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The relative connectivity-tortuosity tensor for conduction of water in anisotropic unsaturated soils.

Peter A.C. Raats¹, Z. Fred Zhang², Andy L. Ward², and Glendon W. Gee²

¹Department of Mathematics and Computer Science, Eindhoven University of Technology, Eindhoven, The Netherlands

²Pacific Northwest National Laboratory, Richland WA, USA

Abstract

The hydraulic conductivity of unsaturated anisotropic soils has recently been described with a tensorial connectivity-tortuosity (TCT) concept. We present a mathematical formalization of the connectivity-tortuosity tensor, assuming that its principal axis coincide with those of the hydraulic conductivity tensor at saturation. The hydraulic conductivity of such unsaturated anisotropic soils is given as the product of a scalar variable, the symmetric connectivity tortuosity tensor, and the hydraulic conductivity at saturation. The influence of the degree of saturation on hydraulic conductivity is illustrated for four well defined synthetic soils through radial plots of the hydraulic conductivity scalar and of the reciprocal hydraulic resistivity scalar, both as function of the saturation. The resulting curves are ellipses. The eccentricity of these ellipses is a measure of the degree of anisotropy of the soil at the particular saturations.

Introduction

Anisotropic soils occur widely in nature and provide an interesting challenge to describe mathematically. Significant work has been done to describe saturated anisotropic soils (e.g., Scheidegger, 1954, 1956; Maasland, 1957; Raats, 1965; Bear, 1972; Dullien, 1979), but unsaturated anisotropic soils largely remain an enigma. Mualem (1984) proposed a conceptual model to quantify saturation-dependent soil anisotropy. The soil was assumed to consist of

numerous thin parallel layers having different hydraulic properties. Variation of the hydraulic conductivity at saturation among the layers was described by a probability density distribution. The model indicated that the degree of anisotropy of unsaturated soil may vary considerably from its value at saturation. However, the model may only be applicable to stratified soils. Theoretical analysis based on stochastic methods (Yeh et al., 1985) suggests that in a steady flow field the anisotropy of a stratified heterogeneous soil should increase as the mean pressure head and water content of the soil decrease. Using the stochastic approach, Polmann et al. (1991) presented a generalized model to account for tension-dependent anisotropy. They adopted a Gardner (1958) exponential relationship between the hydraulic conductivity and the pressure head, and further assumed that the horizontal correlation scale was much larger than the vertical correlation scale. However, application of the Polmann et al. (1991) model requires the knowledge of the variance of $\ln K_s$, with K_s being the hydraulic conductivity at saturation, the correlation between hydraulic parameters, and the vertical correlation length. Typically, such information is not readily available.

Recently, Zhang et al. (2003) proposed a tensorial connectivity-tortuosity (TCT) concept to describe the hydraulic conductivity of anisotropic unsaturated soil. The TCT concept assumes that, in the hydraulic conductivity of unsaturated anisotropic soil, the anisotropy is not merely expressed by a proportionality to the hydraulic conductivity at saturation, but also by three connectivity-tortuosity coefficients $L_i = (L_1, L_2, L_3)$, corresponding to the three principal directions. This TCT concept was tested using synthetic soils with four levels of heterogeneity and four levels of anisotropy. The results show that, while the soil water retention curves are dependent on soil heterogeneity but independent of direction, the connectivity-tortuosity coefficients are functions of both soil heterogeneity and direction. The TCT model can accurately describe the hydraulic functions of anisotropic soils and can be easily introduced into commonly used relative permeability functions for use in numerical solutions of the flow equation. Zhang et al. (2003) regarded the connectivity-tortuosity coefficient as a tensor, which suggest that $L_i = (L_1, L_2, L_3)$ be the components of a symmetric connectivity-tortuosity tensor in the three principal directions. However, there is no mathematical basis for such a tensorial character, this despite the fact that the triple (L_1, L_2, L_3) reflects directional dependence. In this paper we show that, nevertheless, a *relative connectivity-tortuosity tensor* can be defined, namely

