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Reduced Order Modelling of RLC-networks using an SVD-Laguerre based method

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Abstract. With interconnect increasingly contributing to the electrical behaviour of integrated circuits, both by higher frequencies and smaller dimensions, it becomes increasingly important to incorporate its behaviour into simulations of ICs. This can be done rather elegantly by summarizing interconnect behaviour into a compact or reduced order model which is then co-simulated with the circuit. A similar approach can be used in the case of more conventional printed circuit boards. The SVD-Laguerre algorithm proposed by Knockaert and De Zutter [4] can be used for this purpose. In this paper, we describe an efficient implementation of the algorithm for multiple inputs, and show how the mathematical reduced order models can be translated into realizable circuit elements.

1 Introduction

To increase their performance, the characteristic dimensions of ICs and printed circuit boards (PCBs) are decreased and will decrease even further in the future. Higher speed makes the effect of higher frequency modes on the interconnect more important. Therefore, the analysis of signal propagation on the interconnect system is important. However, this requires the solution of Maxwell's equations which is rather demanding from the point of view of computation times. In addition, accurate modelling leads to large systems which can hardly be used in conventional circuit simulations.
To be able to work with models for interconnect structures, a technique known as reduced order modelling is employed. This class of mathematical techniques is able to reduce the sizes of models while preserving their essential features. Classical techniques in this area are the asymptotic waveform evaluation (AWE) method and the Pade-via-Lanczos (PVL) method. The latter is an efficient and robust implementation of the former. Recently, a new reduction method was proposed by Knockaert and De Zutter [4]. We will take a closer look at this method and will show how this method can be used to make realizable circuits.

The paper is built up as follows. In section 2, we briefly show how the discretized Maxwell equations lead to an RLC model for the interconnect system. Then, in section 3, the concept of transfer function is introduced, relating the area of reduced order modelling to concepts used in systems and control engineering where frequent use is made of state space models. Section

4 discusses several points which are of interest when using reduced order modelling techniques in the context of ICs and PCBs. In section 5 the SVD-Laguerre method is explained, whereas in section 6 the efficient treatment of multiple inputs is presented. The translation of the mathematical results to a realizable circuit is discussed in section 7. In the last section, some numerical results are given.

2 Discretization procedure

The modelling of interconnect systems has gradually developed over the years. For DC situations interconnect can be modelled as a short, but as losses and inductances are becoming more important increased use is made of representations using RLCG circuits. It is sufficient to restrict ourselves to the case in which the components R, L, G, and C are frequency independent.
Both ICs and PCBs can be modelled by (large!) RLC-circuits. These models can be obtained via a discretization of the Maxwell equations. As an illustration of this, we will very briefly review how this is done in [7].
To calculate the electromagnetic fields in an electronic system, the Maxwell equations must be solved:

$$\begin{aligned} \nabla \times \mathbf{E} &= -\tfrac{\partial \mathbf{B}}{\partial t} & \mathbf{J} &= \sigma \mathbf{E} \\ \nabla \times \mathbf{H} &= \mathbf{J} + \tfrac{\partial \mathbf{D}}{\partial t} & \mathbf{B} &= \mu \mathbf{H} \\ \nabla \cdot \mathbf{B} &= 0 & \mathbf{D} &= \epsilon \mathbf{E} \\ \nabla \cdot \mathbf{D} &= \rho \end{aligned}$$

After introducing a magnetic vector potential $\mathbf{A}$ and an electric scalar potential φ the system can be rewritten as follows:

$$\begin{aligned} (\Delta + k^2)\mathbf{A} &= -\mu \mathbf{J}, \\ \nabla \cdot (\epsilon \nabla \varphi) + \epsilon k^2 \varphi &= -\rho, \\ \mathbf{J} = \sigma \mathbf{E} &= \sigma(-\nabla \varphi + i\omega \mathbf{A}), \\ \nabla \cdot \mathbf{J} - i\omega \rho &= 0 \end{aligned} \tag{1}$$

with suitable boundary conditions. This system is discretized using a boundary integral method [7] making frequent use of Green's functions. The final discrete system can then be written into a form which is familiar to IC and PCB designers:

$$\begin{aligned} (\mathbf{R} - i\omega \mathbf{L})I - \mathbf{P}V &= 0, \\ -\mathbf{P}^T I + i\omega \mathbf{M} Q &= 0 \\ \mathbf{M}^T V - \mathbf{D} Q &= 0 \end{aligned} \tag{2}$$

Here, the elements of the vector V are the potentials of the elements. The vector I consists of the current through the edges. Q contains the weights of the surface charge density, therefore its elements are the charges of the elements of the circuit.

