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# Analytical expressions for the conductance noise measured with four circular contacts placed in a square array

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In the ideal case, noise measurements with four contacts minimize the contribution of the contact interface. There is a need to characterize conductance noise and noise correction factors for the different geometries provided with four contacts, as already is the case for resistivity measurements with van der Pauw structures. Here, we calculate the noise correction factors for two geometries with a pair of sensors and a pair of current driver electrodes placed in a square array. The first geometry investigated is a very large film compared to the distance  $L$  between four circular electrodes, which are placed in a square array far away from the borders of the film. The second is a square-shaped conductive film with side length  $L$  and provided with four quarter-circle corner contacts with radius  $l$ . The effect of the conductance noise in the film can be observed between current free sensors in a four-point measurement or between current carrying drivers in a two-point measurement. Our analytical expressions are based on approximations to solve the integrals  $\iint (J \cdot \tilde{J})^2 dA$  and  $\iint |J|^4 dA$  for the voltage noise measured across a pair of sensors,  $S_{VQ}$ , and across the drivers,  $S_{VD}$ , respectively. The first and second integrands represent the squared dot product of the current density and adjoint current density and the modulus of the current density to the fourth power, respectively. The current density  $J$  in the samples is due to the current  $I$  passing through the driver contacts. The calculated expressions are applicable to samples with thickness  $t \ll l \leq 0.1L$ . Hence, the disturbances in the neighborhood of the sensors on  $J$  and of the drivers on  $\tilde{J}$  are ignored. Noise correction factors for two- and four-point measurements are calculated for sensors on an equipotential (transversal noise) with the driver contacts on the diagonal of a square and for sensors next to each other on one side of the square with the drivers next to each other on the other side of the square (longitudinal noise). In all cases the noise between the sensors is smaller and less sensitive to the contact size  $2l/L$  than the noise between the drivers. The ratio  $S_{VQ}/S_{VD}$  becomes smaller with smaller contact radius  $l$ . Smaller sensors give a better suppression of interface noise at the contacts. But overly low  $2l/L$  values result in overly high resistance between the sensors and too strong a contribution of thermal noise at the sensors. Therefore, equations are derived to calculate the current level needed to observe  $1/f$  conductance fluctuations on top of the thermal noise. The results from the calculated analytical expressions show good agreement with experimental results obtained from the noise in carbon sheet resistance and numerical results. Transversal noise measurements on a square sample with corner contacts are recommended to characterize the  $1/f$  noise of the layer. This is due to the increased current densities in the sample compared to the open structure, which result in easier detection of the  $1/f$  on top of the thermal noise. © 2007 American Institute of Physics. [DOI: 10.1063/1.2434942]

## I. INTRODUCTION

The general idea to use four contacts for sheet resistance and conductance noise measurements is to minimize the unwanted contact interface contribution. The four-point (in line) probe has been used successfully to quickly measure resistivity and  $1/f$  noise in thin layers.<sup>1</sup>

Here, we investigate the possibility of measuring the sheet resistance and conductance noise in a layer with four contacts placed in a square array far from the borders on an

open structure denoted by  $O$  and in a square layer with four quarter-circle contacts in the corners, denoted by  $B$  for the bordered square structure. The details are as follows: (i) an “open” structure with the four circular contacts with radius  $l$  in a square array with side  $L$ , which are placed far away from the borders, and (ii) a square-shaped sample with side  $L$  and quarter-circle corner contacts with radius  $l$ .

The geometries and studied cases together with their code are shown in Figs. 1(a) and 1(b). We introduce a code for geometry and the use of the four contacts. This code has three parameters and starts with  $O$  or  $B$  to denote the open geometry in Fig. 1(a) or the bordered (square) geometry in

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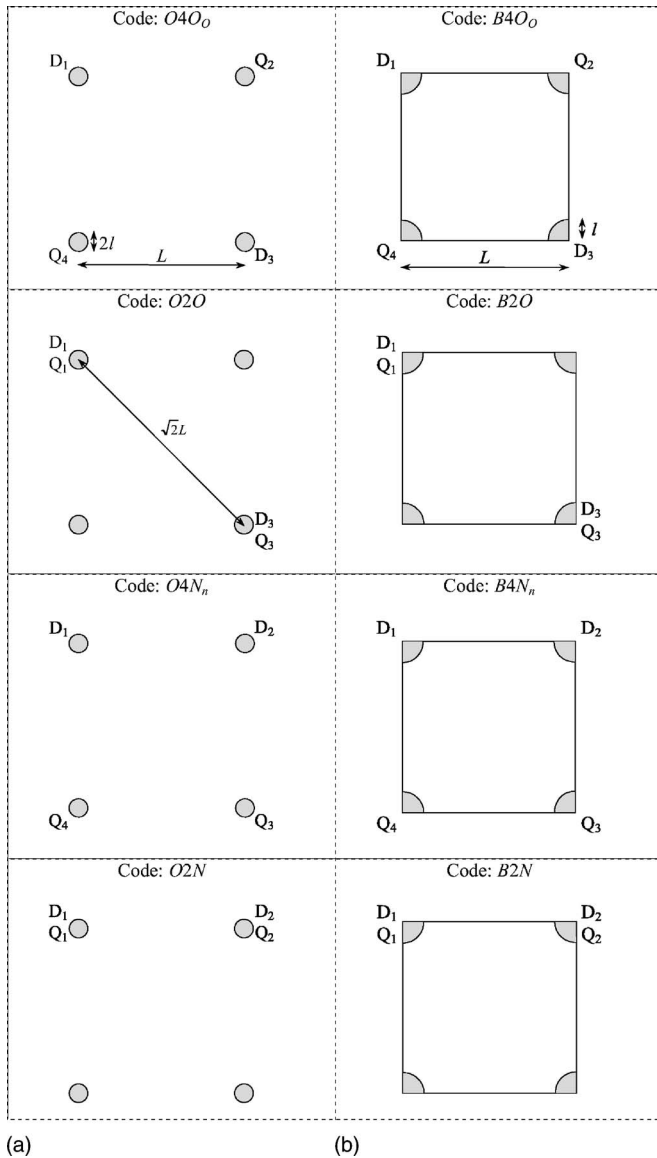


FIG. 1. (a) Open structure with square probe array and code indicating the investigated contact configuration. (b) Bordered square with quarter-circle corner contacts and code indicating the investigated contact configuration.

