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# Cancellation of Linear Intersymbol Interference for Two-Dimensional Storage Systems 

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#### Abstract

This paper discusses the cancellation of linear intersymbol interference (ISI) in two-dimensional (2-D) systems. It develops a theory for the error rate of receivers that use tentative decisions to cancel ISI. It also formulates precise conditions under which such ISI cancellation can be applied effectively. For many 2-D systems, these conditions are easily met, and therefore the application of ISI cancellation is of significant interest. The theory and the conditions are validated by simulation results for a 2-D channel model. Furthermore, results for an experimental 2-D optical storage system show that, for a single-layer disk with a capacity of 50 GB , a substantial performance improvement may be obtained by applying ISI cancellation.


Index Terms-Cross-talk cancellation, intersymbol interference cancellation, residual intersymbol interference, two-dimensional optical storage.

## I. Introduction

STEADILY increasing storage densities are a clear trend in storage systems. Increasing amounts of intersymbol interference (ISI) are a consequence of this trend. A possible technique to deal with this ISI is ISI cancellation. In this technique tentative decisions are used as input to an interference canceller, which attempts to remove those linear or nonlinear ISI components that are not expected by the main bit detector. These unexpected ISI components are denoted in the remainder as residual ISI (RISI) components. In general, RISI components originate from the fact that the equalizer is not able to perfectly shape the ISI structure induced by the storage channel into the ISI structure expected by the bit detector (defined by a so-called target response). The part of the RISI components that originates from symbols subsequent to the current one (i.e., from future symbols), is denoted as precursive ISI.

The general structure of an ISI cancellation scheme is depicted in Fig. 1 for a one-dimensional (1-D) system. The equalized signal $y_{k}$ is a distorted and noisy version of the recorded bits $a_{k}$. Based on tentative decisions $\hat{a}_{k}$ with respect to these recorded bits $a_{k}$, the interference canceller generates an estimate of the RISI contained in the equalized signal $y_{k}$. To make the scheme causal, $y_{k}$ is delayed until tentative decisions are available for all symbols that induce precursive RISI. The RISI estimate is subtracted from this delayed version of $y_{k}$ to produce a signal that is ideally free from RISI. This signal is used as input of the main detector. This main detector produces the final decisions $\hat{\hat{a}}_{k}$. The benefits of the ISI cancellation technique are: simplicity, ability to handle both linear as nonlinear RISI components and absence of loops (i.e., ISI cancellation can be pipelined).

In general, ISI cancellation works effectively if two conditions are fulfilled. First, all the cancelled RISI components should have a relatively small magnitude (with respect to the magnitude of the target-response components). Second, errors

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Fig. 1. General structure of an ISI cancellation scheme.
that affect the main and the tentative decisions measured at the same instant should be statistically independent. In practice the latter condition is fulfilled if the cancelled RISI components originate from symbols that have sufficient "temporal separation" from the current symbols [1]. In a 1-D system, the number of small RISI components tends to be limited and for this reason the performance gain of ISI cancellation is usually small. As a result ISI cancellation is seldom used in practical systems because the substantially increased complexity is not justified by the marginally increased performance.

Besides the trend of increasing storage densities, there is also a general trend of increasing data rates. The development of two-dimensional (2-D) storage systems fits with this trend and permits exploitation of parallelism. The parallelism is achieved by packaging data in a group of adjacent tracks or rows and by parallel processing of these tracks [2]. The physical proximity of the tracks causes 2-D ISI during readout. The main topic of this paper is the use of linear ISI cancellation in 2-D systems to deal with 2-D RISI components. The 2-D ISI cancellation technique presented in this paper is general and can be applied to a variety of 2-D systems: MIMO, holographic storage, pageoriented optical memories, patterned magnetic media, and 2-D optical storage.

In this paper, we argue that the application of ISI cancellation is of significant interest for 2-D systems. A first argument is the fact that the number of small RISI components is increased considerably with respect to the 1-D case. As a result, the performance gain by applying the cancellation technique will also increase. A second argument is the fact that only little additional complexity may be required for the application of ISI cancellation techniques. Because 2-D detection is often accomplished
by several iterations of smaller detection units to avoid the complexity of a full 2-D Viterbi detector [3], [4], decisions of one of these smaller detection units can be used as tentative decisions by the canceller. As a result no additional bit detectors need to be implemented to produce these tentative decisions. Summarizing these arguments, with a limited additional complexity (only the interference canceller needs to be added, not an additional detector) ISI cancellation in a 2-D system may improve performance significantly.

In this paper, the attention is limited to the cancellation of linear ISI. This limitation is reasonable as in general the linear components account for the bulk of the total RISI and moreover in [1] the cancellation of nonlinear ISI components is shown to be ineffective.

An experimental 2-D optical storage system, called TwoDOS, is used to illustrate the performance improvement. For TwoDOS, a partial response maximum likelihood (PRML) receiver with a stripe-wise Viterbi detector (SWVD) was developed [5]. This SWVD performs two consecutive detection iterations. As a result, the outputs of the first iteration can be used as tentative decisions in an ISI cancellation scheme. The application of linear 2-D ISI cancellation in the PRML receiver improves the performance of the system significantly at very limited additional complexity.

In Section II, a historical overview of ISI cancellation schemes for 1-D systems is given. Also in Section II, existing reception techniques for 2-D systems are discussed. In Section III, the 2-D ISI cancellation scheme is proposed and analyzed. Finally, in Section IV, experimental results of linear 2-D ISI cancellation are presented for the TwoDOS system. These experimental results show a substantial performance improvement by applying linear 2-D ISI cancellation.

## II. Overview of ISI Cancellation

In 1-D systems, the adaptive canceller usually employed in echo cancellation was first applied to adaptive equalization by Gersho and Lim [6]. Their work was the extension of early work by Proakis [7]. They developed a cancellation structure that achieves the optimal performance, i.e., the isolated-pulse, matched filter reception. For this to happen, cancellation should be based on the actual data, i.e., on data without decisions errors. Because of the noncausal nature of the cancellation structure, it is necessary to use tentative decisions to synthesize the precursive RISI components. This cancellation structure is sometimes referred to as a two-stage equalizer. The first stage produces tentative decisions which are used by the cancellation filter while the second stage produces final decisions based on the equalizer output signal after cancellation of the RISI components. Gersho and Lim suggest the use of a linear equalizer (LE) as the first-stage equalizer. Significant signal-to-noise ratio (SNR) gains are observed using the canceller compared to an LE. A theory has been worked out on data-aided equalization techniques including linear, decision feedback, and canceller based equalizers [8]. In both [6] and [8], the analysis was based on the mean-square error criterion and ideal (i.e., correct) decisions are assumed. Wesolowski [9], [10] showed that the error performance of the canceller critically depends on the performance of
the first-stage equalizer. Therefore, a decision feedback equalizer was proposed as the first-stage equalizer. The replacement of the LE by a decision-feedback equalizer (DFE) yields a moderate improvement in performance. Based on the assumption that the final decisions will be better than the tentative decisions, it has been argued that replacing tentative decisions by final decisions to synthesize the postcursive RISI contribution (RISI originating from past symbols), will improve performance [11].