the tensor $\mathbf{T}(S_e, L_i)$, with principal components $T_i = S_e^{L_i} = (S_e^{L_1}, S_e^{L_2}, S_e^{L_3})$. We present a mathematical formalization of this connectivity-tortuosity tensor, assuming that its principal axis coincide with those of the hydraulic conductivity tensor at saturation. The hydraulic conductivity tensor of such unsaturated anisotropic soils is shown to be the product of a scalar saturation dependent variable, the connectivity-tortuosity tensor $\mathbf{T}(S_e, L_i)$, and a hydraulic conductivity tensor at saturation.

Expression for the hydraulic conductivity tensor for unsaturated soils.

In the context of the Richards equation, the relationships among the volumetric water content θ , the pressure head h , and the hydraulic conductivity K define the hydraulic properties of a soil. Different classes of soils have been identified using different functions approximating the physical properties. Two groups of parametric expressions describing the hydraulic properties for isotropic soils are:

- A group yielding flow equations that can be solved analytically, in most cases as a result of linearization following one or more transformations;
- A group that is favored in numerical studies and to a large extent shares flexibility with a rather sound basis in Poiseuillean flow in networks of capillaries.

With regard to the second group, following Hoffman-Riem et al (1999; see also Raats, 1993), Zhang et al (2003) observe that the hydraulic conductivity characteristic is commonly defined by an expression of the form

$$K = K_s S_e^L A(S_e, \beta, \gamma), \quad (1)$$

where K_s is the hydraulic conductivity at saturation, L is a lumped parameter accounting for connectivity and tortuosity, S_e is the effective saturation defined by

$$S_e = (\theta - \theta_r) / (\theta_s - \theta_r), \quad (2)$$

where θ_s is the volumetric water content at saturation and θ_r is the residual volumetric water content, and $A(S_e, \beta, \gamma)$ is defined by

$$A(S_e, \beta, \gamma) = \frac{\int_0^{S_e} (h^{-\beta} dS_e)}{\int_0^1 (h^{-\beta} dS_e)}^\gamma, \quad (3)$$

where β and γ are empirical constants.

The water retention curve $h(S_e)$ is a relationship between the scalar variables pressure head h and effective saturation S_e , and Zhang et al (2003) assumed that for anisotropic soils it is still described by a scalar relationship, such as the Brooks and Corey (1966) or the van Genuchten (1980) relationship. This assumption automatically implies particular expressions for the scalar variable $A(S_e, \beta, \gamma)$ defined by (3). Zhang et al (2003) assumed that the volumetric flux vector \mathbf{f} of the water in an unsaturated soil is given by:

$$\mathbf{f} = -\mathbf{K}(S_e)\nabla(h - z) = -\mathbf{K}(S_e)\nabla H, \quad (4)$$

where z is the vertical coordinate taken positive downward, $H = h - z$ is the total head, and $\mathbf{K}(S_e)$ is the hydraulic conductivity tensor. The volumetric flux \mathbf{f} and the driving force ∇H being vectors, it follows from the so-called quotient law (McConnell, 1957) that $\mathbf{K}(S_e)$ is a second order tensor.

For the hydraulic conductivity characteristic $\mathbf{K}(S_e)$ of an anisotropic unsaturated soil, Zhang et al (2003) assumed that there exist at each location three principal directions $i = 1, 2, 3$, for each of which apply expressions analogous to equation (1):

$$K_i(S_e) = K_{is} S_e^{L_i} A(S_e, \beta, \gamma), \quad (5)$$

where, corresponding to the three principal directions $i = 1, 2, 3$, $K_i(S_e) = (K_1(S_e), K_2(S_e), K_3(S_e))$ are the components of the symmetric hydraulic conductivity tensor at saturation S_e , $K_{si} = (K_{s1}, K_{s2}, K_{s3})$ are the components of the symmetric hydraulic conductivity tensor at

