Calibration of material parameters for the Chang-Mander model for unconfined concrete

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Abstract. The current basic OpenSees distribution includes several uniaxial models for concrete. Among them, the model proposed by Chang and Mander in 1994 offers a comprehensive setting applicable both to confined and unconfined concrete, by a proper selection of material parameters. The model offers the possibility to smoothly combine Tsai equation, for the first part of the curve, with a linear branch for the final part. This option is useful to model spalling of unconfined concrete while keeping the smoothness of the curve.
Two basic parameters of the Chang-Mander model for compression, denoted by n and r, govern initial and post-peak stiffness of the Tsai equation, respectively. Besides them, there is a further parameter, denoted by α, which received less attention in the literature and determines the position of the switch between nonlinear and linear parts of the curve.
In the first part of this work, the calibration of the parameters n and r is discussed in some detail. In the second part, the problem of the calibration of α is analyzed. Changes in the value of α may produce large variations in the evaluation of the spalling strain for unconfined concrete. After some comparative analyses with existing models, a simple expression to calibrate α parameter is finally proposed.

Keywords: Unconfined concrete, Chang-Mander model, spalling strain.

1 Introduction

Modeling of Reinforced Concrete (RC) structural elements by means of fiber beam-column elements offers an extremely convenient compromise between accuracy and computational cost for the nonlinear analysis of RC structures and can be implemented in OpenSees by exploiting the force-based formulation and spreading the plasticity along each member. A crucial point for practical applications of this approach is the choice of the material laws and properties which determine the uniaxial stress-strain laws of the fibers. To this end, a large amount of material models can be found in the literature for both confined and unconfined concrete under uniaxial compression [1]. A comprehensive review has been given in [2].
Concrete spalling is an important phenomenon which received a great deal of attention since it can significantly influence ultimate structural behavior as well as durability and serviceability. In [3], a review of the main works available in the literature on the subject can be found. In [4], an analytical method to determine the compressive strain of

the nearby longitudinal reinforcing bar corresponding to cover concrete spalling is proposed. The main aim of the work was to give a simple and reliable method to predict the buckling length of longitudinal reinforcing bars and the spalling of cover concrete in RC members. The effect of lateral deformation of longitudinal bars is quantitatively evaluated and incorporated in the simulation of cover concrete spalling. In [5], the mechanism of concrete spalling is investigated for high and normal strength concrete. In the proposed finite element (FE) model, cover spalling is simulated by setting the elastic modulus of the cover elements to a low value once a threshold tension strain is reached at the cover-core interface, with the threshold tension strain evaluated from experimental data.

The material model proposed by Chang and Mander [1] describes the hysteretic behavior of both unconfined and confined normal and high strength concrete, subjected to cyclic tension and compression loading. Differently from the previous models, they considered the degradation effects induced by partial cycles and pre-existing cracks that can close and open during the loading loops.

Among the several options available in the literature, Tsai function [6] gained considerable diffusion and was selected by Chang and Mander as basis for the development of the model proposed in [1] which is implemented in the current distributions of OpenSees. Two basic parameters of the Chang-Mander model for compression, denoted by n and r, govern initial and post-peak stiffness of the Tsai equation, respectively. Besides them, there is a further parameter, denoted by α, which received less attention in the literature and determines the position of the switch between nonlinear and linear parts of the curve.

In the first part of this work, the calibration of the parameters n and r is discussed in detail. Concerning the parameter n, this is directly correlated to the initial elastic modulus E_c and the peak compressive strain ε_{cp}. After a review and a comparison of various empirical formulas available to correlate E_c and ε_{cp} to the compressive strength f_{cp}, a simple expression for n is obtained in terms of f_{cp}. In the second part, the calibration of α is discussed. Changes in the value of α may produce large variations in the evaluation of the spalling strain for unconfined concrete. After some comparative analyses with existing models, calibration of α is proposed in such a way that the switch occurs at the inflection point of the Tsai curve, which cannot be easily computed in closed form. Numerical evaluation of the inflection point for various types of concrete is then carried out and, finally, a simple expression to precisely calibrate α as a function of compressive strength f_{cp} is proposed for practical material parameters determination.

