Model order reduction for nonlinear problems in circuit simulation

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Electrical circuits usually contain nonlinear components. Hence we are interested in MOR methods that can be applied to a system of nonlinear Differential-Algebraic Equations (DAEs). In particular we consider the TPWL (Trajectory PieceWise Linear) and POD (Proper Orthogonal Decomposition) methods. While the first one fully exploits linearity, the last method needs modifications to become efficient in evaluation. We describe a particular technique based on Missing Point Estimation.

1 Introduction

Simulation for nanoelectronics requires that eventually circuit equations can be coupled to electromagnetics, to semiconductor equations, and to heat transfer. Model Order Reduction (MOR) is a means to speed up simulation of large systems. Existing MOR techniques have to be generalized to be applicable to the resulting system of (Partial) Differential-Algebraic Equations (DAEs, PDAES). Also nonlinearity and parameterization have to be taken into account. They are the key points to make MOR applicable in industrial simulation and occur in three related MOR topics: behavioral modeling (for use in system design), nonlinear and parameterized MOR (for use in the main simulations), and approximation methods using response surface techniques for robust design (for use in optimization).

With respect to nonlinear systems, Proper Orthogonal Decomposition (POD) methods [4] as well as extensions of the balanced truncation methods have recently been developed. In [1, 6] a generalization to DAEs, LSPOD, based on Least Squares techniques was studied. To reduce costs in evaluations Missing Point Estimation (MPE) has been applied. The idea is to build a nonlinear reduced order model by projecting a part of the original equations onto the subspace spanned by the chosen set of basis functions. Let $\mathbf{x} \approx \mathbf{V}\tilde{\mathbf{x}}$ with $\mathbf{V} \in \mathbf{R}^{n\times r}$, $\tilde{\mathbf{x}} \in \mathbf{R}^r$, and $r \leq n$ and assume that we need to evaluate the nonlinear function $\mathbf{f}(\mathbf{x}) \approx \mathbf{V}^T\mathbf{f}(\mathbf{V}\tilde{\mathbf{x}})$. In contrast to evaluate all coordinates of $\mathbf{f}$ we take a selection matrix $\mathbf{P} \in \{0,1\}^{g\times n}$ of full rank with $\mathbf{P}\mathbf{P}^T = \mathbf{I}_g$ and consider $\bar{\mathbf{V}} = \mathbf{P}\mathbf{V} \in \mathbf{R}^{g\times r}$ to approximate $\mathbf{f}(\mathbf{x}) \approx \bar{\mathbf{V}}^T\mathbf{P}\mathbf{f}(\mathbf{P}^T\bar{\mathbf{V}}\tilde{\mathbf{x}})$ with g coordinates of $\mathbf{f}$ (in practise $g \leq 2r$). To minimize the interpolation and orthogonalization error we determine $\mathbf{P}$ to minimize $||(\mathbf{V}^T\mathbf{P}^T\mathbf{P}\mathbf{V})^{-1} - \mathbf{I}_r||$ by applying the iterative version of the greedy algorithm [8]. The technique has been successfully applied to reduce a DAE model of an inverter chain model [1, 6]. The method can also be combined with LSPOD [6].

The same inverter chain model has also been considered in [7] to study Trajectory Piece-Wise Linear (TPWL) [5], combined with "PoorMan's TBR" [3]. The locally linearized systems, created along a typical time-domain trajectory of the original system, are reduced and combined to a weighted global system. The time points for updating the local linearized systems are determined dynamically, based on error control [7].

2 Diode chain model

We consider the diode chain model shown in Fig. 1 (with the parameters I_s, V_T, R, C), described by the following system of DAEs. Here the diode functionality is modelled by the function $g(V_a, V_b)$ and the input function by $U_{\rm in}(t)$.

$$
\begin{aligned}
V_1 - U_{\rm in}(10^9 t) &= 0,\\
i_E - g(V_1, V_2) &= 0\\
g(V_1, V_2) - g(V_2, V_3) - C\tfrac{d}{dt}V_2 - \tfrac{1}{R}V_2 &= 0,\\
&\vdots\\
g(V_{N-1}, V_N) - g(V_N, V_{N+1}) - C\tfrac{d}{dt}V_N - \tfrac{1}{R}V_N &= 0,\\
g(V_N, V_{N+1}) - C\tfrac{d}{dt}V_{N+1} - \tfrac{1}{R}V_{N+1} &= 0,
\end{aligned}
$$

$$
g(V_a, V_b) = \begin{cases} (I_s e^{\frac{V_a - V_b}{V_T}} - 1) & \text{if } V_a - V_b > 0.5\\ 0 & \text{otherwise}\end{cases}
$$

$$
U_{\rm in}(t) = \begin{cases} 20 & \text{if } t \leq 10\\ 170 - 15t & \text{if } 10 < t \leq 11\\ 5 & \text{if } t > 11\end{cases}
$$

