Classical and Keynesian Models of Inequality and Stagnation

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Abstract

This paper studies two formal models of long run growth with a medium-run distributive cycle, both of which feature causal links from the rise in inequality to a deterioration of long run macroeconomic performance. Both versions feature an endogenous income-capital ratio: one through the Keynesian notion of effective demand, the other building on induced bias in technical change. A key focus of the analysis is on the assumptions necessary in both frameworks to generate policy implications consistent with the observed decline of the labor share, the income-capital ratio, and labor productivity growth during the neoliberal era. Importantly, both theories: (a) provide space for mutually reinforcing pro-labor and pro-growth policies in the long run, although they differ in the mechanisms at play in these processes; (b) imply a potential tradeoff between pro-labor policies and growth on one hand, and long-run employment on the other; (c) are consistent with the evidence on the distributive cycle at business cycle frequency.

Keywords: Distributive cycle, induced technical change, labor share, stagnation
JEL Codes: E11, E12, E25, E32, O33, O41

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1 Introduction

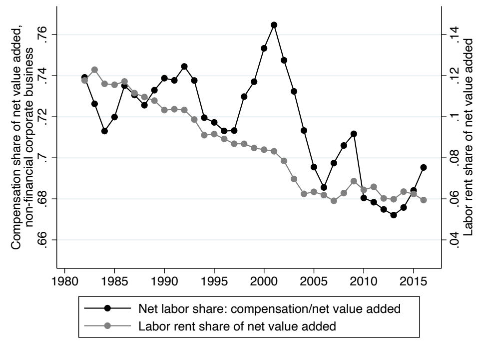
The Great Recession and the slow recovery that followed revived debates about secular stagnation and its relation with distributive variables. The decline in long run macroeconomic performance, however, precedes 2008: as shown in Figure **??**, the economic trajectory of the United States after the Volcker disinflationary shock of the early 1980s is characterized by a decline in the labor share of income, a decline in labor productivity growth, and a decline in the income-capital ratio, the latter defined as the ratio of real value added over net fixed assets. Over the same period, the complement to one of the civilian unemployment rate shows a slightly positive trend, despite its volatility due to two deep recessions: the Volcker shock of the early 1980s, and the Great Recession of 2008.[1] The declining trend in the BEA "headline" labor share in the top-left panel of Figure **??** accelerates in the 2000s, but the surge in the late 1990s is likely driven to a large extent by the top sliver of the wage and salary distribution. **?** have calculated a measure of labor rents as a share of value added (displayed in gray) that shows a monotonically decreasing trend over the neoliberal period. Identifying the fall in worker power as the leading cause, the authors argue that this decline captures a redistribution of rents from labor to capital and, therefore, a decline in the labor share of income.[2]

Another stylized fact of importance for the US economy is the presence of a medium-run *distributive cycle*, that is a counterclockwise movement in the activity-distribution space. Figure **??** displays such cycles for the post-war US macroeconomy. High profitability spurs economic activity—measured either by the income-capital ratio, the output gap, or the employment rate. In turn, the output expansion results in an increase in the labor share; the corresponding reduction in profitability eventually reduces capital accumulation so that activity slows down. The labor share then starts declining, restoring profitability and investment, and the cycle can repeat itself. The first formalization of a distributive cycle is due to **?**, who intended to provide a mathematical description of the Marxian notion of capital-labor conflict. Applied contributions that followed found empirical support for the distributive cycle while adding a role for the Keynesian notion of effective demand (**?????**).

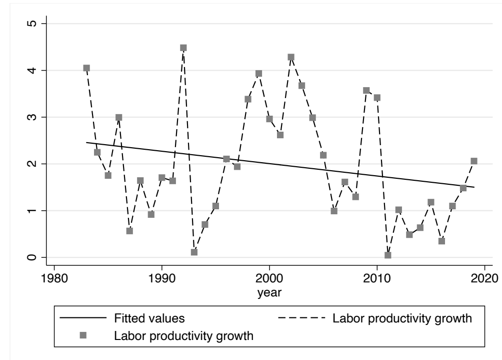
It is a longstanding practice in macroeconomics to separate long run trends from short-to-medium run fluctuations, and to consider business cycles as ultimately driven by exogenous shocks. However, it is highly desirable to have a theoretical framework capable of simultaneously accounting for both sets of stylized facts as endogenous features of an

[1]**?** provide related references, discussion and empirical evidence that the decline in (G-7) growth rates post-2008 is *not* driven by recessionary factors, but stagnationary tendencies that preceded the crisis.

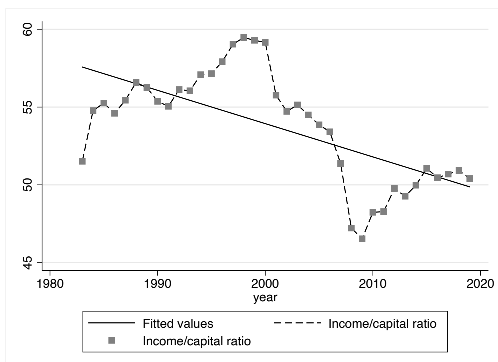
[2]This measure combines estimates of union wage premia, large firm wage premia, and industry wage premia; but it is likely to underestimate the decline in labor rents because it does not include the difficult-to-quantify effect of rising shareholder power in the US economy. However, measurement issues, especially of the labor share and income-capital ratio, are prevalent and must be carefully considered as discussed at length in the literature (see **?**). **?**, for example, suggest that the decline in the labor share results from the switch to treat intellectual property products (IPP) as investment expenditure, although it is unclear how much of IPP-related incomes are actually rents.



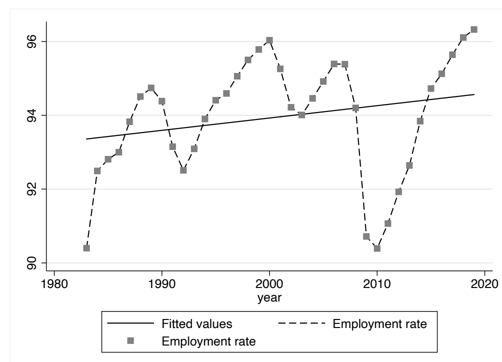
(a) Labor share (%, left axis) and labor rent share (right axis)



(b) Labor productivity growth (%)



(c) VA/NFA (%)



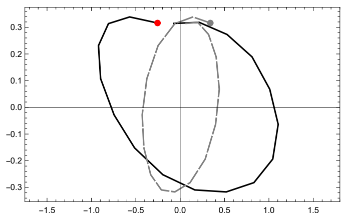
(d) Employment rate (%)

Figure 1: **US macroeconomic trends, 1983-2019.** Data sources: the labor share is the BLS's headline measure (FRED series PRS85006173, percentage share from BLS *Labor productivity and cost measures*), the imputed labor rent share is from **?**; labor productivity growth is FRED series PRS85006091; the income-capital ratio is net value added/net fixed assets (BEA Table 1.14 line 3 for corporate business net value added; BEA Table 6.1 line 2 for current-cost net stock of fixed assets of corporate business); the employment rate is the remainder to one of the civilian unemployment rate ($1-$*UNRATE* from FRED); the output gap is log ratio of real to potential GDP (*GDPC1* and *GDPPOT*, FRED).

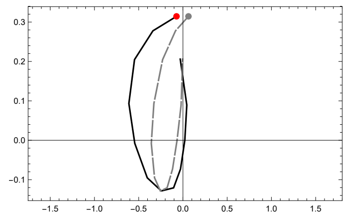
advanced capitalist economy such as the United States. Our goal in this paper is precisely to provide an account of the deterioration of long run economic performance in the Neoliberal era embedded in a medium run distributive cycle. To do so, we build on two strands of heterodox literature: on the one hand, both supply- and demand-driven distributive cycles; on the other hand, the literature on distribution-driven technological change. We build on and find many touchpoints with the innumerable and lasting contributions by the late Peter Flaschel, in particular **??**.

Our contribution is to delineate necessary assumptions, relevant mechanisms, and key results.[3] In both models, a deterioration of labor market institutions and the consequent

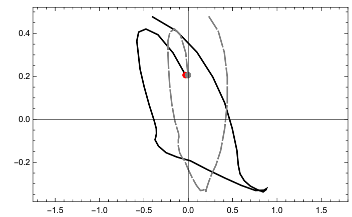
[3]Importantly, and first, we aim to provide an account of the qualitative feature of the US economy in the



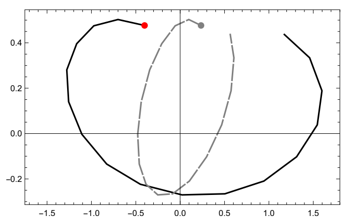
(a) 1953:II–1957:III



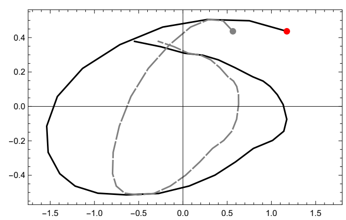
(b) 1957:III–1960:II



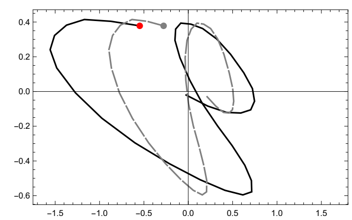
(c) 1960:II–1969:IV



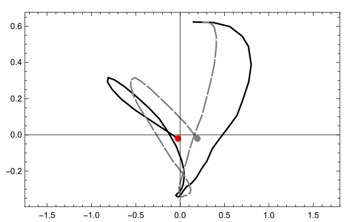
(d) 1969:IV–1973:IV



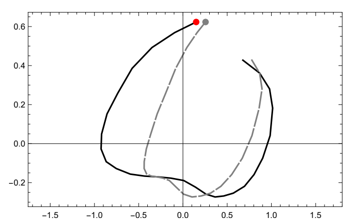
(e) 1973:IV–1981:III



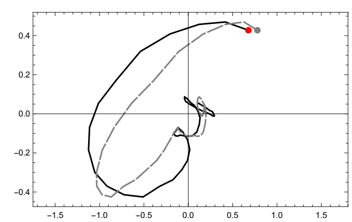
(f) 1981:III–1990:III



(g) 1990:III–2001:I



(h) 2001:I–2007:IV



(i) 2007:IV–2019:IV

Figure 2: **US distributive cycles, 1953:II–2019:IV**. Each panel displays smoothed cycles between employment rate and labor share (dashed) and output gap and labor share (solid) at business cycle frequency, from NBER peak-to-peak. Labor share is on the vertical axis. Starting points are marked with dots. For brevity, cycles prior to 1953 are not shown and the 1980:I peak is subsumed in panel (e). Data sources: Employment rate is the remainder to one of the civilian unemployment rate ($1-$*UNRATE*, FRED), output gap is log ratio of real to potential GDP (*GDPC1* and *GDPPOT*, FRED), and labor share is the BLS's headline measure (FRED series PRS85006173). The applied filter is the maximal overlap discrete wavelet transform; displayed are 4-8 year cycles. For details, see **?** and **?**, p. 203.

increase in inequality leads to rising demand and accumulation in the short run, because both aggregate demand and capital accumulation are *profit-led* in the now-established post-Keynesian terminology. However, long-run macroeconomic performance ultimately worsens, because the declining labor share reduces the pressure to innovate in order to save on labor costs. As a consequence, labor productivity growth, which anchors output growth in the long-run, falls. Moreover, the income-capital ratio must also decline: Harrodian balanced growth requires in the long run that the accumulation rate be equal to the growth

Neoliberal era, and not an accurate quantitative match to the data. Second, we consider changes in the labor markets only as a potential driver of inequality but certainly not the only one. As detailed in the next section, and as suggested by an anonymous reviewer, other factors are important and likely complementary—among these are globalization, economic policy stance, or financialization, to name a few. See also the concluding section for references.

rate of the effective labor force, i.e., the sum of the growth rates of population and labor productivity. An increasing profit share puts upward pressure on the former but downward pressure on the latter, and the decline in the income-capital ratio ensures that the equality is restored.