The combination of a linear ISI canceller combined with an error reducing circuitry, called quantized logical equalizer (QLE), has been proposed in [12]. In [13], through a simulation study, the performance of the canceller was shown to lag well behind the Viterbi Detector and to provide little improvement over a DFE at high recording densities. However for specific low-pass channels, the performance of the canceller was shown to approach that of a maximum-likelihood detector if the input data is constrained using a run-length-limited code [14]. The use of reliabilities produced by the error-correcting decoder in the ISI cancellation mitigates the error propagation [15]. This approach can be generalized to an iterative scheme of alternately cancelling RISI components and decoding. Furthermore, the ISI cancellation technique can be applied to nonlinear channels [11], [16], [17] resulting in SNR gains at the detector input. The corresponding error rate improvements are not shown. In [18] the problem of error propagation is addressed. Error propagation caused by the first stage can degrade the effectiveness of the cancellation technique severely. To mitigate the degradation, nonlinear cancellation combined with trellis coding was proposed. This scheme requires two Viterbi detectors, which leads to an increased complexity.

In general, the cancellation technique can be shown to be effective if the ISI that is being cancelled is "small" and if errors affecting final and tentative decisions are statistically independent [1], [19]. The latter condition basically means that the cancelled RISI components should originate from symbols that have sufficiently "temporal separation" from the current symbols.

In commercial 1-D storage systems, the application of ISI cancellation is limited. Two main reasons can be identified for this limited application: 1) the presence of an auxiliary (Viterbi) detector to produce the tentative decisions will cause a substantial increase in overall complexity and 2 ) the number of "small" ISI components that can be cancelled effectively is in general quite low, and as a result the potential performance improvement is small [1].

For 2-D storage systems, receiver structures have been presented based on linear equalization (LE) [20], [21], decision feedback equalization (DFE) [22]-[24], and iterative detection [25], [26]. The extension of the 1-D ISI canceller to its 2-D equivalent has however not been reported. In this paper, the application of cancellation techniques is described for 2-D storage systems. The major drawbacks of ISI cancellation in 1-D systems become much smaller for 2-D systems. First, due to the 2-D nature, it should be clear that the number of small ISI components is increased considerably with respect to the 1-D case. As a result, the performance gain by applying the cancellation technique will increase. Second, because of complexity issues,


Fig. 2. System model. The assumed channel model is depicted in combination with the ISI cancellation scheme.
bit detection in 2-D systems is hardly if ever accomplished by a full 2-D Viterbi detector [3], [4]. Instead, detection is often accomplished by several iterations of smaller detection units. As the detection process is divided into smaller subprocesses, it is possible to use decisions of one of these smaller subprocesses as tentative decisions in cancellation techniques. As a result, with a very limited additional complexity (only the interference canceller needs to be added), ISI cancellation in a 2-D storage system can yield a significant performance gain.

## III. Linear ISI Cancellation in 2-D Systems

In this section, linear 2-D ISI cancellation is presented and analyzed. The 2-D ISI cancellation scheme together with the assumed channel model is shown in Fig. 2. The signals $\underline{y}_{k}=$ $[y(k, 0) y(k, 1) \cdots y(k, R-1)]^{T}$ where $R$ represents the number of tracks, are output of a linear 2-D channel model. Because of the linearity, the overall 2-D channel response of each track $k$ can be separated into two 2-D responses: one response expected by the detector (the target response) and one response which is undesired in the detector (the undesired RISI response). Therefore the signal $y(k, r)$ belonging to track $r$ can be expressed as the sum of three terms. The first term is the desired partial response signal value (obtained by convolving the target response with the bits); the second term is the undesired RISI value (obtained by convolving the undesired RISI impulse response with the bits); and finally a noise term

$$
\begin{align*}
y(k, r)= & \sum_{p=0}^{\delta-1} \sum_{q=0}^{R-1} g_{0}(p, r-q) a(k-p, q) \\
& +\sum_{p=-\gamma}^{\lambda-1} \sum_{q=0}^{R-1} g_{1}(p, r-q) a(k-p, q)+n(k, r) \tag{1}
\end{align*}
$$

where $g_{0}(p, q)$ is the target response expected by the detectors, $g_{1}(p, q)$ is the RISI impulse response, $a(k, r)$ are the channel inputs $(a(k, r) \in\{-1,1\}), n(k, r)$ are noise samples, and $\delta$ is the length of the target response. Here, we assume all tracks have the same target and RISI response. In total, there are $R \gamma$ precursive RISI components and $R \lambda$ postcursive RISI components. Notice that $g_{1}(p, q)$ is a noncausal impulse response if $\gamma>0$ (precursive RISI). In this section, the noise samples are assumed to be white and Gaussian with variance $\sigma^{2}$ and the noise samples of different tracks are uncorrelated. For every track $r$ an interference cancellation filter with impulse response $c_{r}(p, q)$ generates an estimate of the RISI contained in $y(k, r)$ based on the tentative decisions $\hat{a}(i, j)$ for $i \in[0, R-1]$ and $j \in[k+M-\gamma, k+M+\lambda]$, where $M$ is the delay introduced by
the tentative detector. In general, to effectively cancel all RISI, the response $c_{r}(p, q)$ should be equal to $g_{1}(p, q)$. The RISI estimates of all tracks are subtracted from delayed versions of $\underline{y}_{k}$ (with a delay of $M+\gamma$ symbols such that tentative decisions are available for all precursive RISI components). The resulting signals $\underline{z}_{k}$ are used as inputs of the main detector which produces the final bit decisions $\underline{\hat{\hat{a}}}_{k}$. In this paper, we assume for simplicity reasons that the main and the tentative detector operate based on the same target response $g_{0}(p, q)$. This assumption is however not strictly needed. For example, a configuration is possible where besides $g_{1}(p, q)$ also part of $g_{0}(p, q)$ is cancelled and as a result the main detector operates on a truncated version of $g_{0}(p, q)$ [27].

