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E.M.M. Winands[1,2]

[1]Department of Mathematics and Computer Science
[2]Department of Technology Management
Technische Universiteit Eindhoven
P.O. Box 513, 5600 MB Eindhoven, The Netherlands
{e.m.m.winands}@tue.nl

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Abstract

Polling systems with large deterministic setup times find many applications in production environments. The present note studies the delay distribution in exhaustive polling systems when the setup times tend to infinity. Based on the recently proposed mean value analysis (MVA) for polling systems, a novel simple approach is developed to show that the scaled delay distribution converges to a uniform distribution. Since MVA is not limited to exhaustive polling systems, the analysis of the present note can be readily extended to a wide range of polling systems.

Keywords: polling system, setup times, delay distribution, asymptotics.

1 Introduction

A typical polling system consists of a number of queues, attended by a single server in a fixed order. There is a huge body of literature on polling systems that has continued to grow since the late 1950s, when the papers of [4, 5] concerning a patrolling repairman model for the British cotton industry were published. Polling systems have a wide range of applications in communication, production, transportation and maintenance systems. Excellent surveys on polling systems and their applications may be found in [11, 12, 13] and in [3].

The present note is concerned with the service discipline most commonly used in polling systems, the so-called *exhaustive* policy, which means that a queue must be empty before the server moves on. Unfortunately, the interdependence of the queueing processes prohibits an exact explicit analysis, leading to the need of using numerical techniques to determine performance measures of interest. However, such numerical techniques provide only limited insight into the behavior of the system with respect to its input parameters. In these circumstances, one naturally resorts to asymptotic estimates. In particular, the present note presents an exact asymptotic analysis of the delay distribution in exhaustive-type polling systems when the deterministic setup times tend to infinity. Since the delay obviously grows without bound in such a case, we focus on the scaled delay, i.e., the delay divided by the total setup time per cycle.

From a theoretical point of view, such an analysis is evidently interesting, since it deepens the understanding of the behavior of systems with large setups. From a practical point of view, polling systems with large setup times find a wide variety of applications in production environments. For example, in the *stochastic economic lot scheduling problem* (SELSP), where multiple standardized products have to produced on a single machine with significant setup times, polling system are frequently encountered as modeling tool for (widely-used) fixed-sequence base-stock policies. We refer to [14] for a survey on the SELSP and for a large number of cases of production environments with large setup times. With respect to the distribution of the setup times, it is important to remark that in production environments setup times are typically deterministic due to the nowadays efficient control of production processes.

To the best of our knowledge there exist only two papers in the vast polling literature addressing the problem of large setup times. [7] explores the *descendant set approach* (DSA) in combination with the *strong law of large numbers* (SLLN) for *renewal reward processes* (RRP) to analyze polling systems with deterministic setups and mixtures of exhaustive and gated service. [9] presents a somewhat simpler analysis in the case of an exhaustive system with deterministic setups, where the order of service is determined by a polling table. The main result in both papers is the fact that the scaled delay distribution converges in distribution to a uniform distribution.

The objective of the present note is the development of a new approach to derive the scaled delay distribution for polling systems with increasing deterministic setups. The main building block of our analysis is the recently proposed *mean value analysis* (MVA) of [15], which shows that the scaled intervisit times converge in probability to a constant as the setup times increase to infinity. This result immediately leads to the known asymptotic expression for the scaled delay distribution.

The main contribution of the present note is two-fold. Firstly, we present a novel simple approach to analyze the asymptotic scaled delay distribution. An approach originally developed for the computation of *mean* delays (MVA) is straightforwardly applied to derive the asymptotic result for the complete delay *distribution*. Secondly, although the present note provides in the interest of space only the detailed analysis in case of pure exhaustive service, MVA can be applied directly to systems with pure gated service, mixed exhaustive-gated service, batch arrivals, periodic server routing and discrete time settings (see [15]). This, in turn, implies that for these systems similar limit theorems can be derived straightforwardly when the setups tend to infinity, which extends the class of systems analyzed so far. Furthermore, as MVA opens new ways for the evaluation of an even larger variety of models, it is not inconceivable that the present note is a starting point of the introduction of a general class of systems for which similar limit theorems hold.

The remainder of the present note is organized as follows. Section 2 describes the model in detail, while Section 3 presents the main theorem. Subsequently, Section 4 is mainly devoted to the proof of this theorem. Finally, Section 5 concludes the note.