Thus, in both models outlined in this paper the income-capital ratio is endogenous, in addition to endogenous income shares and employment rate. However, the main mechanisms at play are starkly different. The model of the classical distributive cycle (CDC, hereafter) builds on the notion of induced technical change, discussed in more detail in Section **??**, according to which an increase in the labor share of income spurs productivity-enhancing innovation by profit-maximizing firms aiming to economize on labor costs. As in **?** and **?**, the income-capital ratio adjusts as the result of the firm's profit-maximizing choice. In sharp contrast, at the heart of the Keynesian distributive cycle (KDC hereafter) is the notion of effective demand. Here, quantity adjustments in the income-capital ratio ensure equilibrium in the goods market, but technical change is also endogenous to and increasing in the labor share. Despite these differences, both models imply a long run decline in the income-capital ratio in response to a deterioration in labor market institutions.

The final portion of our argument concerns the relation between the functional distribution of income and long run employment. The Keynesian model features an unambiguous tradeoff between labor-friendly redistribution and the long run employment rate. While the Classical model is in principle ambiguous in this regard, trends in the United States economy during the neoliberal era suggest the distribution-employment tradeoff to be relevant, as displayed in Figure **??**.

We proceed as follows. The next section selectively reviews the related literature. Section **??** outlines the common elements between the two modeling frameworks, while Section **??** and Section **??** present KDC and CDC models and their comparative statics implications, respectively. The following Section **??** illustrates transitional dynamics of both models with numerical simulations. The last section concludes. To foreshadow results, (I) at business cycle frequency, both models imply that income-capital ratio and employment rate lead the labor share, and thus conform to the empirical evidence, discussed in the next section. Further, (II) in steady state of both models, weaker labor market institutions reduce the labor share, the income-capital ratio *and* the growth rate of labor productivity, thus producing key stylized facts of the neoliberal era. Throughout, we discuss in detail the necessary assumptions. Our main contribution thus can be summarized as providing two theories—different in specific mechanism, similar in foundational ideas—that render capital-friendly labor market policies a cause of inequality and stagnation.

2 Selected Literature

Well-known mainstream explanations of the stylized facts presented in Figure **??** can be broadly grouped in supply-side and demand-side accounts, respectively. Among the former, **?** and **?** have proposed an explanation of the rising capital share of income using a standard neoclassical growth model with high elasticity of substitution between capital and labor. An exogenous decline in labor productivity growth or population growth, which anchor the long-run growth rate of the economy, is responsible for an increase in the difference between the rate of return to wealth r and the growth rate g. Accordingly, the capital-income ratio rises (the income-capital ratio falls). If the substitution elasticity exceeds unity, the decline in the income-capital ratio will result in a falling labor share of income. The empirical estimates presented in **?** provided some support for the requirement of an elasticity of substitution above one for a cross-section of countries. Importantly, all of these accounts presuppose full employment at all times: they cannot explain the trend in the (un)employment rate illustrated above, and of course also do not speak to cyclical concerns.

A widely-debated demand side explanation has been proposed by Larry Summers (**??**), who built on insights by Alvin Hansen (**?**). Hansen used the concept of secular stagnation to describe an economy with chronic excess of desired savings over desired investment. Summers argued that the critical factor in this story is that the natural (i.e., full-employment) interest rate is negative. If this is the case, laissez-faire adjustment would imply "a kind of inverse Say's law" (**?**, p. 71) where insufficient demand creates its own lack of supply and a lower level of output, and this process may go on indefinitely (**?**, p.61). Summers refers to changes in the functional distribution only insofar as they affect the aggregate propensity to save in the economy, and not in relation to productivity growth.

The linkages between income shares and macroeconomic performance, both in levels (the income-capital ratio) and growth rates has long been a focus of heterodox macroeconomics. This literature rejects key tenets of neoclassical economics, such as instantaneous smooth factor substitution, marginal productivity pricing and clearing labor markets, as well as the idea of a market for loanable funds whose equilibrium interest rate ensures full employment. Starting with the contributions by **?** and **?**, authors in the neo-Kaleckian tradition have advanced the idea that the income-capital ratio as a proxy for aggregate demand is wage-led through the role of consumption out of wages in boosting effective demand. Depending on model specification, wage-led aggregate demand may translate into wage-led growth, so that the positive effects of redistribution toward labor may not be confined to the level of economic activity, but even the growth rate of the economy. Importantly, the labor share in this literature is either seen as fully exogenous (to economic activity) or determined by an exogenous markup. Labor-friendly redistribution policies, or reductions in product market concentration, would then have positive effects on aggregate demand (see also **??**).

However, the available empirical evidence on the systematic cyclical interactions between measures of economic activity and income shares in the US is not favorable to the neo-Kaleckian model. First, the observed counterclockwise fluctuations in macroeconomic activity *and* the labor share require both variables to be endogenous (**?**). Further, an array of empirical research demonstrates that aggregate demand is *profit-led*, and that activity leads the cycle, contrary to the implications of the neo-Kaleckian model. **???????** provide evidence in favor of profit-led activity and profit-squeeze distribution, and **?** confirm and survey this consensus. **?** have suggested the theoretical possibility of "pseudo-"Goodwin cycles with wage-led demand when a financial cycle is introduced. **?** find evidence that demand is profit-led even when including a financial cycle, thus rebutting their theoretical argument on empirical grounds. Current research concerns the apparent weakening of the mechanisms underlying the Goodwin pattern; see **?** for empirical results on this issue and **?** for related discussion.

That aggregate demand appears to be profit-led has been seen by neo-Kaleckians as the ultimate defeat of progressive redistribution policies. And yet, recent theoretical and applied work in both Classical and Keynesian traditions has advanced the hypothesis that labor-friendly policies may be beneficial to long-run growth through supply forces, i.e. *induced or endogenous technical change* (see **?**, for a comprehensive survey). Policy-driven increases in real wages relative to labor productivity may spur labor-saving innovations that will ultimately generate faster labor productivity growth. The literature builds on seminal contributions by **?** and **?**. **?????** incorporate this mechanism of induced technical change in classical Goodwinian frameworks. Keynesian and structuralist research draws on the underlying idea—that high real unit labor costs spur labor-saving technical change—but does not utilize optimizing, supply-driven model structures (**??**). **??** elaborate the relevant theoretical linkages.

Our contribution also has elements in common with the established literature on the widely-accepted decline in worker power during the Neoliberal era and the effect of high wages on automation and the adoption of new labor-augmenting technologies. With regards to the first strand of literature, **?** and **?** maintain that deregulation in the labor market and a decline in unions' bargaining power might have played a role in reducing the wage share. More recently, **?** argue that labor has been crushed in the recent economic history of the United States: from the decline in private-sector union membership, to the lessening of the established positive relation between wage payments and firm size since the 1980s; to the decline in the wage premium in certain industries such as manufacturing, mining, telecommunications, and utilities; and finally, the declining strength of the link between revenue productivity and wage payments in manufacturing, which serves as a powerful indicator of rent sharing between firms and workers. Our argument is that these developments have also produced a decline in the growth rate of labor productivity because of the lessened incentives by firms to innovate in order to save on labor costs. Similar results can in fact be found in the recent literature on task-based production and automation, where high wages and scarce labor determine strong productivity gains from automation, while productivity gains are small in an environment with low wages and abundant labor (**????**). This positive relation between labor incomes and technical change is not, however, connected to the existence of distributive cycles, or even other cyclical narratives.

In addition, our work relates to recent agent-based literature that has investigated the impact of income inequality and labor market institutions on innovation and economic activity. **?** and **?** extend the "Schumpeter meeting Keynes" (K+S) model to account for labor market institutions. The K+S approach offers a model of growth cycles that combines the Keynesian element of aggregate demand and the Schumpeterian dynamics of evolutionary endogenous growth. **?** show that the decline in unionization can account for the simultaneous slowing of productivity and real wage growth. **?** investigate the effectiveness of alternative labor market policies, and show that either demand-management or passive labor market policies perform better than active labor market policies in mitigating inequality and sustaining long-run growth. In similar fashion, the agent-based stock flow consistent models presented by **?** and **?** integrate endogenous, costly innovations and demand-determined activity levels. They show that both redistribution towards low- and middle-level workers and progressive tax-schemes tend to favor economic growth. None of these contributions, however, connect their investigations to the discussion on secular stagnation and to the existence of distributive cycles. To the best of our knowledge, **?** are the only ones who obtain Goodwin-like cycles within an ABM environment, but contrary to our analysis they do not obtain a clear link between labor market institutions and growth.

On the contrary, distributive cycles have been analyzed within the standard neoclassical framework. Several contributions have emphasized the counter-cyclical nature of the labor share. Multiple explanations have been put forward to explain movements in factors shares, but in most cases the ultimate source of the distributive cycle consists in a technological shock. This can happen directly in competitive real business cycle models, where a stochastic shock to output elasticities affects income shares and output (**??**); otherwise, the effect of the shock can be mediated by economies of scale (**?**) or non-competitive labor and goods markets (**?**). Since technology is exogenous, there is no feedback from functional income distribution to the incentive to innovate so that these models cannot explain the simultaneous decline in the labor share and labor productivity growth, contrary to our contribution. Two recent papers investigate the mutual interplay between the functional income distribution and technical change in endogenous growth models (**??**). They do not, however, analyze the role of labor market institutions.

Finally, our analysis builds on and extends the many fundamental contributions made by the late Peter Flaschel on these topics over his long and illustrious career. First, and as in **??**, our research emphasizes not only policy effects in steady state, but also disequilibrium processes, transitional dynamics and fluctuations around a balanced growth path. Second, and similarly to **??**, we present both supply- and demand-driven models of an extended **?** growth cycle, recognizing the central importance of the framework for heterodox macroeconomics. Third, our focus on the potential trade-off between employment and distribution connects this effort to arguments about labor market institutions and flexicurity advanced in **?**.

3 Common elements

The purpose of this section is to introduce notation, and provide a common framework for the exposition of KDC and CDC models subsequently. Key similarities between the two frameworks pertain to: (i) the production technology, (ii) the class-based savings behavior, (iii) the theoretical core of Goodwin's distributive growth cycle, and (iv) extensions of the latter to endogenize the income-capital ratio.

We begin with the production technology. In both the Keynesian and the Classical model, final output Y is produced using fixed proportions of capital K and labor L. We assume in standard fashion that the labor force N grows exogenously at a rate $n > 0$. Denoting the existing stock of labor-augmenting technologies by A, the output-capital ratio (or, equivalently, income-capital ratio) at full utilization by $\sigma \equiv Y^p/K$, the rate of utilization by $U \equiv Y/Y^p$, and the observed output-capital ratio as $u = \sigma U$, the aggregate production technology is

$$Y = \min\{AL, \sigma UK\}. \tag{3.1}$$

CDC and KDC differ fundamentally in their approach to the income-capital ratio. The CDC model assumes continuous full utilization. For brevity, we set $U = 1$. Accordingly, Section **??** introduces the income-capital ratio directly as σ, which—as will be discussed further below—is determined by technology choices made by firms in the economy. In contrast, the KDC model assumes continuous *under*-utilization of capital. For brevity, we set $\sigma = 1$. Hence, in Section **??** the observed income-capital ratio u is a state variable, and implicitly dominated by demand variation via the rate of utilization U. In short, both frameworks model the realized income-capital ratio, but in CDC the technical coefficient σ becomes endogenous, whereas in KDC the rate of utilization becomes endogenous.

Next, we consider class-based saving behavior and related accounting. In both models, the economy is populated by two classes of households: capitalists own the means of production, receive profit income Π after paying wages to workers, and save a constant fraction of profits denoted by $s_\pi \in (0, 1)$. Workers supply labor to firms, earn a real wage Ω, and do not save. Given that profit-maximization requires firms to set effective capital σUK equal to effective labor AL, with wage share $\psi \equiv \Omega L/Y = \Omega/A$, capitalists' profits will be $\Pi = \sigma U(1 - \psi)K = u(1 - \psi)K$—with $U = 1$ in the Classical model and $\sigma = 1$ in the Keynesian model—and total savings in the economy will be $s_\pi\Pi$. Denoting the profit share as $\pi = 1 - \psi$, the profit rate follows as $r = \pi\sigma U = \pi u$.