In Section III-A, the symbol error rate of a Viterbi detector is analyzed in case RISI is present at the detector input. Subsequently, the symbol error rate of the ISI cancellation scheme of Fig. 2 is analyzed in Section III-B. The effect of error propagation on the overall receiver performance is discussed in Section III-C. Finally, Section III-D illustrates the effectiveness of the ISI cancellation scheme for three simplified channel models.

## A. Probability of Error of a Viterbi Detector in the Presence of RISI

The probability of symbol error of a 2-D Viterbi detector (VD) in the presence of 2-D RISI can be derived following the same approach as presented in [1] for a 1-D system. The differences between the 1-D and 2-D case together with the main conclusions are highlighted here. The probability of symbol error is

$$
\begin{equation*}
P_{e} \leq \sum_{\varepsilon \in E} w_{H}(\varepsilon) P(\varepsilon) \tag{2}
\end{equation*}
$$

where $E$ is the set of all possible 2-D error events $\varepsilon$ in which the null event (no errors) is excluded, $w_{H}(\varepsilon)$ is the number of symbol errors in the error event $\varepsilon$ and $P(\varepsilon)$ is the probability that error event $\varepsilon$ occurs. Also

$$
\begin{equation*}
P(\varepsilon)=P_{1}(\varepsilon) P_{2}(\varepsilon) \tag{3}
\end{equation*}
$$

where $P_{1}(\varepsilon)$ is the probability that the VD selects the path corresponding the error event instead of the path corresponding to the actual data sequence, i.e., $P(a(k, r))<P(a(k, r)+\varepsilon(k, r))$. Furthermore, $P_{2}(\varepsilon)$ is the probability of occurrence of a data sequence $a(k, r)$ that supports the error event $\varepsilon$.

Assume that the path associated with $\varepsilon$ differs from the correct path for $k_{0} \leq j \leq k_{1}$. Define

$$
\begin{align*}
f^{\mathbf{a}}(k, r) & =\sum_{p=0}^{\delta-1} \sum_{q=0}^{R-1} g_{0}(p, r-q) a(k-p, q) \\
i^{\mathbf{a}}(k, r) & =\sum_{p=-\gamma}^{\lambda-1} \sum_{q=0}^{R-1} g_{1}(p, r-q) a(k-p, q) \\
\Delta_{0}^{\varepsilon}(k, r) & =f^{\mathbf{a}^{\varepsilon}}(k, r)-f^{\mathbf{a}}(k, r) \\
\Delta_{1}^{\varepsilon}(k, r) & =i^{\mathbf{a}^{\varepsilon}}(k, r)-i^{\mathbf{a}}(k, r) \tag{4}
\end{align*}
$$

where $f^{a}(k, r)$ is the ideal detector input for track $r$ at time $k$ given a specific data sequence $\mathbf{a}=\left[\underline{a}_{k_{0}-\lambda+1} \cdots \underline{a}_{k_{1}+\gamma}\right], i^{\mathbf{a}}(k, r)$ is the total RISI for
track $r$ at time $k$ and $\mathbf{a}^{\varepsilon}$ is the data sequence according to the error event $\varepsilon$ (i.e., $a^{\varepsilon}(k, r)=,a(k, r)+\varepsilon(k, r)$ ).

The VD will select the wrong path if

$$
\begin{align*}
\sum_{j=k_{0}}^{k_{1}+\delta-1} \sum_{i=0}^{R-1}(y(j, i) & \left.-f^{\mathbf{a}}(j, i)\right)^{2} \\
& >\sum_{j=k_{0}}^{k_{1}+\delta-1} \sum_{i=0}^{R-1}\left(y(j, i)-f^{\mathbf{a}^{\varepsilon}}(j, i)\right)^{2} \tag{5}
\end{align*}
$$

or, equivalently, if

$$
\begin{align*}
& \sum_{j=k_{0}}^{k_{1}+\delta-1} \sum_{i=0}^{R-1} \Delta_{0}^{\varepsilon}(j, i) n(j, i) \\
& \quad>\frac{1}{2} \sum_{j=k_{0}}^{k_{1}+\delta-1} \sum_{i=0}^{R-1} \Delta_{0}^{\varepsilon}(j, i)^{2}-2 i^{\mathrm{a}}(j, i) \Delta_{0}^{\varepsilon}(j, i) \tag{6}
\end{align*}
$$

Let $N$ be the total number of symbols transmitted for a single track and $\mathcal{R}^{N}$ the vector space of $N$-tuples of real numbers. It is convenient to define the following vectors in $\mathcal{R}^{N}$

$$
\begin{align*}
\Phi_{0}^{\varepsilon}(r) & =\left[\Delta_{0}^{\varepsilon}(0, r) \Delta_{0}^{\varepsilon}(1, r) \cdots \Delta_{0}^{\varepsilon}(N, r)\right]^{T} \\
\Phi_{1}^{\varepsilon}(r) & =\left[\Delta_{1}^{\varepsilon}(0, r) \Delta_{1}^{\varepsilon}(1, r) \cdots \Delta_{1}^{\varepsilon}(N, r)\right]^{T} \\
\Lambda(r) & =\left[i^{\mathbf{a}}(0, r) \dot{i}^{\mathbf{a}}(1, r) \cdots i^{\mathbf{a}}(N, r)\right]^{T} \\
n(r) & =[n(0, r) n(1, r) \cdots n(N, r)]^{T} \tag{7}
\end{align*}
$$

Since clearly $\Delta_{0}^{\varepsilon}(k, r)=0$ for $k \geq k_{1}+\delta$ or $k<k_{0}$, we can express the condition for error event $\varepsilon$ to occur as

$$
\begin{equation*}
\sum_{r=0}^{R-1} \Phi_{0}^{\varepsilon}(r) x(r)^{T}>\frac{d(\varepsilon)}{2 \sigma} \tag{8}
\end{equation*}
$$

where $x(r)=(1 / \sigma) n(r)$ and $d(\varepsilon)$ is the Euclidian weight (denoted as Euclidian distance in the remainder of the text) of a particular error event $\varepsilon$ :

$$
\begin{equation*}
d(\varepsilon)=\sum_{r=0}^{R-1}\left\|\Phi_{0}^{\varepsilon}(r)\right\|^{2}-2 \Lambda(r) \Phi_{0}^{\varepsilon}(r)^{T} \tag{9}
\end{equation*}
$$

In the absence of any RISI, $\Lambda(r)=\left[\begin{array}{lll}0 & 0 & \cdots\end{array}\right]^{t}$, and the Euclidian distance reduces to $d_{0}(\varepsilon)=\sum_{r=0}^{R-1}\left\|\Phi_{0}^{\varepsilon}(r)\right\|^{2}$ which is the usual expression for the distance of error event $\varepsilon$.