The state of the diode chain model consists of 302 elements but there is a lot of redundancy. The numerical solution (nodal voltage in each node) on the time interval $[0, 70$ ns$]$ is computed by the Euler Backward method with fixed stepsizes of 0.1 ns. TPWL is able to reduce the model to small sizes with an acceptable error (see Fig. 2 (left)). Most of the time the relative error of TPWL is lower than the chosen error bound $\varepsilon = 0.025$. Furthermore, for higher order reduced models a smaller number

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of linearization points (LP) is used than for the reduced models with lower order, as the local systems with higher orders are more accurate. E.g., for a reduced model of order 100 we have used 42 LPs and for smaller reduced models 60 LPs.
POD (without MPE) is also able to reduce this nonlinear model to size 10. The POD models are, as expected, (much) more accurate than the TPWL ones (see Fig. 2 (right)), but are considerably slower to simulate than the TPWL models (see the corresponding extraction and simulation times in Table 1). POD was significantly speeded up by combining the POD with MPE and by keeping the Jacobian matrices constant as much as possible.

Fig. 1 Schematic of diode chain.

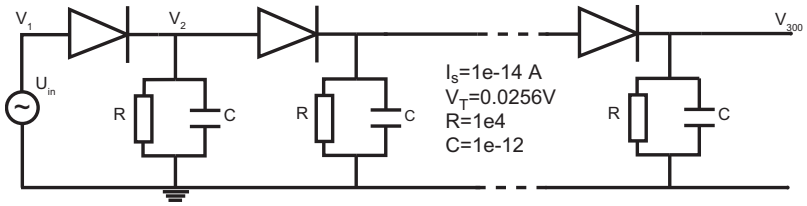
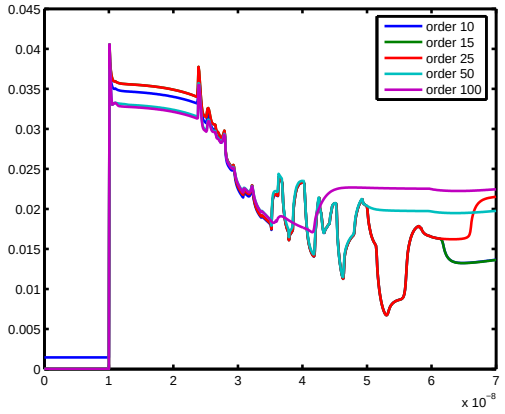
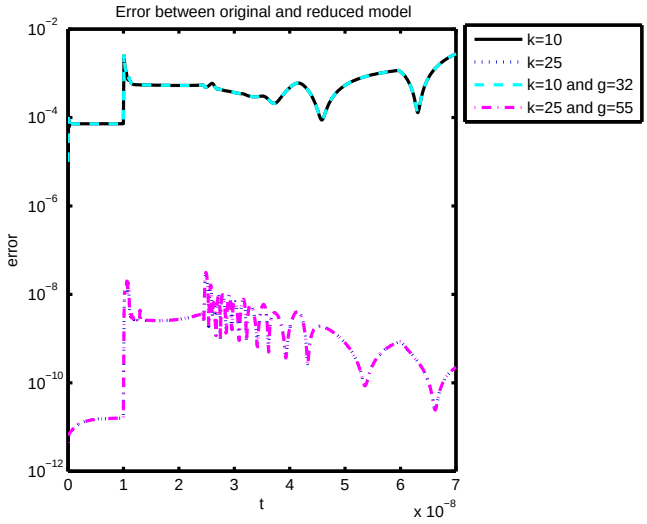


Table 1 Comparison of performances of TPWL and of POD.

Model	r	Extr. time	Sim. time	Model	r	g	Extr. time	Sim. time
Original	302	0	80	POD	10	302	80	87
TPWL	10	290	1.1	POD	20	302	80	102
TPWL	25	285	1.5	POD + MPE	10	32	84	28
TPWL	50	206	2.3	POD + MPE	20	55	89	31

Fig. 2 Numerical results diode chain, showing the errors for TPWL (at the left) and for POD (at the right).





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