saturation ($S_e = 1$), and $L_i = (L_1, L_2, L_3)$ are the connectivity-tortuosity parameters. This means that they assumed $\mathbf{K}(S_e)$ to be a symmetric second order tensor, which in a coordinate system coinciding with the three principal directions can be represented as:

$$\mathbf{K}(S_e) = \begin{vmatrix} K_1(S_e) & 0 & 0 \\ 0 & K_2(S_e) & 0 \\ 0 & 0 & K_3(S_e) \end{vmatrix} = A(S_e, \beta, \gamma) \begin{vmatrix} K_{s1} S_e^{L_1} & 0 & 0 \\ 0 & K_{s2} S_e^{L_2} & 0 \\ 0 & 0 & K_{s3} S_e^{L_3} \end{vmatrix}. \quad (6)$$

Equation (6) can also be written as:

$$\mathbf{K}(S_e) = \begin{vmatrix} K_1(S_e) & 0 & 0 \\ 0 & K_2(S_e) & 0 \\ 0 & 0 & K_3(S_e) \end{vmatrix} = A(S_e, \beta, \gamma) \begin{vmatrix} K_{s1} & 0 & 0 \\ 0 & K_{s2} & 0 \\ 0 & 0 & K_{s3} \end{vmatrix} \begin{vmatrix} S_e^{L_1} & 0 & 0 \\ 0 & S_e^{L_2} & 0 \\ 0 & 0 & S_e^{L_3} \end{vmatrix}. \quad (7)$$

Equation (7) suggests that we can regard $T_i(S_e) = S_e^{L_i} = (S_e^{L_1}, S_e^{L_2}, S_e^{L_3})$ as the principal components of the *relative connectivity-tortuosity tensor* $\mathbf{T}(S_e)$ corresponding to the three principal directions $i=1,2,3$. Note that it is tacitly assumed that the principal axis of the hydraulic conductivity tensor \mathbf{K}_s at saturation and the relative connectivity-tortuosity tensor $\mathbf{T}(S_e)$ coincide. With this interpretation, the hydraulic conductivity $\mathbf{K}(S_e)$ at the effective saturation S_e is given as the product of three factors:

- the scalar variable $A(S_e, \beta, \gamma)$;
- the symmetric relative connectivity-tortuosity tensor $\mathbf{T}(S_e, L_i)$, with principal components $T_i = S_e^{L_i} = (S_e^{L_1}, S_e^{L_2}, S_e^{L_3})$;
- the symmetric hydraulic conductivity tensor \mathbf{K}_s at saturation, with principal components $K_{si} = (K_{s1}, K_{s2}, K_{s3})$:

$$\mathbf{K}(S_e) = A(S_e, \beta, \gamma) \mathbf{T}(S_e, L_i) \mathbf{K}_s. \quad (8)$$

Note that at saturation the relative connectivity-tortuosity tensor $\mathbf{T}(S_e, L_i)$ reduces to the unit second order tensor \mathbf{I} , i.e. $\mathbf{T}(S_e = 1, L_i) = \mathbf{I}$.

With the tensorial nature of the saturation-state dependent hydraulic conductivity tensor established, various other concepts are easily defined, generalizing concepts long known in the context of saturated soils. (see e.g. Scheidegger, 1954, 1956; Maasland, 1957; Raats, 1965; Bear, 1972; Dullien, 1979). In the following we mainly generalize the presentation by Raats (1965, subsection 4.1.6) for saturated anisotropic soils to unsaturated anisotropic soils.

