Multiscale analysis of masonry vaults coupling shell elements to 3D-Cauchy continuum

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Abstract. This study adopts an enhanced multiscale approach to investigate the effects of the damaging process on the structural behavior of masonry vaults with regular texture, in view of their safety assessment. The model, recently developed by the authors, links two different structural models at macro and microscale, exploiting the advantages of each formulation. At the macroscopic level a homogeneous Mindlin-Reissner shell is modeled and its constitutive response is derived by the detailed analysis of a three-dimensional (3D) masonry Unit Cell (UC) studied at microlevel. The UC is considered as the assembly of elastic bricks and damage-plastic zero-thickness interfaces, representative of both mortar and mortar-unit interaction, thus accounting for the actual geometry, arrangement and constitutive response of each constituent material. A Transformation Field Analysis procedure is used to link the two scales, speeding up the numerical simulations. Structural response of a masonry vault under differential settlements is investigated, determining its load-bearing capacity and the damaging path evolving in the structure up to collapse. The reliability of the results is proved by comparison with outcomes derived by detailed micromechanical analysis, interpreting and arguing similarities and differences.

Introduction

Vaulted structures are the key element of many historical and monumental masonry constructions and require a special attention to maintain their structural integrity and preserve cultural identity. Many efforts were devoted to developing efficient procedures, both analytical and numerical, to accurately study the response of masonry curved elements, like arches and vaults. An extensive review specially dedicated to masonry vaults can be found in [1].

The limit analysis is one of the oldest methods, widely adopted in the past [2]. However, recently some interesting contributions proposed modern numerical procedures relying on limit analysis concepts [3].

Among the computational approaches, the discrete element [4] and the finite element method (FEM) are today the most selected to model in detail general geometry, mechanical properties, boundary conditions and accurately reproduce the masonry structural response. Limiting the attention to the FEM, a possible classification of the proposed procedures relies on the scale of the analysis [5] and distinguishes between macromechanical, micromechanical and multiscale approaches [6].

This paper investigates the effects of the damaging process on the structural response of an experimental masonry vault, by adopting the multiscale procedure proposed by the authors in [7]. This relies on a Reduced Order Model (ROM) based on the Transformation Field Analysis (TFA) to address the nonlinear homogenization problem of the regular masonry considered at the microscale level.

Two different models are coupled at the macroscopic structural scale and microscopic material level. Indeed, a thick shell formulation is adopted at the higher level for the vault, whereas a 3D Cauchy continuum is used for the masonry repetitive Unit Cell (UC) at the lower material scale. Here, possible nonlinear damage-plastic mechanisms are only concentrated at the mortar joints modeled as zero-thickness interfaces. The two scales communicate by means of a kinematic map, defined as function of the macroscopic strain measures, that represents the displacement field in the UC, in the framework of a computational homogenization procedure. The overall micromechanical response of the UC is evaluated under the effects of the imposed macroscopic strain components and inelastic strains and proper localization operators are computed. Based on these, the evolution problem of the damage and plasticity at each macroscopic point is solved.

The presented procedure is implemented in the finite element program FEAP and used to study the structural response of the masonry vault under differential settlements, investigating both the global response features and the damaging paths.

Shell-3D multiscale formulation

The multiscale strategy proposed in [7] is used to analyze response of masonry vaults. The model adopts different formulations at the two analysis scales (Fig. 1): at macroscale, a homogeneous thick shell is considered, based on Mindlin-Reissner plate theory. To derive the constitutive response of the shell, at each macroscopic point a UC, modeling the actual masonry texture, is linked. This is studied at microscale adopting a three-dimensional Cauchy continuum. The UC is formed by assembling linear elastic bricks and nonlinear mortar joints, modeled as interfaces. The relationship between tractions, $\mathbf{t}$, and displacement jumps, $\mathbf{s}$, at the interface results as:

$$\mathbf{t}=\mathbf{C}\left[\mathbf{s}-\boldsymbol{\pi}\right]=\mathbf{C}\left[\mathbf{s}-D\left(\mathbf{c}+\mathbf{p}\right)\right] \tag{1}$$

with $\mathbf{C}$ the diagonal stiffness matrix and $\boldsymbol{\pi}$ the inelastic vector accounting for damage, D, unilateral contact, $\mathbf{c}$, and sliding friction, $\mathbf{p}$ [8].

