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The Correspondence between Projective Codes and 2-Weight Codes

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Abstract — The hyperplanes intersecting a 2-weight code in the same number of points obviously form the point set of a projective code. On the other hand, if we have a projective code C , then we can make a 2-weight code by taking the multiset of points $\langle c \rangle \in PC$ with multiplicity $\gamma(w)$, where w is the weight of $c \in C$ and $\gamma(w) = \alpha w + \beta$ for some rational α and β depending on the weight enumerator of C . In this way we find a 1-1 correspondence between projective codes and 2-weight codes. The second construction can be generalized by taking for $\gamma(w)$ a polynomial of higher degree. In that case more information about the cosets of the dual of C is needed. Several new ternary codes will be constructed in this way.

Let C be a projective q -ary $[n, k, d]$ code, with nonzero weights w_1, \dots, w_t . Each subcode D of codimension 1 in C has nonzero weights w_1, \dots, w_t with respective frequencies f_1, \dots, f_t , say, and these frequencies satisfy

$$\sum f_i = q^{k-1} - 1$$

(this follows by counting all nonzero vectors in D), and

$$\sum (n_D - w_i) f_i = n_D (q^{k-2} - 1),$$

where n_D is the effective length of D , that is, the number of coordinate positions where D is not identically zero (this follows by counting the zero entries of all vectors in D).

Since C is projective, we have $n_D = n - 1$ for n subcodes D , and $n_D = n$ for the remaining $(q^k - 1)/(q - 1) - n$ subcodes of codimension 1.

It follows that for arbitrary choice of α, β the sum

$$\sum (\alpha w_i + \beta) f_i$$

does not depend on D but only on n_D , and hence only takes two values.

Fix α, β in such a way that all numbers $\alpha w_i + \beta$ are nonnegative integers, and consider the multiset X (in the projective space PC) consisting of the 1-spaces $\langle c \rangle$ with $c \in C$ taken $\alpha w + \beta$ times, if w is the weight of c . Then X is the point set of a 2-weight code.

For example from a ternary $[16, 5, 9]$ code with weight enumerator $0^1 9^{16} 12^{14} 15^{12}$ we can construct a ternary $[69, 5, 45]$ code with weight enumerator $0^1 45^{210} 54^{32}$ by taking $\alpha = \frac{1}{3}$ and $\beta = -3$.

Now, let C be a 2-weight linear q -ary $[n, k, d]$ code, with nonzero weights u and v . Let X be the corresponding multiset in $PG(k-1, q)$, so that $|X| = n$, and each hyperplane meets X in either $|X| - u$ or $|X| - v$ points. The hyperplanes meeting X in $|X| - u$ points ($|X| - v$ points, respectively) obviously form the point set of a projective code. This construction can be said to be the inverse of the first construction.

For example a ternary $[149, 5, 99]$ code has been proved to have weight enumerator $0^1 99^{222} 108^{20}$ if it exists [2]. The second construction would then yield a projective ternary self-orthogonal $[10, 5, 3]$ code, which cannot exist since 10 is not a multiple of 4.

The first construction can be generalized in the following way: Suppose we have an $[n, k, d]$ code C over $GF(q)$ with nonzero weights w_1, \dots, w_t . Let D be a subcode of codimension 1 of C . Let the frequencies of w_1, \dots, w_t in D be denoted by f_1, \dots, f_t . Then the Pless power moments [3] give us:

$$\sum_{i=1}^t v_i^r f_i = \sum_{j=0}^n B_j \left(\sum_{\nu=0}^r \nu! S(r, \nu) q^{k-1-\nu} \binom{n-j}{n-\nu} \right) - n^r,$$

where $v_i = n - w_i$, B_j is the number of codewords of weight j in the dual of D and $S(r, \nu)$ is a Stirling number of the second kind. Let $p_i(v) = \sum_{s=0}^{t-1} p_s^{(i)} v^s$ be the polynomial that is 0 for $v = v_h$, $h \neq i$ and is 1 for $v = v_i$, ($i = 1, \dots, t$). Consider the set X_i in the projective space PC consisting of 1-spaces $\langle c \rangle$ ($c \in C$) with multiplicity $p_i(v)$, where $w = n - v$ is the weight of c . Then X_i is a projective code that is intersected by D in $(\sum_{h=1}^t p_i(v_h) f_h)/(q-1)$ points. So if we can compute the weight enumerator up to weight $t-1$ of the dual of any codimension 1 subcode of C (which corresponds to a coset of the dual of C), then we can compute the weights in X_i ($i = 1, \dots, t$), using the Pless power moments. Once we know the weight in X_i corresponding to each coset of the dual for every i , we can construct codes by taking the union of some X_i 's.

For example if we take for C the $[12, 6, 6]$ extended ternary Golay code, then we find a ternary $[220, 6, 144]$ and a $[232, 6, 153]$ code, which both improve on the bounds in [1]. If we take for C the ternary $[7, 6, 2]$ code, then we find a $[140, 6, 90]$ and a $[203, 6, 132]$ code, which also improve on the bounds in [1].

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