Both models build on the theoretical core of the distributive cycle, which pits the employment rate $e \equiv L/N = Y/(AN)$ and the labor share as respectively the prey and the predator in a conflictual yet symbiotic relationship. As in **?**, log-differentiation of the very definition of the state variables in this model implies the standard structure as

$$\hat{e} \;=\; \hat{Y} - (\hat{A} + n) \tag{3.2}$$

$$\hat{\psi} \;=\; \hat{\Omega} - \hat{A}. \tag{3.3}$$

At steady state, $\hat{e} = \hat{\psi} = 0$. In consequence, the growth rate of output must be equal to the natural rate of growth. In this sense, both Goodwinian models presented here are labor-constrained: the conflict over the functional distribution of income maintains a constant employment rate in the long run. It follows that the steady state growth rate of output is equal to the growth rate of the effective labor force, or Harrod's natural rate. Importantly, a constant employment rate in steady state does *not* imply full employment: even in the classical version, where Goodwin's core assumes Say's Law, the labor market does not necessarily clear. For future reference, we define the warranted and natural rate:

$$g^w = s_\pi(1 - \psi)\sigma U \tag{3.4}$$

$$g^* = \hat{A} + n. \tag{3.5}$$

Further, both models incorporate the income-capital ratio as a third state variable. As already discussed, in the CDC model, the full capacity income-capital ratio σ becomes endogenous to the evolution of technology implied by the firms' profit-maximizing behavior, whereas in the KDC model the observed income-capital ratio u is determined through a multiplier-accelerator process.

Additionally, both models utilize a linear real wage Phillips curve to describe labor market institutions, and the resulting real wage dynamics. This is in line with Goodwin's original framework, but clearly an abstraction. We assume that the growth rate of the real wage is ultimately determined in the labor market, i.e. real wages are *labor-market led* (**??**). This approach precludes an important role for price dynamics in the goods market. However, the simplified formulation is used extensively in the Goodwinian literature and appears to best fit empirical evidence (see references on p. **??**). As it further aids comparability of the two models put forth here, we stick with this standard formulation.[4]

Finally, in both models we investigate how changes in labor market institutions affect the macroeconomy in the short and the long run. To that end, we introduce the parameter z to capture the quality of labor market institutions. As will be discussed in more detail below, the parameter z enters the real wage Phillips curve in the Keynesian and classical model, but in the latter it additionally affects the innovation possibility frontier (**?**), which constrains

[4]Closely related literature has considered alternatives. **?**, for example, takes nominal wages and labor productivity as given and hence lets the profit share directly (and instantaneously) increase with demand pressure in the goods market. Similarly, some neo-Kaleckian research builds on the notion that the profit share (or mark-up rate) predominantly reflects pricing power, and is therefore determined in the goods market (see **?**, for an important example). **?** also place tendencies for concentration in the goods market, and hence pricing power, at the center of their Steindlian theory of stagnation. Notably, both **?** and **?** include a "reserve army effect," but this does not materialize via a real wage Phillips curve, or "dual" Phillips curves (see **?**, and references therein). Instead, it is modeled as the adverse effect of high employment rates on output growth or accumulation. In light of recent debates concerning concentration, the neo-Goodwinian literature should be extended to more comprehensively consider market power.

the choice of technological improvements made by firms and is carefully described below. By convention, we assume that an increase in z captures a labor-friendly shift in labor market institutions.

4 The Keynesian model

This section presents the cyclical and long-run dynamics of a Keynesian model of the distributive cycle. Effective demand, its main feature, is introduced via an independent output growth function $h = \hat{Y}$ embedded in the laws of motion of income-capital ratio and employment rate. Differently from the Classical model below, changes in the income-capital ratio do not originate in technology and firm's optimization behavior, but in the mechanism driving the equalization of the growth of expenditures (i.e., demand) and capital stock (i.e., supply). This closure of the model allows us, as outlined in the previous section, to simply write the observed income-capital ratio as $u = Y/K$. Note that the steady-state income-capital ratio in both models is constant and in line with Harrod-neutral technical change. Crucially, and differently from the Classical model, the disequilibrium between output growth and capacity growth emerges here due to the independent role of aggregate demand, and it is addressed via quantity-adjustment in u.

At business cycle frequency, the model exhibits profit-led activity and profit-squeeze distribution: that is, demand varies inversely with the labor share, and the labor share increases in the rate of capacity utilization. The former feature emerges from the following mechanisms, discussed in more detail below: first, a rise in the profit share stimulates investment demand and therefore output growth while all along increasing capacity and consequently the warranted growth rate via saving. Second, we consider the employment rate as another measure of economic activity, which, as is common in Keynesian approaches, varies directly with aggregate demand and, therefore, with the income-capital ratio. Finally, we postulate in standard fashion that rising employment increases the real wage (and, hence, *ceteris paribus*, increases the labor share) and squeezes profits. In a nutshell, over business cycles, economic activity leads distribution.

Distributive conflict takes center stage in the long run via the induced technical change effect. This effect allows a labor market shock to the real wage, and therefore to the labor share, to reverberate into the growth rates of labor productivity and output. The endogenous labor productivity growth implies that the natural rate also becomes endogenous and that the long-run growth rate will be affected by aggregate demand.

4.1 Behavioral functions

The model is built on Classical and Keynesian foundations that feature the warranted rate of growth g^w (equation **??**) and three behavioral functions. These describe the growth of output via an independent expenditure function h, endogenous labor productivity growth

a, and the real wage Phillips curve ω:

$$\hat{Y} = h(u, e, \psi), h_u > 0, h_e < 0, h_\psi < 0 \tag{4.1}$$

$$\hat{A} = a(\psi), a_\psi > 0 \tag{4.2}$$

$$\hat{\Omega} = \omega(e, z), \omega_e > 0, \omega_z > 0. \tag{4.3}$$

The partials in equations **??**-**??** can be motivated as follows. First, and broadly in line with **?**'s pioneering work on cyclical growth, h_u is positive: a higher level of demand as proxied by a higher income-capital ratio u leads to an increase in the growth rate of output. This positive sign ($h_u > 0$) also assures that increases in demand u lead to increases in the employment rate e, which in turn is necessary for stability of the overall dynamics of the economy.

Second, a tight labor market captured by a higher rate of employment e makes the expansion of production more costly. **?**, p. 236 motivates the sign for the partial, $h_e < 0$ as a decrease in the desired rate of expansion due to adjustment and turnover costs at high employment rates. The sign can also be motivated with direct reference to **?**'s seminal essay on the "political aspects of full employment:" high employment rates undermine the power of capital, and thus depress expansion plans (see also **??**).

Further, $h_\psi < 0$ represents a Kaleckian link from the functional distribution of income to economic activity. The built-in assumption is that investment demand responds negatively to a higher labor share, *and* does so sufficiently strongly to overcome any positive effects of ψ on consumption expenditures.

The induced technical change effect ($a_\psi > 0$) implies that higher real wages relative to labor productivity trigger efforts to economize on labor costs. As will be seen in Section **??**, this positive relation can be formally micro-founded through the profit-maximizing choice of the direction of technical change by competitive firms (see also Section **??**). Here we model aggregate productivity directly, and simply assume that pressure for labor-saving innovation arises when real unit labor costs are high: $\hat{A} = a(\psi), a' > 0$. For further discussion of this reduced-form approach, see **??**.

Finally, real wage growth responds to the employment rate and the quality of labor market institutions according to a real wage Phillips curve: $\hat{\Omega} = \omega(e, z)$, with $\omega_e, \omega_z > 0$. The profit squeeze arises as the ultimate result of a Keynesian chain of causation: high demand increases the income-capital ratio u, which drives up the employment rate e, which in turn puts upward pressure on real wage growth and hence the labor share ψ. This mechanism is common in neo-Goodwinian models and related empirical applications.[5]

[5]For a discussion of profit-squeeze effects between labor share on the one hand and income-capital ratio or employment rate on the other, see **?**. These authors motivate pressure from u on ψ vs. e on ψ on the basis of insider vs. outsider bargaining, respectively. In our model, only outsider bargaining matters, but the resulting dynamics are qualitatively equivalent. See also the discussion of real wage determination in Section **??**.

4.2 The dynamical system

The model consists of three differential equations describing the evolution of the income-capital ratio, the employment rate, and the wage share. Given the definition of u, e and ψ, log-differentiation provides the following laws of motion for the three state variables:

$$\dot{u} = u(\hat{Y} - \hat{K}) = u[h(u, \psi, e) - s_\pi(1 - \psi)u] \tag{4.4}$$

$$\dot{e} = e[\hat{Y} - (\hat{A} + n)] = e[h(u, \psi, e) - a(\psi) - n] \tag{4.5}$$

$$\dot{\psi} = \psi(\hat{\Omega} - \hat{A}) = \psi[\omega(e, z) - a(\psi)]. \tag{4.6}$$

Equations **??** and **??** have already been discussed in Section **??**. Equation **??** requires further discussion, since various motivations for dynamic adjustment of the income-capital ratio (or rate of utilization) exist in the relevant Keynesian literature. First, recall from Section **??** that full capacity output is $Y^p = \sigma K$. In the Keynesian case, $\sigma = 1$, so that not only $\hat{Y}^p = \hat{K}$ but also $Y^p = K$. Hence, equation **??** models the rate of utilization. Second, Harrod, Domar and Kalecki all emphasized the dual nature of investment: it creates demand (as expenditure, contained in Y), and creates supply capacity (as installed capital K). These theoretical concepts need to be related to the differential equation $\hat{u} = \hat{Y} - \hat{K} = \hat{Y} - \hat{Y}^p$, which is definitionally true.

The formulation in equation **??** assumes that h models the growth rate of *ex ante* expenditures on investment and consumption goods, whereas g^w describes the growth rate of *ex post* installed supply capacity. If the growth of aggregate demand exceeds that of supply, $h > g^w$, *ex post* investment falls short of *ex ante* investment. This in turn implies an increase in the rate of utilization ($\dot{u} > 0$), and (though only implicitly) a decline in inventories.

This approach differs in some respects from the literature. **?**, for example, models the income-capital ratio, and interprets the growth rate of the capital stock as an investment function (see eq. 8, p. 234). The model of **?** describes the utilization rate like ours, but the growth rate of capacity is interpreted as "capital formation," i.e. an accumulation function (see p. 396). This is conceptually similar to Skott, despite other crucial differences. Further, **?**, **?**, Ch. 9, p. 307 and related literature put forth a "dynamic IS-equation" for changes in aggregate demand which would be similar to a linearization of our equation for $\hat{u}$, though these authors additionally include the real interest rate, and not the employment rate.

Across these examples, including the present model, profitability drives accumulation in Keynesian goods markets. Our formulation is concise and tractable, and consistent with the key features of the extant literature. Additionally, it facilitates the straightforward comparison to the CDC model. In the following paragraphs, we characterize steady state and dynamics in more detail.

In a non-trivial steady state, $\dot{u} = \dot{e} = \dot{\psi} = 0$, and the three state variables attain their non-zero steady state levels, indicated by a star. This also implies that $\hat{Y} = h = g^w = g^* = \hat{A} + n$, and $\hat{\Omega} = \hat{A}$. Put differently, in steady state, Harrod's three growth rates equilibrate, and Kaldor's stylized facts are satisfied. Further, and in light of the preceding discussion, the goods market is in equilibrium only when all three state variables have attained positions of rest: $\hat{Y} = \hat{K}$ only when $\dot{u} = 0$, which, given h, can be maintained only when $\dot{e} = \dot{\psi} = 0$, too.

Without further restrictions on functional forms and parameters there are no guarantees of existence and uniqueness of the stationary state. However, if a steady state of the system above exists, it can be characterized in reduced form as follows:

$$u^* = \Phi^u(\boldsymbol{\alpha}, z) \tag{4.7}$$

$$e^* = \Phi^e(\boldsymbol{\alpha}, z) \tag{4.8}$$

$$\psi^* = \Phi^\psi(\boldsymbol{\alpha}, z) \tag{4.9}$$

where, for example, Φ^u denotes that the steady state income-capital ratio u^* is a function of a set of parameters $\boldsymbol{\alpha}$ and the critical parameter z. Crucially, z appears only in the real wage Phillips curve, and directly affects the steady state labor share via equation **??**. Further, the income-capital ratio adjusts to equalize the growth rate of output h with the warranted rate g^w, the employment rate adjusts to equalize the growth rate of output h with the natural rate g^*, and the labor share adjusts to equalize the growth rates of real wage and labor productivity.