## B. Probability of Error of the ISI Cancellation Scheme

It is convenient for the analysis of the error performance of the ISI cancellation scheme to assume that both the main and the tentative detectors of Fig. 2 are VDs. Both VDs are matched to the desired component of the channel (i.e., the target response $g^{0}$ ). An interference canceller with response $c(p, q)=g^{1}(p, q)$ for every track is fed with the decisions of the tentative VD. The probability of error for the cancellation scheme can be expressed as

$$
\begin{equation*}
P_{e} \leq \sum_{\varepsilon \in E, \varepsilon^{\prime} \in E_{0}} w_{H}(\varepsilon) P_{1}\left(\varepsilon, \varepsilon^{\prime}\right) P_{2}\left(\varepsilon, \varepsilon^{\prime}\right) \tag{10}
\end{equation*}
$$

where $E$ is the set of all error events without the null event, $E_{0}$ is the set of all error events including the null event, $w_{H}(\varepsilon)$ is the number of symbol errors in the error event $\varepsilon, P_{1}\left(\varepsilon, \varepsilon^{\prime}\right)$ is the probability that the tentative VD selects the path associated with error event $\varepsilon^{\prime}$ and the main VD selects the path associated with error event $\varepsilon$, and $P_{2}\left(\varepsilon, \varepsilon^{\prime}\right)$ is the probability of the occurrence of a data sequence a that supports both $\varepsilon$ and $\varepsilon^{\prime}$ as possible error events.

The probability $P_{1}\left(\varepsilon, \varepsilon^{\prime}\right)$ can be computed by expressing the conditions for which both VDs make a decision error. The condition for the tentative VD is given by ( 8 ), whereas the condition for the main VD can be obtained by replacing $\Lambda(r)$ in condition (8) by $\Phi_{1}^{\varepsilon^{\prime}}(r)$. Therefore, the conditions for $\varepsilon$ and $\varepsilon^{\prime}$ are

$$
\begin{equation*}
\sum_{r=0}^{R-1} \Phi_{0}^{\varepsilon^{\prime}}(r) x>\frac{d\left(\varepsilon^{\prime}\right)}{2 \sigma} \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
\sum_{r=0}^{R-1} \Phi_{0}^{\varepsilon}(r) x>\frac{d\left(\varepsilon \mid \varepsilon^{\prime}\right)}{2 \sigma} \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
d\left(\varepsilon^{\prime}\right)=\sum_{r=0}^{R-1}\left\|\Phi_{0}^{\varepsilon^{\prime}}(r)\right\|^{2}-2 \Lambda(r) \Phi_{0}^{\varepsilon^{\prime}}(r) \tag{13}
\end{equation*}
$$

and

$$
\begin{equation*}
d\left(\varepsilon \mid \varepsilon^{\prime}\right)=\sum_{r=0}^{R-1}\left\|\Phi_{0}^{\varepsilon}(r)\right\|^{2}+2 \Phi_{1}^{\varepsilon^{\prime}}(r) \Phi_{0}^{\varepsilon}(r) \tag{14}
\end{equation*}
$$

Conditions (11) and (12) define a region in $\mathcal{R}^{N}$ delimited by two $N-1$-dimensional hyperplanes. It is always possible to introduce an orthogonal transformation such that $\Phi_{0}^{\varepsilon}$ and $\Phi_{0}^{\varepsilon^{\prime}}$ lie on a 2-D plane. Then the hyperplanes become simple straight lines and the region can be easily visualized. As a result the joint probability of $\varepsilon$ and $\varepsilon^{\prime}$ can be computed by integrating the 2-D Gaussian density with unit variance

$$
\begin{equation*}
\mathcal{N}(x, y)=\frac{1}{2 \pi} e^{-\frac{1}{2}\left(x^{2}+y^{2}\right)} \tag{15}
\end{equation*}
$$

Three cases are of interest:

- Case I: The two vectors $\Phi_{0}^{\varepsilon}$ and $\Phi_{0}^{\varepsilon^{\prime}}$ are orthogonal. In this case the joint probability of $\varepsilon$ and $\varepsilon^{\prime}$ can be expressed as

$$
\begin{equation*}
P_{1}\left(\varepsilon, \varepsilon^{\prime}\right)=Q\left(\frac{d\left(\varepsilon^{\prime}\right)}{2 \sigma}\right) Q\left(\frac{d\left(\varepsilon \mid \varepsilon^{\prime}\right)}{2 \sigma}\right) \tag{16}
\end{equation*}
$$

- Case II: The same error event occurs in the tentative and the main detectors $\varepsilon=\varepsilon^{\prime}$, and $\Phi_{0}^{\varepsilon}=\Phi_{0}^{\varepsilon^{\prime}}$. Because the two conditions (12) and (11) must be satisfied, the joint probability can be expressed as

$$
\begin{equation*}
P_{1}\left(\varepsilon, \varepsilon^{\prime}\right)=Q\left(\frac{\max \left(d\left(\varepsilon^{\prime}\right), d\left(\varepsilon \mid \varepsilon^{\prime}\right)\right)}{2 \sigma}\right) \tag{17}
\end{equation*}
$$

In general, $d\left(\varepsilon^{\prime}\right)>d\left(\varepsilon \mid \varepsilon^{\prime}\right)$ and hence the probability of error of the ISI cancellation scheme is basically determined by the probability of error of the tentative detector.

- Case III: Vectors $\Phi_{0}^{\varepsilon}$ and $\Phi_{0}^{\varepsilon}$ are neither parallel nor orthogonal. In this case, the integral does not have a closed-form
solution but some tight upper bounds can be computed in many cases of interest.
In Section III-C, the effect of error propagation on the performance of the receiver is studied.