The linking procedure involves the definition of a suitable kinematic map. Indeed, the UC microscopic strain field in the bricks, where the displacements are continuous, is expressed as:

$$\boldsymbol{\varepsilon}=\mathbf{B}\mathbf{E}+\boldsymbol{\varepsilon}^{*} \tag{2}$$

being $\mathbf{E}$ the vector collecting the generalized strains of the shell, $\boldsymbol{\varepsilon}^{*}$ the strain field derived by the periodic displacement perturbation, and $\mathbf{B}$ the matrix governing the kinematic map.

The upscaling process is performed by invoking a generalized Hill-Mandel principle. Accordingly, the shell stresses, $\boldsymbol{\Sigma}$, result as:

$$\boldsymbol{\Sigma}=\frac{1}{A}\int\int_{-t/2}^{t/2}\mathbf{B}^{T}\boldsymbol{\sigma}dx_{3}dA \tag{3}$$

being A the area of the UC mid-plane, t the UC thickness and x_3 the axis running along t. In the right side of Eq. 3, only the brick stresses, $\boldsymbol{\sigma}$, are considered, as the interface terms give null contributions.

The Transformation Field Analysis technique is exploited to determine the homogenized response of the composite material. This consists in the subdivision of the interfaces into regions, called subsets, where the inelastic quantities are assumed as uniform. The effects of the macroscopic strains and that of the inelastic ones in each subset of the UC are determined by preliminary elastic analyses and, then, the results obtained in terms of localization matrices are used during the multiscale analysis to evaluate the constitutive response of the homogenized shell.

The model is implemented in the finite element code FEAP using a nonlocal integral formulation to overcome the mesh-dependency numerical issue.

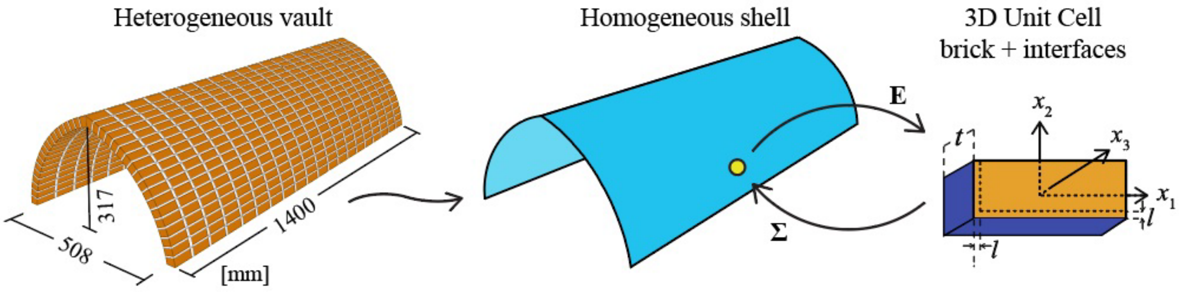

Figure 1: Multiscale modeling approach for masonry vaults.

Masonry vault under differential settlements

The response of the masonry barrel vault experimentally tested in [9] (Fig. 1) is numerically reproduced. In [10], same specimen is tested under vertical differential settlements to investigate the performance of the proposed approach in reproducing the behavior of curved elements. To extend this study to different loading conditions and analyze the response under different stress-strain states, horizontal displacements of the abutments are here considered, as indicated in Fig. 2, simulating the eventual out-of-plane mechanisms of the walls supporting the vault. Fig. 1 shows the adopted UC, made of a single brick, one head and one bed mortar joint. As opposed to the real specimen geometry that assumes running bond texture, for this test stacking bond arrangement is considered, resulting in a simpler and computationally less time-consuming description. In fact, under such loading conditions, structural response is expected to be governed by bed mortar joint opening, which is equally described by both masonry layouts.

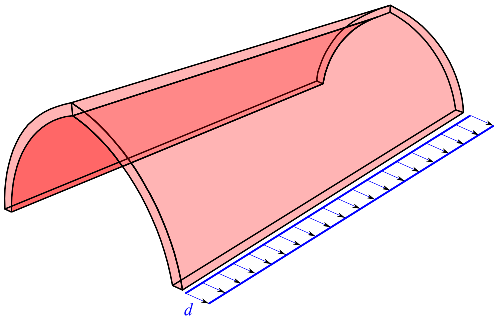

Figure 2: Differential horizontal displacement of the vault abutments.

