Torsion of a crankshaft in a gascompressor

Technische
Universiteit
Eindhoven

Opleiding Wiskunde voor de Industrie Eindhoven

STUDENT REPORT 89-09

TORSION OF A CRANKSHAFT IN A GASCOMPRESSOR

Hugo Kleijer
Hedwig Snijders
Marco van der Veen

November 1989

ECMI

Den Dolech 2
Postbus 513
5600 MB Eindhoven

TORSION OF A CRANKSHAFT IN A GASCOMPRESSOR

Hugo Kleyer
Hedwig Snijders
Marco van der Veen

November 1989

SUMMARY

In this report a mathematical model is developed of an engine driving a single piston via a crankshaft. The aim is to calculate the torsion in the crankshaft when the piston is moving freely. The model is based on a second order ordinary differential equation, which is linearized.

Stable and unstable parameter regions are calculated with the aid of Floquet theory. Applying numerical methods the evolution in time of the torsion is computed.

It turns out that for most parameter values the system is stable, but there is a discrete spectrum of parameter values for which the system is unstable.

The torsion mainly follows the driving force, with a smaller oscillation super-imposed on it. These oscillations result from the undriven eigenfrequencies of the system.

CONTENTS

1. Introduction

2. The Mathematical Model

3. Stability Analysis

4. Analysis of Numerical Results

5. Conclusions

1. Introduction

We study the torsional behaviour of the crankshaft in a gascompressor, which is used e.g. to keep gas under pression in large scaled pipeline systems. Such a crankshaft transmits the rotating motion of a driving engine to pistons which pump gas. A schematic sketch of the compressor is presented in the following picture.

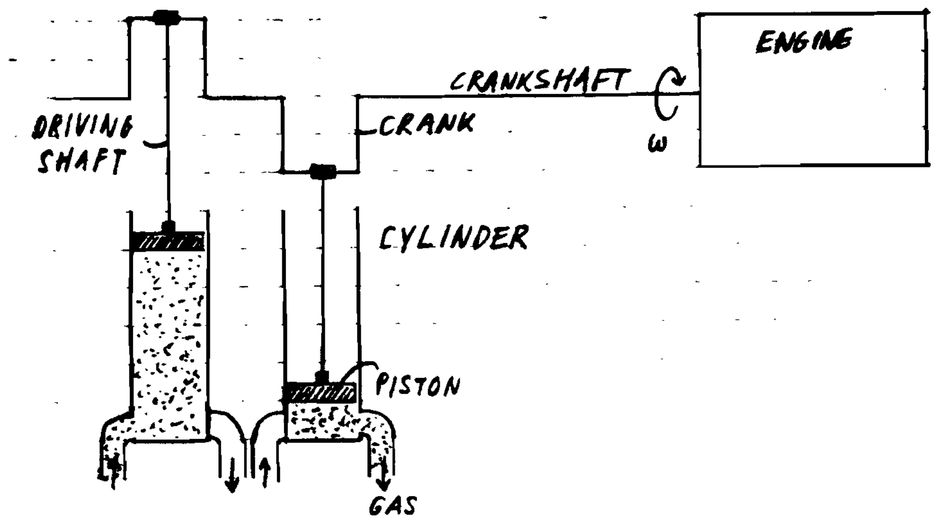

Fig. 1.1.

The pistons perform a transversal motion in a cylinder. By this, gas is pumped in and out of the cylinder. The mass forces and the load on the pistons cause torsion of the crankshaft.

Experience learns that the crankshaft is a vulnerable part of the gascompressor. Clearly the torsion of the crankshaft must not become too large.

In this report we show how the amplitude of the torsional oscillations can be calculated for a given parameter set describing a compressor. In our mathematical model only some aspects of the real system are taken out. However, it contains still the essential features of the phenomenon under consideration. A computer program is developed yielding results for a specific compressor configuration. In this report we use this program as a tool to analyse the qualitative torsional behaviour in more generality.

2. THE MATHEMATICAL MODEL

When trying to develop a mathematical model of a gascompressor we have to make some assumptions. For instance, we assume that all friction forces in the moving parts are negligible and, furthermore, that the driving engine is infinitely strong. I.e. it drives the crankshaft with a constant angular velocity ω. Besides, neither the load nor the torsion of the crankshaft has any influence on the speed of the engine.

We consider a compressor with one piston only. A clear insight of the single piston system is a necessary condition for the analysis of the multiple piston system. Schematically, the single piston system can be depicted as in figures 2.1 and 2.2.

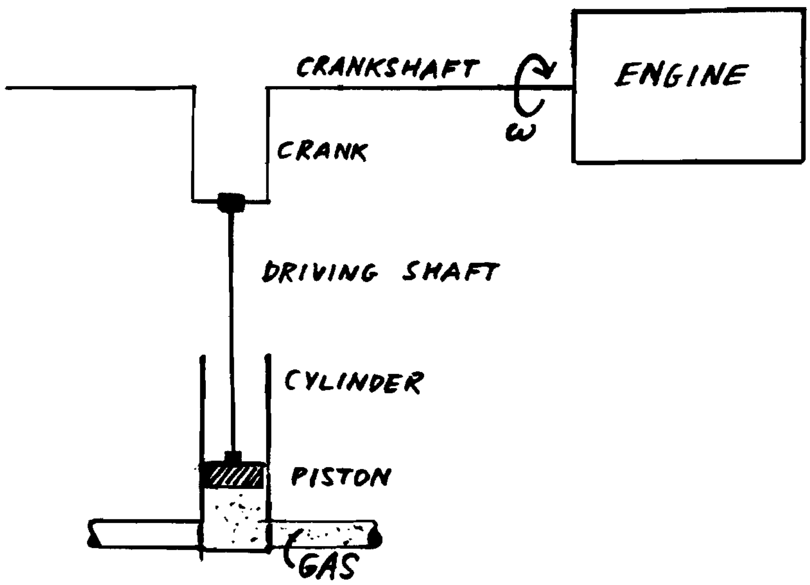

Fig. 2.1.

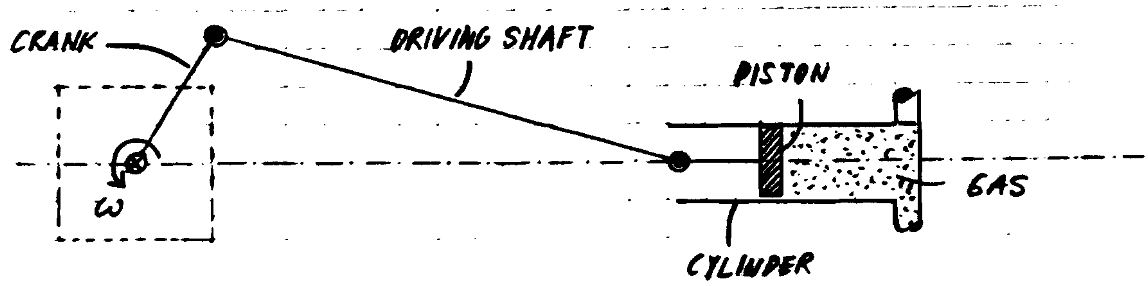

Fig. 2.2.

We have the following characteristic parameter values

moment of inertia of the crank	$I_{crank} = 10\,\mathrm{kg\,m^2}$
mass of the piston	$M_{pi} = 900\,\mathrm{kg}$
mass of the drivingshaft	$M_{ds} = 350\,\mathrm{kg}$
radius of the crankshaft	$R_{cs} = 0.15\,\mathrm{m}$
radius of the crank	$R_c = 0.15\,\mathrm{m}$
length of the drivingshaft	$L_{ds} = 1{,}0\,\mathrm{m}$
angular velocity of the crankshaft	$\omega = 40\,\mathrm{rad}/s$
torsional stiffness of the crankshaft	$q = 5 \cdot 10^6\,\mathrm{Nm}$
force applied on the piston by the gas	F_{gas}, $\lvert F_{gas} \rvert \leq 3 \cdot 10^5\,\mathrm{N}$.

Our numerical analysis only involves the behaviour of the unloaded system, i.e. the system without external gas forces F_{gas}. The analysis of this free system yields a first order prediction for the behaviour of the loaded system. However, in the equations in the next section the role of the gas forces is still explicitly mentioned.

In our model the engine drives the crankshaft with a constant angular velocity ω. Because of the inertia of the system, parts of the crankshaft will be twisted. The angle $\Delta\phi$ of the crankshaft twist is called the torsion. To put it differently, let $\phi(t)$ denote the angle of rotation of the ultimate left hand side of the crankshaft (at the crank, see Fig. 2.1 and 2.2) at time t, then $(\Delta\phi)(t) = \phi(t) - \omega t$.

We assume that the torsion of the crankshaft varies linearly from 0 to $(\Delta\phi)(t)$ going from the ultimate right hand side to the ultimate left hand side of the crankshaft.
If the mean torsion, i.e. $\Delta\phi$ divided by the length of the crankshaft exceeds a certain value, the crankshaft breaks. With the aid of the equation of motion for the system we shall predict the behaviour of $\Delta\phi$.

Our derivation of this equation of motion is based on the Lagrangian L of the system. This quantity is defined as $L = U - V$, where U is the total kinetic energy and V the potential energy. In our case it turns out that L depends explicitly on ϕ, $\dot{\phi}$ and t. Having calculated L the equation of motion follows from the Euler-Lagrange equation

(2.3) $$\frac{\partial L}{\partial \phi} = \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\phi}}\right] + M_{gas}$$

where M_{gas} denotes the external moment caused by the gas pumped in and out of the cylinder. To find the explicit expression for $L(\phi, \dot{\phi}, t)$ we derive expressions for the various energy terms in L.

The torsion generates a moment $M(\Delta\phi) = G\,\Delta\phi$ with G the torsional stiffness. The corresponding potential energy V is given by

(2.4) $$V = \int_0^{\Delta\phi} M(\xi)\,d\xi = \tfrac{1}{2}\,G\,\Delta\phi^2.$$

The total kinetic energy is the sum of the kinetic energies of the different parts of the gascompressor. So

$$U = E_{kin}\,[\text{piston}] + E_{kin}\,[\text{driving shaft}] + E_{kin}\,[\text{crank}] + E_{kin}\,[\text{crankshaft}]$$

or, in abbreviation,

(2.5) $$U = E_{pi} + E_{ds} + E_{crank} + E_{cs}.$$

For convenience we introduce the parameter $\lambda = \frac{R}{L}$ being the ratio of the length of the crank and the length of the driving shaft.
Let V_{pi} be the velocity of the piston and let M_{pi} be its mass. Then, cf. Fig. 2.2,

(2.5.1) $$E_{pi} = \tfrac{1}{2}\,M_{pi}\,V_{pi}^2 = \tfrac{1}{2}\,M_{pi}\,\dot{\phi}^2 \sin^2\phi\,R^2\left[1 + \frac{\lambda\cos\phi}{\sqrt{1-\lambda^2\sin^2\phi}}\right].$$

Let M_{ds} be the mass of the driving shaft and I_{ds} its moment of inertia. Let $\underline{x}(x_1, x_2)$ be the center of gravity of the driving shaft. Take constants α, a, b as pointed out if Fig. 2.3 below $(a+b=1)$.