The hydraulic conductivity vector and hydraulic conductivity scalar

At some point, let \mathbf{n} be a unit vector in some arbitrary direction. The hydraulic conductivity vector $\mathbf{k}_n(S_e)$ associated with the \mathbf{n} direction is defined by:

$$\mathbf{k}_n(S_e) = \mathbf{K}(S_e)\mathbf{n}. \quad (9)$$

The physical significance of $\mathbf{k}_n(S_e)$ becomes evident on dividing (4) by the magnitude $|\nabla H|$ of the driving force ∇H :

$$\frac{\mathbf{f}}{|\nabla H|} = -\mathbf{K}(S_e)\frac{\nabla H}{|\nabla H|} = \mathbf{K}(S_e)\mathbf{n}. \quad (10)$$

Comparison of (9) and (10) shows that

$$\mathbf{k}_n(S_e) = \frac{\mathbf{f}}{|\nabla H|}, \quad (11)$$

i.e., the hydraulic conductivity vector $\mathbf{k}_n(S_e)$ associated with the \mathbf{n} direction is the flux vector of the water resulting from unit driving force $|\nabla H|=1$ in the \mathbf{n} direction. Only for the principal directions, the direction of \mathbf{f} , and hence $\mathbf{k}_n(S_e)$, will coincide with the direction of \mathbf{n} .

The hydraulic conductivity scalar $k_n(S_e)$ associated with the \mathbf{n} direction is defined by:

$$k_{\mathbf{n}}(S_e) = \mathbf{n} \cdot \mathbf{k}_{\mathbf{n}}(S_e) = \mathbf{n} \cdot \mathbf{K}(S_e) \mathbf{n}, \quad (12)$$

or, in a coordinate system coinciding with the principal directions of $\mathbf{K}(S_e)$ in which the direction cosines of the \mathbf{n} direction are denoted by l, m, n :

$$k_{\mathbf{n}}(S_e) = l^2 K_1(S_e) + m^2 K_2(S_e) + n^2 K_3(S_e). \quad (13)$$

The conductivity scalar $k_{\mathbf{n}}(S_e)$ associated with the \mathbf{n} direction is the component of the conductivity vector $\mathbf{k}_{\mathbf{n}}(S_e)$ in the \mathbf{n} direction. Or in view of (11), the conductivity scalar $k_{\mathbf{n}}(S_e)$ associated with the \mathbf{n} direction is the component of the flux in the \mathbf{n} direction resulting from unit driving force in the \mathbf{n} direction. In the context of the saturated case, Bear (1972) refers to $k_{\mathbf{n}}(S_e)$ as the directional hydraulic conductivity in the direction of the flow.

According to (13), a radial plot of $1/\sqrt{k_{\mathbf{n}}(S_e)}$ gives a family of ellipsoids with S_e as a parameter. These ellipsoids have semi-axes $1/\sqrt{K_i(S_e)} = (1/\sqrt{K_1(S_e)}, 1/\sqrt{K_2(S_e)}, 1/\sqrt{K_3(S_e)})$. For stratified soils, such as the synthetic anisotropic soils of Zhang et al. (2003), two of the principal components of the hydraulic conductivity tensor are equal to each other. It is then sufficient to consider the family of ellipses with semi-axes $1/\sqrt{K_i(S_e)} = (1/\sqrt{K_{par}(S_e)}, 1/\sqrt{K_{nor}(S_e)})$, where $K_{par}(S_e)$ and $K_{nor}(S_e)$ are the principal components of the hydraulic conductivity tensor corresponding to the directions parallel and normal to the strata. Figures 1a-d show four families of such ellipses for the four sets of parameters considered by Zhang et al. (2003) in their Figures 4a-d. Note that parameters a, n, K_{sp} , and K_{sn} are similar among the four sets, while L_p decreases and L_n increases from Figs. 1a through 1d. The individual ellipses are labeled by S_e . Larger ellipses are for smaller saturation. The distance of a point on the ellipses to the center represents the magnitude of $1/\sqrt{k_{\mathbf{n}}(S_e)}$ for the \mathbf{n} direction coinciding with the flow direction. The minor axes of the ellipses in Fig. 1 correspond to the principal direction with larger hydraulic conductivity.

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Figure 1. Radial plots of $1/\sqrt{k_n(S_e)}$ as a family of ellipses at different saturations for the four soils of Zhang et al. (2003) in their Figures 4a-d. The numbers on the ellipses are saturations.

