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F O R M A S Y

FOrecasting and recruitment in MAnpower
SYstems

by

J. Wessels and J.A.E.E. van Nunen

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Summary:

In this paper the tools are dveloped for forecasting and recruitment planning
in a graded manpower system. Basic features of the presented approach are:
- the system contains several grades or job categories in which the em-
  ployees stay for a certain time before being promoted or leaving the
  system,
- promotability and leaving rate for any employee depend on time spent in
  the job category and personal qualifications (like education, experience,
  age),
- recruitment is not necessarily restricted to the lowest level in the system,
- several planning aims and restrictions are allowed.

The approach is based on a generalized Markov model for the dynamic behaviour
of an individual employee. On this Markov model a forecasting procedure and
a recruitment-scheduling procedure are based.

1. Introduction:

The subject of this paper is the dynamic behaviour of a graded manpower system.
In particular it will be investigated, how a generalized type of Markov model
for the dynamic behaviour of an individual employee may be used to develop
a forecasting procedure for the behaviour of the system. This forecasting
procedure will be used for the construction of a recruitment-scheduling pro-
cedure.

Any individual employee in a manpower system entered this system with certain
qualifications like education, age, experience. In the course of his stay
in the system he builds up a job history, aging gradually and collecting ex-
perience. If one is going to construct a model for the dynamic behaviour of
an individual, one should bear in mind which qualifications affect this be-
haviour.

In the literature one finds many models in the form of a Markov chain. See
e.g. [1, 3, 4, 14, 12, 7, 8, 9]. In a Markov chain model, the employee is
supposed to jump from one state to another with fixed transition probabilities.
A natural choice for the state concept is grade or job category. However,
this choice induces a model in which transition probabilities (or promotabi-
lity) do not depend on the personal qualifications and the time spent already
in the grade. Using a somewhat more generalized state concept, a Markov model
will be developed for the dynamic behaviour of an individual such that the
influence of personal qualifications and grade age are taken into account.
This model is treated in section 2.

In section 3 the forecasting problem is considered. It is demonstrated how
an efficient forecasting procedure may be based on a well-structured data
system.

The quality of the forecasts is discussed in section 4.

In sections 5 and 6 the model and the forecasting procedures are used for the
construction of recruitment planning procedures. In section 5 a linear pro-
gramming approach is given. Our approach differs from the linear programming
approaches in [1, 4, 11] with respect to the criterion function.

In section 6 a recruitment-scheduling procedure is introduced using the spe-
cific dynamic structure of the system behaviour. For this recruitment sche-
duling procedure recruiting may be allowed for all or for some grades. The
basic procedure may be used for several planning aims and restrictions. The
procedure consists of a blending of forward and backward dynamic programming.
The amount of backwards induction is restricted by side conditions.

In section 7 it is argued why in our opinion the dynamic approach for re-
cruitment planning is preferable to the linear programming approach. Here the
side conditions play an essential role.

In section 8 it is shown how a manpower data system together with the fore-
casting and recruitment planning procedures can form an efficient and flexible
information system for manpower management.

Section 9 is devoted to some remarks and extensions.

Velden and Mr. P. Westenend of the department T.E.O. at Philips, Eindhoven.
The authors' students, Mr. J. Kessels and Mr. F. Esser, have worked out
several parts of this approach for T.E.O. Their work may be found in their
master theses for the Department of Mathematics, Technological University
Eindhoven and in their reports for Philips (all written in Dutch). We are
grateful for their cooperation and the kind hospitality they received at
T.E.O. The authors wish to emphasize that the data used in the examples,
and results as illustrated in some of the figures, are purely artificial.
The figures are only inserted to illustrate which kind of information can
be achieved. The structure of the model and the presentation of the results,
however, has been based on the experience with the case study at Philips.

2. The dynamic behaviour of an individual employee:

In the introduction it was already stated that a straightforward application
of the Markov chain approach leads to a model which is not very realistic.
Bartholomew [1] says: "the basic form of the Markov chain model for social
and occupational mobility is rarely adequate in practice".
The main disadvantage of the basic Markov chain model with grades as states,
is the fact that in reality future developments (promotion, leaving) are af-
fected by much more of the employee's history than his current grade.
The main advantage, however, of a Markov chain approach is its easy process-
ing mechanism which allows simple computation of main characteristics.
Luckily a solution is available in many cases, viz. the main influence from
history on future developments often comes from a relatively small part.
If this is the case, an extension of the concept of state by incorporating
the relevant parts of the history may be of help in order to obtain a rea-
listic and workable Markov model. See also Forbes [8, 9]. This technique,
which is useful in other types of situations as well (see e.g.[15]), will
be used in this section.

Suppose a manpower system has G grades or job categories, we denote them by
1,2,...,G. Promotions are supposed to increase the grade number. The grade
structure may be linear (see fig. 2.1) or branching (see fig. 2.2).
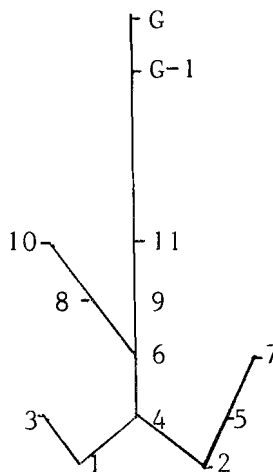
fig. 2.1 : linear grade structure   fig. 2.2 : branching grade structure

In a linear grade structure promotion induces a transition from the actual grade g to grade $g+1$. In a branching grade structure the rising branches indicate the promotion possibilities in a grade.

Suppose the personal characteristics of an employee can be measured by a qualification index and an age index. Let the qualification index run from 1 to Q and the age index from 1 to A.
The age index may be supposed to indicate an age class e.g. 1: 20-24, 2: 25-29, 3: 30-39, 4: 40-60.
The qualification index may indicate educational level and some types of experience. By the way, it may be possible to include experience in the grade structure. A person's qualification index may run through a branching (as in fig. 2.2) or a linear structure.

As a further characteristic for an employee we introduce the grade age $\ell = 0,1,\ldots,L$. With ℓ we denote the number of years an employee is in his current grade.
Hence, when somebody is promoted to grade g or enters the system in grade g he usually gets the grade age 0.

Now summarizing:
a person's state is characterized by four indices g, q, a, ℓ where
 - g is his current grade ($g = 1,\ldots,G$);
 - q is his current qualification index ($q = 1,\ldots,Q$);
 - a is his current age index ($a = 1,\ldots,A$);
 - ℓ is his current grade age ($\ell = 0,1,\ldots,L$).

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Example:

A realistic example appeared to be as follows
- G = 10 (with salary groups as grades) and a linear grade structure;
- Q = 3, with a linear structure and three educational levels as qualifica-
  tions; experience is incorporated in the grade and in the (fictitious)
  educational level;
- A = 3, with 1: 20-34, 2: 35-44, 3: 45-60;
- L = 9, after 9 years in the same grade the grade age is supposed to re-
  main constant.

In this way we get $G \times Q \times A \times (L+1)$ states. In the example $10 \times 3 \times 3 \times 10 = 900$.
It will not be necessary to consider all these states: nobody will be in the
topgrade G with qualification index 1 and age index 1.
The first work is to make a set S_1 of relevant states. This can be achieved
in the following way:
1. Make the set S_0 of all states (g, q, a, ℓ) which can possibly occur in
   practice. In this way unlikely combinations as $(G, 1, 1, \ell)$ and $(1, Q, A, \ell)$
   are thrown away. Furthermore, if for a certain grade g (e.g. an entrance
   grade) it is very unlikely to reach a high gradeage, combinations like
   (g, q, a, L) may be thrown away.
2. lump together all states in S_0 which give the same expectations for the
   future. In the topgrades it may not be necessary to make difference be-
   tween qualification indices. In this way the set S_1 may be constructed
   with a number of relevant states, which is possibly substantially smaller
   than $G \times Q \times A \times (L+1)$.