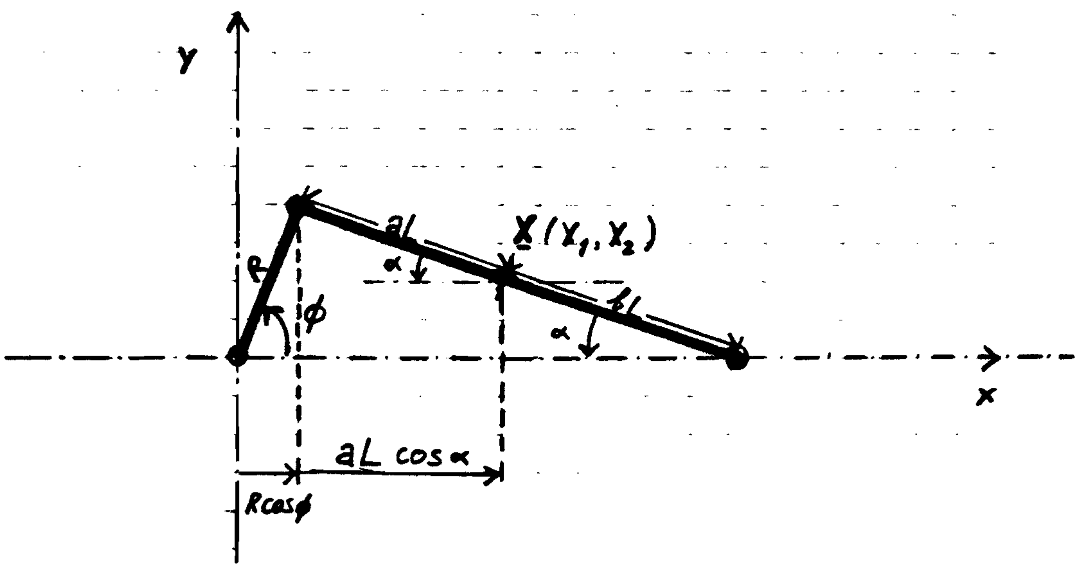

Fig. 2.3.

Then $E_{ds} = \frac{1}{2} M_{ds} \, |\dot{\underline{x}}|^2 + \frac{1}{2} I_{ds} \dot{\alpha}^2$. Now

$$x_1 = R \cos\phi + a L \cos\alpha$$

$$x_2 = b L \sin\alpha$$

and

$$\sin\alpha = \lambda \sin\phi.$$

It follows that $\dot{\alpha} = \dfrac{\lambda(\cos\phi)\dot{\phi}}{\cos\alpha}$, whence

$$\dot{x}_1 = \dot{\phi}\left[R \sin\phi + a L \frac{\lambda^2 \sin\phi \cos\phi}{\sqrt{1-\lambda^2 \sin^2\phi}}\right]$$

$$\dot{x}_2 = \lambda b L \dot{\phi} \cos\phi.$$

Summarizing we get for E_{ds},

$$(2.5.2) \qquad E_{ds} = \frac{1}{2} M_{ds} R^2 \dot{\phi}^2 \left[\left[\sin\phi + \frac{a\lambda \sin\phi \cos\phi}{\sqrt{1-\lambda^2 \sin^2\phi}}\right]^2 + b^2 \cos^2\phi\right] +$$

$$+\frac{1}{2} I_{ds} \dot{\phi}^2 \frac{\lambda^2 \cos^2 \phi}{1-\lambda^2 \sin^2 \phi} =$$

$$= \frac{1}{2} M_{ds} R^2 \dot{\phi}^2 \left[\left[\sin\phi + \frac{a \lambda \sin\phi \cos\phi}{\sqrt{1-\lambda^2 \cos^2 \phi}} \right]^2 + \right.$$

$$\left. + \left[\frac{I_{ds}}{M_{ds} L^2 (1-\lambda^2 \sin^2 \phi)} + b^2 \right] \cos^2 \phi \right].$$

For the crank we have,

$$E_{crank} = \frac{1}{2} I_{crank} \dot{\phi}^2. \tag{2.5.3}$$

Finally we compute the kinetic energy of the crankshaft. It depends on the rotational velocity of the crankshaft and on its moment of inertia I_{cs}. But its rotational velocity is not homogeneous but varies linearly from ω to $\dot{\phi} = \omega + \Delta\dot{\phi}$. So

$$E_{cs} = \frac{1}{2} I_{cs} \frac{1}{l} \int_0^l (\omega + \frac{r}{l} \Delta\dot{\phi})^2 \, dr = \tag{2.5.4}$$

$$= \frac{1}{2} I_{cs} (\omega^2 + \omega \Delta\dot{\phi} + \frac{1}{3} \Delta\dot{\phi}^2).$$

We henceforth assume that $R \ll L$ i.e. $\lambda \ll 1$. Correspondingly, we can linearize the kinetic energy with respect to λ:

$$U \approx \frac{1}{2} \dot{\phi}^2 [M_{pi} R^2 \sin^2 \phi (1 + 2\lambda \cos\phi) + \tag{2.7}$$

$$+ M_{ds} R^2 (\sin^2 \phi + 2 a \lambda \sin^2 \phi \cos\phi + \left[b^2 + \frac{I_{ds}}{M_{ds} L^2} \right] \cos^2 \phi$$

$$+ I_{crank}] + \frac{1}{2} I_{cs} (\omega^2 + \omega \Delta\dot{\phi} + \frac{1}{3} \Delta\dot{\phi}^2).$$

This expression for U can be put in a more transparant form,

$$U = \frac{1}{2} \dot{\phi}^2 I(\phi) + \frac{1}{2} I_{cs} (\omega \Delta\dot{\phi} + \frac{1}{3} (\Delta\dot{\phi})^2) + \frac{1}{2} I_{cs} \omega^2 \tag{2.7'}$$

where $I(\phi)$ denotes the term between rectangular brackets.

Having derived the expression for the Lagrangian, we obtain a differential equation for the torsion in the crankshaft from the Euler-Lagrange equation

$$\ddot{\phi} (I(\phi) + \frac{1}{3} I_{cs}) + \frac{1}{2} I'(\phi) \dot{\phi}^2 + G(\phi - \omega t) - M_{gas}(\phi) = 0. \tag{2.8}$$

We consider the situation that the crankshaft does not break. It follows that $\Delta\phi$ has to remain small. In fact for the parameter values given above we have $\Delta\phi \leq 0.1$ rad. (we observe that the maximally allowed value for $\Delta\phi$ depends also on the length of the crankshaft). Since $\Delta\phi$ is assumed to remain small we can linearize equation (2.8). This way we get the following linear equation of motion for the torsion:

$$\Delta\ddot{\phi}\,(I+\frac{1}{3}I_{cs})+\Delta\dot{\phi}\,(\omega I')+\Delta\phi(\tfrac{1}{2}\,\omega^2 I''-\tfrac{1}{2}\,(I')^2\,\omega^2+G-M_{gas}') \tag{2.9}$$

$$\tfrac{1}{2}\,I'\omega^2-M_{gas}=0.$$

Non dimensionalization yields

$$\dot{\xi}(I+\frac{1}{3}I_{cs})+\dot{\xi}\,I'+\xi\left[\tfrac{1}{2}\,I''-\tfrac{1}{2}\,(I')^2+\frac{G}{\omega^2}-\frac{M_{gas}'}{\omega^2}\right] \tag{2.10}$$

$$+\tfrac{1}{2}\,I'-\frac{M_{gas}}{\omega^2}=0$$

with $\tau=\omega t$, $\xi(\tau)=\Delta\phi(t)$ and

$$I(\tau)=R^2\,[2\,\lambda\cos\tau\,\sin^2\tau\,(M_{pi}+a\,M_{ds})+$$

$$+\sin^2\tau\,(M_{pi}+M_{ds})+$$

$$+\left[\frac{I_{ds}}{M_{ds}\,L^2}+b^2\right]\cos^2 t\,M_{ds}]+I_{crank}.$$

3. Stability analysis

Our stability analysis for the solutions of equation (2.10) is based on Floquet theory. First we present some general theorems and properties.

Our equation, is of the following general type

(3.1) $\ddot{y} + G(t)\dot{y} + H(t)y = RL(t)$

or, equivalently

(3.2) $\dot{\underline{x}} = A(t)\,\underline{x} + \underline{f}(t)$

where

$$\underline{x} = (y, \dot{y})^T,$$

$$A(t) = \begin{bmatrix} 0 & 1 \\ H(t) & G(t) \end{bmatrix}, \quad \underline{f}(t) = \begin{bmatrix} 0 \\ RL(t) \end{bmatrix}.$$

In our case A en $\underline{f}$ are periodic with period T.
First we look at the corresponding homogeneous equation

(3.3) $\dot{\underline{x}} = A(t)\,\underline{x}.$

It is not hard to see that if $\underline{x}(t)$ is a solution of (3.3) then $\bar{x}(t) = \underline{x}(t+T)$ is a solution of (3.3), also. Hence, if $X(t)$ is a fundamental matrix of (3.3) then also $X(t+T)$ is a fundamentaal matrix of (3.3).

Property a.
Let $X(t)$ be a fundamental matrix of (3.3). Then there exists a nonsingular matrix C such that $X(t+T) = X(t)C$ for all $t \geq 0$. The matrix C is referred to as the discrete transition matrix. We clearly have from $C = X^{-1}(0)\,X(T)$:

$$X(t+mT) = X(t)C^m, \quad m \in \mathbb{Z},\ t \in \mathbb{R}.$$

Property b.
Let $X_1(t)$ and $X_2(t)$ be both fundamental matrices of (3.3) with corresponding discrete transition matrices C_1 and C_2. Then the matrices C_1 and C_2 are similar, they have the same eigenvalues and the same Jordon form. These eigenvalues which thus only depend on $A(t)$, are referred to as characteristic multipliers, or Floquet multipliers.

Now the following important theorem holds:

Theorem (Floquet).
The equation $\dot{\underline{x}} = A(t)\underline{x}$ has a non-trivial solution $\underline{x}_\lambda(t)$ with the property that $\underline{x}_\lambda(T+t) = \lambda \underline{x}_\lambda(t)$, $t \in \mathbb{R}$, if and only if λ is a characteristic multiplier.
Let $X(t)$ be a fundamental matrix of (3.3) with $X(0) = I$. Then for every solution $\underline{x}(t)$ of (3.3) we

have

$$\underline{x}(t+T)=\lambda\underline{x}(t) \Leftrightarrow \underline{x}(t)=X(t)\underline{a}$$

$$\text{with } X(T)\underline{a}=\lambda\underline{a}.$$

Remark 1.

The equation (3.3) admits a non-trivial T-periodic solution if and only if it has a Floquet multiplier equal to one.

The T-periodic solutions are of the form $\underline{x}(t)=X(t)\underline{a}$ with $X(T)\underline{a}=\underline{a}$.

Remark 2.

For the solution $\underline{x}_\lambda(t)$ of (3.3) appearing in the above theorem we get

$$\underline{x}(t+mT)=\lambda^m\underline{x}(t), \quad t\in \mathbb{R}, m\in\mathbb{Z}.$$

It follows that if $|\lambda|>1$ then $|\underline{x}(t)|\to 0$ as $t\to\infty$ and, if $|\lambda|<1$, then $|\underline{x}(t)|\to 0$ as $t\to\infty$.

Next we consider the inhomogeneous equation

$$\text{(3.2)} \qquad \dot{\underline{x}}=A(t)\underline{x}+\underline{f}(t).$$

Let $X(t)$ be a fundamental matrix of (3.3) satisfying $X(0)=I$. Then the general solution of (3.2) is

$$\underline{x}(t)=X(t)\,[\underline{x}_0+\int_0^t X^{-1}(s)\underline{f}(s)\,ds]$$

where $\underline{x}_0$ denotes any initial condition.

It follows that if $\underline{x}(t)$ is a solution of (3.2) then also $\underline{x}(t+T)$ is a soluton of (3.2).

We also see that if $\underline{x}(0)=\underline{x}(T)$ then $\underline{x}(t)=\underline{x}(t+T)$, $t\in\mathbb{R}$. Now $\underline{x}(0)=\underline{x}(T)$ if and only if

$$\underline{x}_0=X(T)\,[\underline{x}_0+\int_0^T X^{-1}(s)\underline{f}(s)\,ds], \text{ or}$$

$$\text{(3.4)} \qquad (X(T)-I)\underline{x}_0=\int_0^T X^{-1}(s)\underline{f}(s)\,ds.$$

If the matrix $X(T)-I$ is invertible then equation (3.4) has precisely one solution $\underline{x}_0$. Put differently, if $\lambda=1$ is not a Floquet multiplier of (3.3) then (3.2) has exactly one T-periodic solution.

For our case (3.1) we determined the discrete transition matrix numerically and next calculated its eigenvalues (which are the Floquet multipliers). In order to investigate the dependence of the system on the parameters, we vary only one parameter while keeping the others constant. First, we varied the torsional stiffness G_p of the crankshaft. The remaining parameter values are

$$M_{ds}=350, \ M_{pi}=900, \ \lambda=0.15, \ R=0.15,$$

$I_{cs,0} = 7.5$ and $\omega = 36$.