2 Chang-Mander model for unconfined concrete

When numerical modelling is employed to estimate the structural capacity of a RC structure, to obtain real and accurate predictions, it is fundamental to use constitutive laws which can realistically describe the material behaviour. The full-range stress-strain curves under uniaxial compression are essential for the rational design and analysis of concrete structure members, particularly in full-range non-linear analysis and seismic analysis [7].

Current OpenSees basic distribution includes several uniaxial concrete models like, for example, the well-known Concrete02 based on Kent-Scott-Park [8] and Concrete04 on Popovic curve [9]. Besides them, Concrete07 and ConcreteCM offer implementations of the Chang-Mander model [1]. The various models have similar behavior until the material compressive strength is reached with slight variation on the initial stiffness, but they differ in the post-peak region. In Concrete04 the shape of the curve is fixed by the basic parameters whereas in Concrete02 the slope in the post-peak range can be modified independently by setting the values of ultimate strain and stress, but the softening behaviour is described through a linear branch without constraints on its slope so that the overall curve is typically non-smooth. On the other hand, in the Chang-Mander-based models the softening region is described by a linear curve, like in Concrete02, but with the possibility to choose the point where the switch between the two parts take place and, very importantly, subject to the constraint of smoothness.
Generally speaking, stress-strain laws for concrete can be expressed by nondimensional coordinates normalized with respect to stress and strain at peak:

$$x := \frac{\varepsilon}{\varepsilon_{cp}}, \; y := \frac{\sigma}{f_{cp}} \tag{1}$$

With this notation Popovics [9] function, used by Concrete04, can be expressed as:

$$y = \frac{rx}{r-1+x^r} \tag{2}$$

where r is a parameter that governs the descending branch of the stress-strain constitutive law. Denoting with $n \coloneqq y'(0)$ the nondimensional initial elastic modulus of the material, it follows that:

$$n = \frac{r}{r-1} \tag{3}$$

The dimensional initial modulus E_c can be obtained from the nondimensional one by multiplying by the secant modulus: $E_c = n\, E_{sec}$, hence nondimensional modulus can be obtained as ratio between initial and secant dimensional ones:

$$n = \frac{E_c}{E_{sec}} = E_c \frac{\varepsilon_{cp}}{f_{cp}} \tag{4}$$

Popovics curve has the advantage to describe with a single smooth equation the whole response, however this entails that the slope of the post-peak region is over constrained. Tsai equation generalize Popovic one by considering parameter n as an independent parameter [6]:

$$y = \frac{nx}{1+\left(n-\frac{r}{r-1}\right)x+\frac{x^r}{r-1}} \tag{5}$$

It satisfies $n = y'(0)$, so the initial slope and descending branch can be calibrated independently. Considering Eq. (3), Eq. (5) reduces to Popovic's:

$$y = \frac{rx}{r-1+x^r} \tag{6}$$

which gives:

$$y'(0) = \frac{r}{r-1} \tag{7}$$

Chang-Mander [1] claims that for unconfined concrete the descending branch of the Tsai equation should be followed until a certain nondimensional strain $\alpha > 1$, after which the law should switch to a linear behaviour determined by the slope $y'(\alpha)$. Nondimensional Chang-Mander equation can then be written as:

$$y = \begin{cases} \dfrac{nx}{1+\left(n-\frac{r}{r-1}\right)x+\frac{x^r}{r-1}} & x \leq \alpha \\ max[y(\alpha)+y'(\alpha)(x-\alpha), 0] & x > \alpha \end{cases} \tag{8}$$

Summarizing, the main Chang-Mander parameters necessary to describe the full stress-strain constitutive law are *n, r*, and *α*.