In the example (there remained) 39 relevant (g, q, a)-combinations (after the
lumping) each generating maximally 10 states in S_1 with the relevant grade
ages. Clearly the number of relevant states will depend on the problem under
study.
For administrative simplicity we introduce the state 0 for people who left
the system. Hence leaving coincides with a transition from some state (g, q,
a, ℓ) to 0.
A promotion coincides with a transition from some state (g, q, a, ℓ) to some
state $(g_1, q_1, a_1, 0)$ with $g_1 > g$, $q_1 \geq q$, $a_1 \geq a$.

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If an employee did not leave the system in the course of a year and is not promoted, then he makes a transition from some state (g, q, a, ℓ) to $(g, q, a, \ell+1)$ if $\ell < L$ or if $\ell = L$ he remains in the same state. Observe that q and a represent the situation when the grade was entered. Now our set of states S consists of 0 and the relevant states in S_1.

A Markov chain model for the dynamic behaviour of an individual employee now only requires the specification of the transition probabilities. With the help of the grade structure, the qualification structure, and the preceding remarks on allowed transitions it is possible to design a subset $S(s)$ of S for any state $s \in S$, where $S(s)$ consists of all states which may be reached from s in one transition.

The sets $S(s)$ have e.g. the following properties:

$$S(0) = \{0\},$$
$$(g, q, a, \ell+1) \in S(g, q, a, \ell) \text{ for } \ell < L,$$
$$0 \in S(g, q, a, \ell),$$
$$(g_1, q_1, a_1, \ell_1) \notin S(g, q, a, \ell) \text{ for } \ell < L \text{ if } g_1 < g \text{ or}$$
$$q_1 < q \text{ or } a_1 < a \text{ or } 0 < \ell_1 \leq \ell, \text{ etc.}$$

If $s_1 \notin S(s)$ the transition probability $p(s,s_1)$, the probability of reaching state s_1 in one year if one is now in state s, is by definition equal to zero. Hence

$$p(0,0) = 1 ,$$
$$\text{and for all s : } \sum_{s_1 \in S'(s)} p(s,s_1) = 1 .$$

We will use $p(s,s_1)$ as the transition probability for an "arbitrary" employee in state s to jump in one year to state s_1. An important problem is the estimation of $p(s,s_1)$ with $s_1 \in S(s)$. In the following section this problem will be considered together with the problem of the forecasting of state occupations.

3. Forecasting:

One of the main reasons for the construction of models as in the following section is that they offer the possibility of forecasting the numbers of

employees who will occupy the states in the years to come.

Let us suppose that there are n(s) employees in state s $\in$ S at time t = 0. Let us further suppose (for the moment being), that no recruiting takes place. So, since the state 0 is incorporated in the system, the number of employees in the system is for the moment supposed to be constant, say K. For an arbitrary member of state s, the probability of reaching state s_1 in one year is equal to $p(s,s_1)$. Hence the expected number of employees who start in s and reach s_1 in one year is equal to $n(s)p(s,s_1)$. This result does not require **independence of the individual behaviour as appears from the formal treatment.**

$\equiv$ $X_k(t)$ denotes the state in which the k-th employee is at time t (k = 1,...,K);

$\equiv$ $\delta(s,s_1) := \begin{cases} 0 \text{ if } s \neq s_1 \\ 1 \text{ if } s = s_1 \end{cases}$;

$\equiv$ $N_s(t) := \sum\limits_{k=1}^{K} \delta(s,X_k(t))$ is the number of employees in state s at time t;

$\equiv$ $N_s(0) := n(s)$;

$\equiv$ $\mathbb{E}N_s(t) = \mathbb{E} \sum\limits_{k=1}^{K} \delta(s,X_k(t))$

$= \sum\limits_{k=1}^{K} \mathbb{E}\delta(s,X_k(t))$

$= \sum\limits_{k=1}^{K} \mathbb{P}(X_k(t) = s)$,

where $\mathbb{E}$, $\mathbb{P}$ denote expectation and probability respectively;

$\equiv$ $\mathbb{P}(X_k(t) = s) = \sum\limits_{s'} \mathbb{P}(X_k(t-1) = s')p(s',s)$ for t = 1,2,... ;

$\equiv$ now it is useful to introduce some matrix and vector notation. Since the states may be numbered, N(t) can be viewed upon as the row vector with components $N_s(t)$. $\mathbb{E}N(t)$ is defined as the row vector with components $\mathbb{E}N_s(t)$. P is the matrix with components p(s,s'). $\mathbb{P}_k(t)$ is the row vector with components $\mathbb{P}(X_k(t) = s)$;

$\equiv$ $\mathbb{P}_k(t) = \mathbb{P}_k(t-1)$ P for t = 1,2,... ;

$\equiv$ $\mathbb{P}_k(t) = \mathbb{P}_k(0)P^t$ for t = 0,1,2,... ;

$\equiv$ $\mathbb{E}N(t) = \sum\limits_{k=1}^{K} \mathbb{P}_k(0)P^t = \left[\sum\limits_{k=1}^{K} \mathbb{P}_k(0) \right]P^t = N(0)P^t$.

Hence the expected number of employees in state s at time t is equal to the s-component of $N(0)P^t$, if at time 0 the employees are distributed according to $N(0)$ over the states. This derivation did not depend on the specific problem, it holds for any situation where any one of a number of individuals walks through a finite state space according to a Markov chain. Independent behaviour of the people involved was not supposed either. This is an important feature since independence of promotion behaviour is not very likely in practice. The situation where fixed fractions get a promotion may be nearer to reality than the situation where all individuals in the same state may be promoted independently.

If $N(0)$ and P have been given, it is easy to give a forecast $\hat{N}_s(t)$ for the number $N_s(t)$ employees which occupy the states at time t:

$$\hat{N}(0) := N(0),$$

determine $\quad \hat{N}(t) := \hat{N}(t-1)P, \quad$ for $t = 1,2,\ldots,T.$

Or explicitly: $\hat{N}_s(0) := n(s),$

$$\hat{N}_s(t) := \sum_{s'} \hat{N}_s(t-1)p(s',s) \quad \text{for } t = 1,\ldots,T$$

The quality of these forecasts will be considered in section 4. In the sequal of this section we will first extend the forecasting procedure to the situation with recruitment. Finally the problem of estimating $p(s,s')$ will be considered.

The possibility of leaving the manpower system was already incorporated in the model by the introduction of the state 0. However, we did not incorporate the possibility of new employees who enter the system after $t = 0$.
Suppose, for the moment being, that the numbers of new employees have been given in advance: $R_s(t)$ is the number of employees recruited in state s at time t, $R(t)$ is the row vector with components $R_s(t) \cdot (t = 1,\ldots,T)$, $s \in S$. We suppose throughout this paper that newly recruited employees jump through the state space S with the same transition probabilities $p(s,s')$ as the old ones.
Now the vector of expected numbers of employees in the different states at time t becomes

$$\mathbb{E}N(t) = N(0)P^t + R(1)P^{t-1} + \ldots + R(t-1)P + R(t)$$

$$= N(0)P^t + \sum_{\tau=1}^{t} R(\tau)P^{t-\tau}$$

$$= [\mathbb{E}N(t-1)]P + R(t) .$$

Now we get the following procedure for the determination of $\hat{N}_s(t)$, the forecast of $N_s(t)$:

PROCEDURE (FORECASTING):

$$\hat{N}(0) = N(0) ,$$

determine: $\hat{N}(t) := \hat{N}(t-1)P,$ for $t = 1,2,\ldots,T.$

More explicitly: $\hat{N}_s(0) := n(s)$,

$$\hat{N}_s(t) := \sum_{s_i} \hat{N}_{s_1}(t-1)p(s_1,s) + R_s(t)   \text{for } t = 1,\ldots,T.$$

The numbers $R_s(t)$ may be forecasts of the recruited numbers:
the same forecasting procedures may be used if some independencey conditions
are satisfied e.g. the futural behaviour of employees recruited in state s
at time t is independent of the number of employees recruited in state s at
time t.