3 Transfer Functions, Approximation and Reduced Order Modelling

It is convenient to rewrite RLC models in terms of a state space formulation. Such formulations are of the form

$$\begin{aligned} \mathbf{C}\dot{\mathbf{x}} &= \mathbf{G}\mathbf{x} + \mathbf{B_i}\mathbf{u} \\ \mathbf{y} &= \mathbf{B_o}^T\mathbf{x}, \end{aligned} \tag{3}$$

where $\mathbf{B_i}$ and $\mathbf{B_o}$ are the matrices selecting the input and output, respectively. This reformulation can be done both for MNA formulations of circuits, and for the discretized system of Maxwell equations as derived in the previous section.

An efficient and commonly used way to solve the state space system is via the Laplace transform. Within this methodology, the so-called transfer function is introduced. It is the function $\mathbf{H}(s)$ giving the direct relation between input and output, in the frequency (denoted by s) domain. It is obtained by eliminating the state space vector $\mathbf{x}$. The s-parameter can be considered as the complex frequency $i\omega$. For (3) we have:

$$\mathbf{H}(s) = \mathbf{B_o}^T(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B_i}, \tag{4}$$

such that $\mathbf{y} = \mathbf{H}(s)\mathbf{u}$. A model which approximates the original model, can be called accurate if the transfer function of the original model is approximated well by the transfer function of the approximating model.

As can be understood from the procedure summarized in section 2, the models obtained for interconnect systems on ICs or PCBs consist of (very) large systems of equations. This is not very convenient, and a coupling of these large systems with circuit equations is almost out of the question. With Reduced Order Modelling the original model is replaced by a model which is smaller, but has (approximately) the same properties. There is a danger, however, that some essential properties are lost during the mathematical procedure. Ideally, these properties should be preserved.

In our search for a smaller circuit, describing approximately the same behaviour, an important issue is the preservation of stability and passivity. An RLC-circuit is passive, because it has no active components. Passivity is stronger than stability. A stable circuit, can become unstable when non-linear components are attached to its terminals. In contrast, a passive circuit remains stable under all conditions.

The behaviour of a circuit and the transfer function are uniquely determined by the poles and their associated residue. Poles can be calculated by:

$$\frac{-1}{\sigma(\mathbf{G}^{-1}\mathbf{C})},$$

with $\sigma(\mathbf{G}^{-1}\mathbf{C})$ the eigenvalues of $-\mathbf{G}^{-1}\mathbf{C}$. Because the poles are determining the behaviour of the system, also the poles can be approximated. This is why

methods from the area of eigenvalue approximations are often well-suited for these problems also. Examples of this are the Krylov subspace methods, PVL and PRIMA. In this paper we consider a new Krylov subspace method which is very similar to the others, but with some very attractive properties: SVD-Laguerre [4].

4 Some Theory Behind the Laguerre Method

The idea behind the Laguerre method is that the transfer function can be expanded in terms of Laguerre functions. We consider the Laguerre functions in the s-domain:

$$\Phi_n^\alpha(s) = \frac{\sqrt{2\alpha}}{s+\alpha}\left(\frac{s-\alpha}{s+\alpha}\right)^n \text{ for } n = 0, 1, 2, ... \tag{5}$$

These functions form a uniform bounded orthonormal basis in the frequency domain for $s = i\omega$, with $\omega \in (0, \infty)$, for the space $\mathcal{H}_2$. The transfer function can be expanded in terms of these functions:

$$\mathbf{H}(s) = \frac{2\alpha}{s+\alpha}\mathbf{L}^T \sum_{n=0}^{\infty}\Big((\mathbf{G}+\alpha\mathbf{C})^{-1}(\mathbf{G}-\alpha\mathbf{C})\Big)^n (\mathbf{G}+\alpha\mathbf{C})^{-1}\mathbf{B}\left(\frac{s-\alpha}{s+\alpha}\right)^n \tag{6}$$

