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# Small nonparametric tolerance regions\*

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## Abstract

We present a new, natural way to construct nonparametric multivariate tolerance regions. Unlike the classical nonparametric tolerance intervals, where the endpoints are determined by beforehand chosen order statistics, we take the shortest interval, that contains a certain number of observations. We extend this idea to higher dimensions by replacing the class of intervals by a general class of indexing sets, which specializes to the classes of ellipsoids, hyperrectangles or convex sets. The asymptotic behaviour of our tolerance regions is derived using empirical process theory, in particular the concept of generalized quantiles. Finite sample properties of our tolerance regions are investigated through a simulation study. A real data example is also presented.

**Key words and phrases.** Nonparametric tolerance region, prediction region, empirical process, asymptotic normality, minimum volume set.

**AMS 1991 subject classification.** 62G15, 62G20, 62G30, 60F05.

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