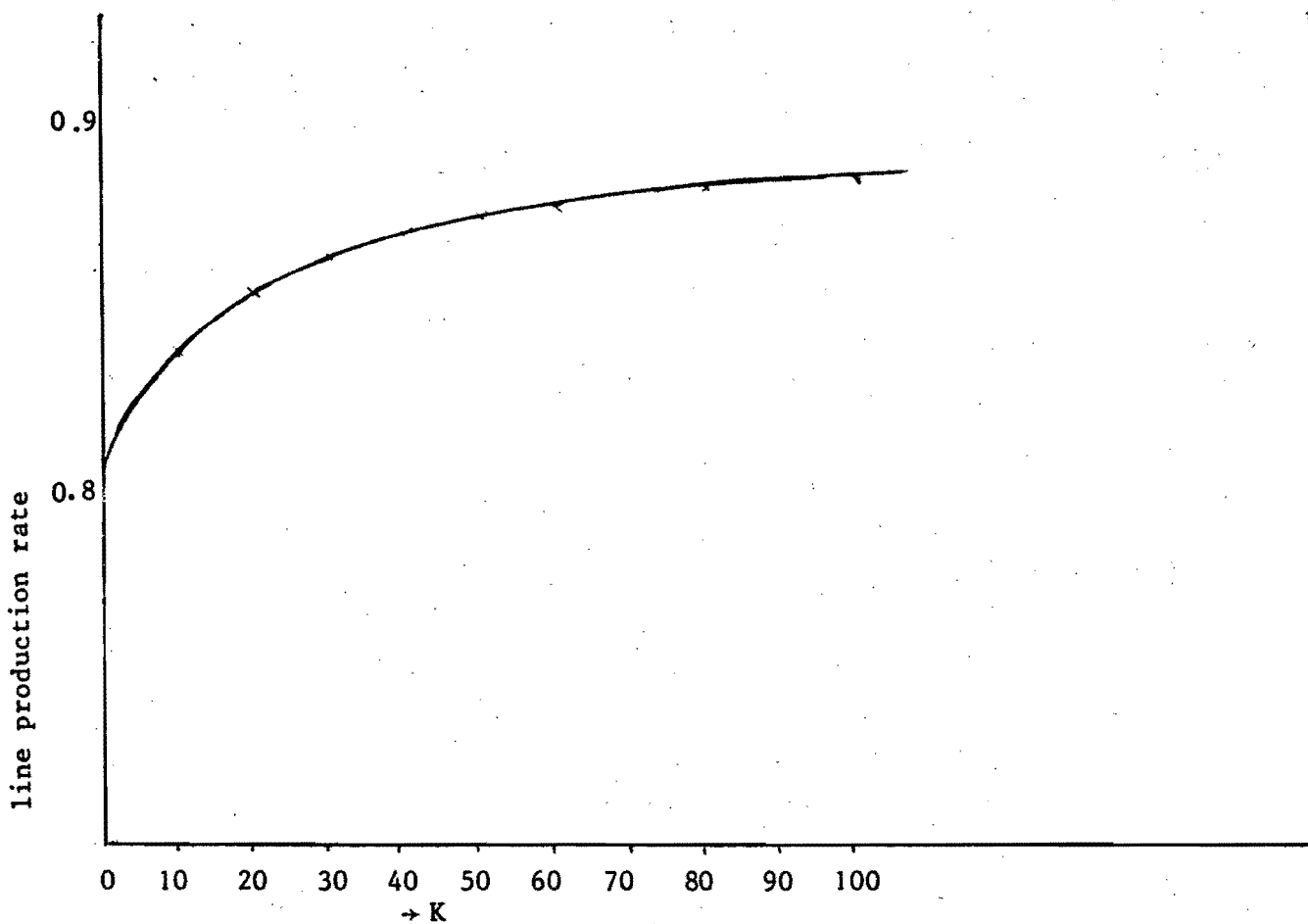
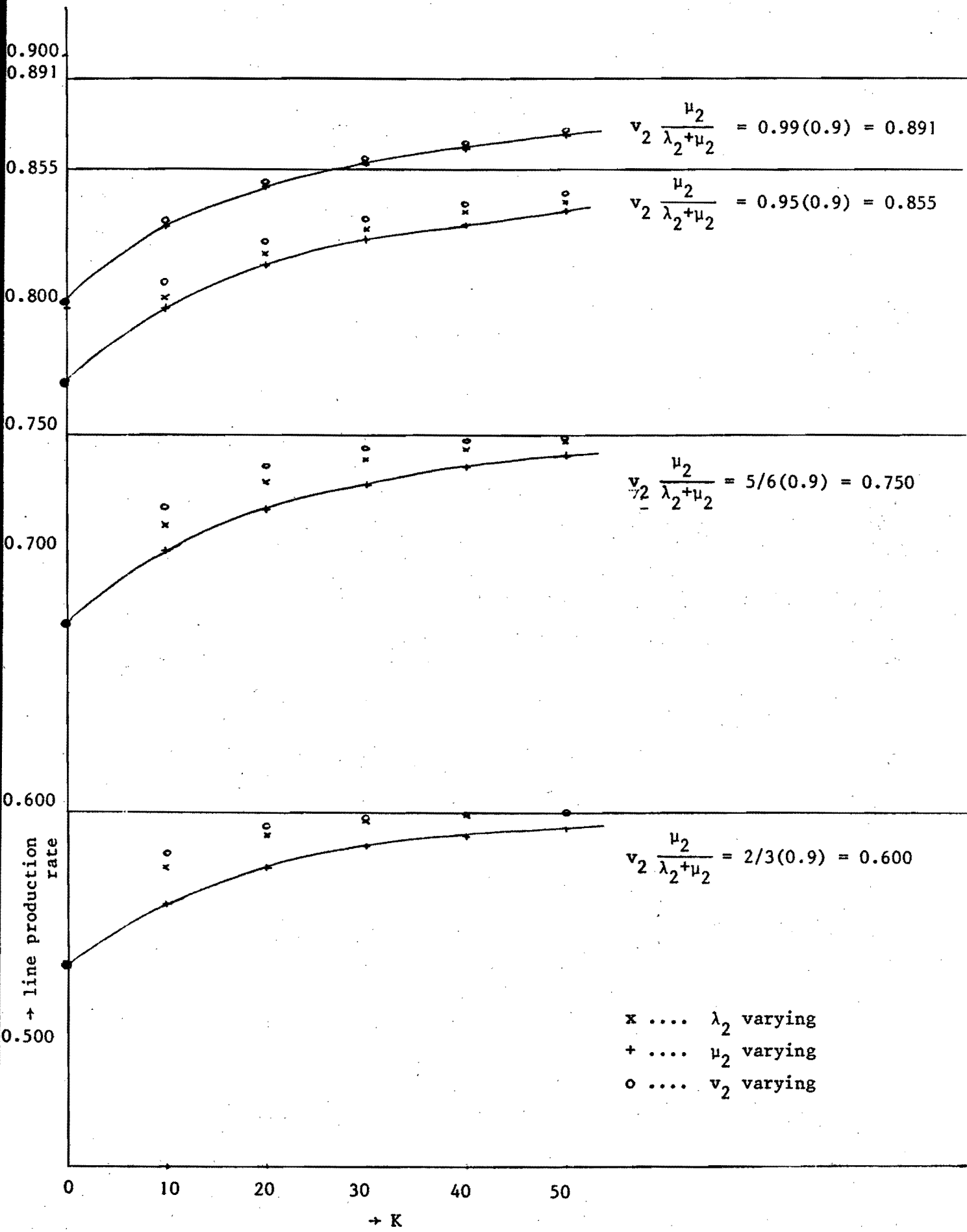