Due to a lack of space we are forced to reference to [3], where the derivation of this expression can be found. The matrices used in this Laguerre expansion can be used to build up Krylov subspaces. An n-dimensional Krylov subspace is defined by:

$$\mathcal{K}_n(\mathbf{b}, \mathbf{A}) = [\mathbf{b}, \mathbf{A}\mathbf{b}, \ldots, \mathbf{A}^{n-1}\mathbf{b}] \tag{7}$$

The main part of the Laguerre method consists of building a Krylov subspace, with the vector $(\mathbf{G} + \alpha\mathbf{C})^{-1}\mathbf{B_i}$ and matrix $(\mathbf{G} + \alpha\mathbf{C})^{-1}(\mathbf{G} - \alpha\mathbf{C})$.

The columns of the Krylov subspace are made orthogonal. In the original article of the SVD-Laguerre method this orthogonalisation is done after all columns of the Krylov-space are created. This is done with a Singular Value Decomposition (SVD). But in repetitive multiplication with a matrix the vectors tend to one dominant direction. To avoid numerical artefacts, we propose to perform orthogonalisation this during the generation of the columns. The system matrices are projected onto this Krylov subspace, spanned by $\mathbf{V}$:

$$\tilde{\mathbf{G}} = \mathbf{V}^T\mathbf{G}\mathbf{V} \quad \tilde{\mathbf{C}} = \mathbf{V}^T\mathbf{C}\mathbf{V} \quad \tilde{\mathbf{B_i}} = \mathbf{V}^T\mathbf{B_i} \quad \tilde{\mathbf{B_o}} = \mathbf{V}^T\mathbf{B_o}$$

For a single input column $\mathbf{B_i}$, the algorithm can be summarized as follows:

Solve $(\mathbf{G} + \alpha\mathbf{C})\mathbf{v}_1 = \mathbf{B_i}$
$\mathbf{v}_1 = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|}$
for j=1,...,k-1
    Solve $(\mathbf{G} + \alpha\mathbf{C})\mathbf{t} = (\mathbf{G} - \alpha\mathbf{C})\mathbf{v}_j$

for $i = 1, ..., j$
$h_{i,j} = \mathbf{v}_i^H \mathbf{t}$
$\mathbf{t} = \mathbf{t} - h_{i,j}\mathbf{v}_i$
end
$h_{j+1,j} = \|\mathbf{t}\|$
$\mathbf{v}_{j+1} = \frac{\mathbf{t}}{h_{j+1,j}}$
end
$\tilde{\mathbf{G}} = \mathbf{V}^T\mathbf{G}\mathbf{V} \quad \tilde{\mathbf{C}} = \mathbf{V}^T\mathbf{C}\mathbf{V} \quad \tilde{\mathbf{B_i}} = \mathbf{V}^T\mathbf{B_i} \quad \tilde{\mathbf{B_o}} = \mathbf{V}^T\mathbf{B_o}$

Although, solving the matrix equation in this equation is quite expensive, it has to be done for one choice of α, so we can for instance invest in an LU-decomposition, to solve the system efficiently. Further note, that the following holds, during the algorithm: $(\mathbf{G} - \alpha\mathbf{C})^{-1}(\mathbf{G} + \alpha\mathbf{C})\mathbf{V}_{k-1} = \mathbf{V}_k\mathcal{H}$ where $\mathcal{H}$ is a Hessenberg matrix. The small matrix $\mathcal{H}^T\mathcal{H}$ can be used to approximate the singular values of the matrix $(\mathbf{G} - \alpha\mathbf{C})^{-1}(\mathbf{G} + \alpha\mathbf{C})$ and can therefor be used in a stopping criterium.

5 The Laguerre Algorithm for Multiple Input

If an RLC-model is considered with more than one input, the matrix $\mathbf{B_i}$ obviously has more than one column. All of these columns describe one specific input. The approximate model should then also allow more inputs and give an accurate approximation for all of these. In fact, the transfer function has become a transfer matrix, and we should have accurate approximations for all entries of this matrix.