Fig. 1(b). We calculate the four-point noise between the sensors and the two-point noise across the drivers. The second parameter in the code is 4 or 2 to denote a four-point noise between the sensors or a two-point noise across the drivers. We will study two contact positions: (i) the noise between opposite contacts on the diagonal of the square and (ii) the noise between contacts next to each other. The third parameter in the code is *Oo* or *Nn* to denote whether the driver and sensor contacts are connected opposite to each other or next to each other. Hence, the code *B4Oo* means that we are discussing a square sample with quarter circle corner contacts and the four-point sensor noise between opposite contacts while the other pair of opposite contacts serves as drivers.

Following from the above, we need noise correction factors relating the observed voltage noise across a pair of opposite or next to each other placed sensor contacts, for example, to the noise one would observe on a rectangular

shaped sample provided with a pair of interface noise-free line contacts. The voltage and voltage noise across a pair of sensors or drivers on planar square resistors provided, for example, with square corner contacts were compared to numerical results obtained from resistor network simulations.<sup>2,3</sup> Here, we present analytical expressions for noise correction factors.

The problem of  $1/f$  noise between a pair of arbitrarily shaped and placed sensors by using a current source passing a current  $I_D$  through another pair of arbitrarily placed and shaped drivers is in principle solved.<sup>4</sup> All calculations are based on solving the integrals  $\iint (J \cdot \tilde{J})^2 dA$  and  $\iint |J|^4 dA$  for four- and two-point noise analysis.<sup>1-6</sup> Instead of offering numerical solutions, we propose approximations that lead to analytical expressions that are valid for  $2l/L \leq 0.2$ .

The four-point  $1/f$  noise between the sensors in the corners of a square sample with side  $L$  is written as<sup>2-4</sup>

$$S_{V_{QX}} = \frac{I_D^2 R_{sh}^2 C_{us}}{fL^2} f_{QX},$$

with  $R_{sh}(\Omega)$  the sheet resistance of the layer,  $f$  the frequency, and  $C_{us}(\text{cm}^2)$  a parameter characterizing the  $1/f$  noise of a layer. The geometry factor  $f_{QX}$  is the dimensionless noise correction factor, taking into account the size and the position of the contacts. The subscript  $Q$  denotes the four-point sensor noise. The subscript  $X$  can be  $O$  for opposite sensors on a diagonal of the square or  $N$  stating that sensors are next to each other on one side of the square.

If a pair of interface noise-free line contacts of length  $L$  is used on opposite sides of a square sample with side length  $L$  (sensors and drivers coincide), then and only then  $f_Q = f_D = 1$  and  $C_{us} = (fS_{VD}/V_D^2)L^2 = (fS_{VQ}/V_Q^2)L^2$  is the conductance noise characteristic of the layer.

The two-point  $1/f$  noise between a pair of drivers  $S_{VD}$  is written as

$$S_{VDX} = \frac{I_D^2 R_{sh}^2 C_{us}}{fL^2} f_{DX}.$$

For each case we calculate in Sec. III, the ratio  $S_{VQ}/S_{VD} = f_{QX}/f_{DX}$ . The ratio is important because small values for  $S_{VQ}/S_{VD}$  point to a good suppression of a possible unwanted interface contact noise. When the contact size is too small,  $2l/L \ll 1$ , then  $S_{VQ}$  becomes hard to detect out of an increasing thermal noise, which is proportional to the resistance between the sensors, which increases with decreasing  $l$ .

In Sec. II, the correction factors for the calculation of the sheet resistance and the calculation of the resistance between a pair of sensors are presented. The latter is used to estimate the thermal noise across the sensors. The  $1/f$  noise figure of merit for a layer,  $C_{us}$ , is briefly discussed.

In Sec. III, we present the noise correction factors. 12 new analytical expressions [2 geometries  $\times$  2 contact cases (opposite and next)  $\times$  3 equations per situation ( $S_{VQ}$ ,  $S_{VD}$ , and  $S_{VQ}/S_{VD}$ )] are derived.

In Sec. IV, the current density at the borders of the contacts and the current are presented as a detection criterion needed to observe the  $1/f$  noise above the thermal noise.

The calculations will be compared with experimental results in Sec. V.

## II. SHEET RESISTANCE $R_{sh}$ , RESISTANCE $R$ BETWEEN CIRCULAR CONTACTS, AND CHARACTERISTIC $1/f$ NOISE $C_{us}$ IN THIN-FILM RESISTORS

### A. Sheet resistance

The sheet resistance  $R_{sh} = \rho/t$ , with  $t$  the thickness of the conductive layer and  $\rho$  its specific resistivity ( $\Omega$  cm). This resistance can be determined from a four-point measurement by taking the ratio of the voltage drop between two sensor contacts next to each other to the current passing through two driver contacts next to each other. The general relation for the modulus of the two-dimensional current density  $J$  in a layer by applying a current source with strength  $I$  through the points  $(-a, 0)$  and  $(a, 0)$  on the  $x$  axis is given by

$$|J(x, y)| = \frac{I}{\pi} \frac{a}{\sqrt{[(x-a)^2 + y^2][(x+a)^2 + y^2]}}. \quad (1)$$

$$F_O = \frac{2\pi}{\int_{-a}^a (x+a)dx[(x+a)^2 + L^2] - \int_{-a}^a (x-a)dx[(x-a)^2 + L^2]} = \frac{2\pi}{\ln 2} = 9.06. \quad (2)$$

Here,  $F_O$  is the correction factor needed to calculate  $R_{sh}$  from experimentally obtained  $V_O$  and  $I_D$ . The subscript  $O$  indicates that it is for the open structure.