## C. Error Propagation in the Receiver Using Tentative Decisions for ISI Cancellation

For each $\varepsilon$, consider the set $E_{\varepsilon}$ of all those events $\varepsilon^{\prime}$ that satisfy the condition

$$
\begin{equation*}
\sum_{r=0}^{R-1} \Phi_{1}^{T}\left(r, \varepsilon^{\prime}\right) \Phi_{0}(r, \varepsilon)=0 \tag{18}
\end{equation*}
$$

This condition is obviously met in case $\Phi_{1}\left(r, \varepsilon^{\prime}\right)=0$ for $r=$ $[0, R-1]$ (the case of no RISI), but also in case $\Phi_{1}\left(r, \varepsilon^{\prime}\right)$ is orthogonal to $\Phi_{0}(r, \varepsilon)$. Therefore, as a result of (14), the distance of event $\varepsilon$ is not affected by the existence of event $\varepsilon^{\prime}$ in the tentative detector. The summation of (10) can then be split into two terms as follows:

$$
\begin{equation*}
P_{e} \leq \mathcal{P}_{1}+\mathcal{P}_{2} \tag{19}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathcal{P}_{1}=\sum_{\varepsilon \in E, \varepsilon^{\prime} \in E_{\varepsilon}} w_{H}(\varepsilon) P_{1}\left(\varepsilon, \varepsilon^{\prime}\right) P_{2}\left(\varepsilon, \varepsilon^{\prime}\right) \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathcal{P}_{2}=\sum_{\varepsilon \in E, \varepsilon^{\prime} \in \bar{E}_{\varepsilon}} w_{H}(\varepsilon) P_{1}\left(\varepsilon, \varepsilon^{\prime}\right) P_{2}\left(\varepsilon, \varepsilon^{\prime}\right) \tag{21}
\end{equation*}
$$

where $\bar{E}_{\varepsilon}$ is defined as the complement of $E_{\varepsilon}$ with respect to $E_{0}$ [as defined in the text following (10)]. The probability $\mathcal{P}_{1}$ represents the error rate of the main VD in case RISI is absent, i.e., the case of ideal cancellation. Following the results presented in Section III-A, this error rate can be expressed as

$$
\begin{equation*}
\mathcal{P}_{1}=\sum_{\varepsilon \in E} w_{H}(\varepsilon) P_{1}(\varepsilon) P_{2}(\varepsilon) \tag{22}
\end{equation*}
$$

where

$$
\begin{equation*}
P_{1}(\varepsilon)=Q\left(\frac{d_{0}(\varepsilon)}{2 \sigma}\right) \tag{23}
\end{equation*}
$$

is the probability of event $\varepsilon$ in the main detector.
The probability $\mathcal{P}_{2}$ represents the error propagation effect caused by the errors made by the tentative detector. The cancellation scheme will be effective if $\mathcal{P}_{2}<\mathcal{P}_{1}$, i.e., bit errors due to error propagation will not significantly determine the overall bit error rate. However if $\mathcal{P}_{2} \geq \mathcal{P}_{1}$, error propagation will mainly determine the overall bit error rate. Here, we analyze individual terms contributing to $\mathcal{P}_{2}$ and determine when they will lead to error propagation.

- Case I: For small RISI values, $d\left(\varepsilon \mid \varepsilon^{\prime}\right)$ will not be significantly smaller than $d_{0}(\varepsilon)$, i.e.,
$\sum_{r=0}^{R-1} \Phi_{1}^{T}\left(r, \varepsilon^{\prime}\right) \Phi_{0}(r, \varepsilon) \ll \sum_{r=0}^{R-1}\left\|\Phi_{0}(r, \varepsilon)\right\|^{2}$ and $\sum_{r=0}^{R-1} \Lambda^{T}(r) \ll \sum_{r=0}^{R-1}\left\|\Phi_{0}(r, \varepsilon)\right\|^{2}$. Therefore

$$
\begin{equation*}
Q\left(\frac{d\left(\varepsilon^{\prime}\right)}{2 \sigma}\right) Q\left(\frac{d\left(\varepsilon \mid \varepsilon^{\prime}\right)}{2 \sigma}\right) \ll Q\left(\frac{d_{0}(\varepsilon)}{2 \sigma}\right) \tag{24}
\end{equation*}
$$

and error events of this type will not cause error propagation. If $\mathcal{P}_{2}$ is dominated by these events, the cancellation scheme will be effective because $P_{e}$ is essentially determined by the error rate $\mathcal{P}_{1}$ (ideal cancellation). The performance of the tentative detector will not influence the overall performance as the performance of ideal cancellation determines the overall performance, i.e., no additional performance improvement can be obtained by improving the reliability of the tentative detector. For this reason a simple detector (e.g., a symbol-by-symbol detector) may be used as tentative detector. The cancellation scheme only ceases to be effective when RISI values tend to become large.

- Case II: The terms $\varepsilon=\varepsilon^{\prime}$ can contribute to $\mathcal{P}_{2}$ only if $\sum_{r=0}^{R-1} \Phi_{1}^{T}\left(r, \varepsilon^{\prime}\right) \Phi_{0}(r, \varepsilon) \neq 0$. For many channels, events satisfying this condition have large distances and as a result will not significantly contribute to $\mathcal{P}_{2}$. However, there are channels where events satisfying this condition do have minimum distance and as a result are the dominating terms. Since usually $d\left(\varepsilon \mid \varepsilon^{\prime}\right)<d\left(\varepsilon^{\prime}\right)$ (in words, cancellation with erroneous decisions is worse than no cancellation at all), the error rate of the cancellation scheme is essentially determined by the error rate of the tentative detector. Therefore, the cancellation will be ineffective.
- Case III: Nonorthogonal error events with minimum or nearly minimum distance will cause error propagation and as a result the cancellation will be ineffective.
To summarize, the main conditions for which ISI cancellation can work effectively are stated:
- the RISI must be small such that the main VD can make relatively reliable decisions even if the tentative detector makes a decision error and such that the tentative detector can make relatively reliable decisions in spite of the RISI;
- errors affecting the main and the tentative detector must be statistically independent.


## D. Examples

In this section, a simple example of a 2-D ISI cancellation scheme will be used to study the effect of error propagation. A system with three adjacent tracks is considered $(R=3)$, where the data symbols in the different tracks form a hexagonal structure. A full 2-D VD is used as tentative and as main bit detector. These detectors operate based on the target response

$$
g_{0}=\left[\begin{array}{cccccccc}
0 & 0 & 0 & c & c & 0 & 0 & 0  \tag{25}\\
0 & 0 & 0 & c & 1 & c & 0 & 0
\end{array}\right]
$$

where the parameter $c=0.3$. For every track $r$, the noise samples $n(k, r)$ are white and Gaussian with variance $\sigma^{2}$. The noise


Fig. 3. Example 1: BERs of the main and the tentative VD are plotted versus SNR together with BERs of ideal cancellation. Both theoretical (t) and simulation (s) results are shown. (a) $\beta / S=0.05$. (b) $\beta / S=0.10$.
samples of different tracks are uncorrelated. Furthermore, SNR is defined as

$$
\begin{equation*}
\mathrm{SNR}=10 * \log \left(\frac{S^{2}}{\sigma^{2}}\right) \tag{26}
\end{equation*}
$$

where $S^{2}=1+6 c^{2}$ is the total received energy per transmitted bit. Three different RISI impulse responses $g_{1}$ are considered.