The Jacobian matrix of this system, evaluated at such a steady state (though without starring of variables, for the sake of brevity), is

$$J^* = \begin{bmatrix} u(h_u - s_\pi(1-\psi)) & uh_e & u(h_\psi + s_\pi u) \\ eh_u & eh_e & e(h_\psi - a_\psi) \\ 0 & \psi\omega_e & -\psi a_\psi \end{bmatrix}. \tag{4.10}$$

We assume $h_u < s_\pi(1-\psi)$ and $|h_\psi| > s_\pi u$, which implies that the income-capital ratio is self-stabilizing; and that it reacts negatively to a rising labor share, which implies a profit-led economy. As a result, the determinant is unambiguously negative ($|J^*| < 0$), which is necessary for dynamic stability. See Appendix **??** for a signed Jacobian matrix, and restrictions on parameters that guarantee local stability. Further, the sign pattern generates relevant cyclical stylized facts (**???**), and the two-dimensional subsystems are consistent with empirically-observed cycles in u, e and e, ψ.[6]

It is worth stressing once again that this three-dimensional model resolves both of Harrod's problems without sacrificing the essential Keynesian property of a long run role for aggregate demand. Importantly, the solution is facilitated by the interaction between the labor constraint and the functional distribution of income in determining the natural rate of growth. Indeed, the crux of the matter is that the growth rate of output equilibrates with the

[6]The u, ψ-cycle emerges only in the three-dimensional system, and is there determined by $\partial\dot{e}/\partial u = eh_u > 0$: the employment rate increases in the income-capital ratio, and then drives the profit squeeze via ω_e. As a referee reinforces, the dynamics of the three-dimensional model, potentially projected into two-dimensional spaces, are what ultimately matters. See **??** for further discussion.

warranted rate of growth (in equation **??**) and, additionally, with the natural rate of growth (in equation **??**). This of course implies also that $g^w = g^*$. At the same time, the labor share is tied to institutions governing real wage bargaining as described by z, so that:

$$\frac{\partial g^*}{\partial z} = a_\psi \frac{\partial \psi}{\partial z} > 0 \Leftrightarrow \frac{\partial g^w}{\partial z} > 0. \tag{4.11}$$

Consider now the effect of an erosion in labor market institutions and bargaining power of workers, captured by a decline in the z-term in the real wage Phillips curve. Recall that $|J^*| < 0$; further, we denote the (i, j)-th minor as $|J_{ij}|$. Cramer's rule then implies

$$\frac{\partial u^*}{\partial z} = -\psi \frac{|J_{31}|}{|J^*|} = h_e(a_\psi + s_\pi u) \frac{eu\psi}{|J^*|} > 0 \tag{4.12}$$

$$\frac{\partial e^*}{\partial z} = \psi \frac{|J_{32}|}{|J^*|} < 0 \tag{4.13}$$

$$\frac{\partial \psi^*}{\partial z} = -\psi \frac{|J_{33}|}{|J^*|} > 0, \tag{4.14}$$

where $|J_{32}|$ and $|J_{33}|$ are straightforward to sign, and $|J_{31}|$ is also unambiguous due to h_e and h_ψ appearing in both columns of the minor. (See Appendix **??** for details.)

Thus, an adverse shock to labor's bargaining power implies an increase in the employment rate, a fall in the steady-state labor share, a fall in the income-capital ratio, and a decline in the long-run growth rate since $\partial g^* / \partial z = a_\psi \partial \psi / \partial z > 0$. In other words, this shock leads to a new steady-state that features higher inequality, lower growth and higher employment. Thus, the model's key variables match the stylized facts of the neoliberal era.

In summary, in this Keynesian model of the distributive cycle, the labor share is linked in steady state to institutions of the labor market, rather than merely technology. In this view, the state, asked to retreat in the face of excessive faith in markets, and the neoliberal labor market, deregulated and deskilled to favor capital, join forces to depress labor share, income-capital ratio and steady state growth, but generate countervailing forces on the employment rate.

4.2.1 Short-run effect of a negative shock to z

It is worthwhile to trace out not only the long-run, but also the short-run effects of a negative shock to z on the economy starting from a steady state equilibrium. The institutional variable affects the dynamics of the system merely through its effect on real wage growth. Given the initial level of labor productivity growth, lower wage growth reduces the wage share: $\dot{\psi} < 0$. The fall in the wage share affects the dynamics of both the employment rate and capacity utilization. It simultaneously raises output growth and decreases labor productivity growth; and both effects contribute to higher labor demand, or $\dot{e} > 0$. The effect on capacity utilization, on the other hand, is ambiguous in principle because the lower wage share increases both output and capital growth. The assumed signs, however, imply that the income capital ratio increases in response to a lower labor share, and the numerical simulations discussed in Section **??** confirm this result.

5 The Classical model

The Classical version of the model builds on the contributions by **?** and **?**, who introduced the induced innovation hypothesis in the Goodwin growth cycle. The main consequence of this integration is that the perpetual cyclical fluctuations in employment rate and labor share are resolved by the induced feedback from the latter to labor productivity growth. In short, while there are fluctuations along the transitional dynamics, the Goodwin steady state becomes ultimately stable. We generalize their contribution with the introduction of labor market institutions, represented by the shift variable z.

While there is no effective demand and firms are always fully utilizing their capacity in this model, the interaction between the labor market and the choice of technology implies that the framework produces endogenous medium run fluctuations in all its state variables—including the income-capital ratio. As such, the Classical version provides a benchmark for modeling endogenous cycles around a balanced growth path even in economies that are *always* supply-constrained. This contrasts sharply with the standard neoclassical approach where cycles are mostly arising from exogenous shocks, as already noted in Section **??**.

5.1 Accumulation, innovation and choice of productivity growth

As discussed in Section **??**, the Classical model assumes continuous full capacity utilization so that $U = 1$ and $u = \sigma$. Accordingly, since all savings are automatically invested, capital accumulation is:

$$\hat{K} = g^w = s_\pi(1 - \psi)\sigma. \tag{5.1}$$

We model innovation by following the induced technical change literature. The innovation possibility frontier (IPF) describes the evolution of technology by defining the set of growth rates of labor and capital productivity freely available to competitive firms. If we let $a = \hat{A}$, we have $a = f(\hat{\sigma}; z)$, with $f(0; \cdot) > 0, f_{\hat{\sigma}} < 0, f_{\hat{\sigma}\hat{\sigma}} < 0$. The frontier is decreasing and strictly concave in order to capture the increasing complexity in the trade-off between labor-augmenting and capital-augmenting innovations. With respect to the relation between the IPF and labor market institutions, two assumptions ensure that labor-friendly policies increase long run growth, the wage share, and the output-capital ratio:

Assumption 1 $\frac{\partial f(0;z)}{\partial z} > 0$*: z positively affects the Harrod-neutral rate of technological progress.*

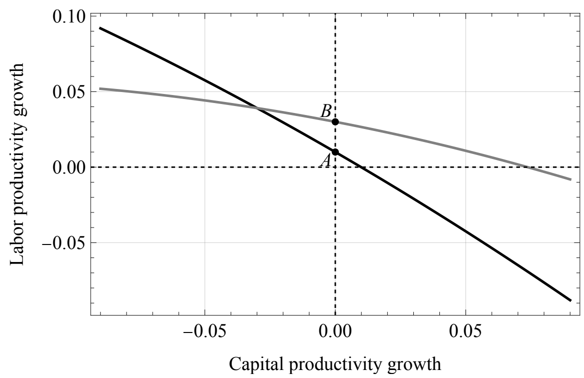


Figure 3: **Innovation Possibility Frontier**. Starting at a baseline Harrod-neutral profile of technical progress (point A), an increase in z under Assumptions **??** and **??** rotates the frontier counterclockwise and makes it flatter at the new, higher Harrod-neutral rate (point B). See Section **??** for the functional form used in order to obtain this plot.

Assumption 2 $\frac{\partial f_{\hat{\sigma}}(0,z)}{\partial z} > 0$*: the slope of the IPF is strictly increasing in* z*.*

Both assumptions capture the political economy of capital-labor conflict in shaping the evolution of technology. The intuition behind Assumption **??** is that the incentives to implement higher labor productivity growth increase when workers' bargaining power rises. Put differently, a more conflictual environment implies stronger pressure for firms to innovate in order to replace labor. Assumption **??**, on the other hand, requires that a higher z renders the IPF flatter, so that the marginal rate of transformation becomes smaller, at the vertical intercept. This captures the other side of the conflict. The intuition is that workers respond to the firm's incentives to economize on labor requirements by rendering the tradeoff between capital-saving and labor-saving technical change harder.

In summary, even though both the position and the shape of the IPF are exogenous in principle, an institutional shock translates into a technological one by affecting the innovation set available to firms. This specification conforms with Classical (and Keynesian) perspectives according to which technology and institutions are fundamentally intertwined.[7] The movement from point A to B in Figure **??** showcases the effect of an increase in z on both the slope and the vertical intercept of the IPF under Assumptions **??** and **??**.

We assume that firms choose the direction of technical change, that is a point ($\hat{\sigma}$, a) on the IPF, in order to maximize the instantaneous rate of unit cost reduction $\psi a + (1-\psi)\hat{\sigma} = \psi f(\hat{\sigma}; z) + (1-\psi)\hat{\sigma}$. The first order condition is

$$-f_{\hat{\sigma}}(\hat{\sigma}; z) = \frac{1-\psi}{\psi}. \tag{5.2}$$

[7]See also **?**, p. 342 for a similar argument applied to investment decisions in a post-Keynesian framework; and the mainstream literature on institutions as a fundamental cause of long run growth, which operationalizes the role of institutions on technology through their effect on total factor productivity (**?**).

Since $f_{\hat{\sigma}\hat{\sigma}} < 0$, capital productivity growth is an inverse function of the wage share, say $\hat{\sigma} = b(\psi, z), b_\psi < 0$. Therefore, labor productivity growth is a direct function of the wage share: $a = f\left[b(\psi, z); z\right], a_\psi > 0$.

5.2 The dynamical system

The three state variables of the economy are capital productivity (or income-capital ratio), the employment rate, and the wage share. The evolution of capital productivity follows from the firm's first order conditions. In order to find the dynamics of the employment rate, we plug $\hat{Y} = \hat{\sigma} + g^w = b(\psi, z) + s_\pi(1 - \psi)\sigma$ and $\hat{A} = f\left[b(\psi, z)\right]$ into equation (**??**). Finally, we need to define real wage growth to track movements in the labor share. In line with the Keynesian model, we assume $\hat{\Omega} = \omega(e, z)$, with $\omega_e, \omega_z > 0$. Accordingly, the dynamical system is:

$$\dot{\sigma} = \sigma b(\psi, z) \tag{5.3}$$

$$\dot{e} = e\left\{b(\psi, z) + s_\pi(1 - \psi)\sigma - (f\left[b(\psi, z); z\right] + n)\right\} \tag{5.4}$$

$$\dot{\psi} = \psi\left\{\omega(e, z) - f\left[b(\psi, z); z\right]\right\} \tag{5.5}$$

The first equation shows that the IPF is solely responsible for the determination of the wage share in the long run. Remembering that x^* denotes the steady state value of variable x, using equation (5.2), and setting $\dot{\sigma} = \dot{e} = \dot{\psi} = 0$, we obtain the non-trivial steady state as described by the three equations:

$$\psi^* = \frac{1}{1 - f_{\hat{\sigma}}(0; z)}, \tag{5.6}$$

$$\sigma^* = \frac{f(0; z) + n}{s_\pi(1 - 1/(1 - f_{\hat{\sigma}}(0; z))}, \tag{5.7}$$

$$\omega(e^*, z) = f(0; z). \tag{5.8}$$

The Jacobian matrix evaluated in steady state (again without starring variables for notational simplicity) is:

$$J = \begin{bmatrix} 0 & e(b_\psi - s\sigma - a_\psi) & es_\pi(1 - \psi) \\ \psi\omega_e & -a_\psi & 0 \\ 0 & \sigma b_\psi & 0 \end{bmatrix}, \tag{5.9}$$

and Appendix A.3 shows the signed Jacobian matrix and provides a proof of the local stability of the steady state.