### 2. Bordered structure (square, B4N)

The sheet resistance in the bordered square structure can be calculated from the van der Pauw results.<sup>7</sup> For small contacts ( $2l/L < 0.2$ ) it still holds<sup>3</sup> that

$$R_{sh} = \frac{V_{QNn}}{I_{DNn}} F_B \quad \text{with } F_B = \frac{\pi}{\ln 2} = 4.53. \quad (3)$$

$F_B = F_O/2$  and the transresistance defined by  $V_{QNn}/I_{DNn}$  in B4Nn is twice the value in O4Nn.

### B. Resistance

We need to know the resistance between a pair of contacts to estimate the thermal noise, which is always in competition with the noise induced by conductance fluctuation. Therefore, we must calculate the resistance values. The general relation for the resistance between two arbitrarily placed contacts is given by  $R = 1/I^2 \iint R_{sh} J^2 dA$ . We find from this relation, for a rectangular sample of width  $W$  and length  $L$  between line contacts, a homogeneous current density  $J = I/W$  and the simple result  $R = LR_{sh}/W$ .

### 1. Open structure O4N

The sheet resistance is calculated following the voltage drop  $V_{QNn}$  between two sensors next to each other and the current  $I_{DNn} = I$  through the other driver contacts,

$$R_{sh} \equiv \frac{V_{QNn}}{I} F_O \Rightarrow F_O = \frac{IR_{sh}}{V_{QNn}}.$$

The sheet resistance from an open structure with four contacts in a square array with  $2l/L \leq 0.2$  is calculated from the line integral of the current density along the shortest path between the sensors at  $(-a, L)$  and  $(a, L)$  with  $a = L/2$ . The electric field component in the  $x$  direction between the two sensors at  $y = L$  is given by  $E_x(x, L) = R_{sh} J_x(x, L)$ .

$$V_{QNn} = - \int_{-a}^a E_x(x, L) dx = - \int_{-a}^a R_{sh} J_x(x, L) dx.$$

For a current source  $I$  injected at  $x, y(-a, 0)$  and leaving at  $x, y(a, 0)$ , we find the following for the  $x$  component of the current density:

$$J_x(x, y) = \frac{I}{2\pi} \left[ \frac{x+a}{(x+a)^2 + y^2} - \frac{x-a}{(x-a)^2 + y^2} \right],$$

### 1. O2N

The resistance between two circular contacts with diameter  $2l$  at a distance  $L$  apart for  $2l/L < 0.2$  is given by<sup>6</sup>

$$R = R_{sh} \frac{\ln(L/l)}{\pi}. \quad (4)$$

### 2. O2O

The distance between the contacts is the diagonal  $\sqrt{2}L$  of the square with side  $L$ , which is  $\sqrt{2}L$ . Hence, with Eq. (4) we find

$$R = R_{sh} \frac{\ln(\sqrt{2}L/l)}{\pi}. \quad (5)$$

### 3. B2N

The modulus of the current density  $J$  at a distance  $r$  around the contact is approximated by  $J = 2I/\pi r$ . Hence, the resistance between two contacts next to each other is given by

$$R = R_{\text{sh}} \frac{4 \ln(L/2l)}{\pi}. \quad (6)$$

The results of Eq. (6) are in fair agreement with the numerical results in Refs. 2 and 3.

#### 4. B2O

The distance between the contacts is  $\sqrt{2}L$ . Using Eq. (6), we find the following for the resistance:

$$R = R_{\text{sh}} \frac{4 \ln(\sqrt{2}L/2l)}{\pi}, \quad (7)$$

which is in good agreement with the numerical results from resistor network simulations for  $2l/L < 0.8$ .<sup>2,3</sup> Hence, the resistance between a pair of contacts in a square sample (*B4Nn* and *B4Oo*) is four times the resistance in an open structure (*O4Nn* and *O4Oo*) with the same  $L$  and  $l$ . The resistance between contacts next to each other is in general at most 20% lower than between opposite contacts for  $l < 0.1L$ .

#### C. $C_{\text{us}}$ a figure of merit for $1/f$ noise in conductive layers

We find the following for the relative voltage  $1/f$  noise across an Ohmic resistance  $R$  when a constant current  $I$  is applied, resulting in average voltage  $V=I\langle R \rangle$ :

$$\frac{S_V}{V^2} = \frac{S_R}{R^2} = \frac{C}{f}. \quad (8)$$

$C$  characterizes the noise independent of bias current  $I$  and frequency  $f$ .  $C$  is dimensionless and inversely proportional to the volume of the resistance. For layers we introduce  $C_{\text{us}} \equiv CWL$  (cm<sup>2</sup>) (Refs. 1–4 and 6) which characterizes the  $1/f$  noise of the layer of unit surface, e.g., 1 cm<sup>2</sup>. The empirical relation of Hooge *et al.* is applicable to homogeneous samples submitted to homogeneous fields,<sup>8</sup>

$$C = \frac{\alpha}{N}, \quad (9)$$

with  $N$  the number of free charge carriers and  $\alpha$  a dimensionless  $1/f$  noise parameter often in the range  $10^{-6} < \alpha < 10^{-3}$ . Then it holds<sup>9</sup> that

$$C_{\text{us}} \equiv CWL = \frac{\alpha}{nt} = \alpha q \mu R_{\text{sh}} \equiv KR_{\text{sh}}, \quad (10)$$

with  $n$  the free carrier concentration,  $t$  the layer thickness,  $\mu$  the mobility, and  $q$  the elementary charge. Here,  $R_{\text{sh}} = \rho/t = 1/q\mu nt$  has been used. The parameter  $K = C_{\text{us}}/R_{\text{sh}}$  can easily be calculated from experimental results without the precondition of homogeneity on a microscopic scale and without the precise knowledge of  $N$ . The mobility and  $N$  are often unknown in cermets and conducting polymers. The  $1/f$  noise characterization in  $K$  values is also applicable to samples with percolation and unknown values for  $N$  or  $\mu$ . This is in sharp contrast with characterizations in  $\alpha$  values.

