

Improved calculation of the steady-state heat conduction from/towards a cylinder in the centre of a slab

Citation for published version (APA):

Laven, J. (1988). Improved calculation of the steady-state heat conduction from/towards a cylinder in the centre of a slab. *International Journal of Heat and Mass Transfer*, 31(11), 2391-2392. [https://doi.org/10.1016/0017-9310\(88\)90170-6](https://doi.org/10.1016/0017-9310(88)90170-6)

DOI:

[10.1016/0017-9310\(88\)90170-6](https://doi.org/10.1016/0017-9310(88)90170-6)

Document status and date:

Published: 01/01/1988

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

TECHNICAL NOTES

Improved calculation of the steady-state heat conduction from/towards a cylinder in the centre of a slab†

J. LAVEN

Eindhoven University of Technology, Laboratory of Colloid Chemistry,
 Postbus 513, 5600 MB Eindhoven, The Netherlands

(Received 17 February 1988 and in final form 4 May 1988)

IN THE literature the Nusselt number, Nu ,‡ for steady-state conduction of heat from/towards a cylinder has been given for several enclosures of a cylinder with diameter D [1-3]. In Fig. 1 these Nu are given for a few similar geometries. Here, the dimension not drawn should be considered as being infinite. As indicated, the Nusselt number for a cylinder in a slit would be larger than that with a quadrangular enclosure. This is implausible. Because the theoretical derivation of Nu could not be traced in the slit case, an estimate for the Nusselt number in the cylinder-slit case will be derived here.

The result given by Carslaw and Jaeger [4] is taken for the stationary temperature distribution ($T(x, y, z)$), as caused by a continuously acting point source in (x_0, y_0, z_0) with strength $q/(\rho C_p)$, where (x, y, z) and (x_0, y_0, z_0) are points in the space between the planes $z = 0$ and L , these planes being kept at temperature $T_2 = 0$ as the reference temperature

$$T(x, y, z) = \frac{q}{\pi\lambda L} \sum_{m=1}^{\infty} \sin\left\{\frac{m\pi z}{L}\right\} \sin\left\{\frac{m\pi z_0}{L}\right\} K_0\left(\frac{m\pi R}{L}\right) \quad (1)$$

where K_0 is the zeroth-order modified Bessel function of the second kind, where $R = \{(x-x_0)^2 + (y-y_0)^2\}^{1/2}$, ρ the density, C_p the specific heat and q the heat generated by the source per time.

The temperature at $(x, 0, z)$ due to a continuous line source at $(0, -\infty < y_0 < \infty, L/2)$ with strength $q/(\rho C_p)$ follows

from integration of equation (1) over y_0

$$T(x, 0, z) = \frac{2q}{\pi\lambda L} \sum_{n=0}^{\infty} \sin\left\{\frac{(2n+1)\pi z}{L}\right\} \sin\left\{\frac{(2n+1)\pi}{2}\right\} \times \int_0^{\infty} K_0\left(\frac{(2n+1)\pi(x^2+y_0^2)^{1/2}}{L}\right) dy_0. \quad (2)$$

Here, q is the amount of heat generated per time per length. With the aid of this expression the temperature has been calculated at three positions at a distance $D/2$ from the line source. These positions are $(0, 0, L/2 + D/2)$, $(D/\sqrt{8}, 0, L/2 + D/\sqrt{8})$ and $(D/2, 0, L/2)$. In the calculations use was made of the integral representation of K_0

$$K_0(x) = \frac{1}{2} \int_0^{\infty} (1/p) \exp\left\{-p - \frac{x^2}{4p}\right\} dp. \quad (3)$$

The results are

$$T(0, 0, L/2 + D/2) = \frac{q}{\pi\lambda} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \cos\left\{\frac{(2n+1)\pi D}{2L}\right\} \quad (4)$$

$$T(D/\sqrt{8}, 0, L/2 + D/\sqrt{8}) = \frac{q}{\pi\lambda} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \times \exp\left\{-\frac{(2n+1)\pi D}{L\sqrt{8}}\right\} \cos\left\{\frac{(2n+1)\pi D}{L\sqrt{8}}\right\} \quad (5)$$

and

$$T(D/2, 0, L/2) = \frac{q}{\pi\lambda} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \exp\left\{-\frac{(2n+1)\pi D}{2L}\right\}. \quad (6)$$

The behaviour of these three expressions is given graphically in Fig. 2. It is clear from this figure that the temperature gradient around a line source in the middle of a slit, at small values of D/L is only dependent on the distance to the source (D/L) and not on the radial direction with respect to the line source. The same holds if one replaces the line source by a cylindrical source with diameter D_i in the centre of the slit,

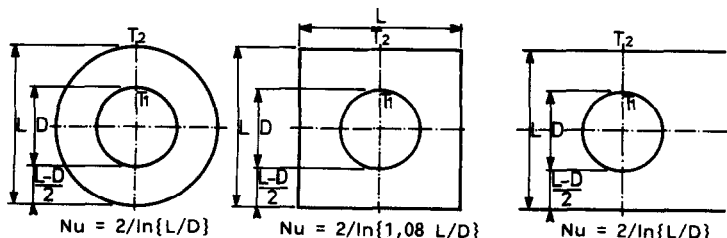


FIG. 1. Several possibilities for symmetrical enclosures of cylinders and corresponding Nusselt numbers according to the literature. The dimension not drawn should be considered to be infinite. The temperatures of the cylinders and of the enclosures are uniform and constant.