The eccentricity of the ellipses measures the degree of anisotropy of the soil at the particular saturations. A more eccentric ellipse at a certain saturation indicates stronger anisotropy at that saturation. For a nearly isotropic soil, as represented in Fig 1a, the ellipses are near concentric circles. Since the hydraulic conductivities corresponding to the principal

directions are a function of saturation, the eccentricity of the ellipses is, in principle, also a function of time. Therefore, even when the direction of the total head gradient remains unchanged, the flow direction may vary with saturation and consequently with time.

The hydraulic resistivity tensor and the inverse relative connectivity-tortuosity tensor.

The hydraulic resistivity tensor $\mathbf{K}^{-1}(S_e)$ is defined as the inverse of $\mathbf{K}(S_e)$

$$\mathbf{K}(S_e)\mathbf{K}^{-1}(S_e) = \mathbf{I}, \quad (14)$$

where \mathbf{I} is the unit second order tensor. In particular at saturation, the hydraulic resistivity tensor \mathbf{K}_s^{-1} is defined by:

$$\mathbf{K}_s\mathbf{K}_s^{-1} = \mathbf{I}. \quad (15)$$

From (14), (8), and (15) it follows that the hydraulic resistivity tensor $\mathbf{K}^{-1}(S_e)$ can be decomposed as:

$$\mathbf{K}^{-1}(S_e) = A^{-1}(S_e, \beta, \gamma) \mathbf{T}^{-1}(S_e, L_i) \mathbf{K}_s^{-1}, \quad (16)$$

where $\mathbf{T}^{-1}(S_e, L_i)$ is the inverse of the relative connectivity-tortuosity tensor defined by:

$$\mathbf{T}(S_e, L_i)\mathbf{T}^{-1}(S_e, L_i) = \mathbf{I}. \quad (17)$$

The physical significance of the hydraulic resistivity tensor $\mathbf{K}^{-1}(S_e)$ becomes clear if one multiplies equation (4) by $\mathbf{K}^{-1}(S_e)$ and uses (14) to obtain:

$$\mathbf{K}^{-1}(S_e)\mathbf{f} = -\nabla(h - z) = -\nabla H, \quad (18)$$

expressing the balance of the driving force $-\nabla(h - z) = -\nabla H$ and the drag force $-\mathbf{K}^{-1}(S_e)\mathbf{f}$.

In analogy with the hydraulic conductivity vector $\mathbf{k}_n(S_e)$ and the hydraulic conductivity scalar $k_n(S_e)$, one can define the hydraulic resistivity vector $\mathbf{r}_n(S_e)$ and the hydraulic resistivity scalar $r_n(S_e)$ by:

$$\mathbf{r}_n(S_e) = \mathbf{K}^{-1}(S_e)\mathbf{n}, \quad (19)$$

$$r_n(S_e) = \mathbf{n} \cdot \mathbf{r}_n(S_e) = \mathbf{n} \cdot \mathbf{K}^{-1}(S_e)\mathbf{n}. \quad (20)$$

The hydraulic resistivity vector $\mathbf{r}_n(S_e)$ associated with the \mathbf{n} direction is the driving force required to produce unit flux in the \mathbf{n} direction. The hydraulic resistivity scalar $r_n(S_e)$ associated with the \mathbf{n} direction is the component of the resistivity vector $\mathbf{r}_n(S_e)$ in the \mathbf{n} direction. In other words, the resistivity scalar $r_n(S_e)$ associated with the \mathbf{n} direction is the component of the driving force in the \mathbf{n} direction needed to produce unit flux in the \mathbf{n} direction.