From a comparison between Au layers, polysilicon, poly-Si Ge, and silicided poly Si, it turned out that  $C_{\text{us}} = KR_{\text{sh}}$  with the surprising result that the factor  $K=5$

$\times 10^{-21}$  cm<sup>2</sup>/Ω for gold was also applicable to the other polycrystalline layers.<sup>9</sup> Therefore, we propose to analyze conductive layers with  $R_{\text{sh}}$ ,  $C_{\text{us}}$ , and  $K = C_{\text{us}}/R_{\text{sh}}$ .  $C_{\text{us}}$  and  $K$  are good figures of merit to characterize the  $1/f$  noise of a layer.  $K > K_{\text{Au}} = 5 \times 10^{-21}$  cm<sup>2</sup>/Ω often points to percolation and current crowding on a microscopic scale. This is the case in carbon sheet resistors, for example, that are used to test the noise correction factors. The  $K$  value =  $2 \times 10^{-13}$  cm<sup>2</sup>/Ω observed in carbon films is about  $4 \times 10^7$  times the value observed in metal films and polycrystalline Si.

### III. NOISE CALCULATIONS FOR THE OPEN AND BORDERED GEOMETRIES

The general noise relation for  $1/f$  noise, between a pair of arbitrarily placed and shaped sensors while the constant current  $I$  is injected through another pair of arbitrarily placed and shaped drivers, is given by Refs. 1–4. This relation was successfully applied in Refs. 10–14. The four-point noise correction factor on a two-dimensional structure is then defined as

$$\begin{aligned} S_{VQ} &= \frac{R_{\text{sh}}^2 C_{\text{us}}}{f} \frac{1}{I^2} \iint (J \cdot \tilde{J})^2 dA \equiv \frac{I^2 R_{\text{sh}}^2 C_{\text{us}}}{f L^2} f_{QX} \Rightarrow f_{QX} \\ &= \frac{L^2 \iint (J \cdot \tilde{J})^2 dA}{I^4}. \end{aligned} \quad (11)$$

Here,  $J$  and  $\tilde{J}$  are the current density and adjoint current density in the film both proportional to  $I$  and with the dimensions of A/cm. The adjoint current density  $\tilde{J}$  in the sample is created by applying the current source  $I$  to the sensor contacts instead of the drivers. If the noise is observed at the driver contacts, then it holds that for two-point measurements  $J = \tilde{J}$  and  $(J \cdot \tilde{J})^2 = (J_x \tilde{J}_x + J_y \tilde{J}_y)^2 = (J_x^2 + J_y^2)^2 = |J|^4$ . Then, the squared value of the dot product equals the modulus of current density to the fourth power. The noise across the drivers and the two-point noise correction factor are given by<sup>5,6</sup>

$$\begin{aligned} S_{VD} &= \frac{R_{\text{sh}}^2 C_{\text{us}}}{f} \frac{1}{I^2} \iint |J|^4 dA \equiv \frac{I^2 R_{\text{sh}}^2 C_{\text{us}}}{f L^2} f_{DX} \Rightarrow f_{DX} \\ &= \frac{L^2 \iint |J|^4 dA}{I^4}. \end{aligned} \quad (12)$$

Here, all analytical expressions for the noise are based on approximate solutions for the integrals in Eqs. (11) and (12). The influences on the current density and adjoint current density in the neighborhood of the non-current-carrying contacts are ignored. Therefore, the validity of the analytical approximations is limited to contacts with  $2l/L \leq 0.2$ .

Some of the four-point problems can be solved by using an approximation of current densities or adjoint current densities around the small contacts by a local homogeneous field  $J_h$  at a fixed angle. We assume that the current density around a small circular contact can be approximated by a

radial current density and that the modulus of the current density  $|J|$  at a distance  $r$  from the center of the contact is given by

$$|J| = \frac{I}{2\pi r}. \quad (13)$$

Then we find the following for the integral in the open structure around one contact:

$$\begin{aligned} \iint (J \cdot \tilde{J})^2 dA &= \int_0^{2\pi} \int_{r_0}^{r_{\max}} \left( \frac{J_h I \cos \theta}{2\pi r} \right)^2 r dr d\theta \\ &= \frac{(J_h I)^2}{4\pi} \ln \frac{r_{\max}}{r_0}, \end{aligned} \quad (14)$$

where  $\theta$  is the angle between the homogeneous current density and the radial  $J$ .

### A. Applied to $O4Oo$

For the open layer with four contacts in the square of side  $L$ , we find the following for the integral around one contact in the case of opposite sensor contacts, if we choose  $r_{\max} = L/2$  and  $r_0 = l$  in Eq. (14):

$$\frac{(J_h I)^2}{4\pi} \ln \left( \frac{L}{2l} \right). \quad (15)$$

The current density at opposite sensor points for a current source  $I$  with opposite drivers is estimated to be a homogeneous current density and is given by Eq. (1) with  $x=0$ ,  $y=a$ , and  $L=\sqrt{2}a$ :

$$J_h = \frac{I}{2\pi a} = \frac{I}{\sqrt{2}\pi L}. \quad (16)$$

Because the  $O4Oo$  problem has a fourfold symmetry around all contacts, we estimate the value of the general integral Eq. (11) to be

$$S_{VQO} = 4 \frac{1}{f^2} \frac{R_{\text{sh}}^2 C_{\text{us}}}{f} \frac{I^2}{2\pi^2 L^2} \frac{I^2}{4\pi} \ln \left( \frac{L}{2l} \right). \quad (17)$$

For  $O4Oo$  this results in the following expressions for the noise and normalized noise at the sensors:

$$\begin{aligned} S_{VQO} &= \frac{I^2 R_{\text{sh}}^2 C_{\text{us}}}{f} \frac{1}{2\pi^3 L^2} \ln \left( \frac{L}{2l} \right) \Rightarrow f_{QO} \equiv \frac{f S_{VQO} L^2}{I^2 R_{\text{sh}}^2 C_{\text{us}}} \\ &= \frac{1}{2\pi^3} \ln \left( \frac{L}{2l} \right). \end{aligned} \quad (18)$$

### B. Applied to $O2O$

For the noise at the driver contacts we apply Eq. (12) and use a twofold symmetry in the integral. We take Eq. (13) as an approximation for the current density around the drivers and take  $r_{\max} \gg r_0 = l$ . This results in the two-point and normalized noise at the drivers:

$$S_{VDO} = \frac{I^2 R_{\text{sh}}^2 C_{\text{us}}}{f} \frac{1}{8\pi^3 l^2} \Rightarrow f_{DO} \equiv \frac{f S_{VDO} L^2}{I^2 R_{\text{sh}}^2 C_{\text{us}}} = \frac{L^2}{8\pi^3 l^2}. \quad (19)$$

The ratio between noise at the sensors and drivers is given by

TABLE I. Approximations for  $J$  and  $\tilde{J}$  at and around the contacts in  $O4Oo$  and  $O4Nn$ .