We now focus on the comparative dynamics of the model with respect to z. Equation (**??**) illustrates that the long run functional distribution of income is fully determined by the slope of the IPF at the steady state. Given our Assumption 1, a positive shock to z

raises the wage share. The IPF and z are also the only determinants of long run growth as $a^* = f(0; z)$. The steady state labor productivity growth is the vertical intercept of the innovation possibility frontier and, given Assumption 1, it moves up with z: see the increase from A to B in Figure 2. The steady state income-capital ratio ensures the equality between the natural $(a + n)$ and the warranted growth rate g^w. Under the assumptions discussed, worker-friendly labor market policies affect the income-capital ratio in two ways. They raise labor productivity growth and, in turn, the natural growth rate, while they harm total saving and the warranted growth rate through the negative impact on the profit share. Equation (**??**) shows that both effects contribute to a rise in the income-capital ratio. The steady state employment rate, on the other hand, stabilizes the wage share dynamics by ensuring that real wage growth equals labor productivity growth. A change in labor market institutions will have an ambiguous effect on the employment rate if z simultaneously raises wage and labor productivity growth. Equation (**??**) shows that employment will fall when $\omega_z(e, z) > f_z(0; z)$, so that a lower employment rate is necessary to slow down wage growth. This seems the relevant case compatible with the empirical evidence discussed in the Introduction.

5.2.1 Short-run Effect of a Negative Shock to z

Let us now follow the effects of a negative shock to z on the economy starting from a steady state equilibrium. As already discussed, the institutional variable affects the dynamics of the system through three separate effects: (a) wage growth; (b) shape, and (c) position of the IPF. The first step consists in understanding what happens to capital productivity growth $\hat{\sigma}$. The firm's optimization implies that equation (**??**) is continuously satisfied. We can totally differentiate it to find $d\hat{\sigma}/dz = b_z = -f_{\hat{\sigma},z}/f_{\hat{\sigma},\hat{\sigma}} > 0$, under Assumption **??** and given the concavity of the IPF. So we know that capital productivity growth decreases on impact after a negative shock to labor market institutions. The effect on labor productivity growth is less obvious at first sight. A movement along the IPF induced by $d\hat{\sigma} < 0$ tends to increase the growth rate a, but the inward shift of the frontier acts in the opposite direction: in fact $da/dz = \frac{\partial f}{\partial \hat{\sigma}} \frac{d\hat{\sigma}}{dz} + \frac{\partial f}{\partial z}$ where the two terms of the sum have opposite signs. We prove in Appendix A.3.1, however, that in order to satisfy (**??**), firms will respond to a decrease in z by initially increasing labor productivity growth while simultaneously decreasing capital productivity growth. This means that on impact technical change is Marx-biased, with negative capital productivity growth while positive labor productivity growth (**?**). Furthermore, wage growth decreases because of the effect of the institutional parameter on the wage-Phillips curve. These conclusions imply that all three state variables of the dynamical system formed by equations (**??**), (**??**) and (**??**) decline on impact: the wage share declines given the joint decrease in real wage growth and increase in labor productivity growth; the income-capital ratio falls as capital productivity growth becomes negative; and, finally, the employment rate decreases due to the simultaneous fall in capital productivity and warranted growth rate and rise in labor productivity growth. This results —which, once again, always holds under the assumptions made in this model— are illustrated in the numerical simulations discussed in the next Section.

6 Transitional dynamics and numerical simulations

This section presents numerical simulations to illustrate the transitional dynamics of both models. First, simulations confirm that both models portray cyclical dynamics of the Goodwin-type: both activity variables (employment rate e and income-capital ratio σU) lead the labor share ψ.

Second, the focus lies on the response of both models to a decline in the parameter z. As previously discussed, this parameter captures characteristics of labor market institutions, and we assume that a decline in z renders these institutions more friendly to capital. Further, simulations demonstrate (i) the effect of a change in z *on impact*, and (ii) the very different effects of a change in z between short run and long run (i.e., steady state). In particular, (i) differs across the two models, since the firms' optimization problem in the classical model requires instantaneous adjustments so that (**??**) remains continuously satisfied. Most importantly for our purposes, (ii) shows that capital-friendly labor market policies lead to a boom in accumulation in the short run, but imply stagnation in the long run.

It should be emphasized that these simulations are merely illustrative. We calibrate the steady states of income-capital ratio, employment rate and labor share to plausible values for an advanced capitalist country such as the US, and assume signs of key coefficients consistent with available empirical evidence (again for the US). However, we are not conducting exercises to match higher moments, or estimate the model.

6.1 Calibration

We begin with an overview of the common elements for both models: the three state variables' steady states, the warranted growth rate and the real wage Phillips curve. Steady state values of income-capital ratio, employment rate and labor share are $u = \sigma U = 0.4$, $e = 0.9$ and $\psi = 2/3$. These roughly correspond to longer run averages for the US macroeconomy. The warranted rate of growth is $g^w = s_\pi(1-\psi)\sigma U$, and equal to realized rate of growth g and natural rate g^* in steady state. We assume that the steady state rate of growth is $g = g^w = g^* = 0.03$, which, given ψ and u implies $s_\pi = 0.225$. Further, the growth rate of the labor force is constant ($n = 0.01$), so that in steady state $\hat{A} = a = 0.02$.

The real wage Phillips curve also appears in both models. It is implemented as a linear function of the employment rate,

$$\omega(e,z) = \omega_0 + ze, \tag{6.1}$$

where $z = 1$ is the critical parameter describing labor market institutions. The calibration exercise leads to $\omega_0 = -0.88$ given e and z, and that at the steady-state $\omega = a = 0.02$.

The key differences are rooted in the different behavioral functions of the CDC and KDC model. The CDC model is built around the innovation possibility frontier (IPF), whereas the KDC model draws on reduced-form functions for output growth h and labor productivity growth a. We discuss these in turn. A functional form for the IPF that satisfies all the necessary assumptions stated previously is

$$\hat{A} = f[b(\psi, z)] = -b^{\beta} - \alpha\frac{b}{z} + za^*, \tag{6.2}$$

where we set $\beta = 2, \alpha = 1/2$. With $z = 1$ and $a^* = 0.02$, this parabola conforms to Harrod-neutral growth in steady state as outlined just above, when $\dot{\sigma} = 0$. Further, a decline in z rotates the IPF clockwise around a point in the North-Western quadrant with negative capital productivity growth and positive labor productivity growth.[8] The first-order condition (**??**) and this functional form for the IPF imply

$$\hat{\sigma} = b(\psi, z) = \left[\frac{1}{\beta}\left(\frac{1}{\psi} - \frac{\alpha}{z} - 1\right)\right]^{\frac{1}{\beta-1}}, \tag{6.3}$$

which fully determines the steady state labor share as $\psi = \frac{z}{z+\alpha} = 2/3$. Note that equation (**??**) determines capital productivity growth, and, after substitution in (**??**), also labor productivity growth. Output growth follows as the sum of capital productivity growth b and the warranted growth rate g^w.

In the KDC model, output growth is determined by the function h in equation (**??**), and labor productivity growth by the function a in equation (**??**). To match the signs assumed for the Jacobian at the steady state, see equation (**??**) and Appendix **??**, we assume linear functions and calibrate these as follows:

$$h = h_0 + h_u u + h_e e + h_\psi = 0.22 + 0.065u - 0.02e - 0.3\psi \tag{6.4}$$

$$a = a_0 + a_\psi \psi = -0.15 + 0.25\psi. \tag{6.5}$$

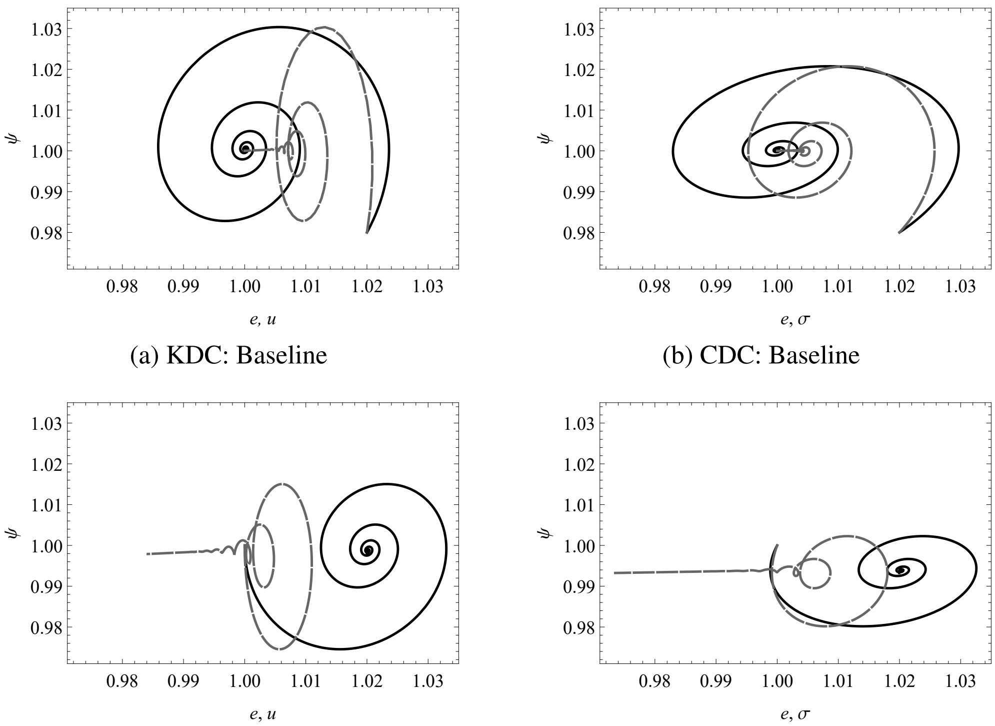
To summarize, we note that with these parameters and in steady state, the Jacobian matrices of both models show the sign patterns as noted in Appendix **??**, and both models feature a pair of complex eigenvalues with negative real parts, and a third real and negative eigenvalue. Accordingly, both models display damped oscillations around their steady state values.

As shown in Appendix **??**, stability of the KDC model depends on several behavioral parameters. In contrast, stability of the CDC model is assured by the structure of the model and the concavity assumption on the IPF. To further explore the KDC parameter constraints derived in Appendix **??**, we conducted a numerical sensitivity analysis that illustrates the effect of key parameters on the eigenvalues of the calibrated model. Results shown in

[8] A visual illustration is provided in Figure **??**: starting from the IPF in grey that intersects the vertical axis at point B, a decline in z produces a clockwise rotation and ultimately a lower Harrod-neutral rate at point A.

Figure ?? clarify that h_u has the narrowest range: theory requires $h_u > 0$, but instability arises not far to the right of zero. Other parameters show significant flexibility.