3 Parameter calibration: *n*, *r*

To set up a rational calibration procedure, it is convenient to relate parameters *n* and *r* to the compressive strength f_{cp}. To this end let us first note that parameter n can be expressed as a function of the peak strain and initial elastic modulus:

$$n = \frac{E_c\, \varepsilon_{cp}}{f_{cp}} \tag{9}$$

Since, in the literature, it is possible to find many relationships for peak strain and even more for elastic modulus in terms of compressive strength, parameter *n* can be conveniently related to f_{cp} as discussed in the next sections.

3.1 Empirical expressions for the initial elastic modulus, *Ec*

There are several empirical equations relating the initial elastic modulus to the compressive strength. Chang-Mander [1] reviewed many of them and proposed a further one. Other possible equations are given also in [10] and [11] (Table 1).

Table 1. Empirical expressions for Young's modulus, E_c.

Authors	Expressions
Chang-Mander [1]	$E_c = 8500\,(f_{cp})^{0.375}$
Model Code 2010 [12]	$E_c = E_{c0}\,\alpha_E \left(\frac{f_{cp}}{10}\right)^{0.3}$
Samani and Attard [10]	$E_c = 0.043\,\rho_c{}^{1.5}\sqrt{f_{cp}}$
Lim & Ozbakkaloglu [11]	$E_c = 4400\,\sqrt{f_{cp}}\,\left(\frac{\rho_c}{2400}\right)^{1.4}$
Noguchi et al. [13]	$E_c = k_1 k_2\, 3.35\, 10^4 \left(\frac{\rho_c}{2400}\right)^2 \left(\frac{f_{cp}}{60}\right)^{1/3}$

The Model Code 2010 (MC) [12] also gives a suitable relationship that contains additional parameters E_{c0} and α_E. The parameter α_E depends on the aggregate type and varies from 0.7 to 1.2, giving a global variation (E_{c0} α_E) from 15100 MPa (lower), for sandstone aggregates, to 25800 MPa (higher), for basaltic or dense limestone aggregates, with a typical value of 22000 MPa (medium), for quartzite aggregates.
The proposals in [10] and [11] include in their expressions the concrete density in kg/m^3 which generally, for normal weight concrete, varies between 2000 and 2400 kg/m^3. Another formula often used for the determination of the initial elastic modulus is given in [13]. In their work, the authors have collected more than 3000 data sets, obtained by using various materials, and have analysed them statistically.

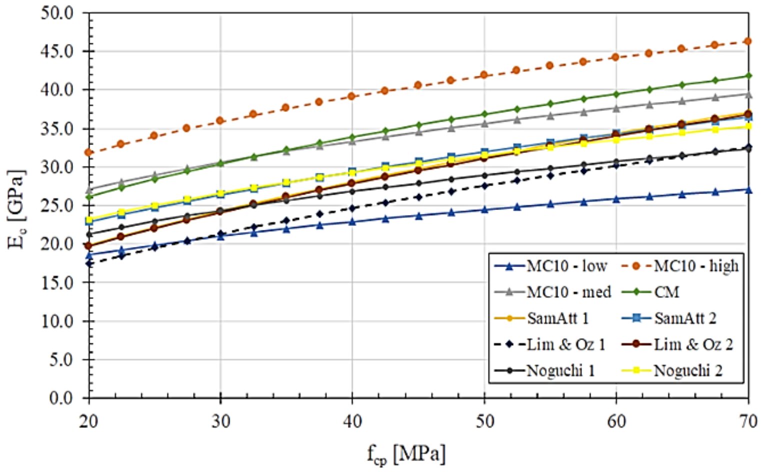


Fig. 1. Comparison of the initial elastic modulus expressions for different concrete compressive strength and different values of concrete density (2200 and 2400 kg/m3): Model Code10 [12], Chang-Mander [1], Samani and Attard [10], Lim and Ozbakkaloglu [11], Noguchi et al. [13].