For practical purposes one is not only or even not primarily interested in
$\hat{N}_s(t)$, but perhaps even more in a forecast for the total number of employees
in grade g at time t. Such a forecast may be obtained by

$$\sum_{q,a,\ell} \hat{N}_{(g,q,a,\ell)}(t) .$$

Other features of interest which can be obtained in an equally simple way
are: the forecasted distribution of people with Q-qualification over the
grades, or the forecasted grade-age distribution for certain grades, or the
qualification distribution for people in certain grades etc. (see section 8).

In order to apply this forecasting procedure in an real situation, it is
necessary to have an estimate for P at one's disposal.
In order to obtain a first estimate for P one may use historical data in the
following way. Suppose there are data for years $-k$, $-k+1$, $\ldots$, -1, 0.
Suppose in year $-t$ there were $N_s(-t)$ employees in state s and $n_{-t}(s,s_1)$ of

them jumped to state s_1 between $-t$ and $-t+1$ ($t = 1,2,...,k$). This gives the
following estimate for $p(s,s_1)$

$$\frac{\sum_{t=1}^{k} n_{-t}(s,s_1)}{\sum_{t=1}^{k} N_s(-t)} .$$

If one wishes to weigh the data with respect to the years from which they
come, one may use the estimate

$$\frac{\sum_{t=1}^{k} w(t)n_{-t}(s,s_1)}{\sum_{t=1}^{k} w(t)N_s(-t)} ,$$

where $w(t)$ is positive ($t = 1,...,k$).

It is not recommendable to use these estimates blindly for the forecasting
procedure. It may occur that some states which will be heavily occupied in
the near future may have had only a few visitors in the recent past. Further-
more, ideas about certain types of promotions may be altering. Therefore it
is necessary to display the results of the data based estimation procedure
for the personnel manager. For this purpose the estimated transition proba-
bilities are not very suitable. Using the estimates one should plot the es-
timated distribution of time spend in a grade for several categories, the
probabilities of certain promotions depending on age etc. In section 8 we
will return to the question what can be done to make the data transparant.
The expertise of the personnel manager is necessary for the verification
and eventual correction of the estimates.

4. The quality of the forecasts:

The quality of the forecasts in section 3 depends on the quality of the
model, on the stochastic character of the model, and on the estimation of
the model parameters. Nobody can reasonably expect general assertions on
on the quality of the model. It is, of course, very important to incorporate
all relevant aspects in the model. Although a general test of model correct-

ness is not very well thinkable, critical aspects of the model can be
tested in a normal way if only sufficient data are available.

So let us take the model for granted and consider the influence of the
randomness on the forecasting quality. Since one cannot hope to forecast
the realization of a random variable exactly, one is already glad with a
small expected quadratic difference:
if the random variable X is forecasted as x, the quality of the forecast
x is

$$\mathbb{E}(X-x)^2 \ .$$

$$\mathbb{E}(X-x)^2 = \text{VAR}(X) + (\mu-x)^2, \text{ where } \mu = \mathbb{E}X.$$

Hence the best forecast for X is $x = \mu$ and its quality is VAR(X).
In de foregoing section we recommended accordingly expectations as forecasts.
For judgment of their quality it would be necessary to get the variances
of the forecasted random variables. For example, if one is interested in the
quality of the forecast for the number of employees in grade 2 after 3 years,
one should compute the variance of

$$\sum_{q,a,\ell} N_{(2,q,a,\ell)}(3) \ .$$

Hence the first step in this problem is the computation of variances and
covariances of the variables $N_s(t)$. Such a computation is not very difficult
under the condition that all employees jump through the states independently
from each other. Under this condition Bartholomew [1] gives the following
recursion formula for t = 1,2,... in the case of no recruitment

$$\text{COV}[N_s(t),N_{s_0}(t)] = \sum_{s_1} \sum_{s_2} p(s_1,s)p(s_2,s_0)\text{COV}[N_{s_1}(t-1),N_{s_2}(t-1)] +$$

$$+ \sum_{s_1} [\delta(s,s_0) - p(s_1,s_0)]p(s_1,s)\mathbb{E}N_{s_1}(t-1) \ ,$$

where $\delta(s,s_0) = \begin{cases} 0 \text{ if } s \neq s_0 \\ 1 \text{ if } s = s_0 \end{cases} \ .$

Since the covariances are equal to zero for t = 0, this formula enables
one to compute the relevant covariances recursively, using the computed
forecasts for $\mathbb{E}N_{s_1}(t-1)$.

For the case with given recruitments the computation of covariances is a straightforward extension if the newly recruited employees jump independently of each other and of the old employees.

The variance of, say, the number of employees in grade 2 after 3 years may now be computed in the following way

$$\text{VAR}[\sum_{q,a,\ell} N_{(2,q,a,\ell)}(3)] = \sum_{q,a,\ell} \sum_{q_1,a_1,\ell_1} \text{COV}[N_{(2,q,a,\ell)}(3), N_{2,q_1,a_1,\ell_1}(3)] \, .$$

Especially important in judging the quality of a forecast is the variation coefficient of the forecasted random variable or the quotient of standard deviation and forecast.

Example:

In an example with the structure of the example in section 2 the variation coëfficient $v_{(g,q,a)}(t)$ (without recruitment) of the number of employees in grade g with qualification index q and age index a for the future points in time t = 1,2,3 have been computed. These variation coëfficients are plotted in figure 5.1 against the forecasted number $\hat{N}_{(g,q,a)}(t)$ of employees with g, q, a at time t = 1. Figure 5.2 shows the variation coëfficients for the employees in the grades ($v_g(t)$ against the forecasted number $\hat{N}_g(t)$ of employees in the grades g at time t = 1.
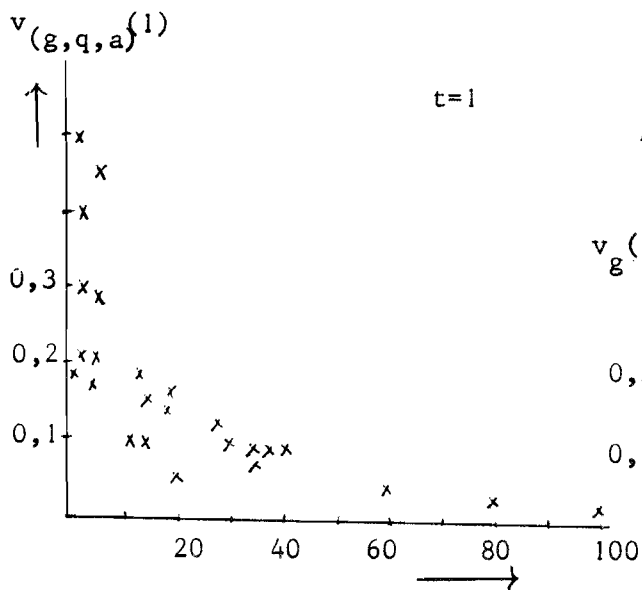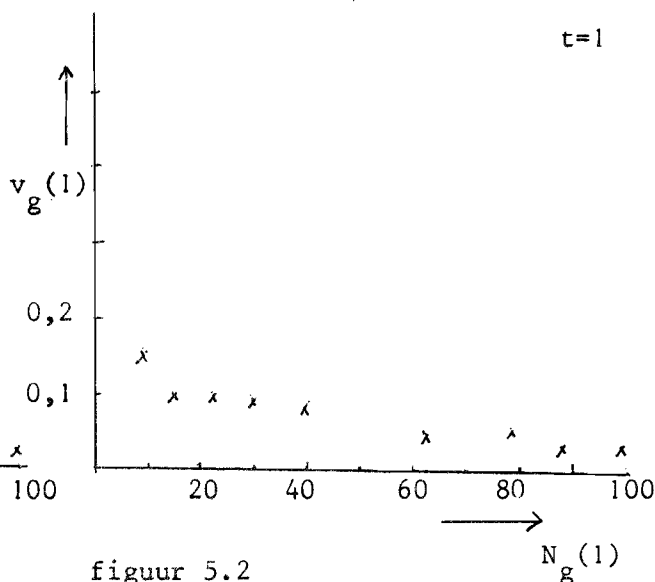
figuur 5.1   figuur 5.2