2 Model

We consider a system with a single server for $N \geq 1$ queues, in which there is infinite buffer capacity for each queue. The server visits and serves the queues in a fixed cyclic order. We index the queues by i, $i = 1, 2, \ldots, N$, in the order of the server movement. All references to queue indices greater than N or less than 1 are implicitly assumed to be modulo N, e.g., queue $N + 1$ actually refers to queue 1. Service at each queue is according to the *exhaustive* policy, which means that a queue must be empty before the server moves on. Customers arrive at all queues according to independent Poisson processes with rates λ_i, $i = 1, 2, \ldots, N$. The service times at queue i are independent, identically distributed random variables with mean $\mathbb{E}[B_i]$ and finite second moment $\mathbb{E}[B_i^2]$, $i = 1, 2, \ldots, N$.

When the server starts service at queue i, a setup time is incurred of which the first and second moment are denoted by $\mathbb{E}[S_i]$ and $\mathbb{E}[S_i^2]$, $i = 1, 2, \ldots, N$, respectively. The mean of the total setup time in a cycle is given by $\mathbb{E}[S] = \sum_{i=1}^{N} \mathbb{E}[S_i]$. For further reference, we introduce the mean residual service time and the mean residual setup time for queue i, $i = 1, 2, \ldots, N$, which can be expressed as $\mathbb{E}[R_{B_i}] = \frac{\mathbb{E}[B_i^2]}{2\mathbb{E}[B_i]}$ and $\mathbb{E}[R_{S_i}] = \frac{\mathbb{E}[S_i^2]}{2\mathbb{E}[S_i]}$, respectively. The occupation rate ρ_i (excluding setups) at queue i is defined by $\rho_i = \lambda_i \mathbb{E}[B_i]$ and the total occupation rate ρ is given by $\rho = \sum_{i=1}^{N} \rho_i$. A necessary and sufficient condition for the stability of this polling system is $\rho < 1$ (see, e.g., [11]). In the remainder of the present note, this stability condition is assumed to hold as we restrict ourselves to steady-state behavior.

The cycle length of queue i, $i = 1, 2, \ldots, N$, is defined as the time between two successive departures of the server at this queue. It is well-known that the mean cycle length is independent of the queue involved and is given by (see, e.g., [11])

$$\mathbb{E}[C] = \frac{\mathbb{E}[S]}{1 - \rho}. \tag{1}$$

Furthermore, the intervisit time I_i for queue i, $i = 1, 2, \ldots, N$, is defined as the time that elapses from the server leaving queue i to finishing the setup again at this queue. Since the server is working a fraction ρ_i of the time on queue i, the mean $\mathbb{E}[I_i]$ of an intervisit period can be expressed as

$$\mathbb{E}[I_i] = (1 - \rho_i)\mathbb{E}[C], \qquad i = 1, 2, \ldots, N.$$

The visit time θ_i of queue i, $i = 1, 2, \ldots, N$, is composed of the service period of queue i, the time the server spends servicing customers at queue i, plus the *preceding* setup time. The mean of a visit period of queue i reads

$$\mathbb{E}[\theta_i] = \rho_i \mathbb{E}[C] + \mathbb{E}[S_i], \qquad i = 1, 2, \ldots, N.$$

We define an (i, j)-period $\theta_{i,j}$ as the sum of j consecutive visit times starting in queue i, $j = 1, 2, \ldots, N$. The corresponding mean is given by

$$\mathbb{E}[\theta_{i,j}] = \sum_{n=i}^{i+j-1} \mathbb{E}[\theta_n], \qquad i = 1, 2, \ldots, N, \qquad j = 1, 2, \ldots, N.$$

Notice that in case $j = 1$ and $j = N$, $\mathbb{E}[\theta_{i,j}]$ is equal to the mean visit period $\mathbb{E}[\theta_i]$ of queue i and the mean cycle length $\mathbb{E}[C]$, respectively. The fraction of the time $q_{i,j}$ the system is in an (i, j)-period equals $q_{i,j} = \frac{\mathbb{E}[\theta_{i,j}]}{\mathbb{E}[C]}$. Moreover, the mean of a residual (i, j)-period is given by

$$\mathbb{E}[R_{\theta_{i,j}}] = \frac{\mathbb{E}[\theta_{i,j}^2]}{2\mathbb{E}[\theta_{i,j}]}, \qquad i = 1, 2, \ldots, N, \qquad j = 1, 2, \ldots, N,$$

with the remark that the second moments $\mathbb{E}[\theta_{i,j}^2]$ are still unknown at this stage.