The compressive strengths of the considered concrete range from 40 to 160 MPa. As a result, a practical and universal equation, which also takes into consideration the types of aggregates and mineral admixtures, is defined. In this equation, ρ_c is in kg/m^3 and f_{cp} is in MPa. The corrective factors k_1 an k_2 are included to account for the variety of the aggregates and of the admixtures. The expression proposed in [13] is well formulated and based of a large amount of experimental data, however, the determination of the corrective factors k_1 and k_2 becomes difficult if no experimental information is available on the concrete mixture of the studied case.
Anyhow in Fig. 1 the relation is plotted in comparison with the other, considering mean values of both factors, k_1 = 1.0 and k_2 = 1.0. In the graph, the series defined as “1” or “2” correspond to the same relationships but different values for the concrete density ρ_c, which is taken equal to 2200 kg/m^3 for the “1” series and 2400 kg/m^3 for the “2” series.

3.2 Empirical relations for peak strain, ε_{cp}

Compressive peak strain of concrete can be related to compressive strength according to various relations. For example, expressions proposed by Eurocode 2 [14], Chang-Mander [1], Samani and Attard [10] or Lim and Ozbakkaloglu [11] can be used, as summarized in Table 2. In particular, the equation proposed in [10] is based on the type of the concrete aggregates and the coefficient c is taken equal to 4.26 for crushed aggregates and equal to 3.78 for gravel aggregates.

Table 2. Empirical relations for peak strain, ε_{cp}.

Authors	Expressions
Chang-Mander [1]	$\varepsilon_{cp} = 0.88 \cdot 10^{-3}\, ln\left(f_{cp}\right)^{0.25}$
Eurocode 2 [14]	$\varepsilon_{cp} = 0.7 \cdot 10^{-3}\left(f_{cp}\right)^{0.31}$
Samani and Attard [10]	$\varepsilon_{cp} = \frac{f_{cp}}{E_c}\frac{c}{\sqrt[4]{f_{cp}}}$
Lim and Ozbakkaloglu [11]	$\varepsilon_{cp} = \frac{{f_{cp}}^{0.225\,k_D}}{1000}\, k_S k_A$
	$k_D = \left(\frac{\rho_c}{2400}\right)^{0.45}, k_S = \left(\frac{152}{D}\right)^{0.1}, k_A = \left(\frac{2D}{H}\right)^{0.13}$

In [11], k_D, k_S and k_A are correcting factors which account for the shape (H and D) of the tested specimen and the density of the employed concrete. These factors are included based on experimental evidence resulting from a huge number of tests carried out on specimens characterized by different geometries, aspect ratios, density, aggregate types and dimensions.

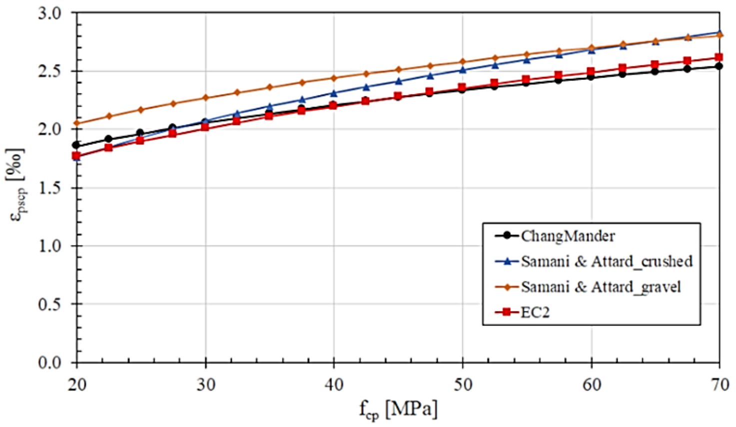


Fig. 2. Comparison of the peak strain expressions for different concrete compressive strengths and aggregate types (crushed and gravel): EC2, Chang-Mander [1], Samani and Attard [10].

In particular, the specimen diameter (D) and aspect ratio (H/D) slightly influence the axial strain at peak i.e., for a given compressive strength, the axial strain at peak compressive stress (ε_{cp}) decreases with either an increase of D or H/D.
In Fig. 2 the above-mentioned expressions are plotted in the same graph for a compressive strength range of 20-70 MPa. The relation given by [11] is not included since it requires additional information regarding the specimen geometry.