Our results are presented in Fig. 3.1 and in Fig. 3.2.

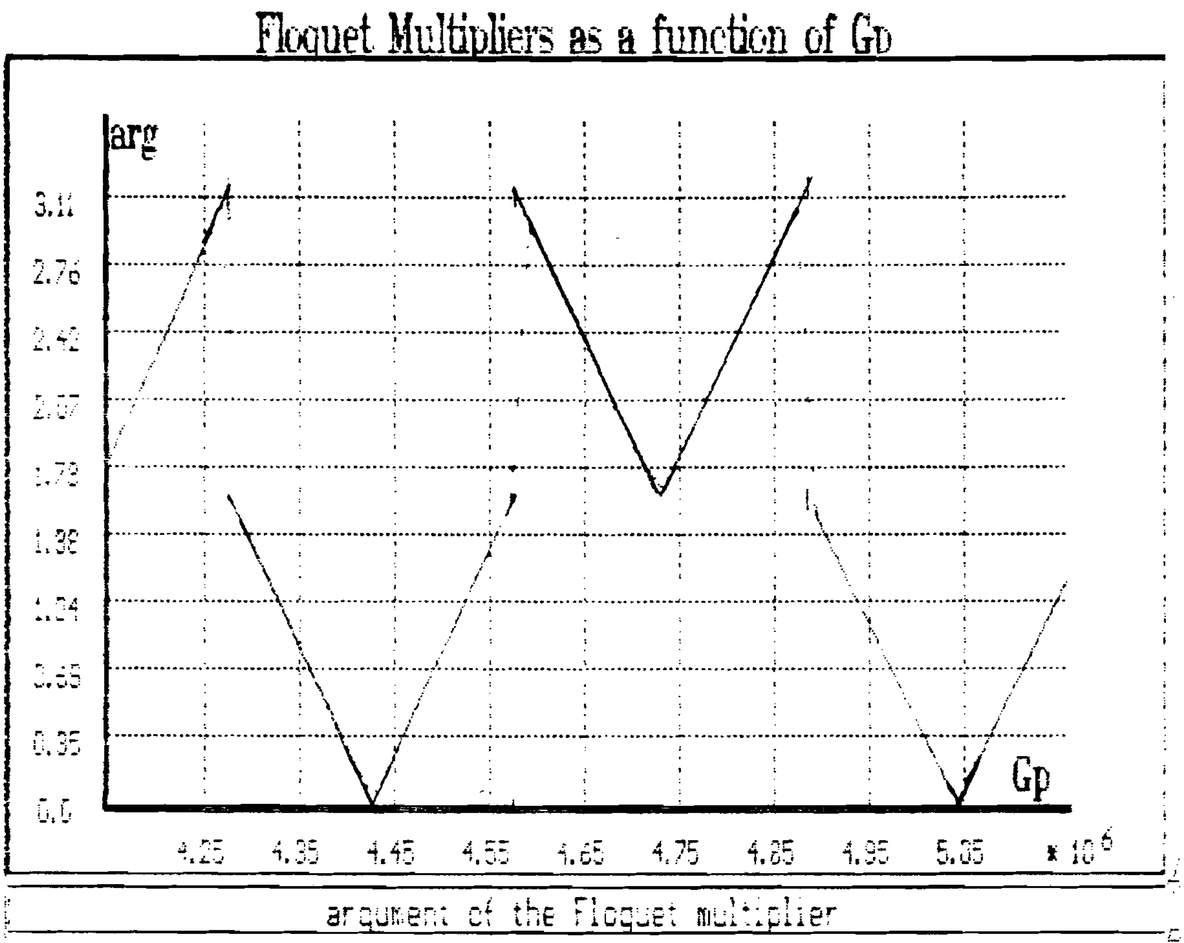

Fig. 3.1. The argument of the Floquet multiplier as a function of the torsional stiffness G_p.

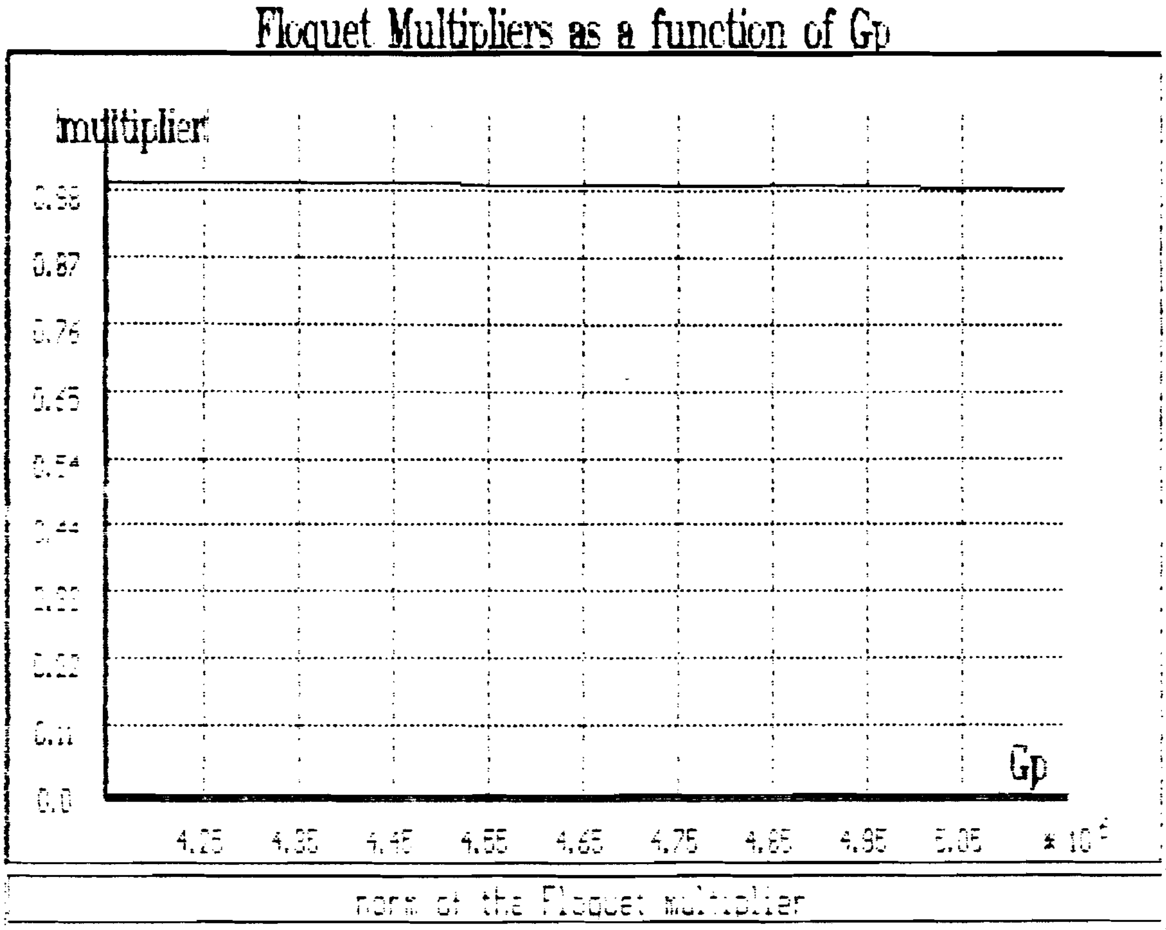

Fig. 3.2. The norm of the Floquet multiplier as a function of the torsional stiffness G_p.

Next, we varied the angular velocity ω of the driving engine. Taking M_{ds}, M_{pi}, λ, R and $I_{cs,0}$ as above and setting $G_p = 5 \cdot 10^6$ we get the following figures.

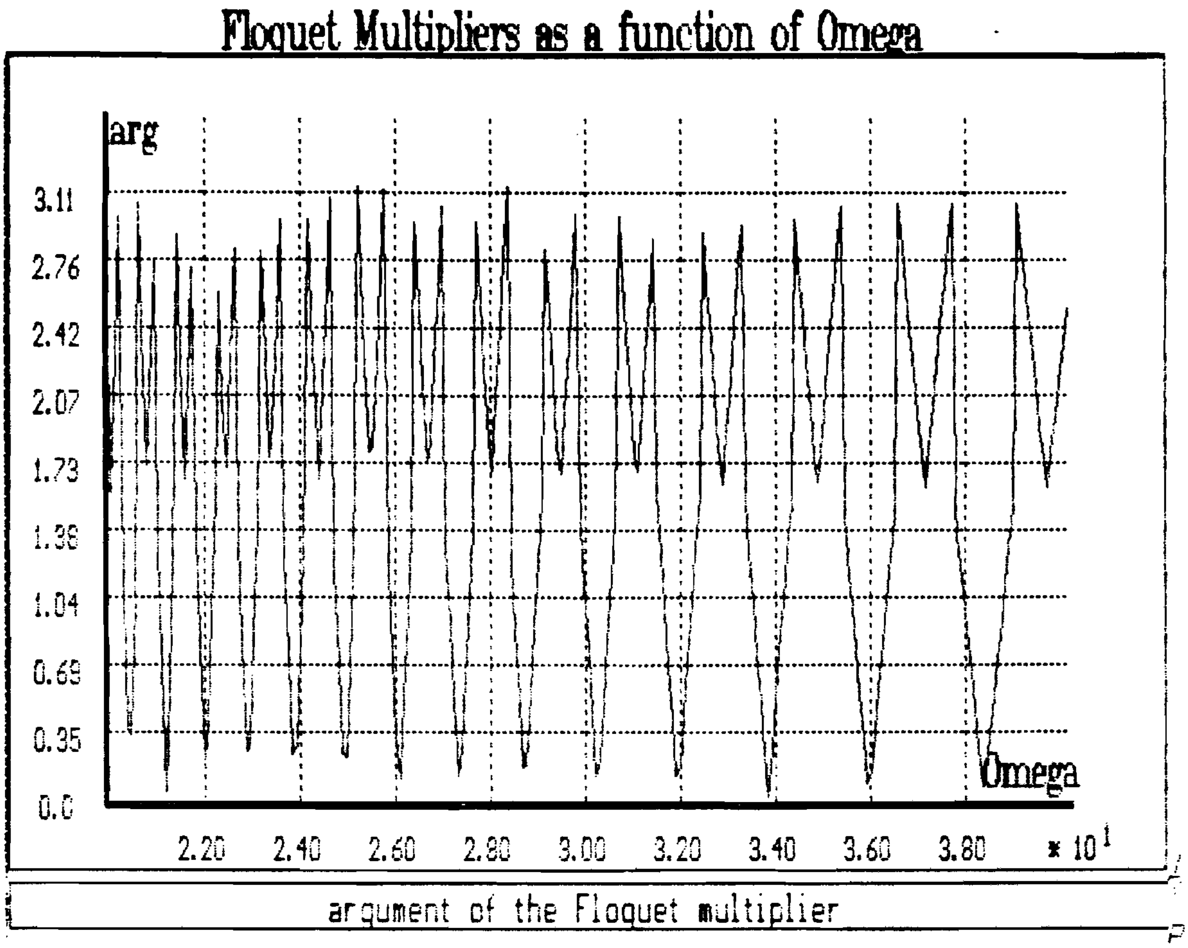

Fig. 3.3. The argument of the Floquet multiplier
as a function of the angular velocity of the driving engine, ω.