The performance measure of interest is the delay W_i of a type-i customer, $i = 1, 2, \ldots, N$, in case the setup times tend to infinity. Since the delay grows to infinity in the limiting case, we focus on the asymptotic scaled delay $\frac{W_i}{\mathbb{E}[S]}$ as $\mathbb{E}[S] \to \infty$, where the ratios of the setup times $a_i = \frac{\mathbb{E}[S_i]}{\mathbb{E}[S]}$ remain constant, $i = 1, 2, \ldots, N$. Of course, our results for the delay distribution can be readily translated into results for the queue length distribution via the distributional form of Little's law [2].

3 Main result

The key result of the present note is the following theorem, which is proved in the next section. Throughout the present note, we take $\xrightarrow{p}$ to represent convergence in probability and $\xrightarrow{d}$ to represent convergence in distribution.

Theorem 3.1. *In case of deterministic setup times, we have for* $i = 1, 2, \ldots, N$*,*

$$\frac{W_i}{\mathbb{E}[S]} \xrightarrow{d} W_i^*, \qquad (\mathbb{E}[S] \to \infty),$$

where W_i^* *is uniformly distributed on* $[0, \frac{1-\rho_i}{1-\rho}]$. □

An intuitive explanation of the uniform distribution emerging in the above theorem is that it represents the position of the server in the cycle, of which the length converges to a constant, on arrival of a tagged customer. For numerical evaluations validating the above limit theorem, we refer to [7] and [9]. Among others, they show via some cases how "fast" the limiting distribution is approached. Furthermore, [9] shows via numerical testing that similar limit theorems carry over to more general systems with, e.g., dynamic visit orders.

From Theorem 3.1 we can perceive the following properties how the asymptotic scaled delay distribution depends on the system parameters.

Property 3.2. *For $i = 1, 2, \ldots, N$,*

1. *W_i^* is independent of the visit order;*

2. *W_i^* depends on the arrival rate and service time distribution of queue i only through the occupation rate ρ_i;*

3. *W_i^* depends on the other queues only through the total occupation rate ρ.*

□

Finally, throughout the present note, for each variable x its scaled counterpart $x/\mathbb{E}[S]$ as $\mathbb{E}[S] \to \infty$ is denoted by x^*.

Remark 3.3. Formula (1) clearly shows that the cycle lengths, and thus the delays, grow without bound both as $\mathbb{E}[S] \to \infty$ and as $\rho \uparrow 1$. The application of MVA in the latter heavy traffic case is the topic of an accompanying paper [8]. However, the limiting behavior in the present note is fundamentally different from that in the heavy-traffic scenario, where the gamma distribution shows up (see [6]). □

4 Proof of main result

In order to prove Theorem 3.1, we use the following well-known relation between the delay W_i and the residual intervisit time R_{I_i} at queue i (see, e.g., [11]),

$$W_i = R_{I_i} + X_i, \qquad i = 1, 2, \ldots, N, \tag{2}$$

where X_i is the delay in an $M/G/1$ queue with arrival rate λ_i and service time with first two moments $\mathbb{E}[B_i]$ and $\mathbb{E}[B_i^2]$. Note that R_{I_i} and X_i are mutually independent.

Since X_i is *independent* of S, we can restrict ourselves to the intervisit times. To study these unknowns, we shift attention to the MVA equations derived in [15] for generally distributed setup times. First of all, [15] derives by a standard application of MVA, i.e., combining the arrival relation with Little's Law, the following equation for $i = 1, 2, \ldots, N$,

$$\mathbb{E}[L_i] = \sum_{n=1}^{N} q_{n,1}\mathbb{E}[L_{i,n}] = \frac{\lambda_i}{1-\rho_i}\Big(\rho_i\mathbb{E}[R_{B_i}] + \frac{\mathbb{E}[S_i]}{\mathbb{E}[C]}\mathbb{E}[R_{S_i}] + (1-q_{i,1})(\mathbb{E}[R_{\theta_{i+1,N-1}}] + \mathbb{E}[S_i])\Big), \tag{3}$$

with $\mathbb{E}[L_{i,n}]$ the mean queue length at queue i at an arbitrary epoch within a visit time of queue n, $i, n = 1, 2 \ldots, N$. However, the unknown $\mathbb{E}[R_{\theta_{i+1,N-1}}]$ form the stumbling block to the straightforward computation of the mean queue lengths via this equation. To obtain the unknowns $\mathbb{E}[R_{\theta_{i+1,N-1}}]$, [15] relates them to $\mathbb{E}[L_{i,n}]$ and derives the following set of equations for these quantities for $i = 1, 2, \ldots, N$, and $j = 1, 2, \ldots, N-1$,