$O4Oo$		$O4Nn$	
$ J_{D1}  \approx \frac{I}{2\pi r}$ (R)	$ \tilde{J}_{D1}  = \frac{I}{\pi\sqrt{2}L}$ (H)	$ J_{D1}  \approx \frac{I}{2\pi r}$ (R)	$ \tilde{J}_{D1}  = \frac{I}{2\pi\sqrt{2}L}$ (H)
$ J_{D3}  \approx \frac{I}{2\pi r}$ (R)	$ \tilde{J}_{D3}  = \frac{I}{\pi\sqrt{2}L}$ (H)	$ J_{D2}  \approx \frac{I}{2\pi r}$ (R)	$ \tilde{J}_{D2}  = \frac{I}{2\pi\sqrt{2}L}$ (H)
$ J_{Q4}  = \frac{I}{\pi\sqrt{2}L}$ (H)	$ \tilde{J}_{Q4}  \approx \frac{I}{2\pi r}$ (R)	$ J_{Q4}  = \frac{I}{2\pi\sqrt{2}L}$ (H)	$ \tilde{J}_{Q4}  \approx \frac{I}{2\pi r}$ (R)
$ J_{Q2}  = \frac{I}{\pi\sqrt{2}L}$ (H)	$ \tilde{J}_{Q2}  \approx \frac{I}{2\pi r}$ (R)	$ J_{Q3}  = \frac{I}{2\pi\sqrt{2}L}$ (H)	$ \tilde{J}_{Q3}  \approx \frac{I}{2\pi r}$ (R)

$$\frac{S_{VQO}}{S_{VDO}} = 4 \left( \frac{l}{L} \right)^2 \ln \left( \frac{L}{2l} \right). \quad (20)$$

### C. Applied to $O4Nn$

The noise between the sensors next to each other is calculated using similar approximations as in  $O4Oo$ . The procedure is again as follows: (i) we calculate the current density at the sensor contacts where the current is not passed through by using Eq. (1) and (ii) we assume that close to these contacts, the current density can be considered homogeneous because it is far from the source. These assumptions lead to the simple solutions given by Eq. (14) for  $\iint (J \cdot \tilde{J})^2 dA$  in Eq. (11). The results for the approximated  $J$  and  $\tilde{J}$  at the contacts are summarized in Table I. The values and expressions for  $J$  and  $\tilde{J}$  are shown for  $O4Oo$  and for  $O4Nn$ . The characters R and H indicate if the current density is assumed to be radial or homogeneous. The cases  $O4Nn$  and  $O4Oo$  show a twofold symmetry in  $J$  or  $\tilde{J}$  but they show a fourfold symmetry in  $J \cdot \tilde{J}$  or  $(J \cdot \tilde{J})^2$  and hence in the integral of Eq. (11). The cases  $O2O$  and  $O2N$ , for example, show a twofold symmetry in  $J$ ,  $|J|^4$ , and in the integral of Eq. (12).

Table I shows that the current densities at the sensors for case  $O4Nn$  are twice as small as for case  $O4Oo$ . Hence, we expect four times less noise between the sensors when  $O4Nn$  is compared to  $O4Oo$ . For the  $O4Nn$  case we have a fourfold symmetry as in  $O4Oo$ . The noise and normalized noise at the sensors are then as follows:

$$\begin{aligned} S_{VQN} &= \frac{I^2 R_{\text{sh}}^2 C_{\text{us}}}{f} \frac{1}{8\pi^3 L^2} \ln \left( \frac{L}{2l} \right) \Rightarrow f_{QN} \equiv \frac{f S_{VQN} L^2}{I^2 R_{\text{sh}}^2 C_{\text{us}}} \\ &= \frac{1}{8\pi^3} \ln \left( \frac{L}{2l} \right), \end{aligned} \quad (21)$$

which is four times lower than for the case of  $O4Oo$  in Eq. (18).

### D. $O2N$

The noise at the driver contacts ( $O2N$ ) is given by Eq. (12) and assuming  $|J_{D1}| = |J_{D2}| = I/2\pi r$  for  $l < r \leq L/2$  with  $L \gg l$ .

$$S_{VDN} = \frac{I^2 R_{sh}^2 C_{us}}{f} \frac{1}{8\pi^3 l^2} \Rightarrow f_{DN} \equiv \frac{f S_{VDN} L^2}{I^2 R_{sh}^2 C_{us}} = \frac{L^2}{8\pi^3 l^2}. \quad (22)$$

Due to the approximation  $L \gg l$ , the result in Eq. (22) is the same as in Eq. (19) for  $O2O$ . The ratio between the noise at the sensor and that across the drivers is as follows:

$$\frac{S_{VQN}}{S_{VDN}} = \left(\frac{l}{L}\right)^2 \ln\left(\frac{L}{2l}\right). \quad (23)$$

### E. B2O

There are some experimental results for the square-shaped samples with square-shaped corner contacts that have been compared with calculations based on numerical simulations with resistor networks.<sup>2,3</sup> Here, we calculate simplified analytical expressions for square-shaped samples with quarter-circle corner contacts. The current density close to a corner contact used as a driver is assumed to be radial in an angle  $\pi/2$  and is approximated as follows:

$$|J(r)| = \frac{2I}{\pi r}, \quad (24)$$

which is four times higher than in an open structure, e.g.,  $O2O$ . The noise across two opposite drivers is calculated with Eq. (12),

$$\begin{aligned} S_{VDO} &= \frac{I^2 R_{sh}^2 C_{us}}{fL^2} \left(\frac{2}{\pi}\right)^3 \left(\frac{L}{l}\right)^2 \Rightarrow f_{DO} \equiv \frac{f S_{VDO} L^2}{I^2 R_{sh}^2 C_{us}} \\ &= \frac{8}{\pi^3} \left(\frac{L}{l}\right)^2. \end{aligned} \quad (25)$$