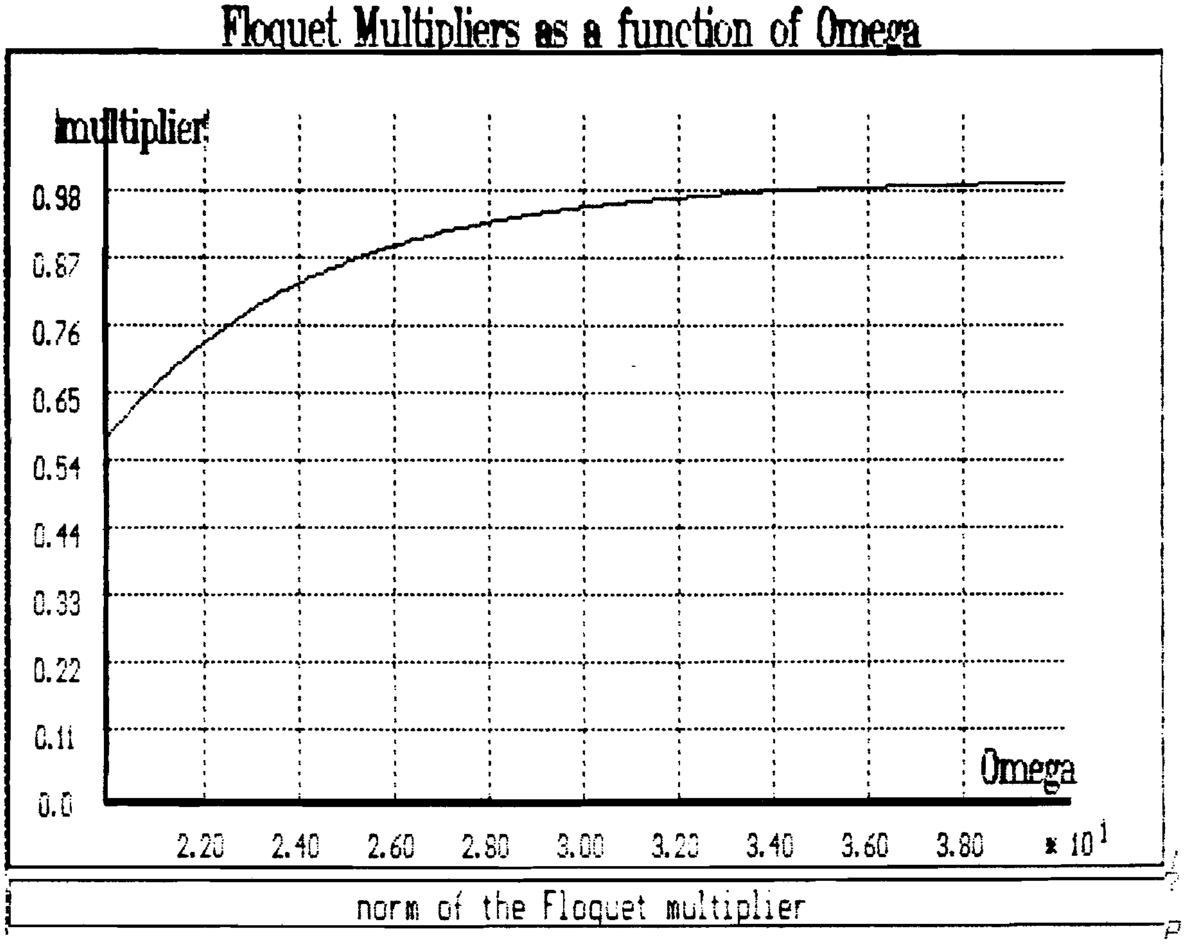

Fig. 3.4. The norm of the Floquet multplier as a function of the angular velocity of the driving engine, ω.

From what we have said before we can draw the conclusion that it may be unwise to choose parameter settings that yield a Floquet multiplier of 1. Near $G_p = 5 \cdot 03 \cdot 10^6$ (see Fig. 3.1) we have such a point, where the Floquet multiplier is exactly 1. So in practice we should avoid this value for G_p.

4. Analysis of Numerical Results

The differential equation describing the torsion of the crankshaft has been written as follows, cf. (2.10) and (3.1)

(4.1) $$\ddot{\xi} + G(t)\dot{\xi} + H(t)\xi = RL(t).$$

Here, ξ denotes the torsion; G, H and RL are functions dependant on t only. We remark that $RL = -2G$. Below we present plots of the functions G, H and RL. For these plots we used the default set of parameters. As we see, the three functions are periodical with the same period as the driving engine, except for H, which has a period half that of the driving engine. The Fourier spectra of G, H and RL show clear peaks, where the peak at 12.8 Hz corresponds to an oscillation frequency which is twice as high as the frequency of revolution of the driving engine ($\omega = 40$ rad/s corresponds to a frequency of about 6.5 Hz). The function G is small compared to H. So we expect that G has little influence on the behaviour of the torsion ξ of the crankshaft.

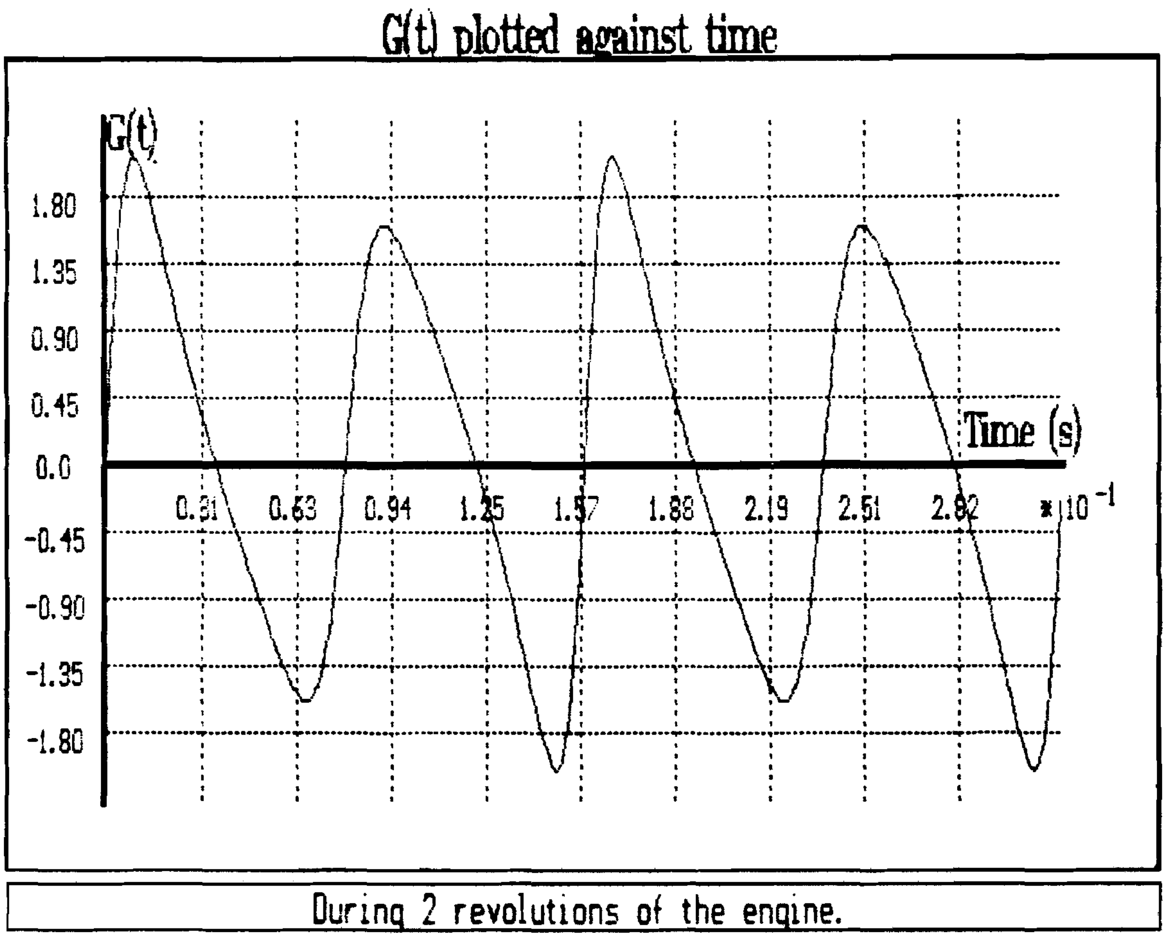

Fig. 4.1. The function $G(t)$ plotted against time.
We remark that $RL(t) = -2\,G(t)$.

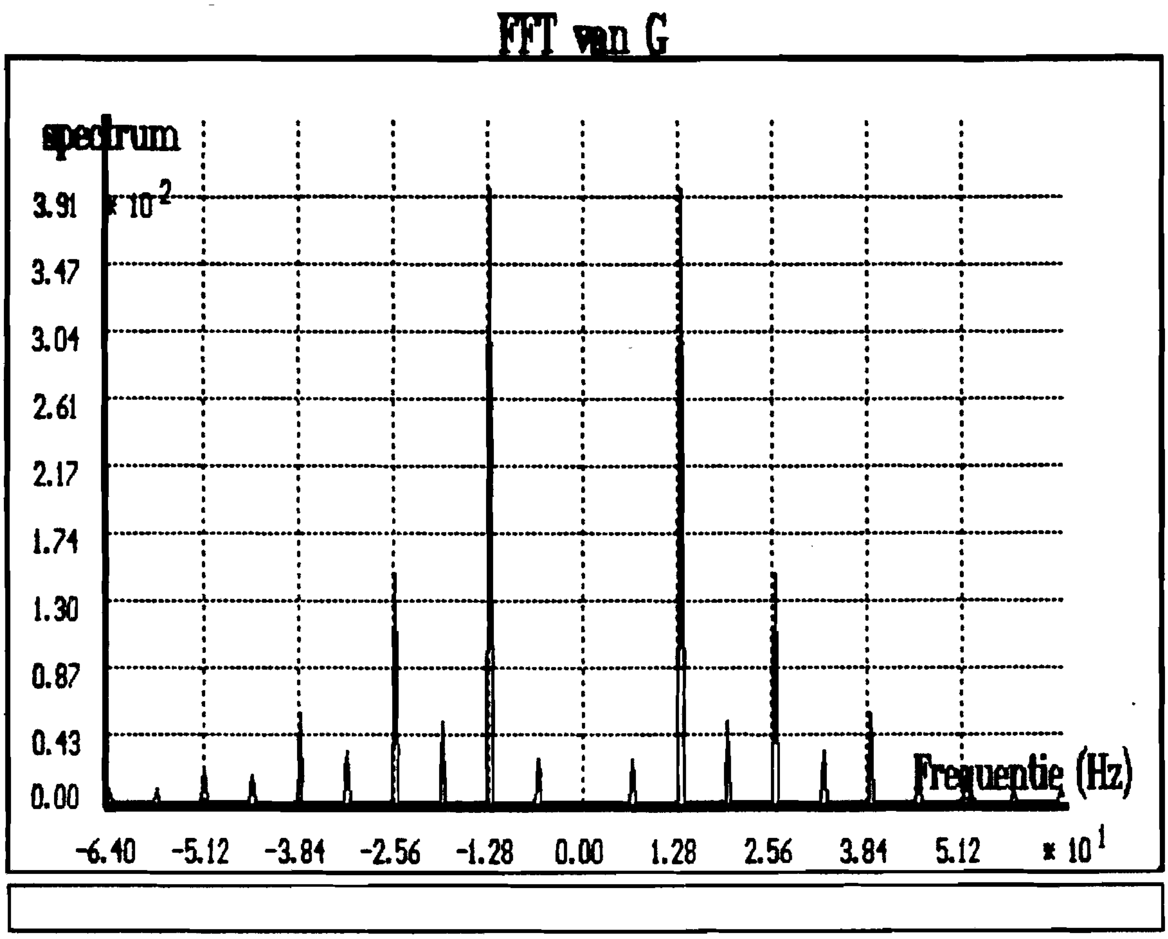

Fig. 4.2. The Fourier spectrum of $G(t)$ and $RL(t)$.