The system in our example consisted at t = 0 of about 800 employees.
As might be expected and also follows from the figures 5.1 and 5.2 the
variation coëfficients are relatively high for the low forecasts. Taking
into account the size of forecasts a measure for the "goodness" of the
forecasts may be found by considering the **weighted** variation coëfficients
as follows:

$$V(t) := \sum_{g,q,a} \frac{\hat{N}_{(g,q,a)}(t)}{\hat{K}(t)} \cdot v_{(g,q,a)}(t) = \frac{\sum_{g,q,a} \sqrt{VAR(N_{g,q,a}(t))}}{\hat{K}(t)},$$

$$W(t) := \sum_{g} \frac{\hat{N}_{g}(t)}{\hat{K}(t)} \cdot v_{g}(t) = \frac{\sum_{g} \sqrt{VAR(N_{g}(t))}}{\hat{K}(t)},$$

where $\hat{K}(t)$ is the forecast for the total number of employees at time t.
In our example we found:

$$V(1) = 0,10 ; \quad V(2) = 0,12 ; \quad V(3) = 0,14$$

$$W(1) = 0,06 ; \quad W(2) = 0,07 ; \quad W(3) = 0,07$$

The variations have been computed under the extra assumption of complete
independent behaviour of all the people involved. Another assumption would
be that percentages of people who get promotion etc. are fixed at the level
corresponding to the transition probabilities. In this case the variances
involved are all equal to zero. The same holds for the variation coefficients.

Reality will be anywhere between these both extremes: complete independence
and fixed promotion rates. It may be conjectured that the variances for both
extremes provide bounds for the variances in the real situation.
As a rule of thumb one might state that a computed variation coëfficient of
0,1 under the independence assumption still gives useful forecasts.

Now some remarks about the influence on the quality of the forecasts caused
by the estimation of the transition probabilities.

Suppose that $\hat{p}(s,s_1)$ is an estimate for $p(s,s_1)$, then the expected number
$n_0(s,s_1)$ of employees going from s to s_1 will be estimated by $n(s)\hat{p}(s,s_1)$,
which gives a quadratic difference with the exact value of

$$n^2(s)[\hat{p}(s,s_1) - p(s,s_1)]^2 .$$

If $\hat{p}(s,s_1)$ is based on historical data in the way as described in section 3, then this quadratic difference has an expectation equal to

$$n^2(s) \; \frac{p(s,s_1)[1 - p(s,s_1)]}{\sum_{t=1}^{k} N_s(-t)} \; .$$

Since the variance of $n_0(s,s_1)$ is equal to $n(s)p(s,s_1)[1 - p(s,s_1)]$, the influence of the estimate may be neglected if $\sum_{t=1}^{k} N_s(-t)$ is substantially larger than $n(s)$.

5. Recruitment planning by linear programming:

In the foregoing sections we were primarily concerned with the forecasting problem in the case of a given recruitment policy. In the sections 5-7 we will investigate the problem of controlling a graded manpower system. In literature many aspects of controlling a graded manpower system have been studied. For an overview see Bartholomew [1, Ch. 4], other references are [3, 4, 5, 6, 10, 11, 12].

In principle a graded manpower system may be controlled by choosing a recruitment policy and a promotion policy. The relative importance of both types of decisional options depends on the situation. In this paper we will consider the promotion policy as fixed and use recruitment as a control variable. However, the described techniques may of course be used to study the consequences of a given (different) promotion policy, viz. by specifying a P-matrix.

So the problem, which is investigated in this section, is the determination of numbers $R_s(t)$ for $s \in S$, $t = 0,1,\ldots,T$, the number of recruited employees in state s at time t. These numbers should satisfy conditions like

$$R_s(t) \geq 0 \quad (s \in S, \; t = 0,\ldots,T) \; ,$$

$$R_s(t) \leq \bar{R}_s(t) \text{ or } \sum_{q,a,\ell} R_{(g,q,a,\ell)}(t) \leq \bar{R}_g(t) \; ,$$

where $\bar{R}_s(t)$ or $\bar{R}_g(t)$ are given upperbounds.
Other conditions might be

$$\hat{N}_s(t) \geq \overline{N}_s(t) \qquad (s \in S, \ t = 0,\ldots,T)$$

$$\hat{N}_s(T) = \overline{N}_s(T) \text{ or } \sum_{q,a,\ell} \hat{N}_{(g,q,a,\ell)}(t) = \overline{N}_g(t) \qquad (s \in S, g \leq G) \ ,$$

where the numbers $\overline{N}_s(t)$ or $\overline{N}_g(t)$ are given.

With these conditions one tries to find a recruitment scheme, which provides a sufficient occupation of the states or grades during the planning period, while it has a desirable state or grade occupation at the end of the period. For the example with the constraints on the states rather than on the grades we get the following linear system in the variables $\hat{N}_s(t)$, $R_s(t)$ ($t = 0,1,\ldots,T$, $s \in S$).

5 ⓐ
$$\begin{cases} 0 \leq R_s(t) \leq \overline{R}_s(t) & t = 0,\ldots,T, \quad s \in S \\ \overline{N}_s(t) \leq \hat{N}_s(t) & t = 0,\ldots,T-1, \quad s \in S \\ \overline{N}_s(T) = \hat{N}_s(T) & s \in S \end{cases}$$

5 ⓑ
$$\begin{cases} \hat{N}_s(0) = n(s) + R_s(0) & s \in S \\ \hat{N}(t+1) = \hat{N}(t)P + R(t+1) & t = 0,\ldots,T-1. \end{cases}$$

Especially the conditions 5 ⓐ may be chosen according to the situation. In practical situations the assumptions on the maximal recruitment will and can in general not be given in terms of restrictions for each state separately but probably in terms of a restriction for a cluster of states. For instance a restriction on the maximal recruitment of employees with a certain qualification or a restriction on the maximal recruitment in a certain grade. Also the restrictions on the minimum occupation at the successive points in time and the requested occupation at time T will in general be given in terms of lowerbound and target respectively for the occupation of a grade.