### F. B2N

The same result is obtained for the noise between two drivers next to each other. Hence,  $S_{VDN}$  and the normalized noise are as in Eq. (25):

$$\begin{aligned} S_{VDN} &= \frac{I^2 R_{sh}^2 C_{us}}{fL^2} \left(\frac{2}{\pi}\right)^3 \left(\frac{L}{l}\right)^2 \Rightarrow f_{DN} \equiv \frac{f S_{VDN} L^2}{I^2 R_{sh}^2 C_{us}} \\ &= \frac{8}{\pi^3} \left(\frac{L}{l}\right)^2. \end{aligned} \quad (26)$$

### G. B4Oo

The noise contribution to opposite sensors mainly consists of four equal contributions due to the fourfold symmetry in  $(J \cdot \tilde{J})^2$ . This can also be observed in the charts of spatial noise distribution in planar resistors with finite contacts.<sup>3</sup> The inner part of the sample does not contribute to the observed noise at the sensors, where  $J \perp \tilde{J}$  approximately holds. We again assume that the current density is more or less homogeneous close to the sensor contacts and stays perpendicular to the diagonal of the square through the sensors. The adjoint current density in the neighborhood of the drivers is for symmetry reasons also assumed to be perpendicular to the diagonal through the drivers and homogeneous in a restricted area around the contacts. The current densities

around the driver contacts and the adjoint current density around the sensor contacts are as usual assumed to be radial and given by Eq. (24). Close to the sensors and drivers we assume

$$|J_Q| = |\tilde{J}_D| = |J_h| = \frac{I}{\sqrt{2}L}, \quad (27)$$

and the approximated analytical expression for  $\iint (J \cdot \tilde{J})^2 dA$  is

$$\begin{aligned} &\int_{\pi/4}^{3\pi/4} \int_l^{L/\sqrt{2}} \left( J_h \frac{2I}{\pi r} \cos \theta \right)^2 r dr d\theta \\ &= J_h^2 I^2 \left( \frac{\pi-2}{\pi^2} \right) \ln\left(\frac{L}{\sqrt{2}l}\right). \end{aligned} \quad (28)$$

Taking  $J_h = I/\sqrt{2}L$  and a fourfold symmetry (four times the contribution around one contact), we find

$$\begin{aligned} S_{VQO} &= \frac{I^2 R_{sh}^2 C_{us}}{fL^2} \frac{2(\pi-2)}{\pi^2} \ln \frac{L}{2l} \\ \Rightarrow f_{QO} &= \frac{f S_{VQO} L^2}{I^2 R_{sh}^2 C_{us}} = \frac{2(\pi-2)}{\pi^2} \ln \frac{L}{\sqrt{2}l}. \end{aligned} \quad (29)$$

The ratio  $S_{VQO}/S_{VD}$  is

$$\frac{S_{VQO}}{S_{VD}} = \frac{\pi(\pi-2)}{4} \left(\frac{l}{L}\right)^2 \ln\left(\frac{L}{\sqrt{2}l}\right) \approx 0.89 \left(\frac{l}{L}\right)^2 \ln\left(\frac{L}{\sqrt{2}l}\right). \quad (30)$$

### H. B4Nn

In contrast to the case with opposite sensors ( $B4Oo$ ), where the inner part of the sample does not contribute to the noise, now the dot product of the vectors in the middle part of the sample contributes most because there  $J \parallel \tilde{J}$  roughly holds. If we assume  $(J \cdot \tilde{J})^2 = (I/L)^4$  in an inner rectangular part of area  $L^2/4$ , then the integral  $\iint (J \cdot \tilde{J})^2 dA$  is estimated to be  $(I/L)^4 L^2/4 = I^4/4L^2$ . The choice is inspired by some results shown in Ref. 3 and the Schwarz-Christoffel mapping<sup>15,16</sup> of the current lines in the structure with two driver contacts next to each other, as shown in Fig. 2. The noise  $S_{VQ}$ , normalized noise, and ratio  $S_{VQ}/S_{VD}$  are

$$S_{VQ} = \frac{I^2 R_{sh}^2 C_{us}}{fL^2} \frac{1}{4} \Rightarrow f_{QN} \equiv \frac{f S_{VQ} L^2}{I^2 R_{sh}^2 C_{us}} = \frac{1}{4} \quad \text{for } 2l/L < 0.2, \quad (31)$$

$$\frac{S_{VQ}}{S_{VD}} = \frac{\pi^3}{32} \left(\frac{l}{L}\right)^2 \approx 0.97 \left(\frac{l}{L}\right)^2. \quad (32)$$

The noise in the square across opposite sensors is about four times the noise between the sensors next to each other, as was already observed for the open structure. This is also confirmed from conformal mapping calculations.<sup>16</sup>

The calculated results for the open structure are shown in Fig. 3. The dotted lines show  $f_Q$ . The full lines with strong negative slope show  $f_D$ . The lines with a positive slope are for  $f_Q/f_D$  for  $O4Nn$  and  $O4Oo$ . Figure 4 is similar to Fig. 3 but for  $B4Oo$ ,  $B2O$ ,  $B4Nn$ , and  $B2N$ . The summary of all