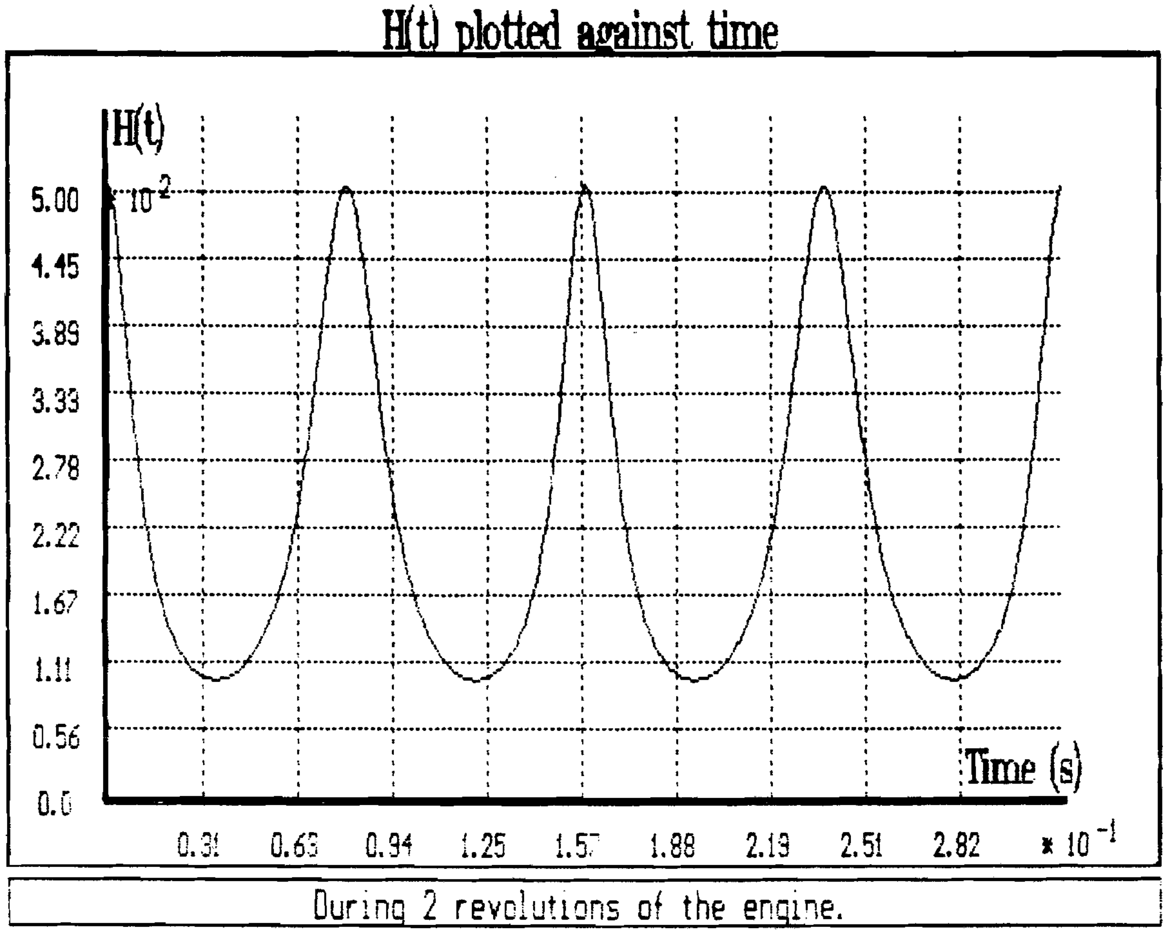

Fig. 4.3. The function $H(t)$ plotted against time.

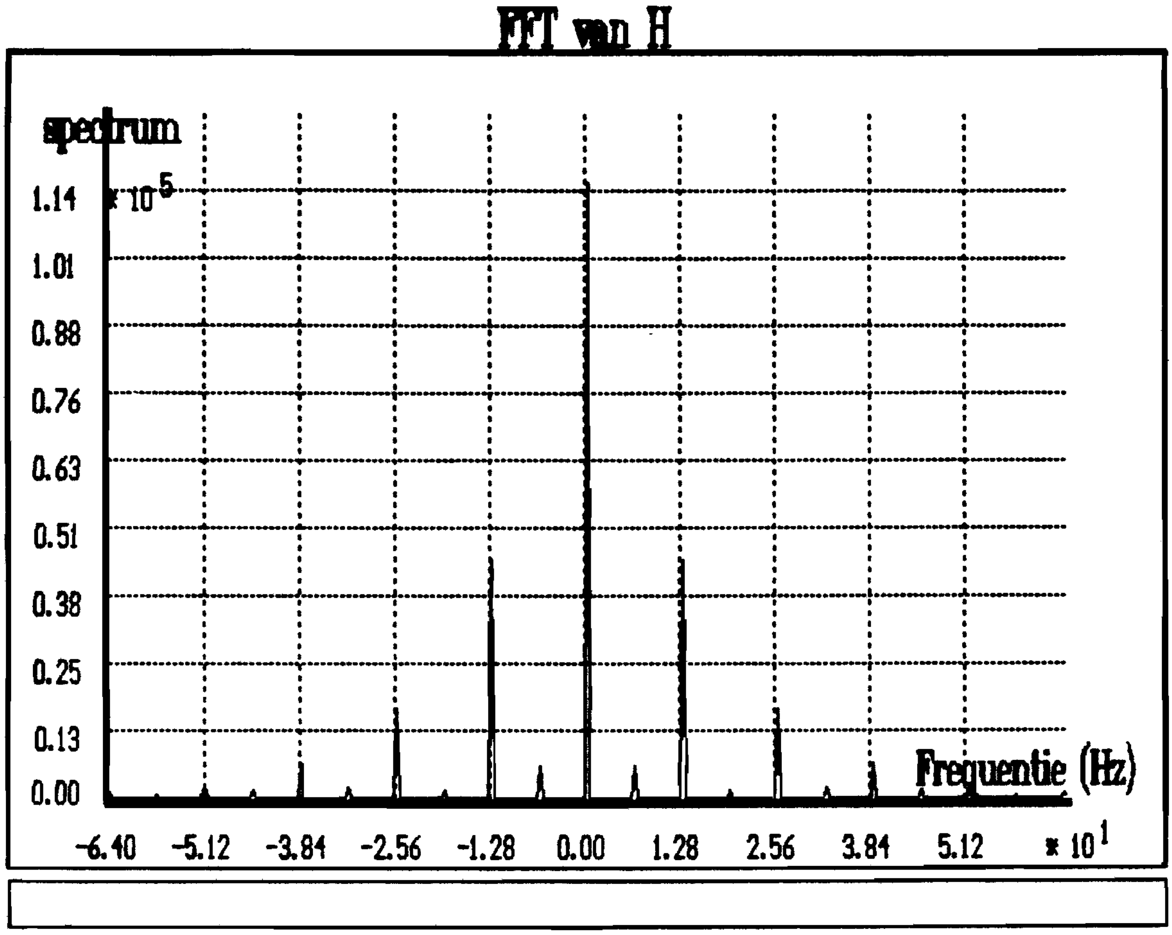

Fig. 4.4. The Fourier spectrum of $H(t)$.

We apply a Runge-Kutta method [1] to the above differential equation to solve it numerically. The convergence of the numerical scheme can be checked in Table (4.1).

dt	t	Φ	ξ	dξ/dt
0.09817	0.0425	1.5708	2.677085E-0003,	8.657284E-0003
0.09817	0.0849	3.1416	1.134168E-0006,	-3.371275E-0003
0.09817	0.1274	4.7124	-2.618490E-0003,	8.717049E-0003
0.09817	0.1698	6.2832	-7.826668E-0007,	-1.267036E-0002
0.04909	0.0425	1.5708	2.666426E-0003,	8.130728E-0003
0.04909	0.0849	3.1416	5.217905E-0005,	-1.351874E-0003
0.04909	0.1274	4.7124	-2.683650E-0003,	8.794694E-0003
0.04909	0.1698	6.2832	7.237811E-0005,	-1.182079E-0002
0.02454	0.0425	1.5708	2.663827E-0003,	8.073006E-0003
0.02454	0.0849	3.1416	7.157798E-0005,	-1.267518E-0003
0.02454	0.1274	4.7124	-2.693681E-0003,	8.966116E-0003
0.02454	0.1698	6.2832	1.045848E-0004,	-1.217703E-0002
0.01227	0.0425	1.5708	2.663518E-0003,	8.070331E-0003
0.01227	0.0849	3.1416	7.285155E-0005,	-1.275614E-0003
0.01227	0.1274	4.7124	-2.693694E-0003,	8.975759E-0003
0.01227	0.1698	6.2832	1.059864E-0004,	-1.222095E-0002

Table (4.1). Using the default of parameters, but taking $\omega = 37$ and $G_p = 4 \cdot 21 \cdot 10^6$, we calculate a numerical solution of equation (4.1). We use ever decreasing time-steps, corresponding to 64, 128, 256 and 512 time steps per revolution of the driving engine.
The symbol *dt* denotes the time step, *t* the time and Φ the angle through which the driving engine rotated during time *t*. The symbol ξ denotes the torsion and *dξ/dt* its time-derivative.

The results of our calculations are depicted in the plots below. They show that the shape of the curve through the (t, ξ) plane vaguely resembles that of the right hand side *RL*(t).

The Fourier spectrum shows very clear peaks, the highest of them at 12.8 *Hz*. Furthermore there are clear peaks at 6.5 *Hz*. All three of these peaks are also found in the spectra of *G*,*H* and *RL*. It seems though, as if most of the peaks above 19.2 *Hz* have canceled.

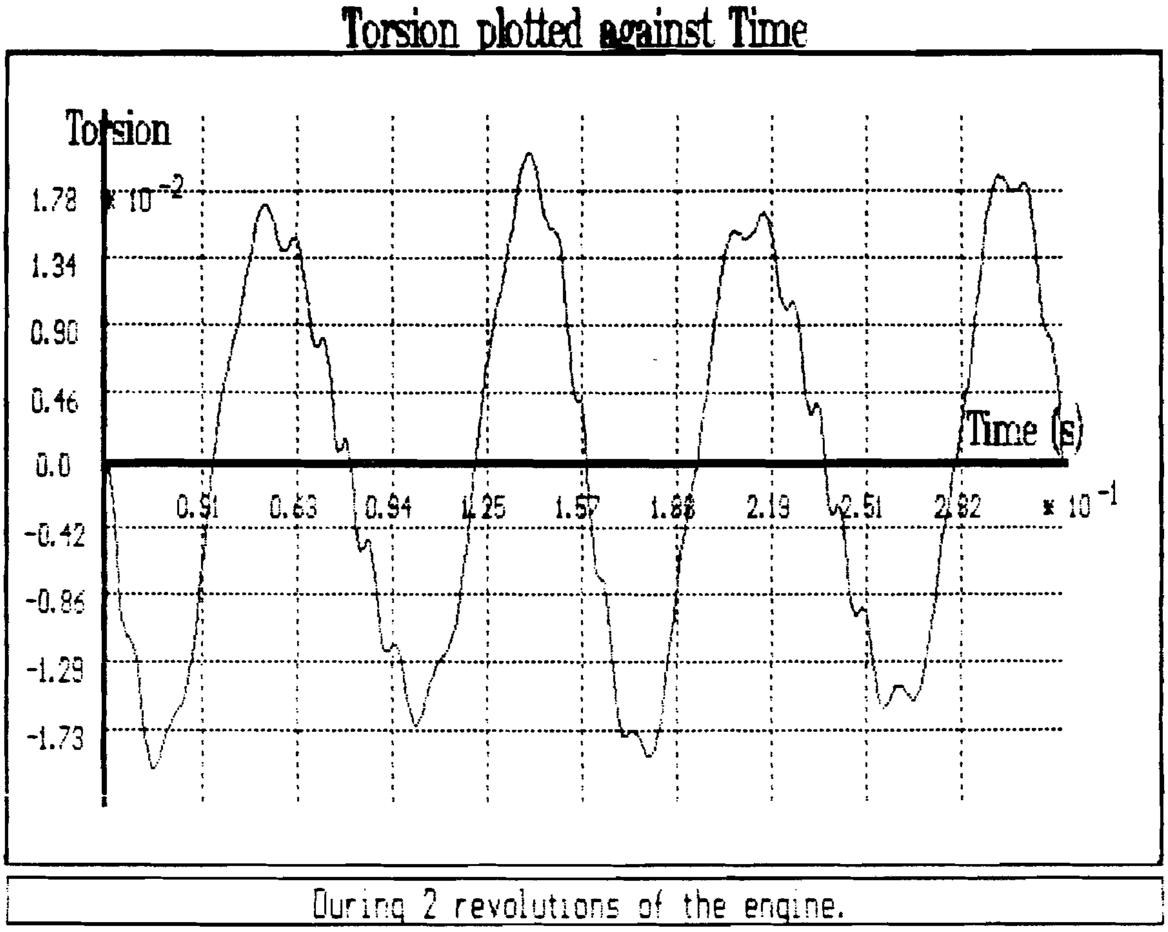

Fig. 4.7. The torsion ξ plotted against time, using the default set of parameters and 256 time steps per revolution of the driving engine. Initial conditions are $\xi = 0$, $d\xi/dt = -0.001$.

