

Two robust methods to compute the current along a straight thin wire

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TWO ROBUST METHODS TO COMPUTE THE CURRENT ALONG A STRAIGHT THIN WIRE

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Abstract: We propose two methods to compute the current along a straight wire segment in a robust manner. In the first we employ the integral equation with exact kernel and a modified forcing function. In the second method, we use the integral equation with reduced kernel in combination with a regularization and filtering procedure. The regularization and filtering are inspired by uniqueness and conditional existence results for the integral equation with reduced kernel, which have been obtained via a renewed interpretation of the integral equations with exact and reduced kernel.

INTRODUCTION

In free space, we consider a straight wire of length L , circular cross section with radius a , and central axis coinciding with the z -axis. For this configuration, the so-called integral equations with exact and reduced kernel, after a Laplace transformation, are obtained as special cases of

$$\left[\partial_z^2 - \frac{s^2}{c_0^2} \right] \int_0^L g(r, z - z', a) I_z(z') dz' = -F_z^i(r, z), \quad (1)$$

where

$$g(r, z, a) = \int_{-\pi}^{\pi} \frac{\exp \left[-(s/c_0) \sqrt{r^2 + a^2 - 2ra \cos(\phi) + z^2} \right]}{4\pi \sqrt{r^2 + a^2 - 2ra \cos(\phi) + z^2}} d\phi, \quad (2)$$

is the angular-averaged Green's function kernel, I_z is the total current along the wire in the z -direction and $F_z^i(r, z)$ is the forcing function, which is related to the z -component of incident electric field. The integral equation with exact kernel is obtained for $r = a$, whereas the integral equation with reduced kernel is obtained by choosing $r = 0$.

The integral equation with reduced kernel, regarded as an integral operator equation, is ill-posed for Lebesgue spaces and for all standard Sobolev spaces $W^{r,p}(0, L)$ with positive and finite r and $1 \leq p < \infty$. This is due to the fact that the reduced kernel is a C^∞ function. The crucial observation is that the operator is *compact* in these function spaces and a compact operator has an unbounded inverse (see Kress [1]). A typical example of the associated numerical problems is given in Figure 1a, which we will discuss later on.

EXACT KERNEL APPROACH

For the integral equation with exact kernel, existence and uniqueness were proven by Jones [2] for excitations that are continuous and have finite derivative with respect to the axial variable. Later, the existence results were extended by Rynne [3–6] to a large class of vector spaces. This means that we can assume that the solution to the integral equation with exact kernel exists. The exact kernel representation corresponds to the angular-averaged scattered field at the mantle of a hollow tube with open end faces. Therefore, the proper forcing function is given by

$$F_z^i(a, z) = s\varepsilon_0 \int_{-\pi}^{\pi} E_z^i(a, \phi, z) d\phi, \quad (3)$$

which differs from the conventional forcing function, which is generally the incident electric field on the axis of the wire. Hence, this approach requires the additional computation of the angular average of the incident field. In view of the space limitations, we do not present numerical results. However, the papers by Rynne have illustrated the capabilities of this approach.

CONDITIONAL EQUIVALENCE

Many existing numerical codes employ the integral equation with reduced kernel. Therefore, it is interesting to further examine the properties of this equation and devise a way to improve the performance of these codes. It was shown by Tijhuis et al. [7] that the integral representation with reduced kernel represents exactly the z -component of the scattered electric field on the axis of the wire due to the current on the mantle. Therefore, in the present context, the most important question is which incident field should be chosen for the forcing function.

We assume that the incident field has a source that lies outside the domain $0 \leq r \leq a$, $0 \leq z \leq L$. Under this assumption, both the incident field and the scattered field, due to the current on the mantle of the wire, are regular inside this domain. To find a forcing function on the axis of the wire, we make the following observations. The thin-wire equation with exact kernel yields a current I_z , owing to the existence results mentioned above, and it is readily shown that the left-hand side of Eq. (1), which we denote as $F_z^s(r, z)$, satisfies

$$\left(\partial_r^2 + \frac{1}{r} \partial_r + \partial_z^2 - \frac{s^2}{c_0^2} \right) F_z^s(r, z) = 0, \quad (4a)$$

$$\begin{cases} F_z^i(a, z) + F_z^s(a, z) = 0 & \text{for } 0 < z < L, \\ F_z^s(r, z) & \text{regular for } 0 \leq r \leq a, 0 < z < L. \end{cases} \quad (4b)$$

The homogeneous solution can be described in terms of the angular-independent TM modes of a hollow cylinder, with unknown amplitudes. In general, these mode amplitudes will not be zero. They are determined by the fields at the end faces, which result after applying appropriate boundary conditions at the end faces at $z = 0$ and $z = L$. Therefore, we immediately obtain existence for the integral equation with reduced kernel for specific values of the amplitudes, at the expense of taking into account the boundary conditions at the end faces. In case the wire is sufficiently thin with respect to the wavelength, all modes are evanescent and therefore take effect only in proximity of the end faces of the wire. This leads to an alternative interpretation of the approximation introduced by Tijhuis et al. [7] that the end effects are negligible. Also, the frequently occurring numerical breakdown near the end faces of implementations of the integral equation with reduced kernel can be understood in this way, since the mode amplitudes are taken identically zero.