6.2 Discussion



(a) KDC: Baseline

(b) CDC: Baseline

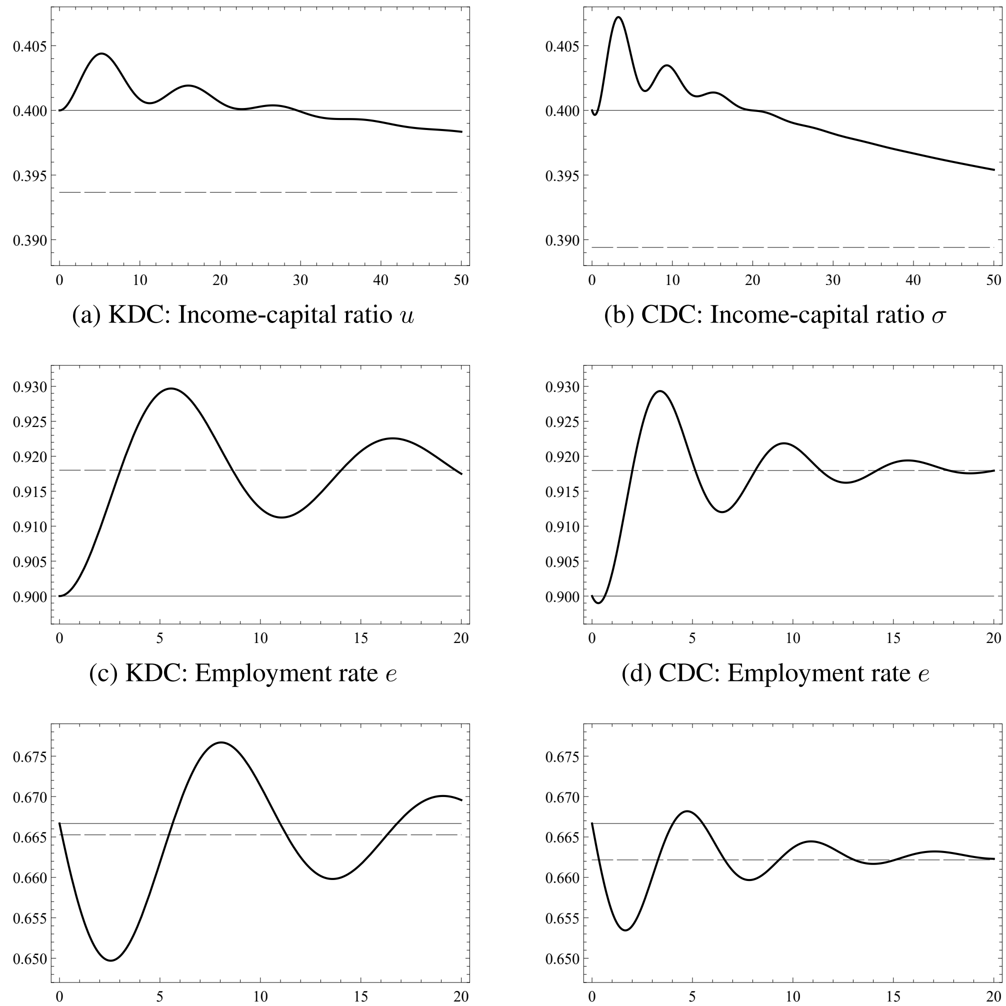
(c) KDC: z-shock

(d) CDC: z-shock

Figure 4: **Simulations: phase trajectories**. Each panel shows simulated phase trajectories, relative to baseline steady state values. Top row represents a baseline simulation, with employment rate e and income-capital ratio (σ or u) 2% above respective steady states on the horizontal axis, and the labor share ψ 2% below steady state on the vertical axis. Bottom row shows trajectories in response to a shock to z, with state variables in steady state at $t = 0$. In all these simulations, the trajectory of employment and labor share is plotted in black; while the trajectory of income-capital ratio and labor share is plotted in dashed gray. See Section **??** for discussion.

This section discusses the results of the simulation exercises on the basis of three different figures. Figure **??** shows trajectories in the phase space. Figures **??** and **??** report time paths of state variables and key growth rates, respectively, in response to an assumed 2% decline in the parameter z. Across all of these figures, KDC model output is shown in the left column of panels, and CDC model output in the right column of panels.

The top row of Figure **??** illustrates convergence of both models from an initial condition out-of-steady-state. The panels show phase trajectories for (e, ψ) and $(\sigma U, \psi)$; to facilitate visualization, the variables are normalized by their respective steady states. The standard



(a) KDC: Income-capital ratio u

(b) CDC: Income-capital ratio σ

(c) KDC: Employment rate e

(d) CDC: Employment rate e

(e) KDC: Labor share ψ

(f) CDC: Labor share ψ

Figure 5: **Simulations: state variables**. Each panel shows simulated phase trajectories for the two model's state variables. Left column represents the KDC, the right column the CDC. The solid (dashed) line indicates baseline (post-shock) steady state. See Section **??** for discussion.

result of a counter-clockwise cycle in activity-labor share space obtains: activity variables (be that the employment rate or the income-capital ratio) lead the labor share; this can be compared to the 4-8 year cycles for the post-war US macroeconomy in Figure **??**.

The bottom row of this figure shows phase trajectories in response to the z-shock, assuming that the model is in steady state at time zero. Two observations stand out. First, the shock to z affects state variables in qualitatively the same manner across the two models. Both the income-capital ratio and the labor share fall, while the employment rate rises. Further, convergence to the new steady state also displays the Goodwin pattern, though the income-capital ratio's cyclical movement ultimately subsides.

The medium term trends of the simulated state variables' response to the decline in z are further visualized in Figure **??**. The solid horizontal line indicates the initial (pre-shock) steady state, and the dashed horizontal line the steady state after the shock. The top row reports the income-capital ratio, which *rises* above the pre-shock steady state for a number of periods, before falling below that level on its path to converge to the new steady state. The labor share (in the third row) declines, and the employment rate (in the second row) rises. This pattern is a critically important facet of the theory laid out in this paper: macroeconomic activity as proxied by the income-capital ratio initially rises following an institutional shift adverse to labor, but ultimately declines towards a lower steady state.

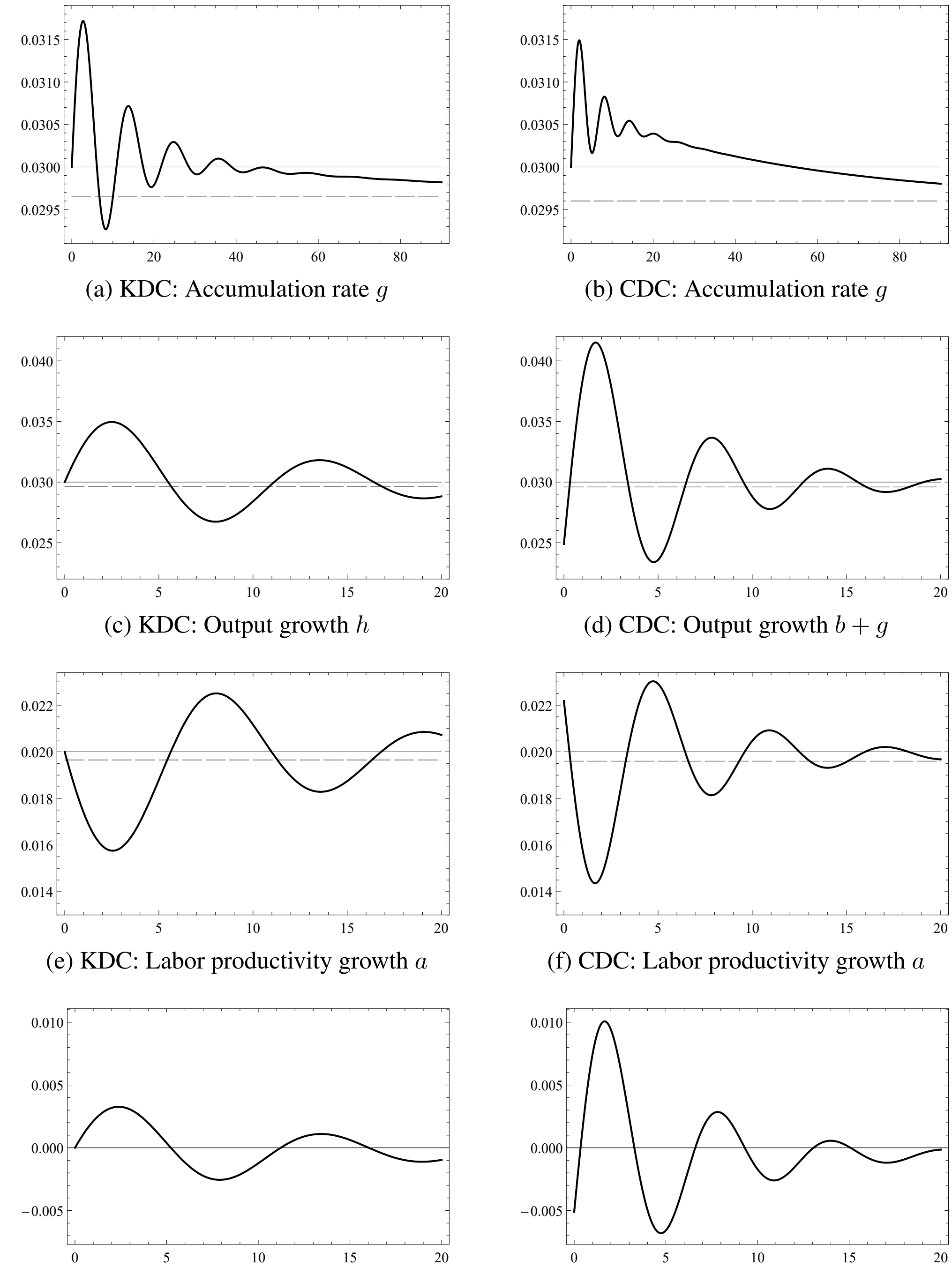
The growth rates in Figure **??** provide further detail on this issue. The first row of panels is important in this regard, reporting the *realized* accumulation rate. In the KDC model, this is the *ex post* rate of accumulation, while *ex ante* investment demand is captured in the h function; see related dicussion in Section **??**. In the CDC model, accumulation always proceeds at the warranted rate. In both models, this rate initially rises above its pre-shock steady state (on average), before stagnating towards the lower post-shock steady state.

The remaining panels in rows 2-4 show the growth rates of output, labor productivity and capital productivity respectively. These shed light, in particular, on the effects of the z-shock *on impact*. For the left column of panels (describing the KDC trajectories), these growth rates all start at their pre-shock values. In contrast, the right column of panels (describing the CDC counterparts) shows a discrete change in these growth rates at $t = 0$. This is due to the fact that *instantaneous* firm optimization forces a jump to the new (rotated) IPF. Once this jump has occurred, movements along the IPF determine the path towards the new steady state. Crucially, as shown in Section **??**, the pattern of technical change is Marx-biased on impact: positive labor productivity growth is coupled with a decline in capital productivity.

7 Conclusion

This paper has advanced two formal models building on Goodwin's seminal distributive cycle to argue that changing labor market institutions in the neoliberal era have played a key role in driving down the labor share, the income-capital ratio, and the growth rates of labor productivity and output. The income-capital ratio is endogenous in both models, but through different mechanisms. In the classical version, the main channel at play is the choice of the direction of technical change at the firm level; while in the Keynesian version, it is the principle of effective demand, and the role of quantity adjustments in the goods market. Despite these differences, both models are fundamentally similar in their portrayal of the relevant short run and long run dynamics. A key objective of this paper was to draw out the specific assumptions necessary to arrive at such a point, and therefore to further debate both on theory and empirical applications.

Both models conform to the empirical evidence at business cycle frequency, i.e. show



(a) KDC: Accumulation rate g

(b) CDC: Accumulation rate g

(c) KDC: Output growth h

(d) CDC: Output growth $b + g$

(e) KDC: Labor productivity growth a

(f) CDC: Labor productivity growth a

(g) KDC: Capital productivity growth b

(h) CDC: Capital productivity growth b

Figure 6: **Simulations: growth rates**. Each panel shows simulated phase trajectories for growth rates implicit in the two model's output. Left column represents the KDC, the right column the CDC. The solid (dashed) line indicates baseline (post-shock) steady state. See Section **??** for discussion.

the Goodwin pattern. Both models also conform to key stylized facts of the neoliberal era, and in particular a decline in the labor share of income coupled with economic stagnation. Additionally, both models imply a potential trade-off between labor-friendly policies aimed at reversing the rise in inequality and the long-run employment rate, which appears to be borne out by US data in the relevant period. In this respect, our frameworks are related to work by **?** on flexicurity: pro-labor (and pro-growth) policies also require compensatory measures to deal with their adverse effects in terms of long-run job destruction.

To conclude, we emphasize that stylized modeling exercises such as our own feature multiple degrees of freedom. As argued before, other factors might be equally salient during the period under consideration and might have contributed to economic stagnation and the decline in the wage share. In particular, the Neoliberal era has been characterized by a rising share of the financial (and real-estate) sector in the economy (**????**), a shift in corporate culture toward the maximization of shareholder value (**?**), growing market power (**?**), monopsony power in the labor market (**????**), and increasing globalization of goods (**?**) and capital markets (**?**). Our focus on adverse shifts in worker power is not meant to dismiss these equally important issues. In fact, our formal modeling exercises have to be considered complementary to the literature emphasizing these other channels.

References

Acemoglu, D. and Autor, D. (2011) Skills, Tasks and Technologies: Implications for Employment and Earnings. Chap. 12 in *Handbook of Labor Economics*, vol. 4, Part B, edited by David Card and Orley Ashenfelter. Elsevier.

Acemoglu, D. and Restrepo P., (2018) The Race Between Machine and Man: Implications of Technology for Growth, Factor Shares, and Employment. *American Economic Review* 108(6): 1488–1542.

Acemoglu, D. and Restrepo, P. (2019) Automation and New Tasks: How Technology Displaces and Reinstates Labor. *Journal of Economic Perspectives*, 33(2): 3–30.

Acemoglu, D., Johnson, S., Robinson, J. (2006) Institutions as a Fundamental Cause of Long Run Growth. In Aghion, P, and Durlauf, S., eds. *Handbook of Economic Growth*. Springer.