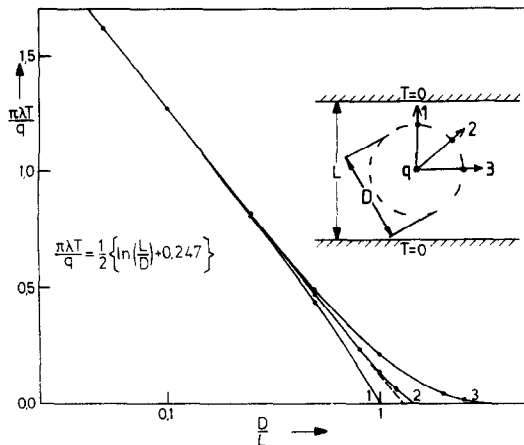


FIG. 2. Steady-state temperature for three positions at a distance $D/2$ from a continuous line source with strength $1/(\rho C_p)$, the source being positioned in the middle between two plates at zero relative temperature.

provided D_1/L is sufficiently small, i.e. < 0.1 . Consequently, the heat transfer is direction independent in that case as well. From the slope of the straight line in Fig. 2 it is concluded that

$$Nu = hD_1/\lambda = q/(\pi\lambda T_1) = 2/\ln\{1.28L/D\}.$$

This result fits much better in the expected order of decreasing Nusselt numbers for a cylinder, a quadrangular tube and a slit as enclosures.

Acknowledgements—The author wishes to thank Prof. Dr H. Janeschitz-Kriegl for his helpful comments.

REFERENCES

1. Y. S. Touloukian, P. E. Liley and S. C. Saxena, *Thermal Conductivity III: Non-metallic Liquids and Gases*, p. 209. IFE/Plenum, New York (1970).
2. W. M. Rohsenow and J. P. Hartnett, *Handbook of Heat Transfer*, pp. 3–121. McGraw-Hill, New York (1973).
3. S. S. Kutateladze, *Fundamentals of Heat Transfer*, p. 93. Arnold, London (1963).
4. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd Edn, p. 423. Clarendon Press, Oxford (1959).

Effect of boundary conditions at the lateral walls on the thermal entry lengths of horizontal CVD reactors

J. E. GATICA,† H. J. VILJOEN‡§ and V. HLAVACEK†

† Department of Chemical Engineering, SUNY at Buffalo, Amherst, NY 14260, U.S.A.

‡ Centre for Advanced Computing and Decision Support, CSIR, P.O. Box 395, Pretoria 0001, Republic of South Africa

(Received 14 January 1988 and in final form 29 April 1988)

1. INTRODUCTION

THE PREPARATION of semi-conductors, insulators and metals from vapor deposition has lately gained increasing importance. In the production of these films for electronic and optical devices, open flow systems have become of particular interest [1]. The interest in flow profiles and possible flow instabilities stems from the need to find conditions which will improve the uniformity of the deposit. Many configurations for CVD reactors have been proposed and are in use nowadays. For instance, the horizontal CVD reactor with its simple configuration constitutes one of the reactors most amenable for the analysis of flow phenomena. Although these reactors are currently used primarily for research and special applications, they still play an important role [2]. Upon entering the heated susceptor region of the reactor, the fluid starts to heat up and will develop a new linear profile (in the absence of natural convection) within its thermal entry length, x_D . It can be shown that thermal instabilities (that is, secondary flows) will be present in this development region for all non-zero Rayleigh numbers. Indeed, two-dimensional numerical results for the GaAs hot-wall reactor showed that rather small temperature differences between successive isothermal zones can cause back-flow

against the imposed forced flow [3]. These secondary flows will be confined to the entrance or will exist throughout the reactor [4]. Numerical studies by Cheng *et al.* [5], Ou *et al.* [6] and Cheng and Ou [7] showed that for the case of large Prandtl number (Pr) fluids, the inclusion of secondary flow always lead to thermal entry lengths which are shorter than lengths which are determined in the Graetz fashion (neglecting secondary flows). In this work we assume that these numerical results will hold qualitatively for gaseous systems.

The importance of knowing this distance, lies among other reasons, on the temperature distribution that will be responsible for the distribution of homogeneous gas-phase reactions, besides governing transport and physical fluid properties. From a mathematical point of view, it is advantageous to know where the linear temperature profile is established since it is the basic state that will be perturbed to find critical values of the Rayleigh number in the developed region.

Several authors treated the limiting case of two infinite horizontal plates. Hsu [8] solved the case of a step increase in heat flux in both the top and bottom plates at $x = 0$. He considered axial conduction and showed that it is important for low Peclet numbers ($Pe < 45$). Hatton and Turton [9] studied the thermal development when the top plate is at ambient temperature and the bottom plate is heated. Cheng and Wu [10] studied the influence of axial conduction on thermal instabilities in the entrance region. Hwang and Cheng [11] considered a similar problem and found that for $Pr \geq 0.7$ the flow is thermally more stable in the thermal

§ Present address: Department of Chemical Engineering, University of Stellenbosch, Stellenbosch 7600, Republic of South Africa.