A uniqueness result for the integral equation with reduced kernel is obtained by employing the same partial differential equation and the regularity results. However, in this case we propagate the solution from the axis of the wire to the mantle of the wire. This yields a unique solution, owing to the uniqueness for the integral equation with exact kernel and the fact that the cylindrical coordinate system is singular in $r = 0$.

REDUCED KERNEL APPROACH

In the above, we have argued that it is possible to construct a proper forcing function for the integral equation with reduced kernel. However, this procedure would destroy the simplicity of the thin-wire equation with reduced kernel and its numerical implementation. Further, even if we were to obtain the mode amplitudes of the forcing function, the integral equation would remain ill-posed, due to the smoothness of the reduced kernel. To circumvent these problems, we propose a regularization and filtering procedure. We employ a penalty function, which is based on the discretized Laplacian. Since the Laplacian induces monotonicity, this penalty function will suppress rapidly varying oscillations.

Our numerical implementation employs subsectional basis and testing functions. To demonstrate the effect of the proposed regularization, we consider a straight thin-wire segment of length $L = 0.5\lambda$ and circular cross-section with radius $a = 5 \cdot 10^{-3}\lambda$, and the incident field is a plane wave, polarized in the z -direction. We have computed the current on the wire using 1000 basis functions. Although the number of basis functions is extremely high for this example, it allows us to clearly demonstrate the effect of regularization.

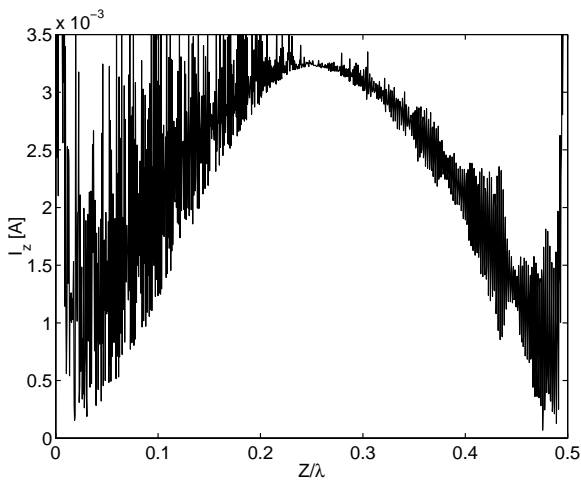


Figure 1a: Modulus of the current distribution versus the position on the wire, with $L = 0.5\lambda$, $a = 5 \cdot 10^{-3}\lambda$, without regularization, using 1000 basis functions. The incident field is a plane wave, polarized in the z -direction.

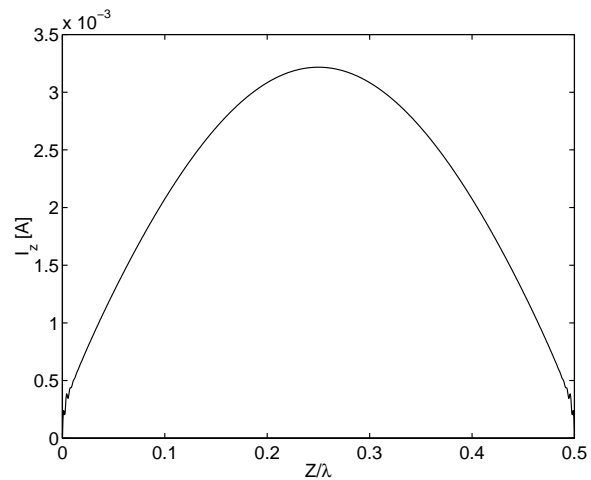


Figure 1b: Modulus of the current distribution versus the position on the wire, with $L = 0.5\lambda$, $a = 5 \cdot 10^{-3}\lambda$, after regularization and filtering, using 1000 basis functions. The incident field is a plane wave, polarized in the z -direction.

First, we have tried to solve the linear system by the conjugate-gradient method without regularization. After 65000 iterations, the residual error was $1.8 \cdot 10^{-3}$ and did not yet meet our termination criterion of 10^{-3} . The result for the current on the wire after 65000 iterations is shown in Figure 1a. Subsequently, we have computed the current from the regularized system with the above choice for the penalty function. The residual error in the conjugate-gradient method reached the termination criterion of 10^{-3} after 391 iterations. We have observed that the regularization drastically improves the convergence of the conjugate-gradient method and that the oscillations are suppressed, except at the end faces. This is due to the fact that the modes have not been taken into account. To improve the results for the current, we have applied post-processing. In particular, we have applied a filter based on local averaging by the stencil $(0.25, 0.5, 0.25)$. This stencil was applied nine times to the current coefficients after regularization. The result of the regularization and filtering procedure is presented in Figure 1b.

CONCLUSIONS

We have described two approaches to obtain robust results for the current on a straight wire segment. The first approach is to use the integral equation with exact kernel with a modified forcing function. As an alternative, we have proposed to use the integral equation with reduced kernel in combination with a regularization and filtering technique. This approach is substantiated by uniqueness and conditional existence results for the integral equation with reduced kernel.

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