Using linear programming it would be possible to compute the set of values for $R_s(t)$, $\hat{N}_s(t)$ satisfying 5 ⓐ, 5 ⓑ such that the total expected salary bill over the planning period is minimal. If the salary for an arbitrary employee in state s is C(s) per year the criterion function becomes (see [1]).

5 ⓒ $\sum_s (\sum_{t=0}^{T} \hat{N}_s(t)) C(s)$.

An objection against this problem formulation is that the system 5 ⓐ , 5 ⓑ
does not necessarily have solutions, e.g. if the target load $\overline{N}(T)$ cannot be
reached. This objection can be met by removing the condition

$$\overline{N}_s(T) = \hat{N}_s(T) \text{ from 5 ⓐ}$$

and adding a distance function like

$\sum_s W_s |\hat{N}_s(T) - \overline{N}_s(T)|$ to 5 ⓒ , where W_s are given nonnegative weihts.

With this modifications the problem "minimize 5 ⓒ subject to 5 ⓐ and 5 ⓑ "
may still be solved by the solution of a linear programming problem (with a
larger number of variables as before).
If one uses the similar modification for the conditions $\overline{N}_s(t) \le \hat{N}_s(t)$ one
gets a linear programming problem (with a tremendous number of variables) which
always possesses a feasible solution.
Not only the size of the linear programming problem gives difficulties, it will
also be difficult to weigh out the salary costs and the weighting of target
differences in the criterion function.
In section 7 some more aspects of this approach will be discussed together
with the same aspects of a dynamic planning approach.

6. Dynamic planning of recruitment:

In this section the same recruitment problem will be considered as in section
5. However, a more straightforward approach, in which the dynamic character of
the model is used in a more direct way, will be presented here. So the problem
is in fact again:

> Determine $R_s(t)$ (t = 0,...,T s $\in$ S) subject to conditions like
>
> (5.a) and (5.b)

We assume that bounds have been given for the grade recruitments and occupa-
tions together with a target for the grade occupation at time T, i.e.

$$0 \leq R_g(t) = \sum_{q,a,\ell} R_{(g,q,a,\ell)}(t) \leq \overline{R}_g(t) \quad (g = 1,\ldots,G \; ; \; t = 0,\ldots,T)$$

$$\overline{N}_g(t) \leq \hat{N}_g(t) = \sum_{q,a,\ell} \hat{N}_{(g,q,a,\ell)}(t) \quad (g = 1,\ldots,G \; ; \; t = 0,\ldots,T-1)$$

$$\overline{N}_g(T) = \hat{N}_g(T) = \sum_{q,a,\ell} \hat{N}_{(g,q,a,\ell)}(T) \quad (g = 1,\ldots,G) \; .$$

The first step will be to give forecasts for the situation without recruitment.

STEP 1 : choose $R_s(t) = 0$ ($s \in S$, $t = 0,1,\ldots,T$)

compute $\hat{N}_s(t)$ ($s \in S$, $t = 0,\ldots,T$) .

As a consequence of STEP 1 also $R_g(t) = 0$ for $g = 1,\ldots,G$, $t = 0,\ldots,T$.
The second and third step investigate the deviations of the solution without recruitment from the given bounds and targets.

STEP 2 : compute $\hat{N}_g(t) := \sum_{q,a,\ell} \hat{N}_{(g,q,a,\ell)}(t)$ $g = 1,\ldots,G$, $t = 0,\ldots,T$.

STEP 3 : compute $d_g(t) := \max\{0, \overline{N}_g(t) - \hat{N}_g(t)\}$ $g = 1,\ldots,G$, $t = 0,\ldots,T-1$.

$d_g(T) := \overline{N}_g(T) - \hat{N}_g(t)$ $g = 1,\ldots,G$.

If some of these deviations are negative (only $d_g(T)$, $g = 1,\ldots,G$, can be negative) there is no feasible solution. This is met by altering the conditions.

STEP 4 : If $d_g(T) < 0$, $\overline{N}_g(T) := \hat{N}_g(T)$, $d_g(T) := 0$.

The changes in $\overline{N}_g(T)$ are listed.

If $d_g(t) = 0$ for $g = 1,\ldots,G$, $t = 0,\ldots,T$ the problem is solved

with the recruitment policy $R_g(t)$.

If not all deviations $d_g(t)$ are zero after STEP 4 we should try a more active recruitment policy. If not all $d_g(0)$ are equal to zero the only way to satisfy the requirements is by recruiting $d_g(0)$ employees in grade g at time 0.

STEP 5 : $\tilde{R}_g(0) := \min\{d_g(0), \overline{R}_g(0)\}$ $g = 1,\ldots,G$,

$\tilde{R}_g(t) := 0$ $g = 1,\ldots,G, t = 1$

$\overline{N}_g(0) := \min\{\overline{N}_g(0), \hat{N}_g(0) + \tilde{R}_g(0)\}$.

changes in $\overline{N}_g(0)$ are listed.

We will assume, in this section, that the system satisfies an additional number of conditions. We believe that, in a number of practical situations, these conditions will be fullfilled if the grades are defined in an adequate way. Moreover, most of the conditions may be weakened or altered as will be indicated in the sequal. However, it might be necessary to adapt the solution technique as will be described in the sequal of this section. The additional conditions unables us to use a coarse model for the determination of the recruitment policy.

First we assume that the grade structure is hierarchical.

We further assume that using the knowledge about the states in which people may enter the system it will be possible to derive from the original model the probabilities $q_{gg_1}(t)$ for finding an arbitrary employee t years after his recruitment in grade g, in grade g_1. For instance if each grade contains only one state in which people may be recruited, say state $(g, q_0, a_0, 0)$, $q_{gg_1}(t)$ may be found from:

(6.1) $q_{gg_1}(t) = \sum_{q,a,\ell} p^t((g,q_0,a_0,0),(g_1,q,a,\ell))$

where $p^t((g,q_0,a_0,0),(g_1,q,a,\ell))$ is the corresponding element of the matrix P^t. If there are more states in grade g in which employees can be recruited it should be investigated whether it is possible to define $q_{gg_1}(t)$ as a weighted sum of terms as appear in the right hand side of (6.1).

Finally we assume that the transition probabilities $q_{gg}(t)$ satisfy:

(6.2) $\dfrac{q_{gg}(T-t)}{q_{gg}(T-t-k)} \leq q_{gg}(k)$, $0 \leq t \leq T$, $k \leq T-t$ $g = 1,\ldots,G$.

This final assumption will be discussed later on in this section.

After executing STEP 5 it should be computed how the newly recruited em-
ployees $R_g(0)$ will affect the grade occupation in the rest of the planning
period. This gives new deviations from target and bounds in the following
way:

STEP 6 : $\hat{N}_g(t) := \hat{N}_g(t) + \sum_{k=0}^{t} \sum_{g_1} \tilde{R}_{g_1}(k) \, q_{g_1 g}(k)$ $g_1 = 1,\ldots,g$ $t = 0,\ldots,T$

$R_g(t) := R_g(t) + \tilde{R}_g(t)$ $g = 1,\ldots G$ $t = 0,\ldots,T$

$\tilde{R}_g(t) := 0$ $g = 1,\ldots,G$ $t = 0,\ldots,T$

Execute STEP 3 .

Now STEP 4 has to be executed again in order to adapt unattainable targets.
Next we might choose for $g = 1$ the number of employees to be recruited at
$t = 1$ equal to $d_1(1)$. If $d_1(1) > \overline{R}_1(1)$ one might solve the bottleneck by an
increase of $R_1(0)$, if possible. Otherwise the bound $\overline{N}_1(1)$ is adjusted. If
this procedure leads to a violation of the targets at $t = T$ it is again tried
to solve the bottleneck by an increase of $R_1(0)$. This can be done by executing
STEP 7 and STEP 8 with $t = 1$ and $g = 1$.