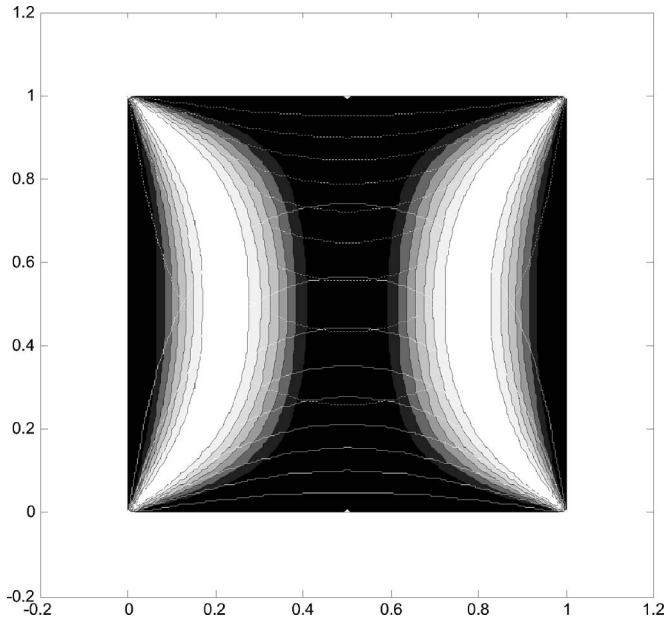


FIG. 2. Picture for the  $B4Nn$  configuration of the current lines (full white lines) and adjoint current density (dashed white lines), which are symmetric to the horizontal line  $y=0.5$ . The equipotentials are perpendicular to the current lines. The values of the dot product of  $(J \cdot \tilde{J})^2$  are given in gray. The white areas have a negligible contribution to the integral and the very dark areas contribute strongly to the integral.

calculated expressions for the noise factors  $f_Q$ ,  $f_D$ , and  $f_Q/f_D$  is shown in Table II. The analytical expressions for the noise factors are defined as in the Introduction and Eqs. (11) and (12):

$$S_{VX} = \frac{I_D^2 R_{sh}^2 C_{us}}{fL^2} f_X. \tag{33}$$

In a four-point measurement, the subscript  $X=Q$  indicates that the noise is measured across the sensors  $S_{VQ}$ . In a two-point noise measurement across the drivers,  $S_{VD}$  is observed and  $X$  denotes  $D$ .

There is a twofold symmetry for the two-point noise problem and a fourfold symmetry for the four-point problem. In the cases  $O4Oo$  and  $B4Oo$  there is even rotation symmetry over  $90^\circ$  in the shape of the areas of high and low noise

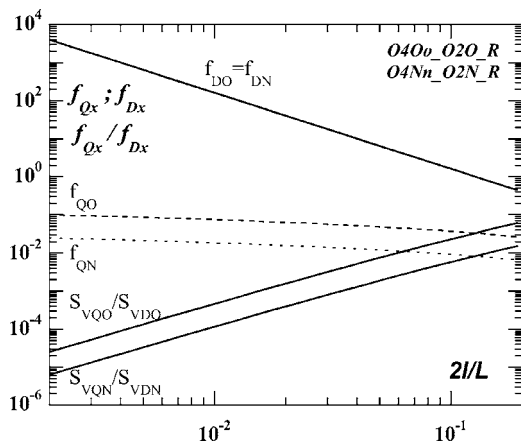


FIG. 3. Geometry correction factors and ratios for the open structure with square array probe vs  $2l/L$ .

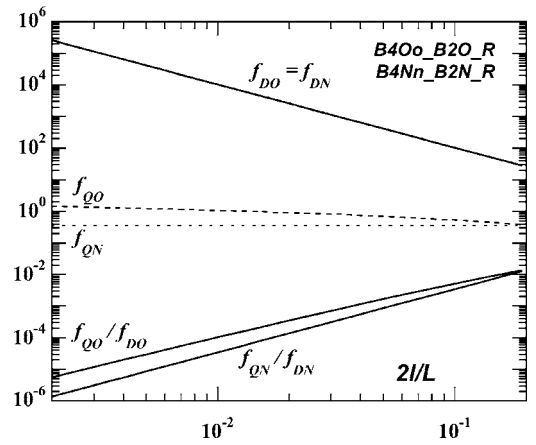


FIG. 4. The geometry correction factors and ratios for the square sample with corner contacts vs  $2l/L$ .

contributions.<sup>3</sup> The  $n$ -fold symmetry in  $\iint (J \cdot \tilde{J})^2 dA$  or  $\iint |J|^4 dA$  is shown in the final column of Table II.

#### IV. CRITICAL CURRENT TO OBSERVE $1/f$ NOISE ABOVE THERMAL NOISE AT THE SENSORS

We investigated appropriate structures with four contacts to characterize the  $1/f$  noise of a conductive layer without the contributions of possible contact interface noise. But the  $1/f$  noise between a pair of sensors is often much smaller than that between the drivers for a geometry with a good suppression of possible interface noise contributions (small  $2l/L$  values). A small contact radius results in a higher resistance between the sensors and a higher thermal noise, which is in competition with the  $1/f$  noise. In the  $O4Nn$  case, for example, we have  $R \approx (R_{sh}/\pi) \ln(L/l)$  and the following detection criterion holds:

$$S_{VQ} = \frac{I^2 R_{sh}^2 C_{us}}{fL^2} \frac{1}{8\pi^3} \ln \frac{L}{2l} > 4kTR_{sh} \frac{1}{\pi} \ln \frac{L}{2l}. \tag{34}$$

This means that the dissipated power density  $I^2 R_{sh}/L^2$  must be large enough,

$$\frac{I^2 R_{sh} C_{us}}{L^2 f 8\pi^2} > 4kT. \tag{35}$$

The local value of the dissipated power density at the rim of the contact must stay below a critical value  $P_e$  ( $W/cm^2$ ) to avoid (i) excessive heating and (ii) nonlinearity because Ohm's law is not applicable beyond a critical field.

The highest current density is at the rim of contacts and is given by  $J=I/2\pi l$  for the open structure. The highest power density at that spot then is  $P_e=J^2 R_{sh}=(I/2\pi l)^2 R_{sh}$ . Hence,

$$\frac{4kTL^2 f 8\pi^2}{R_{sh} C_{sh}} \leq I^2 \leq 4\pi^2 P_e \frac{l^2}{R_{sh}}, \tag{36}$$

$$\frac{32\pi^2 kTfL^2}{C_{us}} < I^2 R_{sh} \leq 4\pi^2 P_e l^2. \tag{37}$$

In order to reach the detection conditions we need the following:



TABLE II. Summary of analytical expressions for the open and bordered square structures.