Since $r_n(S_e)$ is a scalar, the reciprocal hydraulic resistivity scalar $k_n^*(S_e) = r_n^{-1}(S_e)$ exists, and is, according to (20), given by

$$k_n^*(S_e) = \frac{1}{\mathbf{n} \cdot \mathbf{K}^{-1}(S_e)\mathbf{n}}, \quad (21)$$

or, in a coordinate system coinciding with the principal directions of $\mathbf{K}(S_e)$ in which the direction cosines of the \mathbf{n} direction are denoted by l, m, n :

$$k_n^*(S_e) = \left(\frac{l^2}{K_1(S_e)} + \frac{m^2}{K_2(S_e)} + \frac{n^2}{K_3(S_e)} \right)^{-1}, \quad \text{or} \quad \frac{1}{k_n^*(S_e)} = \frac{l^2}{K_1(S_e)} + \frac{m^2}{K_2(S_e)} + \frac{n^2}{K_3(S_e)}. \quad (22)$$

Equation (22) can also be written as

$$\frac{\left(l\sqrt{k_n^*(S_e)} \right)^2}{K_1(S_e)} + \frac{\left(m\sqrt{k_n^*(S_e)} \right)^2}{K_2(S_e)} + \frac{\left(n\sqrt{k_n^*(S_e)} \right)^2}{K_3(S_e)} = 1. \quad (23)$$

The reciprocal resistivity scalar $k_{\mathbf{n}}^*(S_e) = r_{\mathbf{n}}^{-1}(S_e)$ associated with the \mathbf{n} direction is the magnitude of the flux in the \mathbf{n} direction produced by a driving force whose component in the \mathbf{n} direction is of unit magnitude. In the context of the saturated case, Bear (1972) refers to $k_{\mathbf{n}}^*(S_e)$ as the directional hydraulic conductivity in the direction of the gradient.

According to (23), a radial plot of $\sqrt{k_{\mathbf{n}}^*(S_e)}$ gives a family of ellipsoids with S_e as a parameter. These ellipsoids have semi-axes $\sqrt{K_i(S_e)} = (\sqrt{K_1(S_e)}, \sqrt{K_2(S_e)}, \sqrt{K_3(S_e)})$. For stratified soils, such as the synthetic anisotropic soils of Zhang et al. (2003), two of the principal components of the hydraulic conductivity tensor are equal to each other. It is then sufficient to consider the family of ellipses with semi-axes $\sqrt{K_i(S_e)} = (\sqrt{K_{par}(S_e)}, \sqrt{K_{nor}(S_e)})$, where $K_{par}(S_e)$ and $K_{nor}(S_e)$ are the principal components of the hydraulic conductivity tensor corresponding to the directions parallel and normal to the strata. Figures 2a-d show four families of such ellipses for the four sets of parameters considered by Zhang et al. (2003) in their Figures 4a-d. Again, the individual ellipses are labeled by S_e . Smaller ellipses are for smaller saturations. The distance of a point on the ellipses to the center represents the magnitude of $\sqrt{k_{\mathbf{n}}^*(S_e)}$ for the \mathbf{n} direction coinciding with the hydraulic gradient. Contrary to Fig. 1, the minor axes of the ellipses in Fig. 2 correspond to the principal direction with smaller hydraulic conductivity.

Again, the eccentricity of the ellipses measures the degree of anisotropy of the soil at the particular saturations. Using the capillary tube network model and computer simulation, Bear et al (1987) appear to have been the first to introduce the two directional hydraulic conductivities in the context of the unsaturated case. Corresponding to our Figure 2, in their Figure 6 ellipses are shown for a soil for which $K_1 > K_2$ for $S_e < 0.87$, $K_1 = K_2$ for $S_e = 0.87$, and $K_1 < K_2$ for $S_e > 0.87$, where $(K_1(S_e), K_2(S_e))$ is the S_e -dependent pair of principal values of the hydraulic conductivity. However, Bear et al. (1987) did not try to describe the S_e -dependence mathematically.

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Figure 2. Radial plots of $\sqrt{k_n^*(S_e)}$ as a family of ellipses at different saturations for the four soils of Zhang et al. (2003) in their Figures 4a-d. The numbers on the ellipses are saturations.