3.3 Proposed expression for *n*

The comparisons discussed in previous sections shows that, even if several different relationships are available, each one depending on different parameters, the obtained values for E_c and ε_{cp} show a relatively good agreement.
Combining the relationships proposed in the Model Code 2010 for the initial elastic modulus and in the Eurocode 2 for the peak strain, the following expression for the parameter n of the Chang-Mander model as a function of the compressive strength:

$$n = 3.5083 \cdot 10^{-4} E_{c0} \left(f_{cp}\right)^{-0.39} \tag{10}$$

The chosen combination (MC10 and EC2) offers a considerable modelling flexibility as it contains the additional parameter E_{c0} which allows the formula to accommodate various types of concrete. If the mean value of E_{c0}= 22000 MPa is used, Eq. (10) gives:

$$n = 7.7183 \left(f_{cp}\right)^{-0.39} \tag{11}$$

which is similar to the expression proposed by Chang-Mander:

$$n = 7.2 \left(f_{cp}\right)^{-0.375} \tag{12}$$

Concerning the parameter r, Chang-Mander proposed the following expression:

$$r = -1.9 + \frac{1}{5.2} f_{cp} \tag{13}$$

which gives a good approximation for the first part of the post-peak curve of unconfined concrete, for various values of the compressive strength. This relation is to be considered as an alternative to the Popovic's expression:

$$r = \frac{n}{n-1} \tag{14}$$

where r is constrained to n so to provide a good approximation for confined concrete.
Summarizing, the following expressions can be used for the determination of the Chang-Mander concrete model:

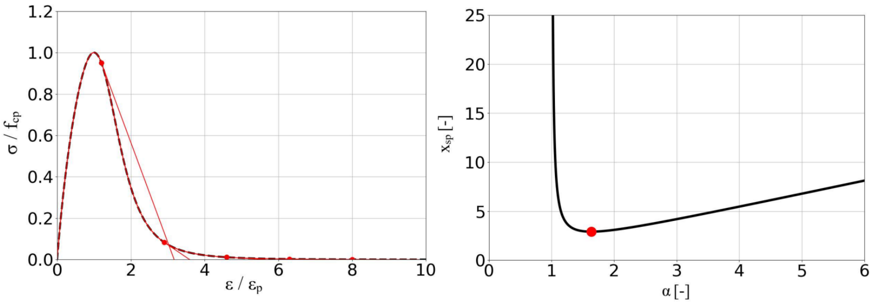
$$n = 3.5083 \cdot 10^{-4} E_{c0} \left(f_{cp}\right)^{-0.39}; \qquad r = -1.9 + \frac{1}{5.2} f_{cp} \tag{15}$$

Employing the above-mentioned expression for the n and r parameters in the Tsai equation, constitutive laws can be obtained for plain and confined concrete.

4 Parameter calibration: *α* for unconfined concrete

The third parameter of the Chang-Mander model that can be modified to better describe the material law is the switch strain α which determines the spalling strain x_{sp} corresponding to the point where concrete exhibits zero residual stress. Generally speaking, parameter α can take any value bigger than 1, however, as clearly shown in Fig. 3a, it is important to consider that even small variations of the parameter can yield strong changes in the corresponding spalling strain.

For example, if $\alpha = 1$, since the tangent at the peak is horizontal, an infinite spalling strain $x_{sp} = \infty$, is obtained. For greater values of α, x_{sp} first decreases until the inflection point and then increases again. Fig. 3b shows a typical variation of x_{sp} as a function of α with a clear indication of the existence of a minimum. Comparison of Fig. 3b with the corresponding curves shows that the minimum value in the curve corresponds to a switch at the inflection point.



(a) *(b)*

Fig. 3. (a) Influence of the switch parameter in Chang-Mander model; (b) spalling strain as a function of the switch parameter in Chang-Mander model (f_{cp} = 38 MPa, E_{c0}= 22000 MPa).