STEP 7 : If $d_g(t) \leq \overline{R}_g(t)$, $\tilde{R}_g(t) := d_g(t)$ else

$\tilde{R}_g(t) := \overline{R}_g(t)$

$\tilde{R}_g(t-k) := \max\{0, \min\{\overline{R}_g(t-k) - R_g(t-k), \dfrac{d_g(t) - \sum_{\ell=0}^{k-1} \tilde{R}_g(t-\ell) q_{gg}(\ell)}{q_{gg}(k)}\}$

for $k = 1,\ldots,t$.

Execute STEP 6 .

$\overline{N}_g(t) := \min\{\overline{N}_g(t), N_g(t)\}$, changes in $\overline{N}_g(t)$ are listed.

Now it might occur that the target $\overline{N}_1(t)$ is violated i.e. $d_1(T) < 0$. This
means that there is a discrepancy between the lowerbound $\overline{N}_1(1)$ and the target
$\overline{N}_1(T)$. We try to solve this bottleneck by recruiting at an earlier time in
grade 1.

STEP 8 :   If $d_g(T) < 0$ define $k := \max\{\ell < t \mid R_g(\ell) \neq \bar{R}_g(\ell)\}$,

choose $\tilde{R}_g(k) \geq 0$ such that:

(1) $\tilde{R}_g(k) \leq \bar{R}_g(k) - R_g(k)$

(2) $\tilde{R}_g(k) \leq \dfrac{R_g(k+1)}{q_{gg}(1)}$, $\tilde{R}_g(k+1) := -\tilde{R}_g(k) \cdot q_{gg}(1)$

(3) $\tilde{R}_g(k) \cdot q_{gg}(T-k) - \tilde{R}_g(k+1) q_{gg}(T-k-1)$ minimal but $\geq d_g(T)$.
   Execute STEP 6, if $R_g(k+1) = 0$ execute STEP 8 with $t = t-1$, else
   execute STEP 8, Execute STEP 4 .

We may proceed in this way by executing STEP 7 and STEP 8 for $g = 1$ and
$t = 2,3,\ldots,T$ respectivily. Then we go to $g = 2$ consecutive computation for
$t = 1,2,\ldots,T$ gives the results for grade 2. Now it might be that $d_2(T) > 0$
and $R_2(t) = \bar{R}_2(t)$ for all $t = 0,\ldots,T$. This problem is tried to be met in
a similar way as it was done for $\bar{N}_1(T)$ (STEP 8) by recruiting in grade 1 at
earlier times. However, the target $\bar{N}_1(T)$ is keep fixed. Then grade 3 may be
treated in a similar way, etc.

Finally the methode yields the requested recruitments and forecasted grade
occupations, together with a list of corrections of the bounds and the targets
in the original problem. This solution may be used for a discussion on the
recruitmnet policy and the restraints and targets. The fact that this method
does not try at any price to give a solution for the original problem is very
essential and one of the main differences with the linear programming approach.
This will be considered in section 7.

If we try to use dynamic programming in its standard (backward) form as solution
technique for solving the recruitment planning problem we would have been
confronted with a tremendous number of "states".
In order to avoid this difficulty we made a more explicit use of the structure
of the problem, viz. the grade structure. Using this structure it was possible
to construct a procedure which applies forward dynamic programming as much as
possible and only applies backward dynamic programming to a limited depth
(provided by the side conditions) and as seldom as possible.

It should be emphasized again, that in fact we treat only an example. This holds for the problem and the solution technique as well, viz. the depth of backtracking.

For example, which $\overline{N}_g(t)$ are lowerbounds and which are targets may depend on the situation. The condition as given in (6.2) is imposed to ensure that by executing STEP 8 already satisfied lowerbounds remain satisfied. Since in practical problems STEP 8 will be used in general only a very few times (6.2) may in fact be weakened. (6.3) was satisfied in the example which we considered.

As far as the solution technique concerns it depends on the practical requirements whether it might be simplified or not. For instance in a <u>very</u> simplified form STEP 7 and 8 might be omitted.

On the other hand more sophisticated versions of the technique of this section may be constructed. For instance in our approach a **violation of the targets** $\overline{N}_{g_0}(T)$ that could arise by satisfying the bounds $\overline{N}_g(t)$ for $g < g_0$ is simply met by correcting the target $\overline{N}_{g_0}(T)$. But the solution technique might be constructed such that the bottlenecks are tried to be met by recruiting at earlier times in preceding grades.

In the dynamic planning approach the size of the sophistication can in fact be chosen depending on the problem under study.

7. <u>Dynamic planning versus linear programming</u>:

The first reason for the investigation of alternative recruitment planning procedures was the size of the linear programming problem in section 6. Although it would have been possible to replace the linear programming formulation by a smaller one, we preferred a more direct approach for the following reasons.

a. it is very difficult to propose a satisfactory criterion, especially one which weighs the distance form the targets and the costs. By our dynamic approach the total salary bill is guarded while the target is approximated. This gives the possibility to decide, while knowing what the consequences are for both salary bill and targets.

b. as pointed out in section 5, the first linear programming problem may not be feasible, whereas both linear programming problems may be such that

very expensive solutions are proposed in order to reach the targets as
good as possible. For instance, the linear programming solution may
propose to recruit many employees in grade g at an early time in order
to satisfy a deficit of one person in the target $\overline{N}_{g+1}(T)$. In the dynamic
approach such effects can be avoided.

c. as mentioned in section 6, the list of corrections is very useful as a
basis for evaluation. Post optimal analysis in linear programming gives
less information if some of the restraints are interfering.

d. the dynamic programming approach offers as it were, in a natural way, an
invitation for conversational use by the concerned departments. This might
result in a better rejection of the model and the solution technique as
well on the problem under study.

8. Conclusions:

In this section it will be described how the tools developed so far may be
used, and we will illustrate this with some figures.

a. data base: a data base is necessary containing the data of the employees
and of the former employees about their experiences in the manpower system
over a certain number of years. This data base should be arranged in such
a way that one can obtain statistical information of the following types
by standard procedures.

a.1. statistical information on the actual situation in the manpower system,
like

$n(g,q,a,\ell) :=$ the number of employees with grade g qualification index q,
age index a and grade age ℓ,

$n(g,q,a,\ell)$ plotted as a function of ℓ for fixed g,q,a.

$n(g,\ell) := \sum_{q,a} n(g,q,a,\ell)$ plotted as a function of ℓ for fixed g. (see figure 8.1).

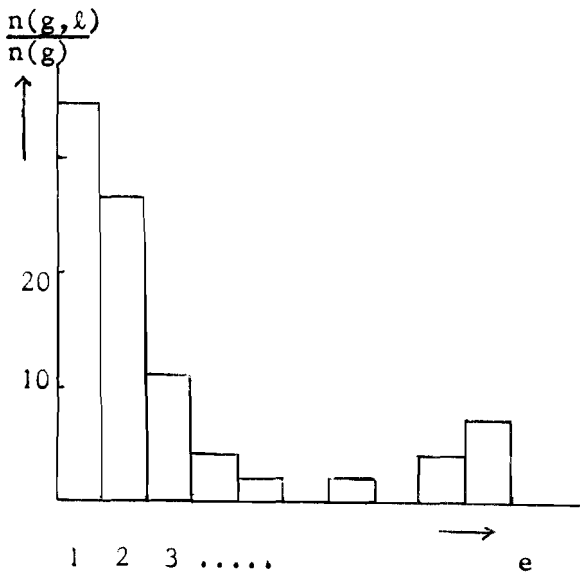$n(g,a) := \sum_{q,\ell} n(g,q,a,\ell)$ plotted as a function of a for fixed g.