Fig. 1.

In most practical cases the two production units will not be identical. In fig. 2 and table 1 the effect of the buffer capacity is shown for the cases

- a.  $\lambda_2 > \lambda_1 = 0.01$ ,  $\mu_2 = \mu_1 = 0.09$ ,  $v_2 = 1$ ,  $\lambda_2$  varying
- b.  $\lambda_1 = \lambda_2 = 0.01$ ,  $\mu_2 < \mu_1 = 0.09$ ,  $v_2 = 1$ ,  $\mu_2$  varying
- c.  $\lambda_1 = \lambda_2 = 0.01$ ,  $\mu_1 = \mu_2 = 0.09$ ,  $v_2 < 1$ ,  $v_2$  varying

The net production rate of  $P_1$  is  $\frac{\mu_1}{\lambda_1 + \mu_1} = 0,9$ . The parameters are chosen such that the cases a, b and c can be compared for the same net production rate of  $P_2$ ,  $v_2 \cdot \frac{\mu_2}{\lambda_2 + \mu_2}$ .



$\lambda_2$	$\mu_2$	$v_2$	$v_2 \frac{\mu_2}{\lambda_2 + \mu_2}$	K						
				0	10	20	30	40	50	$\infty$
a. 0.0110	0.09	1	0.891	0.802	0.833	0.849	0.858	0.864	0.869	0.891
b. 0.01	0.0817	1	0.891	0.802	0.832	0.848	0.857	0.863	0.868	0.891
c. 0.01	0.09	0.00	0.891	0.802	0.835	0.850	0.859	0.865	0.870	0.891
a. 0.0153	0.09	1	0.855	0.770	0.804	0.821	0.831	0.838	0.842	0.855
b. 0.01	0.0590	1	0.855	0.770	0.800	0.817	0.827	0.833	0.838	0.855
c. 0.01	0.09	0.95	0.855	0.770	0.810	0.826	0.835	0.841	0.845	0.855
a. 0.0300	0.09	1	0.750	0.675	0.714	0.731	0.740	0.744	0.747	0.750
b. 0.01	0.0300	1	0.750	0.675	0.704	0.720	0.730	0.737	0.741	0.750
c. 0.01	0.009	0.83	0.750	0.675	0.721	0.737	0.744	0.747	0.749	0.750
a. 0.0600	0.09	1	0.600	0.540	0.578	0.591	0.597	0.599	0.600	0.600
b. 0.01	0.015	1	0.600	0.540	0.564	0.578	0.587	0.591	0.596	0.600
c. 0.01	0.09	0.67	0.600	0.540	0.584	0.595	0.598	0.600	0.600	0.600

Table 1.

The loss of production due to a lack of storage capacity is in the first place a function of the buffer capacity and the net production rate of  $P_2$  (the net production rate of  $P_1$  being given). But if the difference between both net production rates increases it becomes more and more necessary to distinguish between the cases a, b and c.

For instance,  $\lambda_2 = 0.01$ ,  $\mu_2 = 0.03$ ,  $v_2 = 1$  and  $\lambda_2 = 0.01$ ,  $\mu_2 = 0.09$ ,  $v_2 = 0.83$  give the same net production rate of  $P_2$  (0.750), but in the first case one needs a buffer capacity of 20 units to realize the same line production rate as in the second case with a buffer capacity of 10 units.

For all parameter settings a buffer has the most effect in case c and the least in case b.

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