In the literature, few expressions can be found to calculate the spalling strain; only in [15] some indications are given. However, after consideration of typical experimental data, it turns out that a possible way to calibrate the switching strain while keeping reasonable values of the spalling strain is to set α in such a way that the switch between the two branches takes place at the inflection point ε_i, where experimental curves show a change of sign in the curvature. The location of inflection point of the descending branch has been investigated by many authors who proposed various empirical expressions based on regression analyses from experimental data, some of which are summarized in Table 3.

The expression proposed in [11], shown in the third row of Table 3, is more complicated and it is valid also in the case of confined concrete (having peak strength equal to f_{cc}^{*}) where a residual compressive resistance ($f_{c,res}$) is observed and considered. When plain concrete is analysed, $f_{cc}^{*} = f_{cp}$ and $f_{c,res} = 0$. Hence, the above equation can be rewritten as:

$$\alpha_{opt} = \left(10 \ f_{cp}^{-0.47}\right) \left(\frac{\rho_c}{2400}\right)^{0.4} \tag{16}$$

Using one of the above expressions for the spalling strain to calibrate parameter α generally leads to the non-smoothness of the curve since the slope constraint is violated. It turns out therefore that, to get a reliable formula utilizable with the Chang-Mander model, a specific expression for the inflection point of the Tsai equation is needed.
Although the second derivative of the Tsai equation can be easily computed, its zeros are hard to find in closed form. On the other hand, it is possible to compute in closed form the spalling nondimensional strain as a function of α:

$$x_{sp}(\alpha) = \alpha - \frac{y(\alpha)}{y'(\alpha)} = \frac{r(1-\alpha^r)-n(r-1)}{(r-1)(1-\alpha^r)}\alpha^2 \tag{17}$$

from which we get:

$$x'_{sp}(\alpha) = \frac{y(\alpha)y''(\alpha)}{[y'(\alpha)]^2} \tag{18}$$

Hence a sufficient condition to get a minimum value of x_{sp} is the vanishing of the second derivative of y, that is, the inflection point of the descending branch of the Tsai function. Namely $y''^{(\alpha)} = 0$ implies $x'_{sp}(\alpha) = 0$ a minimum.
On the other hand, inflection point can be easily computed for Popovic's equation as $x_{flex} = \sqrt[r]{r+1}$ but for Tsai equation this seems not to be possible in closed form. However, it can be solved numerically as proposed in the next section.

Table 3. Expression considered for the evaluation of the inflection point α_{opt}.

Authors	Expressions
Su and Bei [7]	$\alpha_{opt} = 3.86 - 0.54 \ln f_{cp}$ $\alpha_{opt} = 6.20 - 1.12 \ln f_{cp}$
Samani and Attard [10]	$\alpha_{opt} = 2.76 - 0.35 \ln f_{cp}$
Lim and Ozbakkaloglu [11]	$\alpha_{opt} = \begin{pmatrix} 2.8 \left(\frac{f_{c,res}}{f^*_{cc}}\right) f_{cp}^{-0.12} + \\ 10 \left(1 - \frac{f_{c,res}}{f^*_{cc}}\right) f_{cp}^{-0.47} \end{pmatrix} \left(\frac{\rho_c}{2400}\right)^{0.4}$

4.1 Proposed expression for spalling strain

Using the relations proposed in Sect. 3 for the parameters n and r, it is possible to compute numerically the spalling strain as a function of α for various values of the compressive strength (Fig. 4).
For each value of the compressive strength, there is a minimum value of the spalling strain, $x_{sp,opt}$, taking place at a value α_{opt} which, as discussed above, coincides with the inflection point of the descending branch.

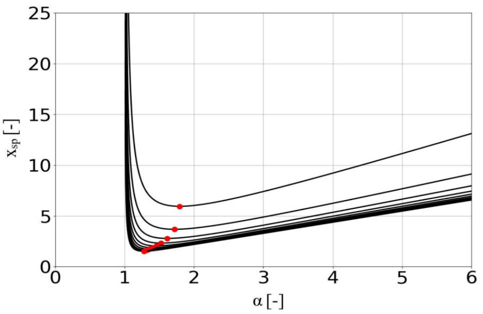


Fig. 4. Comparison of the obtained switch parameter for different values of compressive strength ranging from 20 to 70 MPa (initial E_{c0} = 22000 MPa).