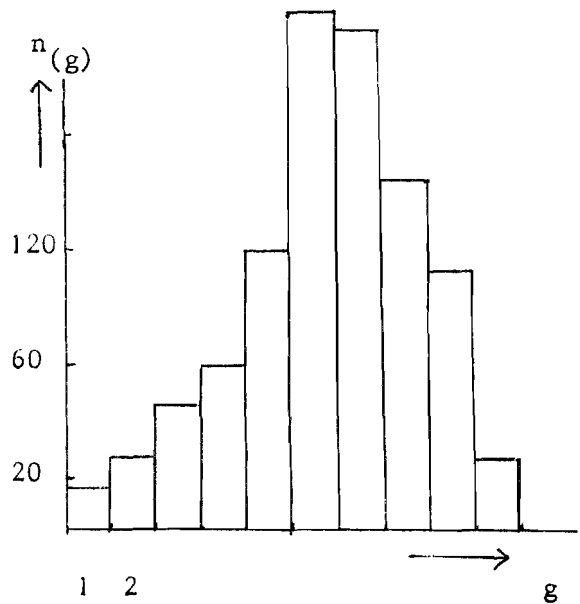$n(q,a) := \sum_{g,\ell} n(g,q,a,\ell)$ plotted as a function of a for fixed q.

$n(g) := \sum_{q,a,\ell} n(g,q,a,\ell)$ plotted as a function of g. (see figure 8.2)

$n(a) := \sum_{g,q,\ell} n(g,q,a,\ell)$ plotted as a function of a.

etc.

figuur 8.1

figuur 8.2

a.2. statistical information on transitions in the recent past, like

$n_{-t}(s,s_1)$:= the number of employees who were at time $-t$ in state s
and jumped to state s_1 in the next year.

$N_s(-t)$:= $\sum_{s_1} n_{-t}(s,s_1)$ = the number of employees who were at time $-t$
in state s.

$\sum_{q,a,\ell} N_{(g,q,a,\ell)}(-t)$ = the number of employees with grade g at time $-t$.
This may be plotted for fixed t as a function of g or for
fixed g as a function of t.

The same might be asked for

$100 \cdot \sum_{q,a,\ell} N_{(g,q,a,\ell)}(-t)(\sum_{s \neq 0} N_s -t)^{-1}$, where s = 0 is the state where
former employees dwell.

$$\frac{\sum_{t,q,a,q_1,a_1} n_{-t}(g,q,a,\ell_0;g_1,q_1,a_1,0)}{\sum_{\ell,t,q,a,q_1,a_1} n_{-t}(g,q,a,\ell;g_1,q_1,a_1,0)}$$

plotted as a function of ℓ_0 for fixed g and g_1, being the grade age
distribution of the people who are promoted from grade g to g_1.

The number of employees that left the system at time -t.

The age distribution over the past years (see figure 8.3).

etc.
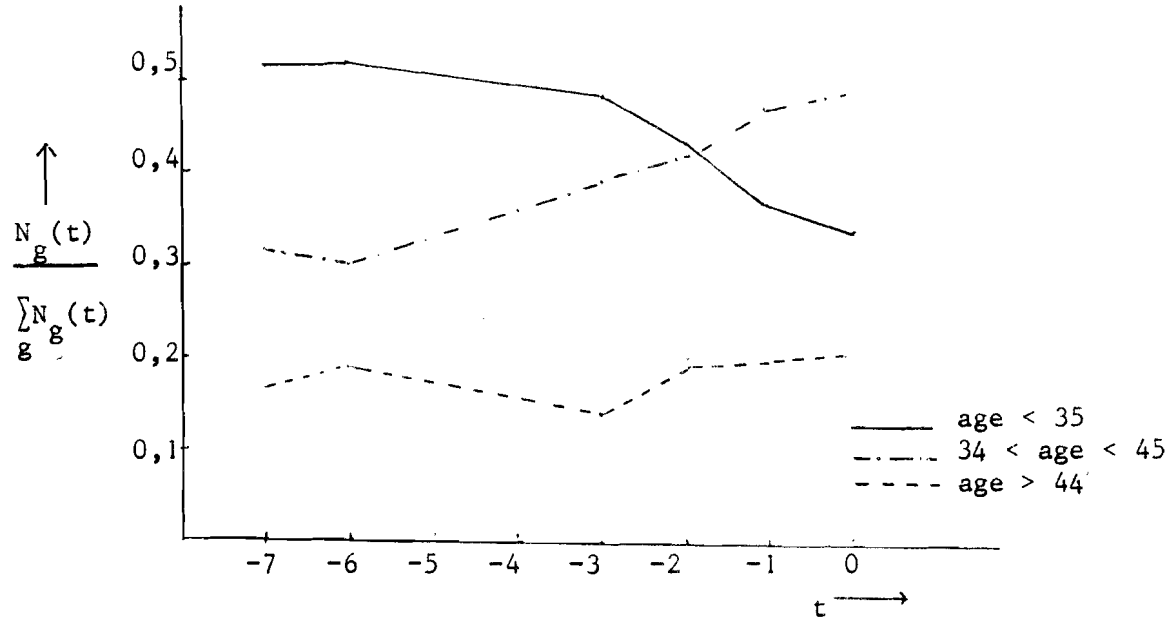
figuur 8.3

In this way a data base and some standard statistical procedures based on the model approach already give much information about the current situation, the behaviour of the system in the past, and trends in this behaviour. Furthermore such a data base and a couple of statistical procedures are necessary for the forecasting and recruitment planning procedures.

b. forecasting: the forecasting procedure consists essentially of four parts, viz. the estimation of transition probabilities from historical data, correction of the estimates after evaluation, the proper forecasting procedure, and the forecasting information part.

b.1. estimation from historical data may use some statistical procedures for the data base in the following way
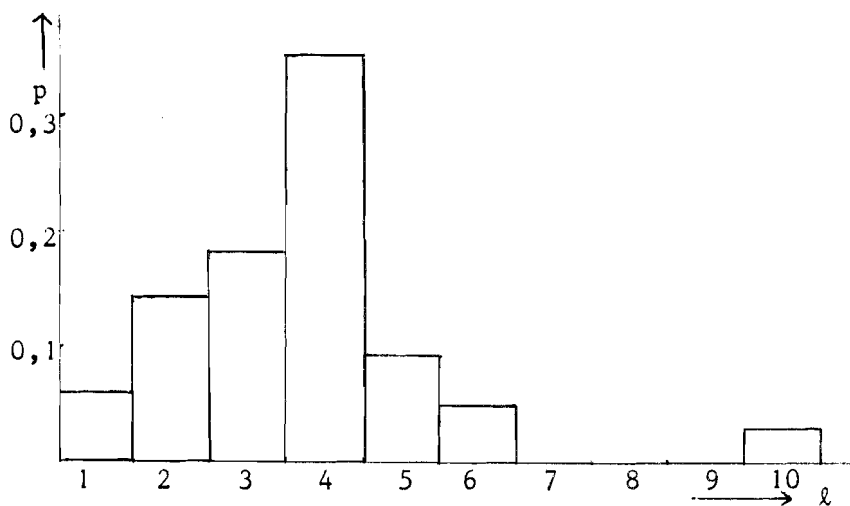
PROCEDURE (ESTIMATION):

compute $n_{-t}(s,s_1)$ for all s,s_1 and $t = 1,\ldots,k$.

compute $N_s(-t)$ for all s and $t = 1,\ldots,k$.

$$p(s,s_1) := \frac{\sum_{t=1}^{k} n_{-t}(s,s_1)}{\sum_{t=1}^{k} N_s(-t)} \quad \text{for all } s,s_1.$$

Figure 8.4 shows the probability (p) for an arbitrary employee with qualification index q and age index a to leave (g,q,a) ℓ years after arriving in (g,q,a).

figuur 8.4

b.2. correction of the estimates after evaluation may proceed in the following way. The estimates $p(s,s_1)$ and a number of relevant plots as may be computed using a.2 are evaluated with responsible personnel managers with respect to the question whether these probabilities reflect the expected behaviour for the future or not. These discussions should lead to corrected values for $p(s,s_1)$, which will be used in the proper forecasting procedure.

b.3. for the proper forecasting procedure one needs the matrix P consisting of components $p(s,s_1)$ resulting from b.1 and b.2; furthermore one needs the current situation reflected by n(s) (see a.1) and estimates of recruitments $R_s(t)$ in state s at time t.