$$\lambda_i\mathbb{E}[R_{\theta_{i+1,j}}] = \sum_{n=i+1}^{i+j} \frac{q_{n,1}}{q_{i+1,j}}\mathbb{E}[L_{i,n}], \tag{4}$$

$$\mathbb{E}[R_{\theta_{i,1}}] = \frac{1}{1-\rho_i}\left(\mathbb{E}[L_{i,i}]\mathbb{E}[B_i] + \frac{\rho_i\mathbb{E}[C]}{\mathbb{E}[\theta_{i,1}]}\mathbb{E}[R_{B_i}] + \frac{\mathbb{E}[S_i]}{\mathbb{E}[\theta_{i,1}]}\mathbb{E}[R_{S_i}]\right), \tag{5}$$

and for $j = 2, 3, \ldots, N-1$,

$$\mathbb{E}[R_{\theta_{i,j}}] = \frac{q_{i,1}}{q_{i,j}} \left(\frac{\mathbb{E}[R_{\theta_{i,1}}]}{\prod_{n=1}^{j-1}(1-\rho_{i+n})} + \sum_{n=1}^{j-1} \frac{\mathbb{E}[S_{i+n}] + \mathbb{E}[L_{i+n,i}]\mathbb{E}[B_{i+n}]}{\prod_{m=n}^{j-1}(1-\rho_{i+m})} \right) + (1 - \frac{q_{i,1}}{q_{i,j}})\mathbb{E}[R_{\theta_{i+1,j-1}}], \tag{6}$$

with as unknowns the mean residual (i,j)-periods $\mathbb{E}[R_{\theta_{i,j}}]$ and the mean conditional queue lengths $\mathbb{E}[L_{i,n}]$. The set (3)-(6) can be solved numerically, and, subsequently, the unconditional mean queue lengths $\mathbb{E}[L_i]$ and the mean delays $\mathbb{E}[W_i]$ can be computed. We refer to [15], for more details.

Unfortunately, the set (3)-(6) has no closed-form solution for general parameter settings. However, if the setup times are deterministic, we have $\mathbb{E}[S_i^2] = \mathbb{E}[S_i]^2$ and, thus, $\mathbb{E}[R_{S_i}] = \frac{\mathbb{E}[S_i]}{2}$, $i = 1, 2, \ldots, N$. If we now divide both sides of this set by $\mathbb{E}[S]$ and let $\mathbb{E}[S] \to \infty$, we obtain for $i = 1, 2, \ldots, N$, and $j = 1, 2, \ldots, N-1$,

$$\sum_{n=1}^{N} q_{n,1}\mathbb{E}[L^*_{i,n}] = \frac{\lambda_i}{1-\rho_i}\Big(\frac{(1-\rho)a_i^2}{2} + (1-q_{i,1})(\mathbb{E}[R_{\theta^*_{i+1,N-1}}] + a_i)\Big), \tag{7}$$

$$\sum_{n=i+1}^{i+j} \frac{q_{n,1}}{q_{i+1,j}}\mathbb{E}[L^*_{i,n}] = \lambda_i \mathbb{E}[R_{\theta^*_{i+1,j}}], \tag{8}$$

$$\mathbb{E}[R_{\theta^*_{i,1}}] = \frac{1}{1-\rho_i}\left(\mathbb{E}[L^*_{i,i}]\mathbb{E}[B_i] + \frac{\mathbb{E}[S_i]}{\mathbb{E}[\theta_{i,1}]}\frac{a_i}{2}\right), \tag{9}$$

and for $j = 2, 3, \ldots, N-1$,

$$\mathbb{E}[R_{\theta^*_{i,j}}] = \frac{q_{i,1}}{q_{i,j}} \left(\frac{\mathbb{E}[R_{\theta^*_{i,1}}]}{\prod_{n=1}^{j-1}(1-\rho_{i+n})} + \sum_{n=1}^{j-1} \frac{a_{i+n} + \mathbb{E}[L^*_{i+n,i}]\mathbb{E}[B_{i+n}]}{\prod_{m=n}^{j-1}(1-\rho_{i+m})} \right) + (1 - \frac{q_{i,1}}{q_{i,j}})\mathbb{E}[R_{\theta^*_{i+1,j-1}}]. \tag{10}$$

Remark that terms like $q_{i,j}$ and $\frac{\mathbb{E}[S_i]}{\mathbb{E}[\theta_{i,1}]}$ represent fractions of time, obviously *independent* of (the limit of) S.