1) Example 1: In this example, the RISI impulse response $g_{1}$ is defined as ( $g_{1}$ is aligned with $g_{0}$ defined above)

$$
g_{1}=\left[\begin{array}{cccccccc}
0 & 0 & \beta & 0 & 0 & \beta & 0 & 0  \tag{27}\\
0 & 0 & \beta & 0 & 0 & 0 & \beta & 0 \\
0 & 0 & \beta & 0 & 0 & \beta & 0 & 0
\end{array}\right]
$$

The different RISI components originate from symbols adjacent to the symbols that have nonzero coefficients in the target response $g_{0}$. The theoretical bit error rates (BERs) of the main and the tentative VD are plotted versus SNR in Fig. 3. In the left plot, $\beta / S=0.05$ and in the right plot $\beta / S=0.1$. Furthermore, the BER in case ideal cancellation is performed $\left(\mathcal{P}_{1}\right)$, is also shown in Fig. 3.

These theoretical results are validated by simulation results. For low SNRs, the theoretical BER of the tentative VD is not very accurate. This can be explained by the fact that in the simulation (as in reality) only one error event can occur for a single data pattern while the theoretical BER is calculated by summing the errors of all possible error events for a single data pattern. Furthermore, it must be noted that the theoretical and the simulated results for the main VD do not match well for large values of $\beta / S$. This is due to the fact that all tentative error events $\varepsilon^{\prime}$ that lead to case II and case III situations (nonorthogonal error events) are not taken into account because of the computational complexity and especially for large values of $\beta / S$ these error events may have small Euclidian distances. As a result, the theoretical BER of the main VD is not a very accurate estimate of the actual BER. The BERs presented in Fig. 3 show that the ISI cancellation scheme does not achieve the performance of ideal cancellation even for small values of $\beta / S$. For this example, the temporal separation between the symbols causing the RISI and the detected symbols is very limited and as a result the ISI cancellation will suffer from error propagation. However, the BER of the main VD is better than the BER of tentative VD. This can

a)

b)

Fig. 4. Example 2: BERs of the main and the tentative VD are plotted versus SNR together with BERs of ideal cancellation. Both theoretical (t) and simulation (s) results are shown. (a) $\beta / S=0.05$. (b) $\beta / S=0.10$.
be explained by the fact that due to the large number of RISI components, cancellation with a limited amount of erroneous decisions is better than no cancellation at all. As a result, impressive gains in BER are observed for both small and large RISI amplitudes despite the error propagation. Concluding, ISI cancellation for this RISI impulse response substantially improves the BER even though error propagation due to nonorthogonal error events prevents the system to achieve ideal cancellation performance.
2) Example 2: In this example the RISI impulse response $g_{1}$ is defined as

$$
g_{1}=\left[\begin{array}{cccccccc}
\beta & 0 & 0 & 0 & 0 & 0 & 0 & \beta  \tag{28}\\
\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

Theoretical and simulated BERs of the main VD, the tentative VD, and ideal cancellation are plotted in Fig. 4 versus SNR. For this RISI impulse response, the symbols causing RISI have sufficient temporal separation such that the vectors $\Phi_{0}(\varepsilon)$ and $\Phi_{0}\left(\varepsilon^{\prime}\right)$ are orthogonal for the error events with minimum distance. As a result error propagation is limited and the BER of the ISI cancellation scheme is almost equal to the BER of ideal cancellation. The ISI cancellation scheme ceases to work efficiently if the value $\beta / S$ becomes too large. In this case also nonminimum distance error events will cause error propagation and as a result the BER deteriorates. But the obtained BER improvement is so impressive (the SNR gain amounts to 5 dB at $\mathrm{BER}=10^{-2}$ ) that cancellation of RISI components with large amplitudes is nevertheless very valuable.
3) Example 3: In the previous examples, the RISI impulse response represented amplitude distortions. In this example, phase distortion is treated. The RISI impulse response $g_{1}$ is defined as

$$
g_{1}=\left[\begin{array}{cccccccc}
0 & 0 & \beta & 0 & 0 & \beta & 0 & 0  \tag{29}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

Theoretical and simulated BERs of the main VD, the tentative VD, and ideal cancellation are plotted in Fig. 5 versus SNR. Despite the fact that the error events of the main and the tentative VD are not always orthogonal, the performance of the ISI cancellation scheme is not severely affected by error propagation.


Fig. 5. Example 3: BERs of the main and the tentative VD are plotted versus SNR together with BERs of ideal cancellation. Both theoretical (t) and simulation (s) results are shown. (a) $\beta / S=0.05$. (b) $\beta / S=0.10$.


Fig. 6. Example of hexagonal structure for TwoDOS with $R=7$. The configuration of stripe-wise Viterbi detection is also shown for this example: two iterations of stripe detectors are used to detect the bits of the broad spiral.

From the results shown in Fig. 5, it is clear that the ISI cancellation scheme works efficiently in case RISI is present in the form of phase distortion.

The results presented in these examples clearly show that the conditions for effective cancellations are easily met in 2-D systems. Even if error propagation becomes an issue, the performance improvement is significant in all cases. The examples show that ISI cancellation is effective for amplitude and phase distortion and as a result ISI cancellation will also be effective for a combination of both distortions.

## IV. Experimental Results for TwoDOS

In the TwoDOS system, bits are stored on a hexagonal lattice [28]. In contrast with conventional optical recording (CD, DVD and BD), where the bits are stored in a single spiral (a 1-D sequence of bits), in TwoDOS the bits are organized in a so-called broad spiral. The broad spiral contains a number $R$ of bit tracks, stacked upon each other to form a hexagonal structure (see Fig. 6). Adjacent rotations of the broad spiral are separated by a guard band consisting of a bit track without any pits. The data is read out with an array of $R$ laser spots arranged such that each spot is centered on one of the bit tracks within the broad spiral. A multi-spot photo detector integrated circuit is used to generate a so-called high-frequency (HF) signal for every bit track.