Code	$f_X$	Ratio $S_{VQ}/S_{VD}=f_Q/f_D$	Presented in	Symmetry
<i>O4Oo</i>	$f_{QO} = \frac{1}{2\pi^3} \ln\left(\frac{L}{2l}\right)$	$\frac{f_{QO}}{f_{DO}} = 4\left(\frac{l}{L}\right)^2 \ln\left(\frac{L}{2l}\right)$	Fig. 3	4
<i>O2O</i>	$f_{DO} = \frac{1}{8\pi^3} \left(\frac{L}{l}\right)^2$			2
<i>O4Nn</i>	$f_{QN} = \frac{1}{8\pi^3} \ln\left(\frac{L}{2l}\right)$	$\frac{f_{QN}}{f_{DN}} = \left(\frac{l}{L}\right)^2 \ln\left(\frac{L}{2l}\right)$	Fig. 3	4
<i>O2N</i>	$f_{DN} = \frac{1}{8\pi^3} \left(\frac{L}{l}\right)^2$			2
<i>B4Oo</i>	$f_{QO} = \frac{2(\pi-2)}{\pi^2} \ln\left(\frac{L}{\sqrt{2}l}\right)$	$\frac{f_{QO}}{f_{DO}} = \frac{\pi(\pi-2)}{4} \left(\frac{l}{L}\right)^2 \ln\left(\frac{L}{\sqrt{2}l}\right)$	Fig. 4	4
<i>B2O</i>	$f_{DO} \cong \frac{8}{\pi^3} \left(\frac{L}{l}\right)^2$			2
<i>B4Nn</i>	$f_{QN} = 0.25$	$\frac{f_{QN}}{f_{DN}} = \frac{\pi^3}{32} \left(\frac{l}{L}\right)^2$	Fig. 4	4
<i>B2N</i>	$f_{DN} \cong \frac{8}{\pi^3} \left(\frac{L}{l}\right)^2$			2

- (i) small  $L$  and not too small  $l$ ,
- (ii) samples with a high  $C_{us}$ , which are of course easier to characterize, and
- (iii) low frequency  $f$ .

Hence, there is a compromise to choose between good interface noise suppression and thermal noise problems.

## V. EXPERIMENTAL RESULTS

The experimental results were obtained on carbon sheets. Figure 5 shows the observed spectra for *B2N*, *B2O*, *B4Oo*, and *B4Nn* with  $2l/L=0.2$  for a driver current of  $17.24 \mu\text{A}$ . The upper curves show the two-point noise measurements (*B2N* and *B2O*), the middle curves show the four-point results (*B4Oo* and *B4Nn*), and the bottom curve shows the thermal noise between the sensors for  $I=0$ . There is no strong difference between two-point noise across the drivers next to each other or opposite on a diagonal of the square. The observed four-point noise on a diagonal is higher than that across contacts next to each other. The calculated values predict a factor of 1.5; here a factor of 2 is observed. The

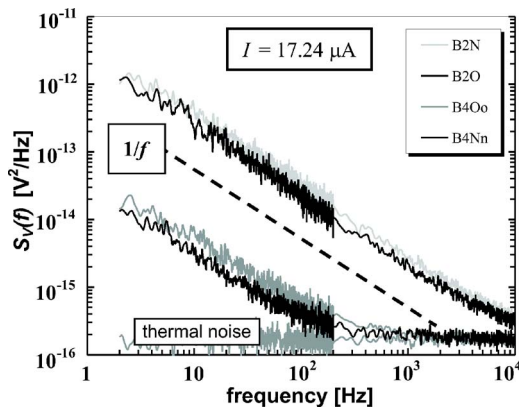


FIG. 5.  $1/f$  spectra obtained from a square carbon sample with  $2l/L=0.2$ .

discrepancy can be due to a small anisotropy problem in the sample and the approximations made in the calculations. The ratio  $f_D/f_Q$  is about a factor of 100, as roughly predicted for  $2l/L=0.2$  (see Fig. 4).

## VI. CONCLUSIONS

Considering Table II and the results shown in Figs. 3 and 4 for *O4Oo*, *O2O*, *O4Nn*, *O2N*, *B4Oo*, *B2O*, *B4Nn*, and *B4N*, the noise across the sensors (opposite or next to each other) is much less sensitive to the contact size than the noise across the drivers. This makes the four-point analysis much more reliable than the two-point measurement for characterizing the layer.

The noise across the drivers is  $4^3$  times larger in the square structure (*B2O* or *B2N*) than in the open structure (*O2O* or *O2N*) because the current density in the corner of a square is four times higher and  $\iint J^4 dA$  is integrated over a quarter of the area. Hence,  $4^4/4=4^3$ .

The noise across the sensors in the opposite position in the open structure (*O4Oo*) is four times larger compared to that across sensors next to each other (*O4Nn*). This is because the current density at the opposite sensor position is twice that for the sensors next to each other. A similar trend between *B4Oo* and *B4Nn* is observed in the bordered structure.

The noise in the four-contact configuration is proportional to  $\iint (J \cdot \tilde{J})^2 dA$  and hence proportional to  $\tilde{J}$  in the neighborhood of the sensors. In all cases we observe a weak dependence of  $S_Q$  on the contact size  $l$  (see Figs. 3 and 4) and a strong dependence of  $S_{VD}$  on  $l$  ( $\propto 1/l^2$ ). This makes the four-contact measuring method between opposite contacts on a square with corner contacts the most appropriate to investigate  $C_{us}$  and  $R_{sh}$ .

The calculated ratio  $S_{VQ}/S_{VD}=f_Q/f_D < 1$  for the two types of geometry, just as was observed earlier<sup>3</sup> on geometries that are invariant for rotations of  $90^\circ$ .

Low values for  $S_{VQ}/S_{VD}$  in general mean good driver contact noise suppression, but overly small values of  $l$  lead to overly high values for the thermal noise at the contacts. This can make the measurement of the  $1/f$  noise across the sensors impossible.

From the comparison of results obtained with a linear four-point probe<sup>1</sup> on an open structure with noncollinear square array probe, we observe the following:  $S_{VQ}$  in a linear four-point probe is 2.5 times higher than that for the  $O4Nn$  case and is  $5/8$  times the value obtained in case  $O4Oo$ .

The experimental results show good agreement with experimental verifications.

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