The scaled set (7)-(10) does have a closed-form solution given by (which can easily verified by substitution),

$$\mathbb{E}[L^*_{i,i}] = \frac{\lambda_i}{2}(\mathbb{E}[C^*] - \mathbb{E}[\theta^*_{i,1}] + \frac{\mathbb{E}[S_i]}{\mathbb{E}[\theta_{i,1}]}\mathbb{E}[\theta^*_{i+1,N-1}]), \qquad i = 1, 2, \ldots, N, \tag{11}$$

$$\mathbb{E}[L^*_{i,i+n}] = \lambda_i(\mathbb{E}[\theta^*_{i+1,n-1}] + \frac{1}{2}\mathbb{E}[\theta^*_{i+n,1}]), \qquad i = 1, 2, \ldots, N, \quad n = 1, 2, \ldots, N-1, \tag{12}$$

$$\mathbb{E}[R_{\theta^*_{i,j}}] = \frac{1}{2}\mathbb{E}[\theta^*_{i,j}], \qquad i = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, N-1. \tag{13}$$

where, by definition, $\mathbb{E}[C^*] = \frac{1}{1-\rho}$ and $\mathbb{E}[\theta^*_{i,j}] = \sum_{n=i}^{i+j-1}(\frac{\rho_n}{1-\rho} + a_n)$. We refer to Remark 4.2 for an intuitive explanation of the solution (11)-(13).

It is easily verified from (13), that

$$\mathbb{V}\text{ar}[\theta^*_{i,j}] = 0, \qquad i = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, N-1.$$

Since an immediate consequence of Chebyshev's inequality (see, e.g., [10]) is that a random variable with zero variance follows a deterministic distribution, we have for $i = 1, 2, \ldots, N$, and $j = 1, 2, \ldots, N-1$,

$$\frac{\theta_{i,j}}{\mathbb{E}[S]} \xrightarrow{p} \mathbb{E}[\theta^*_{i,j}] = \sum_{n=i}^{i+j-1} (\frac{\rho_n}{1-\rho} + a_n), \qquad (\mathbb{E}[S] \to \infty).$$

By definition, this leads for $i = 1, 2, \ldots, N$ to

$$\frac{I_i}{\mathbb{E}[S]} \xrightarrow{p} \sum_{n=i+1}^{i+N-1} (\frac{\rho_n}{1-\rho} + a_n) + a_i = \frac{1-\rho_i}{1-\rho}, \qquad (\mathbb{E}[S] \to \infty).$$

Subsequently, we recall the following well-known relation between the *cumulative distribution function* (CDF) $F_{I_i}(\cdot)$ of I_i and the *probability distribution function* (PDF) $f_{R_{I_i}}(\cdot)$ of R_{I_i} for $i = 1, 2, \ldots, N$, (see, e.g., [10])

$$f_{R_{I_i}}(\cdot) = \frac{1 - F_{I_i}(\cdot)}{\mathbb{E}[I_i]},$$

which, in combination with the fact that convergence in probability implies convergence in distribution, leads to for $i = 1, 2, \ldots, N$,

$$\frac{R_{I_i}}{\mathbb{E}[S]} \xrightarrow{d} \frac{1-\rho_i}{1-\rho} U, \qquad (\mathbb{E}[S] \to \infty),$$

where U is a uniform random variable on $[0, 1]$. Using (2) and recalling that $X^*_i \xrightarrow{p} 0$ as $\mathbb{E}[S] \to \infty$ completes the proof. □

Remark 4.1. The astute reader has already noticed that we use the assumption of deterministic setup times only in the derivation of the closed-form solution (11)-(13) of the scaled MVA equations (7)-(10). In fact, we know that the MVA equations form a set of linear equations, where the $\mathbb{E}[R_{S_i}]$ only show up in the right-hand sides. Due to continuity of linear transformations, we can extend the result of Theorem 3.1 - and, thus, the results of [7] and [9] - by replacing the assumption of deterministic setup times by the following, less restrictive, one that $\mathbb{E}[R_{S_i}]$ approaches $\frac{1}{2}\mathbb{E}[S_i]$ as $\mathbb{E}[S] \to \infty$, $i = 1, 2, \ldots, N$. Since $\mathbb{E}[R_{S_i}]$ can be rewritten as, for $i = 1, 2, \ldots, N$,

$$\mathbb{E}[R_{S_i}] = \frac{1}{2}\mathbb{E}[S_i]\Big(1 + \frac{\mathbb{V}\mathrm{ar}[S_i]}{\mathbb{E}[S_i]^2}\Big),$$

this means that the *variance* of the setup times should remain *finite* in the limiting case of *infinite* setup times (or, at least, $\frac{\mathbb{V}\mathrm{ar}[S_i]}{\mathbb{E}[S_i]^2} \to 0$ as $\mathbb{E}[S_i] \to \infty$). □