Discussion

The hydraulic conductivity scalar $k_{\mathbf{n}}(S_e)$ and the reciprocal hydraulic resistivity scalar $k_{\mathbf{n}}^*(S_e)$ can be regarded as the directional hydraulic conductivities corresponding to the \mathbf{n} direction. In general the two directional hydraulic conductivities are not equal to each other, i.e.

$$k_{\mathbf{n}}^*(S_e) = \frac{1}{\mathbf{n} \cdot \mathbf{K}^{-1}(S_e) \mathbf{n}} \neq \mathbf{n} \cdot \mathbf{K}(S_e) \mathbf{n} = k_{\mathbf{n}}(S_e). \quad (24)$$

For the saturated case, Scheidegger (1954) originally assumed the equality of the two directional hydraulic conductivities $k_{s\mathbf{n}}$ and $k_{s\mathbf{n}}^*$. Later Maasland noticed and Scheidegger admitted the error (Scheidegger, 1956). In his classic treatment of soil anisotropy and land drainage, Maasland (1957) only discusses the directional hydraulic conductivities $k_{s\mathbf{n}}^*$. Dullien (1959) and Bear (1972) discussed both $k_{s\mathbf{n}}$ and $k_{s\mathbf{n}}^*$, as did Carslaw and Jaeger (1959, section 1.20) for the analogous process of thermal conduction.

In principle, the two directional hydraulic conductivities $k_{\mathbf{n}}(S_e)$ and $k_{\mathbf{n}}^*(S_e)$ can be measured as follows (cf. Carslaw and Jaeger, 1959):

- The saturation dependent, directional hydraulic conductivity $k_{\mathbf{n}}(S_e)$ associated with the \mathbf{n} direction can be determined by cutting a plane slice of the soil, so that its normal is in the \mathbf{n} direction, and measure the hydraulic conductivities as a function of S_e by applying a sequence of appropriate total head gradients across it.
- The saturation dependent, directional hydraulic conductivity $k_{\mathbf{n}}^*(S_e)$ associated with the \mathbf{n} direction can be determined by cutting in the \mathbf{n} direction a narrow tube of material and measure its hydraulic conductivity as a function of S_e .

Jump conditions at interfaces between different unsaturated isotropic soils have been discussed by Raats (1972, 1973). The refraction of streamlines and equipotentials at interfaces between saturated anisotropic soils is analyzed in detail for the 2-dimensional case in Raats (1972) and for the 3-dimensional case in Raats (1973). The analysis for the 3-dimensional case for saturated soils shows that whereas the planes of incidence and refraction of the hydraulic gradient ∇H always coincide, the planes of incidence and refraction of the flux coincide only in some very special cases (Raats, 1973). It appears that the swirly streamlines found

computationally by Hemker (2001), Bakker and Hemker and (2002), Hemker and Bakker (2002), and Hemker et al (2004) for flow in layered, anisotropic aquifers are directly related to non-coincidence of planes of incidence and refraction of the flux. Although in principle generalization to unsaturated anisotropic soils is straightforward, computational and observational implementation for 3-dimensional cases with different principal directions at the two sides of the interface still presents quite a challenge.

Conclusions

We presented a mathematical formalization of the tensorial connectivity-tortuosity (TCT) concept of Zhang et al. (2003), assuming that the principal axis of the relative connectivity-tortuosity tensor coincide with those of the hydraulic conductivity tensor at saturation. The hydraulic conductivity of such anisotropic unsaturated soils is given as the product of a scalar variable, the symmetric relative connectivity-tortuosity tensor, and the hydraulic conductivity at saturation. The influence of the degree of saturation on hydraulic conductivity is illustrated for four well defined synthetic soils through radial plots of the hydraulic conductivity scalar and of the reciprocal hydraulic resistivity scalar, both as function of the saturation. The resulting curves are ellipses. The eccentricity of these ellipses is a measure of the degree of anisotropy of the soil at the particular saturations.

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