Amadeo, E.U. (1986) Notes on Capacity Utilisation, Distribution and Accumulation. *Contributions to Political Economy*, 5: 83–94.

Azar, J., Huet-Vaughn, E., Marinescu, I., Taska, E., Von Wachter, T. (2019) Minimum wage effects and labor market concentration. *NBER Working Paper* No w26101.

Barbosa-Filho and Taylor (2006) Distributive and Demand Cycles in the US Economy: A Structuralist Goodwin Model. *Metroeconomica*, 57(3): 389–411.

Barrales, J., Mendieta-Muñoz, I., Rada, C., Tavani, D., von Arnim, R. (2022) The Distributive Cycle: Evidence and Current Debates. *Journal of Economic Surveys*, 36(2): 468–503.

Barrales, J., and von Arnim, R. (2017) Longer-run Distributive Cycles: Wavelet Decompositions for the US, 1948-2011. *Review of Keynesian Economics*, 5(2): 196–217.

Basu, D. and Foley, D. K. (2013) Dynamics of Output and Employment in the US Economy, *Cambridge Journal of Economics*, 37(5): 1077-1106.

Basu, D. and Gautham, L. (2019) What is the impact of an exogenous shock to the wage share? VAR results for the US economy, 1973–2018, *UMASS Amherst Economics Working Papers*, No. 2019-08.

Bental, B. and Demougin, D. (2010) Declining labor shares and bargaining power: An institutional explanation, *Journal of Macroeconomics*, 32(1): 443-456.

Boldrin, M., Wang, Y. and Zhu, L. (2019) The cyclical behavior of factor shares, *Meeting Papers, Society for Economic Dynamics Journal of Macroeconomics*, Number 1208.

Brock, E., and , S. Dobbelaere, S. (2006) Has international trade affected workers bargaining power?, *Review of World Economics*, 142(2): 233-266.

Caiani, A., Russo, A., and Gallegati, M., (2018a) Are higher wages good for business? an assessment under alternative innovation and investment scenarios. *Macroeconomic Dynamics*, 24(1): 191-230.

Caiani, A., Russo, A., and Gallegati, M., (2018b) Does Inequality Hamper Innovation and Growth?, *Journal of Evolutionary Economics*, 29(1): 177–228.

Checchi, D. and Garcia-Peñalosa, C., (2010) Labour Market Institutions and the Personal Distribution of Income in the OECD, *Economica*, 77(307): 413-450.

Chiarella, C. and Flaschel, P. (2000) *The Dynamics of Keynesian Monetary Growth*. Cambridge, UK.

Colciago, A. and Rossi, L. (2015) Firm dynamics, endogenous markups, and the labor share of income. *Macroeconomic Dynamics*, 19(6): 1309-1331.

De Loecker, J., Eckhout, J. (2020) The rise of market power and the macroeconomic implications. *Quarterly Journal of Economics*, 135(2): 561–644.

Delli Gatti, D., Gaffeo, E., Gallegati, G., Giulioni, G., Kirman, A., Palestrini, A., Russo, A. (2006) Complex Dynamics, Financial Fragility and Stylized Facts, Lecture Notes in Economics and Mathematical Systems. In: Beckmann, M., Knzi, H. P., Fandel, G., Trockel, W., Basile, A., Drexl, A., Dawid, H., Inderfurth, K. (ed.),*Artificial Economics*, 127-135.

Diallo, M.B., Flaschel, P., Proaño, C. R. (2011) Reconsidering the dynamic interaction between real wages and macroeconomic activity. *Research in World Economy*, 2(1).

Dosi, G., Freeman, R.B., Pereira, M.C., Roventini, A., and Virgillito, M.E. (2021) The impact of deunionization on the growth and dispersion of productivity and pay. *Industrial and Corporate Change*, 30(2): 377-408.

Dosi, G., Pereira, M.C., Roventini, A., and Virgillito, M.E. (2019) What if supply-side policies are not enough? The perverse interaction of flexibility and austerity. *Journal of Economic Behavior and Organization*, 162: 360-388.

Dutt, A. K. (1984) Stagnation, Income Distribution and Monopoly Power. *Cambridge Journal of Economics*, 11(1): 75–82.

Flaschel, P. (1993) *Macrodynamics: Income Distribution, Effective Demand, and Cyclical Growth*. Peter Lang.

Flaschel, P. (2009) *The Macrodynamics of Capitalism: Elements for a Synthesis of Marx, Keynes, and Schumpeter*. Springer.

Flaschel, P. (2015) Goodwin's MKS System: a Baseline Macro Model. *Cambridge Journal of Economics*, 39(3): 1591-1605.

Flaschel, P., Franke, R., Proaño, C. R. (2006) Wage-Price Dynamics and Income Distribution in a Semi-Structural Keynes-Goodwin Model, *Structural Change and Economic Dynamics*, 17: 452-465.

Flaschel, P., Franke, R., Semmler, W. (2008) Kaleckian investment and employment cycles in post-war industrialized economies, *NSSR Working Paper*, January 2008.

Flaschel, P. and Luchtenberg, S. (2013) *Roads to Social Capitalism*. Routledge.

Flaschel, P. and Skott, P. (2006) Steindlian models of growth and stagnation, *Metroeconomica*, 57(3): 303-338.

Foley, D.K. (2003) Endogenous Technical Change with Externalities in a Classical Growth Model. *Journal of Economic Behavior and Organization*, 52(2):167-189.

Foley, D.K. (2011) The Political Economy of US Output and Employment 2001-2010. SCEPA Working Paper 2011-5, The New School for Social Research.

Foley, D.K.; Michl. T. and Tavani, D. (2019) *Growth and distribution*, Harvard University Press.

Goodwin, R. (1967) A Growth Cycle. In: C. H. Feinstein, ed. *Socialism, Capitalism, and Economic Growth*. Cambridge.

Gordon, R. (2015) Secular stagnation: A supply-side view. *American Economic Review: Papers and Proceedings*, 105(5): 54-59.

Growiec, J., McAdam, P., Muck, J. (2018) Endogenous labor share cycles: Theory and evidence. *Journal of Economic Dynamics and Control*, 87: 74-93.

Hansen, A. (1939). Economic Progress and Declining Population Growth. *American Economic Review*, 29(1): 1-15.

Jayadev, A., Rodriguez, F. (2013). The declining labor share of income. *Journal of Globalization and Development*, 3(2): 1-18.

Julius, A.J. (2005) Steady State Growth and Distribution with an Endogenous Direction of Technical Change. Metroeconomica, 56(1): 101-125.

Kalecki, M (1943) Political aspects of full employment.

Karabarbounis L., Neiman B. (2014) The Global Decline of the Labor Share. *The Quarterly Journal of Economics*, 129(1): 61-103.

Kennedy, C. (1964) Induced bias of technology and the theory of distribution. *The Economic Journal*, 74(295): 541-547.

Koh, D. and Santaeulàlia-Llopis, R. and Zheng, Yu (2020) Labor share decline and intellectual property products capital. *Econometrica*, 88(6): 2609-2628.

Lazonick, W. and O'Sullivan, M., (2000) Maximizing shareholder value: a new ideology for corporate governance. *Economy and Society* 29(1): 13-25.

Li, Mengheng and Mendieta-Muñoz, I. (2020) Are long-run output growth rates falling? *Metroeconomica*, 71: 204-234.

Manning, A. (2003) *Monopsony in Motion*. Princeton, NJ: Princeton University Press.

Manning, A. (2011) Imperfect competition in labor markets. *Handbook of Labor Economics*, 4: 973-1041.

Mendieta-Muñoz, I., Rada, C., Santetti, M. and von Arnim, R. (2020) The US labor share of income: What shocks matter? *Review of Social Economy*.

Mendieta-Muñoz, I., Rada, C., & von Arnim, R. (2021) The Decline of the US Labor Share Across Sectors. *Review of Income and Wealth*, 67(3): 732-758.

Michl, T. and Tavani, D. (2022) Path dependence and stagnation in a classical growth model, *Cambridge Journal of Economics*, 46(1):195-218.

Naidu, S., Posner, E., Weyl, E. (2018) Antitrust remedies for labor market power. *Harvard Law Review*, 132(2): 536-601.

Palley, T. (2007) Financialization: What it is and Why it Matters. PERI Working paper No. 53, University of Massachusetts Amherst.

Percival, D.B. and A.T. Walden (2000) *Wavelet Methods for Time Series Analysis*, Cambridge, UK: Cambridge University Press

Piketty, T. (2014) *Capital in the XXI Century*. Belknap.

Piketty, T., Zucman G. (2014) Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010. *The Quarterly Journal of Economics*, 129(3): 1255-1310.

Proaño, C.R., Flaschel, P., Ernst, E., Semmler, W. (2006) Disequilibrium Macroeconomic Dynamics, Income Distribution and Wage-Price Phillips Curves: Evidence from the U.S. and Euro Area. *Macroeconomic Policy Institute (IMK)*, Working Paper 04/2006.

Rada, C. (2012) Social security tax and endogenous technical change in an economy with an aging population. *Metroeconomica*, 63(4): 727-756.

Rada, C., and Kiefer, D. (2015) Profit-Maximizing Goes Global: The Race to the Bottom. *Cambridge Journal of Economics*, 39: 1333-1350.

Rada, C., Santetti, M., Schiavone, A. and von Arnim, R. (2021) Classical and Keynesian vignettes on secular stagnation: From labor suppression to natural growth. University of Utah Department of Economics Working Paper Series No. 2021-05.

Rada, C., Schiavone, A. and von Arnim, R. (2022) Goodwin, Baumol & Lewis: How structural change can lead to inequality and stagnation. *Metroeconomica*, https://doi.org/10.1111/meca.12390.

Rios-Rull, J.-V. and Santaeulalia-Llopis, R. (2010) Redistributive shocks and productivity shocks. *Journal of Monetary Economics*, 57(8): 931-948.

Rowthorn, R. (1981) Demand, Real Wages, and Economic Growth. *Thames Papers in Political Economy*, Autumn, 1-39.

Setterfield, M. (2021) Whatever happened to the 'Goodwin pattern'? Profit squeeze dynamics in the modern American labour market. *Review of Keynesian Economics*, forthcoming.

Shah, A., and Desai, M. (1981) Growth cycles with induced technical change. *Economic Journal*, 91(364): 1006-10.

Shao, E. and Silos, P. (2014) Accounting for the cyclical dynamics of income shares. *Economic Inquiry*, 52(2): 778-95.

Skott, P. (1989) Effective demand, class struggle and cyclical growth, *International Economic Review*, 30(1): 231-247.

Skott, P. (2017) Weaknesses of wage-led growth. *Review of Keynesian Economics*, 5(3): 336-359.

Skott, P., and Ryoo, S. (2008) Macroeconomic Implications of Financialization *Cambridge Journal of Economics*, 32(6): 827-862.

Stansbury, A., and Summers, L. (2021) The Declining Worker Power Hypothesis: an Explanation for the Recent Evolution of the American Economy. *Brookings Papers on Economic Activity*, Spring 2020.

Stiglitz, J., Tucker, T., Zucman, G. (2020) The Starving State. *Foreign Affairs*, January/February 2020.

Stockhammer, E., Michell, J. (2017) Pseudo-Goodwin Cycles in a Minsky Model. *Cambridge Journal of Economics*, 41(1):105-25.

Storm, S. and Naastepad, C.W.M. (2012) *Macroeconomics Beyond the NAIRU*, Harvard University Press.

Summers, L. H. (2014) US Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound. *Business Economists*, 49 (2): 65-73.

Summers, L. H. (2015) Demand-Side Secular Stagnation. *American Economic Review*, 105 (5): 60-65.

Tavani, D., Flaschel, P., Taylor, L. (2011) Estimated Nonlinearities and Multiple Equilibria in a Model of Distributive-Demand Cycles. *International Review of Applied Economics*, 25(5): 519-538.

Tavani, D., Zamparelli, L. (2017) Endogenous Technical Change in Alternative Models of Growth and Distribution. *Journal of Economic Surveys*, 31(5): 1272-1303.

Taylor, L. (1983) A Stagnationist Model of Economic Growth. *Cambridge Journal of Economics*, 9(4): 383-403.

von Arnim, R. and Barrales, J. (2015) Demand-driven Goodwin cycles with Kaldorian and Kaleckian features, *Review of Keynesian Economics*, 3(3): 351-373.

von Weizsäcker, K.C. (1966) Tentative Notes on a Two-Sector Model with Induced Technical Change. *Review of Economic Studies*, 33(3): 245-251.