A PRML receiver has been built for TwoDOS [5], [29]. It consists of a bit detector preceded by an adaptive equalizer, an adaptive DC compensator, an AGC, and a timing recovery loop. A 2-D VD performs joint bit detection on all bit tracks. To reduce the complexity of a full-fledged 2-D VD, the VD is divided into smaller processing units (called stripe VD). Each stripe VD covers a limited number of bit tracks (so-called stripes with a typical height of 2 or 3 bit tracks). This detection configuration is called a stripe-wise Viterbi detector (SWVD) [30] and is shown


Fig. 7. Estimated amplitudes (normalized with respect to $S$ ) of the RISI impulse response at the detector input for $r=3$. The $x$ axis is the tangential direction and the $y$ axis is the radial direction (centered around track $r=3$ ). Both axis are scaled in terms of $a_{\mathrm{bit}}$, where $a_{\mathrm{bit}}$ is the distance between two bits measured on the disk ( $a_{\text {bit }}=138 \mathrm{~nm}$ for a 50-GB disk).
in Fig. 6 together with the hexagonal structure. The SWVD consists of two detection iterations, where every iteration consists of stripe VDs which are organized in a V-shape. The first iteration is performed by stripe VDs ( $\mathrm{V}_{00}$ up to $\mathrm{V}_{06}$ in the figure) each covering two bit tracks, which results in a stripe VD with 16 states. The second iteration consists of stripe VDs ( $\mathrm{V}_{10}$ up to $\mathrm{V}_{14}$ in the figure) each covering three bit tracks which results in a stripe VD with 64 states. In every iteration, the binary output from a first stripe VD is passed to a next stripe VD to be used as side-information in the branch metric calculations [30]. The outputs of the first iteration are used as side-information in the second iteration.
Electronic beam recorded disks with a capacity of 50 GB (single layer) are placed in an experimental read-out system to produce experimental replay signals (for these disks $R=7$ ). The read-out is conducted under relatively favorable conditions (no scratches, no dropouts, limited amount of dust). The angle of the disk with respect to the laser beam can be varied in a controlled way to identify the performance of the system (BER) for varying angles (denoted as tilt angles). Subsequently, the replay signals are digitized and are applied to the TwoDOS receiver. In the TwoDOS receiver, an ISI cancellation scheme is implemented that uses the outputs of the first iteration as tentative decisions. The impulse response $c_{r}$ of the interference canceller is estimated using an identification scheme. This identification scheme estimates the RISI impulse response at the detector input in a data-aided way (using a training sequence). This RISI impulse response of the central track $(r=3)$ is shown in Fig. 7 for $-1.0^{\circ}$ of radial tilt. The RISI components are limited in amplitude $([-0.04,0.04])$. In this case, there are more than 20 RISI components with a significant amplitude ( $>0.01$ ). Hence, application of 2D ISI cancellation might be very beneficial. In the equivalent 1-D case, there would be only four or five significant RISI components and ISI cancellation would not be beneficial. Furthermore, RISI originating from symbols with limited temporal separation from the symbols of the target response (in the figure dots with indices $x \in\{-1,0,1\}, y \in$
$\{-0.866,0,0.866\})$ is nonnegligible. Based on the latter observation, ISI cancellation will suffer from error propagation and as a result the performance of ideal cancellation will not be achieved. In Section IV-A, the performance of ISI cancellation is discussed in case a SWVD is used that consists of two detection iterations. The reliability of the tentative decisions can be improved by inserting an additional detection iteration. This topology with three detection iterations is discussed in Section IV-B. Another method to improve the reliability of the tentative decisions is the application of cross-talk cancellation before the first detection iteration. This topology with cross-talk cancellation in combination with two and three detection iterations is discussed in Section IV-C.

## A. SWVD With Two Detection Iterations

The results of the experimental system with an SWVD that consists of two detection iterations are shown in Fig. 8. In this figure, together with the BER of tentative decisions (outputs of the first iteration), the BER after the second iteration is plotted versus the radial tilt angle for three different topologies: 1) no ISI cancellation; 2) ISI cancellation based on the outputs of the first SWVD iteration (denoted as decision-directed, DD cancellation); and 3) ISI cancellation based on the actual bits written on the disk (denoted as data-aided, DA cancellation which is clearly not applicable in practical systems but serves as reference for ideal cancellation). The first topology (no ISI cancellation) is the detection topology described in [3]. The performance of this topology will be used as reference to judge the performance of the different topologies.

The application of ISI cancellation is beneficial for this experimental system. The BER at nominal conditions (no radial tilt) is improved from $8.7 \times 10^{-5}$ to $3.9 \times 10^{-5}$. Also, the so-called bath tub curve (the BER versus tilt angles) has broadened, i.e., higher tilt angles can be allowed to achieve the same performance. For example, at a given $\mathrm{BER}=10^{-4}$, the allowed margins for radial tilt are improved from $\left[-0.2^{\circ}, 0.2^{\circ}\right]$ to $\left[-0.3^{\circ}, 0.5^{\circ}\right]$. Hence, ISI cancellation nearly doubles the allowed tilt margins for this experimental system. These results show that still a substantial amount of RISI is left at the input of the tentative detector. The comparison of the results of the DA and the DD ISI cancellation shows that although the ISI cancellation scheme considerably improves the performance, it does not reach the performance of ideal cancellation. This performance gap between DA and DD ISI cancellation indicates that error propagation is an issue for this kind of RISI impulse response.

## B. SWVD With Three Detection Iterations

By improving the reliability of the tentative decisions used in the cancellation scheme, error propagation may be lowered and as a result the overall performance may be enhanced. One way to improve this reliability is the insertion of an additional detection iteration with 2-track stripe VDs. This insertion will not substantially increase the overall complexity as an iteration with the 3-track stripe VDs is much more complex than an iteration with 2-track stripe VDs. As a result a topology with three detection iterations is used: first, two iterations of 2-track stripe VDs and, finally, one iteration of 3-track stripe VDs. Before the


Fig. 8. BER for different radial tilt angles (in degrees). BER is calculated for different receiver topologies: without ISI cancellation, with ISI cancellation based on the tentative decisions (DD ISI Cancel) and with ISI cancellation based on the actual data (DA ISI cancel).
second and before the third iteration interference cancellers are applied to cancel RISI components based on the decisions of respectively the first and the second iteration. The results of the detector with three iterations are presented in Fig. 9. These results should be compared with the results of the detector with two iterations (see Fig. 8).

By comparing the results of these two detectors, a couple of conclusions can be drawn.

- The insertion of an additional iteration of 2-track stripe VDs improves the reliability of the tentative decisions by a factor of 10 .
- Even without any ISI cancellation the additional detection iteration improves the tilt margin from $\left[-0.2^{\circ}, 0.2^{\circ}\right]$ to $\left[-0.3^{\circ}, 0.4^{\circ}\right]$.
- The BER of the final decisions by applying DD ISI cancellation is improved by inserting an additional iteration. The tilt margin in this case is improved from $\left[-0.3^{\circ}, 0.5^{\circ}\right]$ to $\left[-0.5^{\circ}, 0.7^{\circ}\right]$.
- Even for the configuration with three detection iterations, error propagation is still an important issue. This can be seen by comparing the results of the DA and the DD cancellation schemes. Especially for higher tilt angles, the performance gap can be explained by the fact that the RISI components with limited temporal separation from the target response have a large amplitude such that error propagation is enhanced (see Fig. 7). For small tilt angles, the amplitude of these RISI components is so small that error propagation is very limited and as a result, the DD ISI cancellation scheme (almost) achieves the performance of ideal cancellation (DA ISI cancellation).
Summarizing, the insertion of an additional iteration of 2-track stripe VDs improves the performance of the ISI cancellation scheme substantially.