PROCEDURE (FORECASTING):

compute $N_s(0) = n(s)$ for all s

$\hat{N}(0) := N(0)$

compute successively $\hat{N}(t) := \hat{N}(t-1)P + R(t)$ for t = 1,...,T.

b.4. the forecasting information part gives the consequences of the
forecasts for the future behaviour of the system. This can be achieved
by the use of the statistical procedures for the data base. They may
be used to give plots and tables for the expected situation in the
coming years (age distribution for certain grades etc.) and they may
be used to provide statistical information about the expected transitions
and the expected trends.

In this way the forecasting procedure and the data base with statistical
procedures give much information about present past and future. The fore-
casting procedure can be used with $R_s(t) = 0$ i.e. no recruitment. Or it
may be used with an extrapolation of the past recruitment policy.. Or it
may be used with planned recruitment policy.
Furthermore these procedures may be used for a recruitment planning procedure.

c. recruitment planning: a recruitment planning procedure based on the ideas
   developed in section 6 consists of the following three parts.
   c.1. forecasting with $R_s(t) = 0$ using the forecasting procedure in b.3
   and everything needed for that.
   c.2. a recruitment scheduling procedure as described in section 6, which
   finds recruitment numbers for a set of optimal bounds and targets like
   the following:

   $\equiv$ lower bounds for $\hat{N}_g(t)$ ($t = 0,...,T-1$), targets for $\hat{N}_g(T)$,
      upper bounds for $R_g(t)$ ($t = 0,...,T$).

   $\equiv$ growing of all or some lowerbounds for $\hat{N}_g(t)$ ($t = 0,...,T$)
      with a fixed percentage.

The procedure fills deficits for $g = 1$ at the times $t = 0,...,T$ success-
ively, then the same is done for $g = 2$ etc. The way in which deficits
are filled is open for several options, the same holds for the priority
of the bounds and targets. Figure 8.6 gives the expected grade occupation
after recruitment at $t = 0$ and $t = 4$, while figure 8.7 gives the number
of employees to be recruited in each grade at time $t = 0$. The dotted
lines in figure 8.6 indicate the targets. In our example the planning
horizon was $T := 4$.
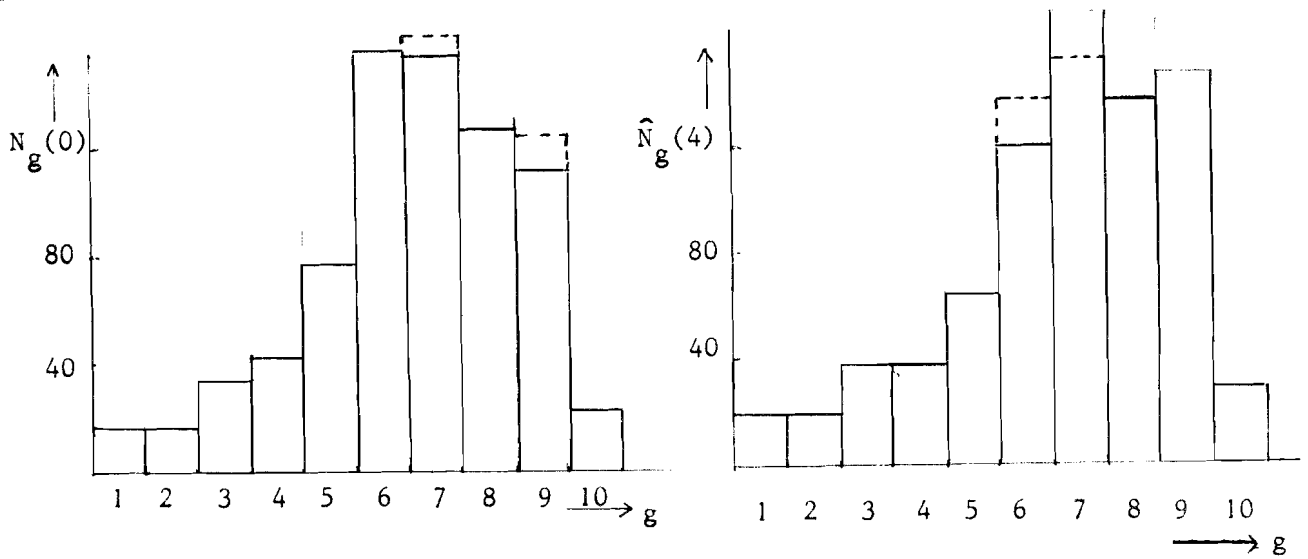
figuur 8.6
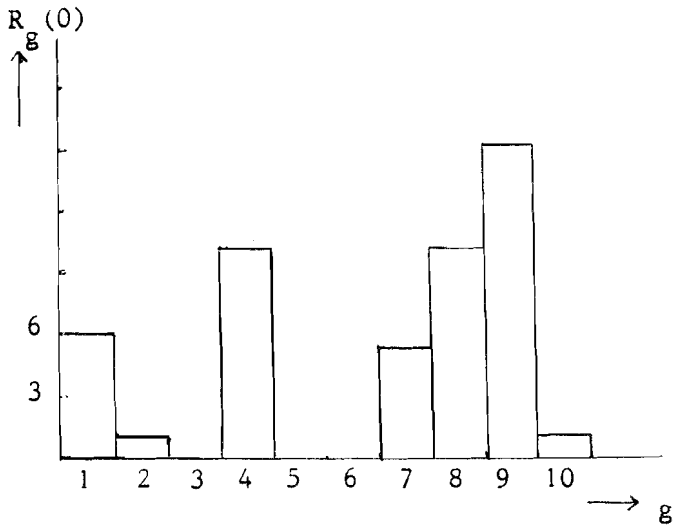
figuur 8.7

c.3. recruitment planning evaluation. After the recruitment planning
procedure an evaluation is necessary, since it is very well possible
that bounds and targets appear to be less worthwhile than thought
before. An evaluation may very well lead to a new run of the recruit-
ment planning procedure with new bounds and targets.

9. Remarks and extensions:

In this paper we did not strive after a complete treatment. Especially the
recruitment planning procedure is open for many extensions. Beside the fact
that several conditions may be weakened also the procedure itself may be
adapted. Furthermore it is not essential to apply the recruitment planning.
for grades only.. Any structure in the state set is hierarchical is allowed.

Points in such a structure might consist for instance of a composition of
grades or alternatively of grade and qualification index.
In order to make use of the system it is less important to have a very general
set of options, than it is to have an easy access to options. Ideally one
may "play" with different options in a conversational way.

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