Multiple inputs implies that the Krylov subspaces are also larger. For example, if two inputs are considered, the Krylov subspaces have a dimension which is twice as large compared with the subspaces generated for one input:

$$\mathcal{K}_n(\mathbf{B_i}, \mathbf{A}) = [\underbrace{\mathbf{B_i}}_{2}, \underbrace{\mathbf{A}\mathbf{B_i}}_{2}, \ldots, \underbrace{\mathbf{A}^{n-1}\mathbf{B_i}}_{2}] \tag{8}$$

Projecting onto these larger subspaces leads to system matrices which are correspondingly larger. Hence, the reduction obtained is less, and we have to be very careful with the number of columns generated. To find an appropriate space which contains the information needed for several inputs, we propose the following algorithm:

For every input column $\mathbf{B}_m$
Calculate $\mathbf{t} = (\mathbf{G} + \alpha\mathbf{C})^{-1}\mathbf{B}_m$
Make this vector orthogonal the already existing columns
Do k iterations of the Laguerre algorithm
every columns is put orthogonal to every other
end
end
Project onto the Krylov-space

Because every column is treated independently and every generated column is made orthogonal to the others, we expect the Krylov subspace to contain less redundant information.

6 A Disadvantage of Reduction Methods

The Kirchhoff's current laws and the branch equations, describing the RLC-model, can be derived directly from the state space formulation. Unfortunately, after reduction this is not possible anymore. For instance, if a state space vector $\mathbf{x}$ is used, in general it will consist of branch currents and node voltages. After projecting this vector onto a Krylov subspace, the rows have lost their physical meaning. The input and output terminals are kept, but the others may disappear. All well-know reduction methods (AWE [1], PVL [2], PRIMA [5]) suffer from this problem. This includes the Laguerre method presented in the previous section(s).

The problem mainly consists of the fact that we cannot make use of a circuit simulator in a direct way. Furthermore, starting from a model consisting of resistances, inductances and capacitors, it is desirable to have a reduced system which also consists of realizable or even passive components. This can not be done directly. However, there is a way to solve this problem via the Laguerre method, and we shall present this now.

Reconsider the Laguerre expansion given before:

$$\mathbf{H}(s) = \frac{2\alpha}{s+\alpha} \sum_{n=0}^{\infty} \mathbf{L}^T \Big((\mathbf{G}+\alpha\mathbf{C})^{-1}(\mathbf{G}-\alpha\mathbf{C}) \Big)^n (\mathbf{G}+\alpha\mathbf{C})^{-1}\mathbf{B} \left(\frac{s-\alpha}{s+\alpha} \right)^n \quad (9)$$

The advantage of this formulation is that the s parameter is not part of the inversion process for large matrices anymore. The parts of this equation which depend on s can be represented by small filters. These filters are shown in Figure 1.

The voltages implied by these filters have to be multiplied by a factor $\mathbf{L}^T((\mathbf{G}+\alpha\mathbf{C})^{-1}(\mathbf{G}-\alpha\mathbf{C}))^n(\mathbf{G}+\alpha\mathbf{C})^{-1}\mathbf{B}$ for every n and then added, in order to obtain the weighted summation. The circuit shown in Figure 2(a) stops at n terms. But this series does converge very slow, so all elements, or at least many elements in the sum must be taken into account. This can be done by implementing a loop of filters, as shown in Figure 2(b). This realizable circuit can be implemented in a circuit simulator. We used `Pstar` which is the Philips proprietary circuit simulation programme.

7 Experimental Results

We applied the proposed algorithm to a PEEC model, and to some PCB examples which were modelled as RLC-circuits. The PEEC method gives an approximation for the behaviour of interconnect, the method was developed