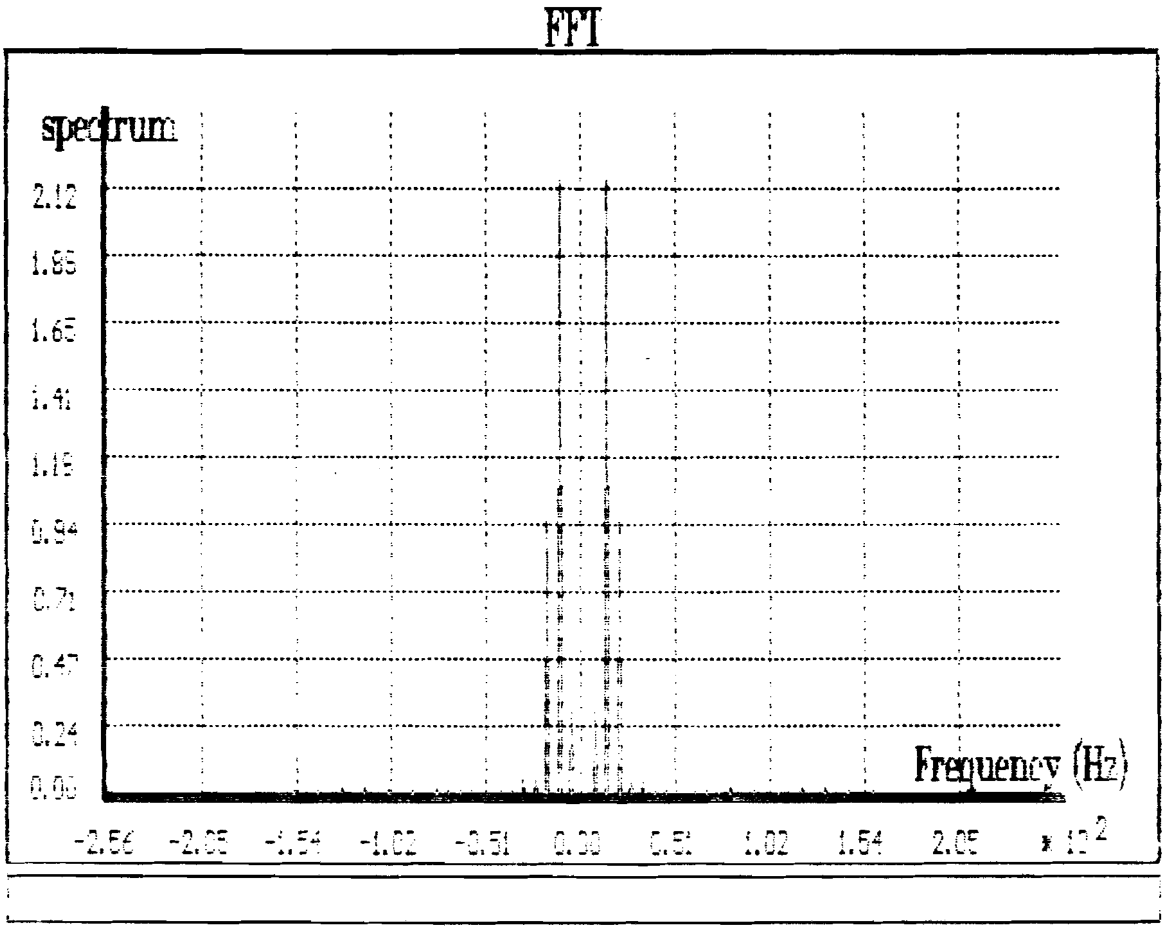

Fig. 4.8. The Fourier spectrum of the torsion ξ.

The hypothesis that $G(t)$ plays only a minor role can be verified by putting $G(t) \equiv 0$ in (4.1) and then again computing the torsion ξ from the resulting equation. The plot in Fig. 4.9 indeed shows that there is only a small difference with the actual $G \neq 0$-case. The amplitude only slightly increases but the shape of the curve in the (t, ξ) plane hardly changes.

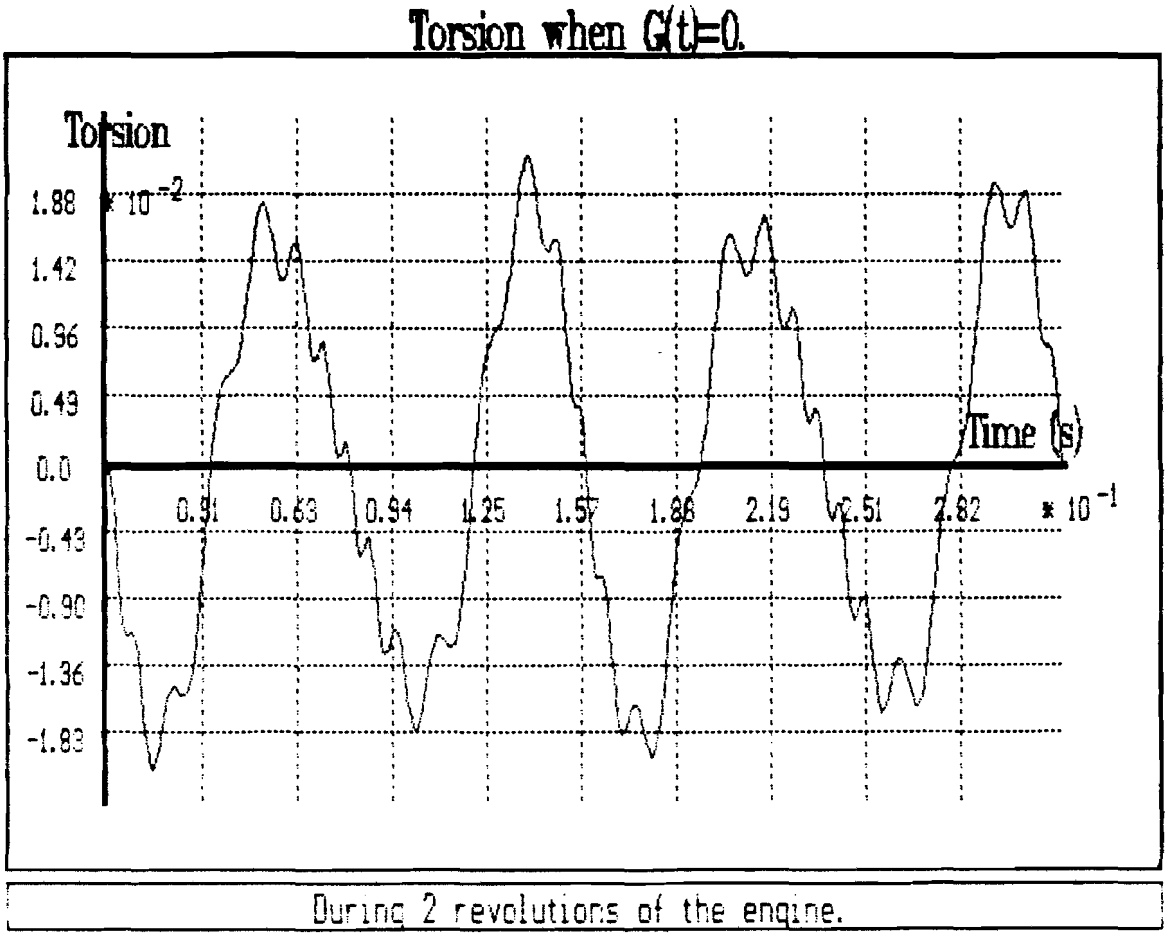

Fig. 4.9. The torsion ξ if $G(t)$ is set to zero.

In real world it is still common practice to carry out computations with averaged coefficients in the ODE. If we average e.g. the dominant function H on the left hand side, we find, cf. Fig. 4.10, that the difference with the original (t, ξ)-curve is very big. We see that the somewhat unsmooth shape of the (t, ξ)-curve in Fig. 4.7 and 4.9 now transforms into a curve consisting of a small, fast oscillation superimposed on a large, slower oscillation.

The overall amplitude increases significantly. The fast oscillations stem from the undriven eigen-frequencies of the system with $RL \equiv 0$ as can be seen in Fig. 4.11 and Fig. 4.12.