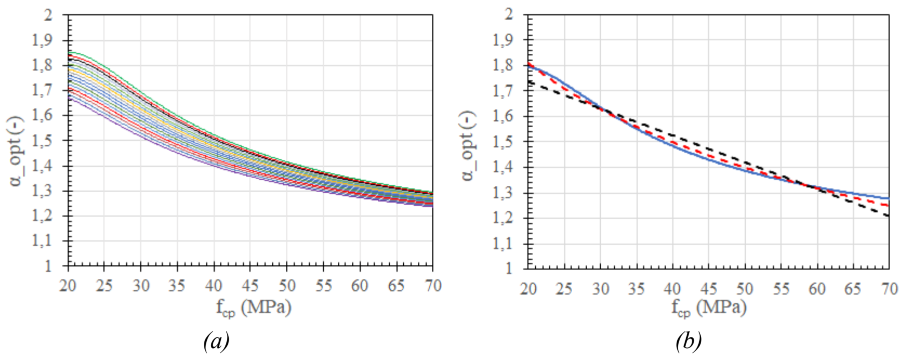


Fig. 5. (a) Plot of the optimal inflection point for different values of E_{c0}, ranging from 15100 MPa to 25800 MPa; (b) proposed curve (solid blue) for the determination of the optimal inflection point as a function of compressive strength (E_{c0}=22000 MPa) and corresponding linear (dashed black) and logarithmic interpolation (dashed red).

In Fig. 5a, α is plotted as a function of the strength for various values of the parameter E_{c0} = 22000 MPa. The figure shows that the various curves have a similar shape which can be approximated employing both linear ($\alpha_{opt} = 1.9473 - 0.0105\,(f_{cp})$) and logarithmic curves ($\alpha_{opt} = 3.151 - 0.448\ln(f_{cp})$), as exemplified in Figure 5b.

Finally, from the above discussion, simple expressions of the optimal switch parameter can be obtained either in linear or logarithmic form (Table 4). In both cases a two-parameters expression is obtained, with the parameters clearly depending on the parameter E_{c0}.

In Fig. 6, the predictions of the proposed formulas are compared with the above-discussed empirical formula for the normalized strain corresponding to the inflection point. As previously indicated, the series defined as “1” or “2” correspond to the same relationships but different values of specific parameters. In [9], the density ρ_c varies and is equal to 2200 kg/m^3 for the “1” series and 2400 kg/m^3 for the “2” series, while in [7], the series “1” corresponds to 10 mm aggregate size and the series “2” to 20 mm aggregate size.

Table 4. Proposed expression for spalling strain.

	Expressions
Linear expression	$\alpha_{opt} = A - B\left(f_{cp}\right)$
Logarithmic expression	$\alpha_{opt} = C - D\ ln\left(f_{cp}\right)$

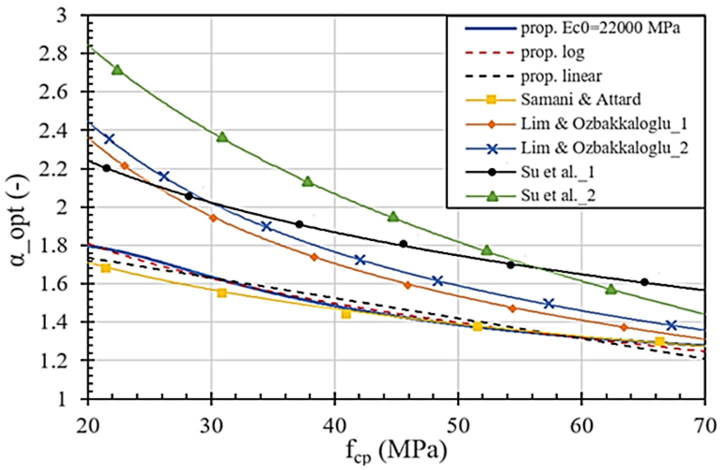


Fig. 6. Optimal value for parameter α as a function of the concrete compressive strength, comparison between the proposed expression and those obtained from the literature.