Fig. 3 shows the global response curve of the vault in terms of total horizontal reaction of the fixed abutment (outward direction assumed as positive) versus imposed horizontal displacement. Blue curve refers to the solution obtained with the proposed model by dividing the UC into 16 subsets, i.e. 8 for the head and 8 for the bed joint, uniformly spaced across the masonry thickness [10]. This solution well agrees with that obtained by a reference micromechanical model (dashed red curve) made of linear elastic 8-node finite elements for the bricks and nonlinear zero-thickness interfaces for mortar joints, proving that the proposed model can correctly predict the vault nonlinear structural behavior.

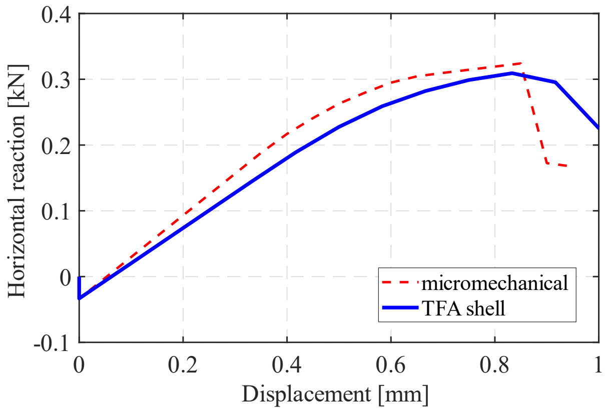

Figure 3: Global response of the vault under differential horizontal displacement of the abutments.

As expected, under the imposed displacement, vault walls uniformly bend inducing opening of the bed mortar joints at the abutments, on the extrados side, and at the key, on the intrados side. The vault deformed shape at $d = 0.85$ mm is shown in Fig. 4, for the two models. For the proposed shell model, contour plot indicates the values of the damage occurring in the bed mortar joints, averaged over the thickness of the UC. Damage appears more diffused than in the micromechanical model due to the homogenization and nonlocal nature of the multiscale model.

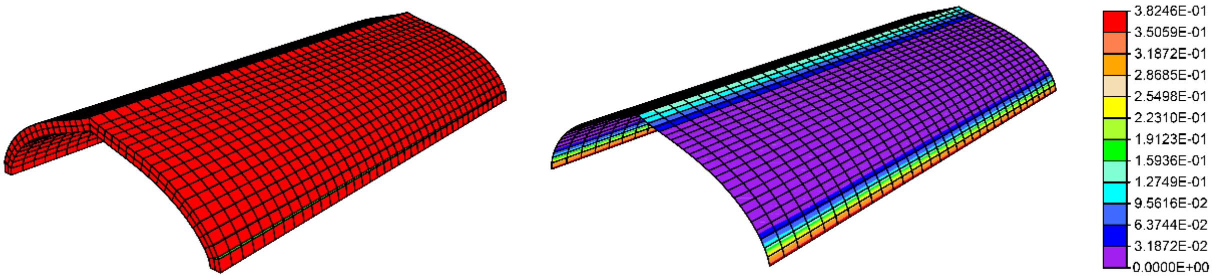

Figure 4: Deformed shape of the vault: micromechanical FEM (left) and multiscale model with damage distribution (right).

Conclusions

The study presented investigated the structural response of masonry vaults, representative elements of many components of the world cultural heritage, mainly focusing on the effects of damage evolution on the response global features. A multiscale procedure, relying on a reduced order model, was adopted, that consider a thick shell model at the structural scale and a Cauchy continuum at the real material scale. Here, the assumption of nonlinear damage-plasticity mechanisms only concentrated at mortar joints turned out to be accurate enough and suitable to reproduce the main characteristics of the masonry vault response, under simple distributions of the ground settlements.

Moreover, the use of the TFA technique, based on the assumption of piece-wise uniform distribution of nonlinearities over the mortar joints at the UC level, has allowed to avoid the high computational burden related to a full FE^2 multiscale procedure. Indeed, the obtained numerical results, as compared to those evaluated by a cumbersome micromechanical analysis, were accurate and reliable. Of course, the adopted method cannot describe localized cracking processes, although giving a correct indication of the region where damage concentrates but in a more diffused fashion.

Therefore, the multiscale model adopted, due to its computational efficiency and stability, has resulted to be a very promising tool and encourages to extend its application to other types of masonry elements characterized by different, more complex, geometries and fabric, as well as different boundary conditions. Further developments will be devoted to enhancing the description of the nonlinearity distributions in the TFA procedure to overcome the limits related to the piece-wise uniform assumption. Moreover, also the possible introduction of strengthening interventions will be accounted for, by properly modifying the presented multiscale approach.

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