## C. Cross-Talk Cancellation

In the detection process, not all bits are defined by the branch in the trellis of a stripe VD: some of them lie either within the


Fig. 9. BER for different radial tilt angles (in degrees). BER is calculated for different receiver topologies with three detection iterations: without ISI cancellation, with ISI cancellation based on the tentative decisions (DD ISI cancel) and with ISI cancellation based on the actual data (DA ISI cancel).


Fig. 10. Single-sided version of the cross-talk cancellation scheme.
bit-track immediately above the stripe or within the bit-track immediately below the stripe. These bits are considered as the sideinformation that is required for the stripe VD. In the SWVD, side-information is taken from the array of most recent bit decisions. However, in the first iteration no bit decisions are available yet. This means that side-information is either not present or has a very poor quality for example when it is generated by simple threshold detection. This lack of side-information is reflected in the fact that BER of the first detection iteration is quite low. This can be seen in Fig. 9, where a BER improvement of a factor of 10 can be observed between an iteration of 2-track stripe VDs without and with side-information (the tentative decisions of respectively the first and the second iteration).

A possible alternative that enables us to avoid using unreliable side-information is cross-talk cancellation (XTC). Here, we will start the discussion with a single-sided version of the conventional XTC scheme as shown in Fig. 10, i.e., only using one side-row for compensation of cross talk.

The filtering is performed using a finite impulse response (FIR) filter. The taps of this filter are denoted $f(p, r+1)$, and are adapted using a least mean square (LMS) algorithm based on some suitable criterion. We can write the XTC scheme as

$$
\begin{equation*}
\tilde{y}(k, r)=y(k, r)-\sum_{p} f(p, r+1) y(k-p, r+1) \tag{30}
\end{equation*}
$$

with $y(k, n)$ the detector input signal of row $r$ at time instant $k$. Signal $\tilde{y}(k, r)$ is the compensated signal.


Fig. 11. Scheme showing the first iteration of the stripe-wise detection including cross-talk cancellation. The white arrows indicate where the compensated signals are used. Note that the compensated signals are only used at the boundaries with low certainty.

Several criteria are possible to update the filter taps. For a practical implementation we have chosen to minimize the mean square error, where the error is taken as the difference between the actual result of cross-talk cancellation and the signal that we would expect after ideal cross-talk cancellation based on the target response. In [31], it is derived that a data aided LMS algorithm to minimize this squared error can be replaced by a nondata aided zero forcing (ZF) algorithm by scaling the resulting filter coefficients and adding a DC-term to the output signal. The big advantage of this scheme is that it does not need any preliminary decisions. The ZF based algorithm is applied in the first iteration of the stripe-wise detection at the low-certainty boundary according to the diagram in Fig. 11.

The experimental results where XTC was applied before the first iteration, are shown in Fig. 12. In the left part of the figure, BERs are shown for an SWVD with two detection iterations and in the right part of the figure an SWVD with three detection iterations was used. Three different topologies were employed: 1) no ISI cancellation; 2) DD ISI cancellation; and 3) DA ISI cancellation.

Based on Figs. 12 and 9, the following observations can be made.

- The reliability at the output of the first iteration with XTC is not as good as the one obtained after a second iteration of 2-track stripe VDs without XTC: $3.7 \times 10^{-3}$ with respect to $1 \times 10^{-3}$. This difference can be explained by the fact that the detection with XTC does not take the signal energy of the cancelled track into account during the detection process.
- The performance of the DD ISI cancellation scheme for topology with two detection iterations and XTC is comparable with the performance obtained by applying three detection iterations with in between DD ISI cancellation and without XTC (tilt margin are respectively $\left[-0.55^{\circ}, 0.8^{\circ}\right]$ and $\left.\left[-0.5^{\circ}, 0.7^{\circ}\right]\right)$. For this reason, the topology with two iterations in combination with XTC is preferable to the topology with three detection iterations, because the latter topology is more complex.
- For the SWVD with both two and three iterations, DD ISI cancellation almost achieves ideal cancellation performance for small tilt angles. As the tilt angle becomes larger, the amplitude of the RISI components increases and as a result error propagation is invoked. This error propagation causes the performance of the ISI cancellation scheme to deviate from the performance of ideal cancellation.
By applying XTC, an additional detection iteration of 2-track stripe VDs and ISI cancellation in between every detection iteration the tilt margin is improved from $\left[-0.2^{\circ}, 0.2^{\circ}\right]$ (see Fig. 8)


Fig. 12. BER for different radial tilt angles (in degrees). BER is calculated for different receiver topologies with XTC: without ISI cancellation, with ISI cancellation based on the tentative decisions (DD ISI cancel) and with ISI cancellation based on the actual data (DA ISI cancel). (a) Two detection iterations. (b) Three detection iterations.
to $\left[-0.7^{\circ}, 0.85^{\circ}\right]$. Summarizing, in all different receiver topologies (two or three detection iterations, with or without XTC), the application of ISI cancellation consistently improves the performance of the receiver substantially.

## V. CONCLUSION

In this paper, we have studied the application of linear ISI cancellation in 2-D systems. A first argument in favor of ISI cancellation is the fact that the number of small RISI components increases considerably with respect to the 1-D case. As a result, the performance gain by applying the cancellation technique also increases. A second argument is the fact that only little additional complexity may be required for the application of ISI cancellation. Because 2-D detection is often accomplished by several iterations of smaller detection units to avoid the complexity of a full 2-D Viterbi detector, decisions of one of these smaller detection units can be used as tentative decisions by the canceller. As a result, no additional bit detectors need to be implemented to produce these tentative decisions. Summarizing
these arguments, with a limited additional complexity (only the interference canceller needs to be added, not an additional detector), ISI cancellation in a 2-D system may improve performance significantly. Experimental results based on the read-out of a 50-GB single layer-disk were provided for an experimental 2-D optical storage system. These results show that the application of ISI cancellation nearly doubles the allowed tilt margin at a BER of $10^{-4}$. Furthermore, by applying an additional detection iteration or by applying cross-talk cancellation to improve the reliability of the tentative decisions, the allowed tilt margin increases even further.

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