The comparison shows that, among the various available expressions, the one proposed by Samani-Attard is the one that yields the best agreement with the one here obtained from the Tsai equation.

5 Conclusion

In the present work, a simple procedure to calibrate the main material parameters of the Chang-Mander uniaxial model for unconfined concrete has been proposed.
By combining the expressions proposed by EC2 and MC10 for peak strain and initial elastic modulus, a formula for the parameter *n* as a function of the compressive strength is obtained first. Then, after a numerical optimization, formulas for the parameter α corresponding to a smooth switch at the inflection point are obtained. Referring to the average value E_{c0} = 22000 MPa and to the case of linear approximation for α, the following simple expressions can be used to calibrate the three main parameters of Chang-Mander model in compression

$$n = 7.7183\left(f_{cp}\right)^{-0.39};\quad r = -1.9 + \frac{1}{5.2} f_{cp}\ ;\quad \alpha_{opt} = 1.9473 - 0.01055\ f_{cp} \quad (19)$$

The proposed expression for α can be directly used in the OpenSees implementations to get a smooth switch between the two branches of the curve.

References

1. Chang, G. A., & Mander, J. B. Seismic energy-based fatigue damage analysis of bridge columns: Part I-Evaluation of seismic capacity (p. 222). Buffalo, NY: National Center for Earthquake Engineering Research (1994).
2. Sima, J. F., Roca, P., & Molins, C. Cyclic constitutive model for concrete. Engineering structures, 30(3), 695-706 (2008).
3. Moccia, F., Ruiz, M. F., & Muttoni, A. Spalling of concrete cover induced by reinforcement. Engineering Structures, 237, 112188 (2021).
4. Dhakal, R. P., & Maekawa, K. Reinforcement stability and fracture of cover concrete in reinforced concrete members. Journal of structural engineering, 128(10), 1253-1262 (2002).
5. Foster, S. J., Liu, J., & Sheikh, S. A. Cover spalling in HSC columns loaded in concentric compression. Journal of Structural Engineering, 124(12), 1431-1437 (1998).
6. Tsai, W. T. Uniaxial compressional stress-strain relation of concrete. Journal of Structural Engineering, 114(9), 2133-2136 (1988).
7. Su, R. K., & Bei, C. The effect of coarse aggregate size on the stress-strain curves of concrete under uniaxial compression. Hkie Transactions, 15(3), 33-39 (2008).
8. Scott BD, Park R, Priestley MJN (1982) Stress-strain behavior of concrete confined by overlapping hoops at low and high strain rates. ACI J
9. Popovics, S. A numerical approach to the complete stress-strain curve of concrete. Cement and concrete research, 3(5), 583-599 (1973).
10. Samani, A. K., & Attard, M. M. A stress–strain model for uniaxial and confined concrete under compression. Engineering Structures, 41, 335-349 (2012).
11. Lim, J. C., & Ozbakkaloglu, T. Stress–strain model for normal-and light-weight concretes under uniaxial and triaxial compression. Construction and Building Materials, 71, 492-509 (2014).
12. Taerwe, L., & Matthys, S. Fib model code for concrete structures 2010. Ernst & Sohn, Wiley (2013).
13. Noguchi, T., Tomosawa, F., Nemati, K. M., Chiaia, B. M., & Fantilli, A. P. A practical equation for elastic modulus of concrete. ACI Structural Journal, 106(5), 690 (2009).
14. EN 1992-1-1 (English): Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings [Authority: The European Union Per Regulation 305/2011, Directive 98/34/EC, Directive 2004/18/EC] (2004).
15. Orakcal, K. Nonlinear modeling and analysis of slender reinforced concrete walls. University of California, Los Angeles (2004).