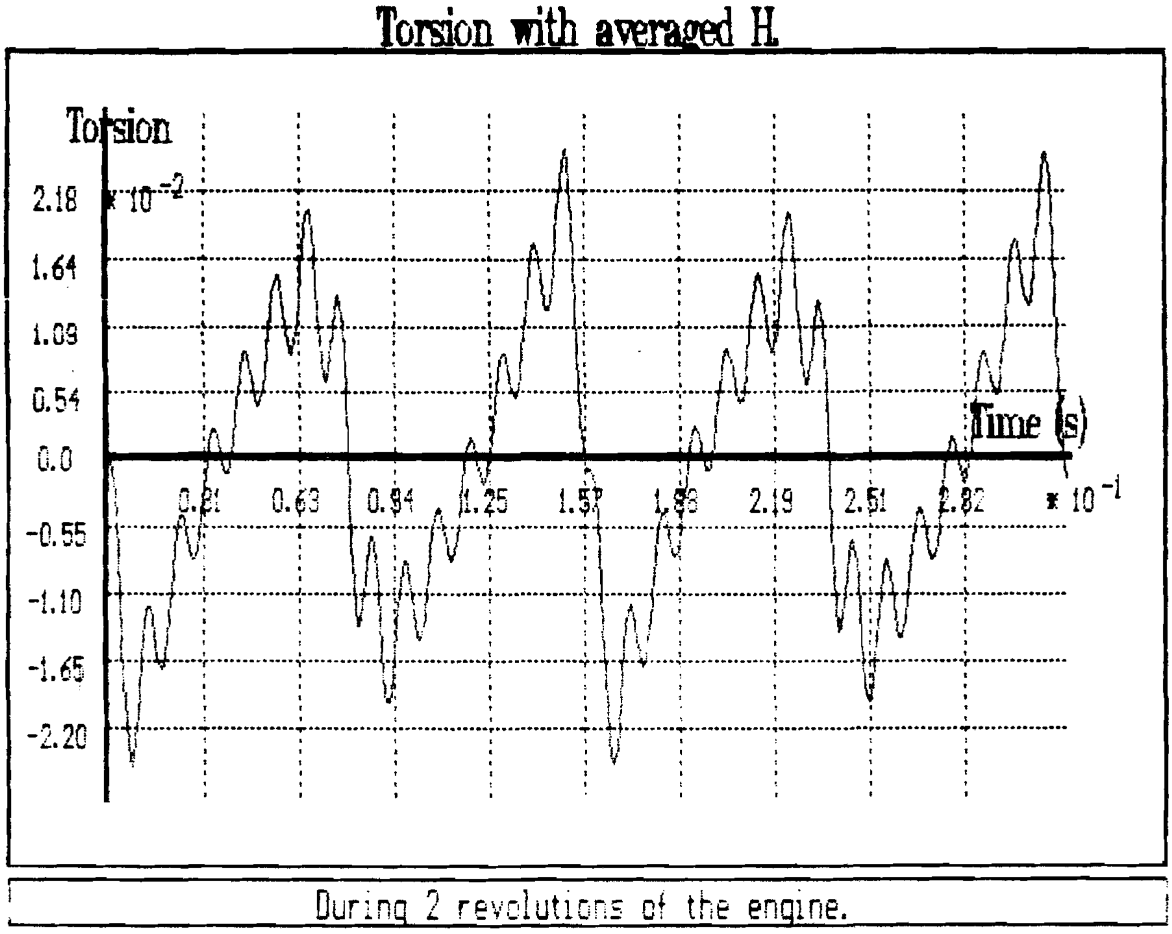

Fig. 4.10. The torsion ξ plotted against time when averaging $H(t)$

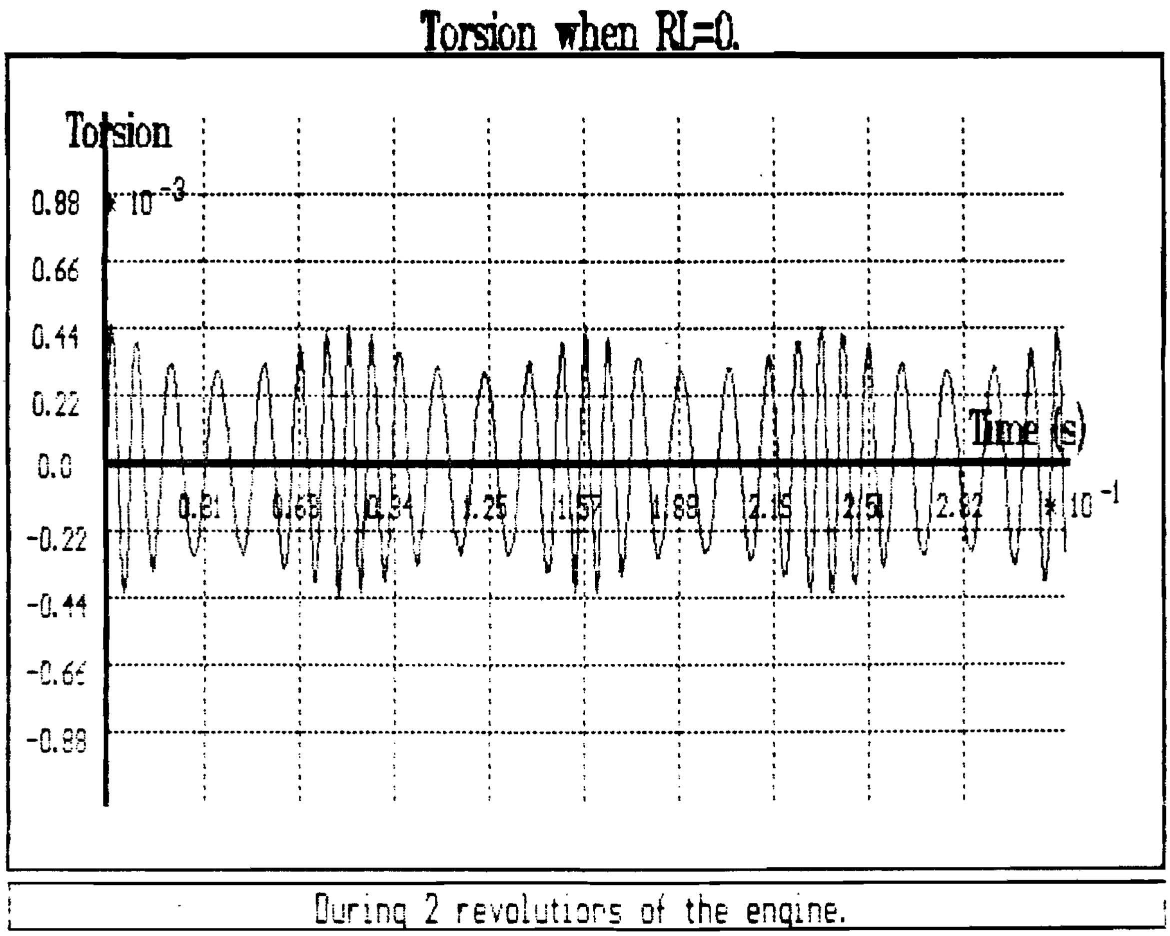

Fig. 4.11. The torsion ξ plotted against time
when setting $RL(t)$ equal to zero.

Averaging H obviously has an effect on the undriven eigenfrequencies of the system. Frequency modulations are canceled out whereas amplitude modulations increase. If we take both RL and G zero and moreover average H, both frequency and amplitude modulation cancel out.

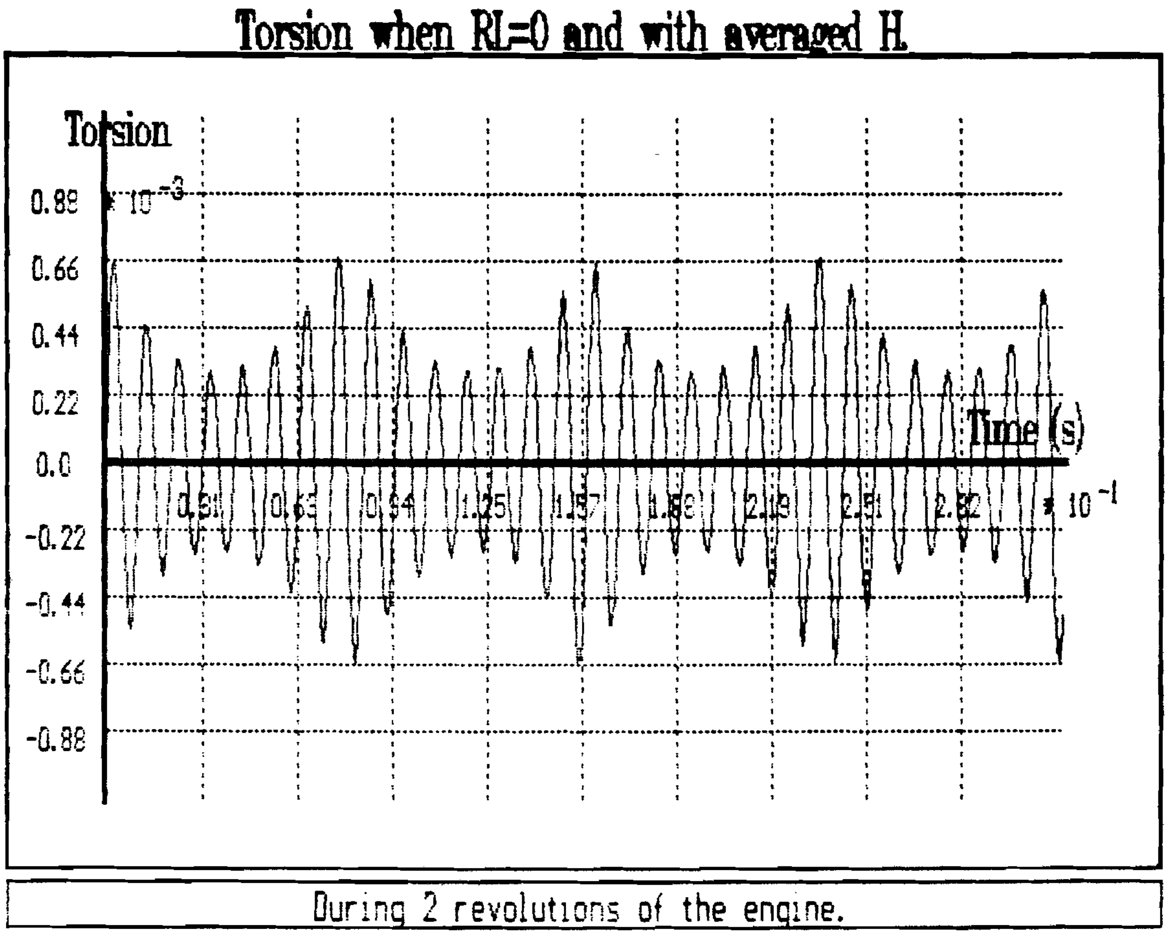

fig. 4.12. The torsion ξ plotted against time when averaging H(t) and setting RL(t) equal to zero.

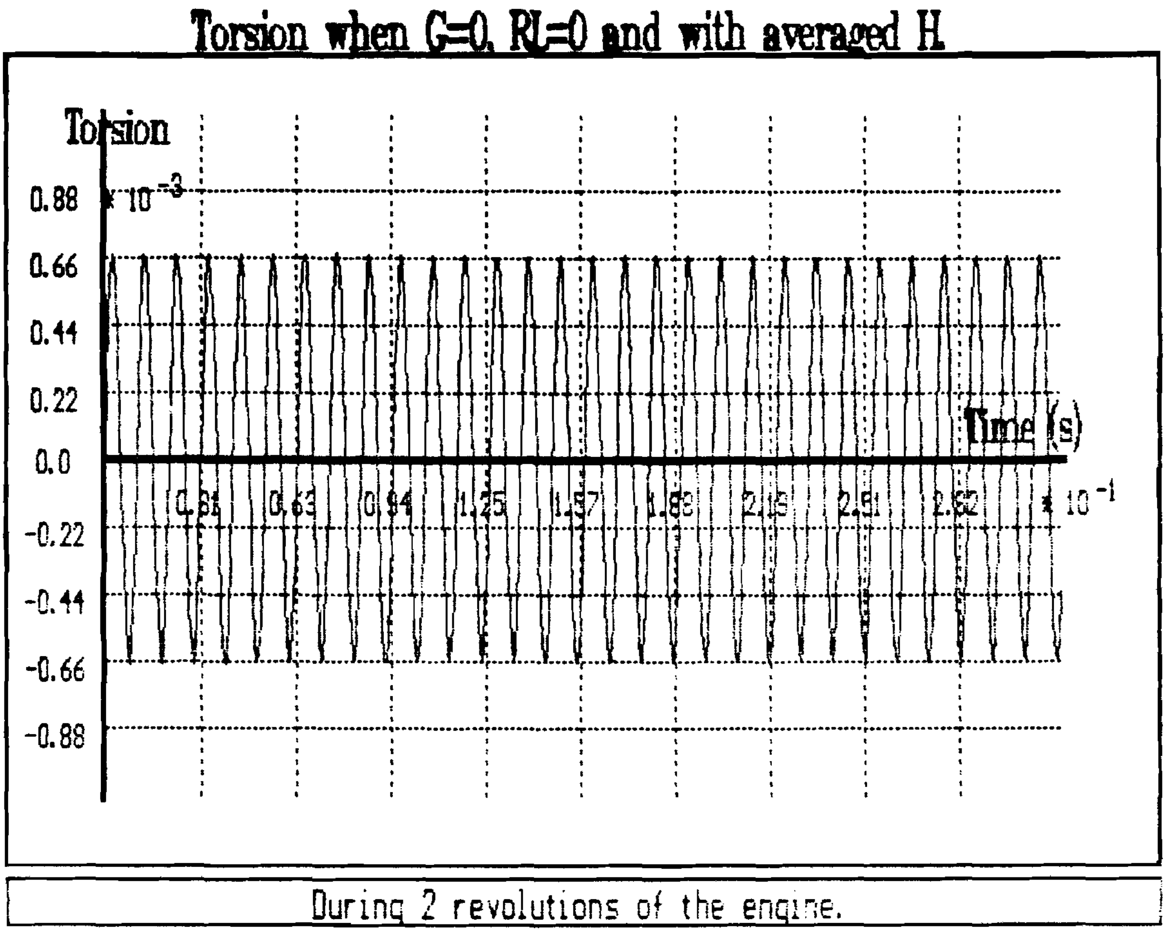

Fig. 4.13. The torsion ξ plotted against time
when averaging $H(t)$ and setting $RL(t) = 0$, and $G(t) = 0$.

In the undriven eigenfrequencies, $G(t)$ obviously brings in the amplitude modulation whereas $H(t)$ brings in the frequency modulation.

If we take a crankshaft with a smaller torsional stiffness we expect the torsion to be larger. That this is indeed the case can be checked in Fig. 4.14, where the torsional stiffness is 10 times smaller than the default value. Although the torsion is still periodical in time, it reaches levels at which the crankshaft will break. Indeed, the crankshaft can only withstand torsion up to values of about 0.1 radials and with this stiffness the torsion exceeds this value of maximum amplitude. So in practice it is not a stable situation we encounter, although in theory it is.