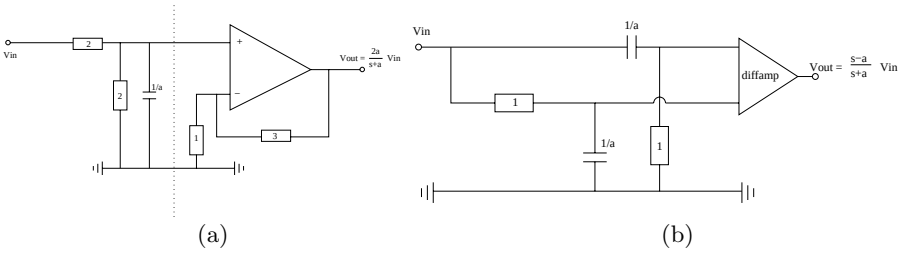


Fig. 1. (a) The filter representing $\frac{2\alpha}{s+\alpha}$, (b) The filter representing $\frac{s-\alpha}{s+\alpha}$

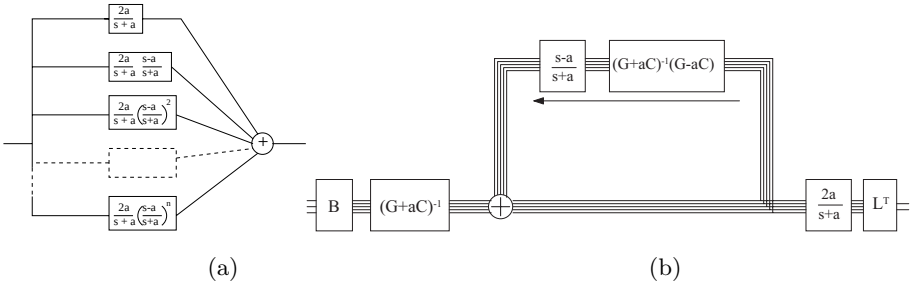
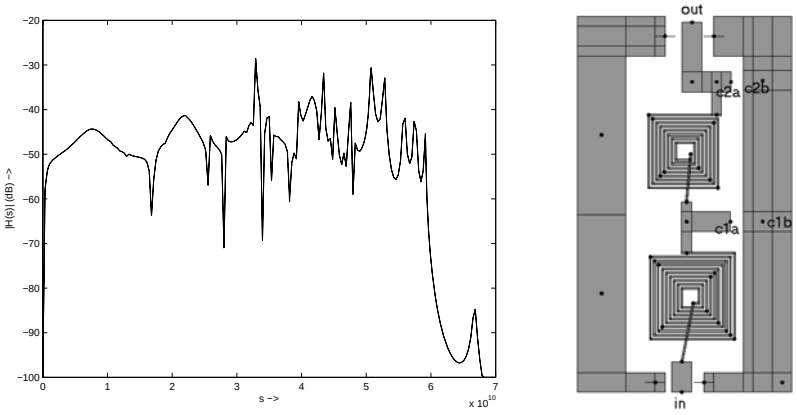


Fig. 2. (a) The filter circuit, (b) The loop circuit

by Ruehli [6]. The PEEC model we used is a nice example, because the graph of the transfer function is rather intricate and hard to approximate. The proposed Laguerre algorithm can approximate this example very well. In Figure 3(a) the approximation of the Laguerre algorithm with $\alpha = 5\ 10^{10}$ and $q = 92$ is shown. For the given frequency range no difference can be observed. Of course we have to be careful with this result. An approximation in the frequency domain does not guarantee a good approximation in the time domain. Transient analysis should be applied to be sure, that the result is accurate.

The other example is not chosen for its complexity, but to show that it is possible to combine our filter realization with non-linear components. We consider a PCB board (see Fig. 3(b)) which, after discretization, can be described by system matrices of 460×460 entries. This representation can be reduced with the proposed method. We used a reduced order model consisting of system matrices of size 60×60, in order to get approximation up to 1 GHz. The loop filter representation of these kind of models were implemented in Pstar and combined with each other and other component. The preliminary results are fine. Sometimes, (as yet unexplained) strange behaviour is observed for lower frequencies. Further, some practical issues have to be solved.



(a) (b)

Fig. 3. (a) The transfer function of the PEEC model: the original and the Laguerre approximation, for $\alpha = 5\ 10^{10}$ and $q = 92$(b) The PCB used in the second example

8 Conclusion

We have shown a modified implementation of the SVD-Laguerre algorithm. We are now able to deal with multiple input in an efficient way and we orthogonalize during the proces. The algorithm is stable and passive and leads to an accurate solution.

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