Remark 4.2. The explicit solution of the MVA equations represented by (11)-(13) has an intuitively appealing interpretation, certainly worth mentioning. That is, in the case of increasing deterministic setup times the polling system converges to a deterministic cyclic system with deterministic service times $\mathbb{E}[B_i]$ and continuous demand rates λ_i, $i = 1, 2, \ldots, N$. The visit times and conditional lengths emerging in (11)-(13) are precisely equal to the corresponding quantities in such a deterministic cyclic system. This explanation also clearly indicates the difficulties arising in system with increasing *stochastic* setup times, since it is certainly not obvious how such a polling system behaves in the limit. □

Remark 4.3. The present note demonstrates that MVA makes the asymptotic analysis of the delay distribution strikingly simple in case the setup times tend to infinity. The underlying reason for this is the fact that MVA explicitly gives, as by product, the second moments of the visit times, intervisit times and cycle lengths. In the case of deterministic setup times tending to infinity, MVA particularly reveals that the scaled intervisit times converge in probability to a constant immediately leading to the result of Theorem 3.1. □

Remark 4.4. The derivation of the results presented here, particularly the closed-form solution of the MVA equations, relies on the assumption that the setups are deterministic (see also Remark 4.1). However, in heavy-traffic the impact of higher moments of the setup times on the delay distribution vanishes (see [6]). By incorporating the heavy traffic results for the variance of the intervisit times derived in our accompanying paper [8], this implies that, as ρ tends to 1, the results of the present note can be easily extended to generally distributed setup times (see, also, [9]). □

Remark 4.5. In [15] it is observed that the mean residual cycle lengths $\mathbb{E}[R_{C_i}] = \mathbb{E}[R_{\theta_{i+1,N}}]$ - recall that a cycle starts at a departure epoch of the server from queue i - satisfy (6) as well. An important observation is that whereas the mean cycle lengths do not depend on the queue at which the cycle starts, the mean residual cycle lengths generally differ. In case of exhaustive service, the following well-known linear relation between the mean delays and the mean residual cycle lengths $\mathbb{E}[R_{C_i}]$ is known (see, e.g., [1]),

$$\mathbb{E}[W_i] = (1 - \rho_i)\mathbb{E}[R_{C_i}], \qquad i = 1, 2, \dots, N.$$

In the past, highly accurate approximations for the exhaustive service policy have been developed based on the assumption that the mean residual cycle lengths are identical for all queues, i.e., $\mathbb{E}[R_{C_i}] = \mathbb{E}[R_C]$ (see [1]). In this context, recall that the proof of Theorem 3.1 shows that the cycle lengths converge to a constant and that they are, thus, completely independent of the queue involved. In other words, the present note proves that the assumption of equality of the mean residual cycle lengths is asymptotically exact when the setup times tend to infinity. □

5 Conclusions

The present note has presented a limit theorem for exhaustive polling systems with increasing deterministic setups, which was proved directly by incorporating MVA for polling systems. Furthermore, it gives a recipe for the derivation of similar limit theorems in other polling systems for which MVA is available. Therefore, the scope of applicability of the technique presented here hinges on the availability of MVA for polling systems. At this moment, MVA is available for polling systems with pure exhaustive and gated service, mixed exhaustive-gated service, batch arrivals, periodic server routing and discrete time settings (see [15]). An interesting, and challenging, topic for further research would be the classification of a general class of polling systems for which MVA can be applied and thus, among others, similar limit theorems can be derived.

Finally, in the present note it is assumed that the arrival processes are Poisson. Intuitively, one would however expect that also in case of general arrival processes the cycle lengths converge to a constant when the setup times tend to infinity. Unfortunately, the techniques used throughout this note rely heavily on the Poisson assumption. Extending our work to general arrival processes is, thus, a very interesting topic for further research.

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