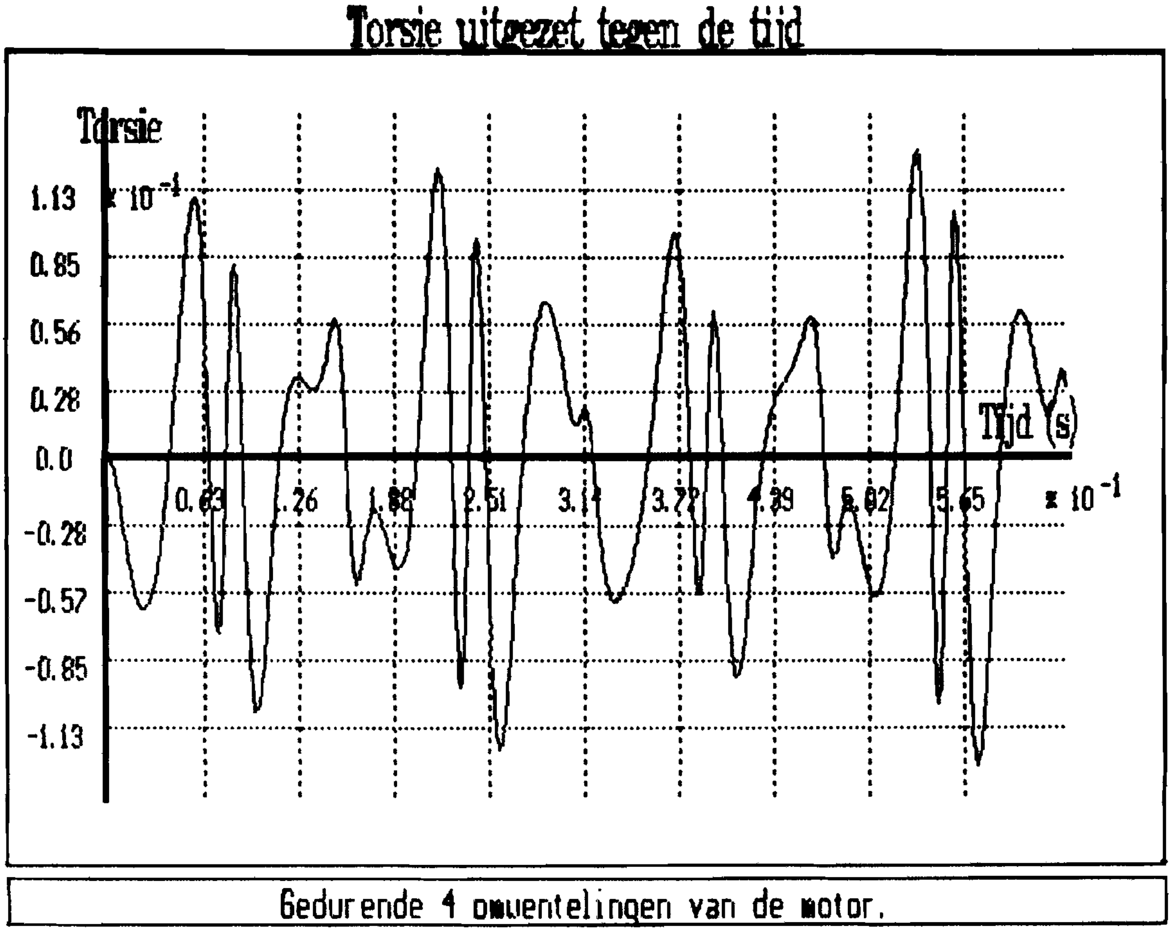

Fig. 4.14. The torsion ξ plotted against time when G_p is taken $5 \cdot 0 \cdot 10^5$, ten times smaller than the default value.

How the time evolution of the torsion depends on the initial conditions can be seen in Fig. 4.15 and Fig. 4.16. If we initially take $\xi = -0.001$ and $d\xi/dt = 0$, we see from Fig. 4.15 that there are hardly any changes noticeable. There is only a slight increase in the overall amplitude if we compare with Fig. 4.7.

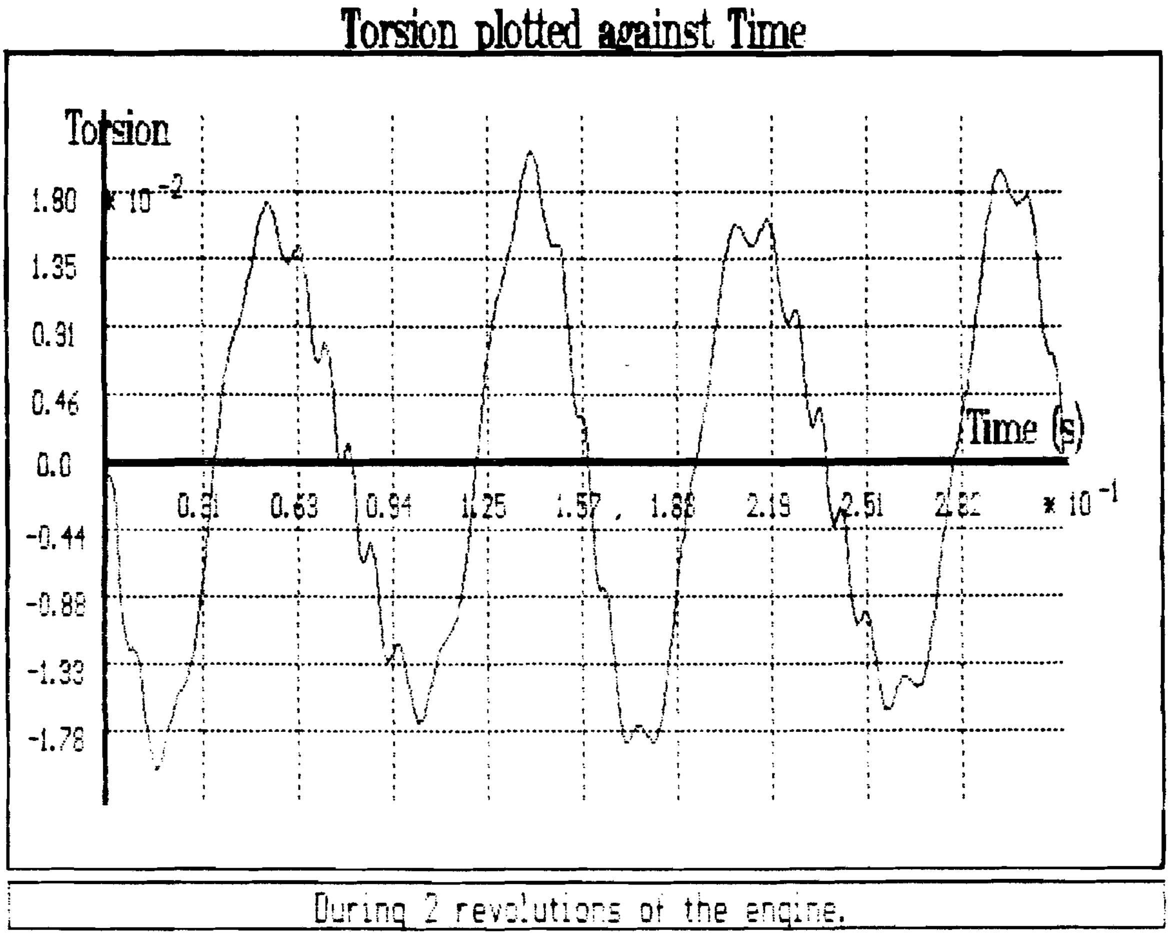

Fig. 4.15. The torsion ξ plotted against time, with initial conditions $\xi = -0.001$ and $d\xi/dt = 0$.

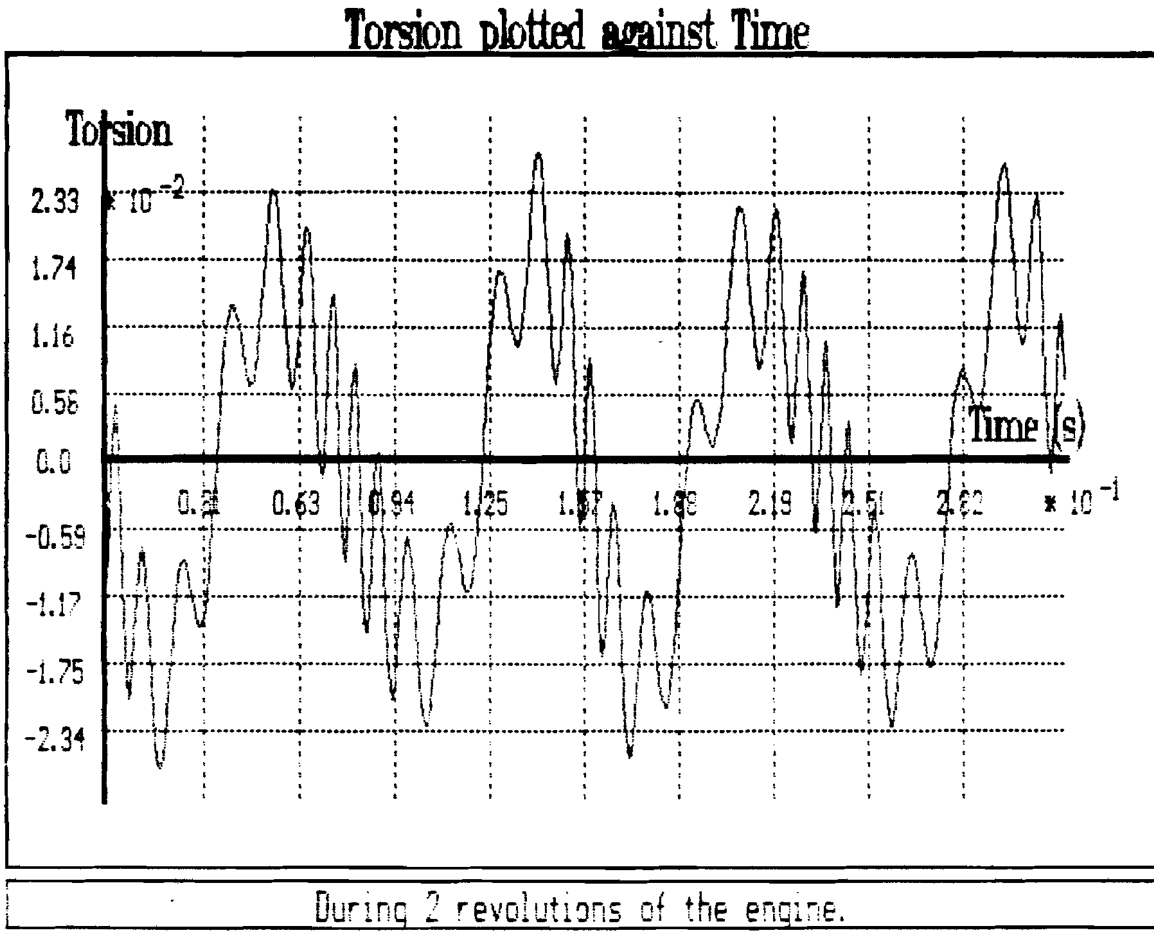

Fig. 4.16. The torsion ξ plotted against time, with initial conditions $\xi = -0.01$ and $d\xi/dt = 0$.

If we take $\xi = -0.01$ at time $t = 0$, the amplitude of the fast oscillations increases, resulting in a higher overall-amplitude as well.

We would expect the amplitude of the oscillation to blow up if we take G_p near to one of the values where a Floquet multiplier of 1 occurs. However, in our simulations we have not detected these phenomena.

5. Conclusions

We modelled the unloaded driven single piston system. We derived a second order ordinary differential equation for the torsion of the crankshaft. This differenital equation is linearized and solved numerically with the help of a Runge-Kutta method. Parameter regions of stability and instability are computed with the help of Floquet theory. We let vary the torsional stiffness of the crankshaft and the rotational speed of the driving engine.
In both cases we see the same behaviour of the Floquet multiplier as a function of the varied parameter. It leads to a discrete spectrum of parameter values where the system is possibly unstable.

The time evolution of the system is computed and analyzed. It turns out that the torsion mainly follows the driving force. A small, fast oscillation is superimposed on it, resulting from the eigenfrequencies of the undriven system. We get a better insight in the role of the time dependant coefficients in the differential equation by varying them and subsequently studying the torsion.

The results show that the default parameter set, which is used in practice, is a good choice.

Literature

[1] An introduction to numerical analysis, K.E. Atkinson, J. Wiley, 1978.

[2] J. Molenaar, TUE Eindhoven, IWDE, Private communication.