Young, A. T. (2004) Labor's share fluctuations, biased technical change, and the business cycle. *Review of Economic Dynamics*, 7 (4): 916-931.

Zamparelli, L. (2015) Induced Innovation, Endogenous Technical Change and Income Distribution in a Labor-Constrained Model of Classical Growth. *Metroeconomica*, 66(2): 243-262.

Zeira, J. (1998) Workers, Machines, and Economic Growth. *Quarterly Journal of Economics*, 113(4): 1091-1117.

Zipperer, B and Skott, P. (2011) Cyclical patterns of employment, utilization, and profitability, *Journal of Post-Keynesian Economics*, 34(1):26-57.

A Appendix

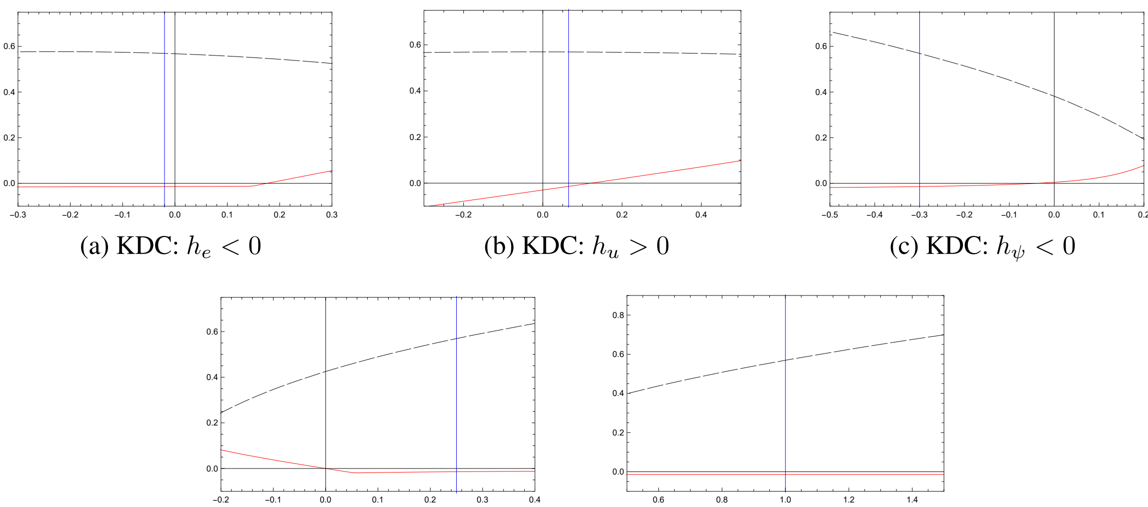
A.1 Local stability conditions

The Routh-Hurwitz conditions for local stability of three dimensional systems involve analyzing the Jacobian matrix J evaluated at the steady state. If we let $|J_{ij}|$ be the minor of J obtained by deleting the ith row and jth column, the conditions are:

$$Tr(J) \quad < \quad 0 \tag{A.1}$$

$$\begin{aligned} |J_{11}|+|J_{22}|+|J_{33}| &> 0 && \text{(A.2)}\\ |J| &< 0 && \text{(A.3)}\\ -Tr(J)(|J_{11}|+|J_{22}|+|J_{33}|)+|J| &> 0. && \text{(A.4)} \end{aligned}$$

A.2 Keynesian model



(a) KDC: $h_e < 0$ (b) KDC: $h_u > 0$ (c) KDC: $h_\psi < 0$

(d) KDC: $a_\psi > 0$ (e) KDC: $\omega_e > 0$

Figure 7: **KDC simulations: Parameters & stability**. Each panel illustrates sensitivity of model stability to a parameter, reported along the horizontal axis and denoted in the panel caption with the assumed sign. Each panel reports three values: The red line is the largest real part of the three eigenvalues. The dashed black line is the (positive) imaginary part of the pair of complex eigenvalues. The vertical blue line is the value of the parameter chosen in the simulations.

We reproduce the Jacobian matrix (equation **??**), and include the sign pattern implied by assumptions stated in the main text:

$$J^* = \begin{bmatrix} u(h_u - s_\pi(1-\psi)) & uh_e & u(h_\psi + s_\pi u) \\ eh_u & eh_e & e(h_\psi - a_\psi) \\ 0 & \psi\omega_e & -\psi a_\psi \end{bmatrix} \tag{A.5}$$

The corresponding sign pattern is then:

$$\begin{bmatrix} - & - & - \\ + & - & - \\ 0 & + & - \end{bmatrix}$$

Under these assumptions, inequalities **??** and **??** hold. In fact, $Tr(J) = u(h_u - s_\pi(1-\psi)) + eh_e - \psi a_\psi < 0$; and $|J_{11}|+|J_{22}|+|J_{33}| > 0$ since $|J_{11}| = -e\psi(h_e a_\psi + \omega_e(h_\psi - a_\psi)) > 0$, $|J_{22}| = -u(h_u - s_\pi(1-\psi))\psi a_\psi > 0$, and $|J_{33}| = -ues_\pi(1-\psi)h_e > 0$. If j_{ij} is the ith row and jth column element of J, **??** requires $j_{11}|J_{11}| - j_{21}|J_{21}| = j_{11}j_{22}j_{33} - j_{33}(j_{11}j_{22} - j_{21}j_{12}) + j_{21}j_{23}j_{13} < 0$. From the Jacobian, $j_{11}j_{22}j_{33} < 0$ and $j_{21}j_{23}j_{13} < 0$, while $-j_{33}(j_{11}j_{22} - j_{21}j_{12}) = ues(1-\psi)h_e < 0$. Hence, inequality **??** holds.

Inequality **??** is more difficult to ascertain, but a sufficient condition for it to hold is $-(eh_e - \psi a_\psi) > uh_u$: Rearranging gives $(j_{11} - Tr(J))|J_{11}| - j_{21}|J_{22}| - Tr(J)(|J_{22}| + |J_{33}|) > 0$, where $j_{11} - Tr(J) = -(j_{22} + j_{33}) = -Tr(J_{11})$. Substituting and distributing gives:

$$\underbrace{-Tr(J_{11})|J_{11}|}_{I}\underbrace{-j_{21}|J_{22}|}_{II}\underbrace{-Tr(J)|J_{22}|}_{III}\underbrace{-Tr(J)|J_{33}|}_{IV} > 0 \tag{A.6}$$

These terms I–IV can be signed, and a sufficient condition for **??** to hold is $I + II > 0$. Rearranging gives:

$$\underbrace{[Tr(J_{11}) + uh_u]}_{+/-}\underbrace{h_e a_\psi}_{-} + \underbrace{\omega_e}_{+}[\underbrace{(Tr(J_{11}) + uh_u)}_{+/-}\underbrace{h_\psi}_{-} - \underbrace{Tr(J_{11})a_\psi}_{-} + \underbrace{uh_u s_\pi u}_{+}]$$

which is positive if

$$Tr(J_{11}) + uh_u < 0 \Leftrightarrow -(eh_e - \psi a_\psi) > uh_u. \tag{A.7}$$

While this is only sufficient, it is straightforwardly interpreted: the stabilizing elements along the trace have to outweigh the destabilizing element h_u. In particular, h_u appears in the law of motion of the employment rate, and there can lead to violation of the fourth Routh-Hurwitz inequality.[9]

Figure **??** illustrates the stability conditions of the Keynesian model discussed above in the context of the numerical simulations of Section **??**. We emphasize again that these simulations are only illustrative. Key coefficients and calibrated variables are consistent with available empirical evidence (for the US), but we replicate the sign pattern of the Jacobian above, rather than match higher moments or estimate the model. The available empirical evidence, discussed in Sections **??** and **??**, supports this calibration (discussed in Section **??**) as broadly plausible.

[9]Note further that the fourth Ruth-Hurwitz inequality ensures that the real parts of a potential pair of complex eigenvalues are negative. Numerical simulations confirm that increases in h_u can lead to a Hopf bifurcation as the real parts pass through zero from below. A stable limit cycle emerges (with linear behavioral functions). Details are available upon request.

A.3 Classical model

The Jacobian matrix of this system, evaluated at the steady state (without starring of variables, for brevity) is

$$J = \begin{bmatrix} 0 & e(b_\psi - s\sigma - a_\psi) & es_\pi(1-\psi) \\ \psi\omega_e & -a_\psi & 0 \\ 0 & \sigma b_\psi & 0. \end{bmatrix} \tag{A.8}$$

And its sign pattern is:

$$\begin{bmatrix} 0 & - & + \\ + & - & 0 \\ 0 & - & 0 \end{bmatrix}$$

Inequality **??** is satisfied as $Tr(J) = -a_\psi < 0$. Remembering $b_\psi < 0$, **??** is also verified since $|J| = e\sigma\psi s(1-\psi)\omega_e b_\psi < 0$. Furthermore, since $|J_{11}| + |J_{22}| = 0$, **??** holds, too: $|J_{33}| = -e\psi\omega_e(b_\psi - s\sigma - a_\psi) > 0$. Similarly to the Keynesian model, (**??**) requires some more work. Remembering $a_\psi = f_{\hat{\sigma}} b_\psi$, we find $-|J_{33}| + |J|/Tr(J) = e\psi\omega_e(b_\psi - s\sigma - a_\psi) - e\sigma\psi s(1-\psi)\omega_e b_\psi / a_\psi = e\psi\omega_e\left(b_\psi - s\sigma - a_\psi - s\sigma(1-\psi)/f_{\hat{\sigma}}\right))$. Further, recall that $-f_{\hat{\sigma}} = \frac{1-\psi}{\psi}$, we find $-|J_{33}| + |J|/Tr(J) = e\psi\omega_e\left(b_\psi - s\sigma - a_\psi + s\sigma\psi\right) = e\psi\omega_e\left(b_\psi - s\sigma(1-\psi) - a_\psi\right) < 0$.

A.3.1 Short-run adjustment of labor productivity growth

Let us prove that the profit-maximizing choice of labor productivity growth is a negative function of z *on impact*. Start at $t = 0$ in steady state and consider a shock that takes the state of the labor market from z_0 to z_1, with $z_0 > z_1$. In steady state, labor productivity growth is $f(0; z_0) \equiv \bar{a}$. Define $\hat{\sigma}^\bullet$ as the level of capital productivity growth such that $f(\hat{\sigma}^\bullet; z_1) = f(0; z_0) = \bar{a}$. Notice that since f is increasing in z and decreasing in $\hat{\sigma}$, it follows that $\hat{\sigma}^\bullet < 0$. Let us now move to the slope of the f function in the two points $(\hat{\sigma}^\bullet, z_1)$ and $(0, z_0)$. Under the assumption $\frac{\partial f_{\hat{\sigma}}(0,z)}{\partial z} > 0$, at the given level of labor productivity growth $\bar{a}$ the after shock labor productivity growth is a steeper function of $\hat{\sigma}$ (in absolute terms); hence $f_{\hat{\sigma}}(\hat{\sigma}^\bullet; z_1) < f_{\hat{\sigma}}(0; z_0)$. Notice that using **??** the steady state equilibrium requires $f_{\hat{\sigma}}(0; z_0) = -\frac{1-\psi}{\psi}$. Define $\hat{\sigma}^{\bullet\bullet}$ as the after shock optimal level of capital productivity growth, that is $\hat{\sigma}^{\bullet\bullet}$ such that $f_{\hat{\sigma}}(\hat{\sigma}^{\bullet\bullet}; z_1) = f_{\hat{\sigma}}(0; z_0) = -\frac{1-\psi}{\psi}$. Since $f_{\hat{\sigma}\hat{\sigma}} < 0$, and $f_{\hat{\sigma}}(\hat{\sigma}^\bullet; z_1) < f_{\hat{\sigma}}(0; z_0) = -\frac{1-\psi}{\psi}$, then $\hat{\sigma}^{\bullet\bullet} < \hat{\sigma}^\bullet$. Finally, given $f_{\hat{\sigma}} < 0$, we have $f(\hat{\sigma}^{\bullet\bullet}; z_1) > f(\hat{\sigma}^\bullet; z_1) = f(0; z_0)$. The drop in z raises labor productivity growth.