Dissipative light bullets in externally driven multimode Kerr cavity with parabolic 3D potential

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Abstract: We study the formation of dissipative light bullets in externally driven multimode GRIN fiber cavity with chirped pulse pumping. Numerical simulations show the generation of stable bullets, with a spatiotemporal shape controlled via the pump chirp.

1. Introduction

Light bullets (LBs), also known as spatiotemporal solitons, are elusive wave packets that may propagate in nonlinear media. LBs involve light localization both in the plane transverse to the propagation direction, and in time [1]. In single-pass (i.e., conservative) optical systems, LB formation may be expected from a counter-balancing between diffractive and dispersive effects on the one hand, and Kerr nonlinearity on the other hand. However, conservative LBs normally suffer from *wave collapse*, so that their propagation is unstable [2]. Wave collapse can be arrested by different mechanisms, e.g., by means of a parabolic refractive index potential. A typical example of a parabolic potential is provided by multimode optical systems, such as graded-index (GRIN) fibers [3,4].

By introducing a 3D spatio-temporal parabolic potential, here we reveal that robust (with respect to propagation instabilities) LB can be found. Furthermore, by controlling the temporal component of the 3D potential, we demonstrate that it is possible to reshape the LBs, e.g., from a perfect sphere into a localized tube.

2. Mean-field model

In the mean-field approximation, the dynamics of the electric field envelope $A(x,y,\tau,t)$ propagating within the cavity is governed by the modified dimensionless Lugiato-Lefever (LL) equation with a 3D parabolic potential

$$\frac{\partial A}{\partial t} = \mathrm{i}\left(\nabla_\perp^2 + \frac{\partial^2}{\partial \tau^2}\right)A - \mathrm{i}\left(x^2+y^2+C\tau^2\right)A + \mathrm{i}|A|^2A - (1+\mathrm{i}\delta)A + P, \tag{1}$$

where δ is the cavity detuning parameter, P is the pumping rate, $\nabla_\perp^2 = \partial_{xx} + \partial_{yy}$ is the Laplace operator describing spatial diffraction, (x^2+y^2) is the transverse parabolic index profile (or spatial potential), and $C\tau^2$ originates from synchronous phase modulation or linearly chirped pulsed pump driving [5]. Here the chirp coefficient C controls the curvature of the temporal potential.

3. Results: effects of temporal potential on LBs shaping

Let us focus on the implications that the temporal potential may have on the formation and shape of LBs. Figure 1 shows different LB solutions for $P=3$, $\delta=3$, and various values of C. This solution was obtained through direct numerical simulations of Eq. (1), when considering as initial condition a 3D Gaussian profile with unit value for both its amplitude and width. We observed that the convergence to the LB state is very fast. Simulations over a long time scale demonstrate the stability of the LBs.

The spatiotemporal shape of the LBs can be adjusted by tuning the strength of the chirp coefficient C. In Fig. 1(a), by setting the parameter $C=1$ we obtain a perfectly spherical 3D LB. The field intensities $|A(x,y=0,\tau)|^2$ and $|A(x,y,\tau=0)|^2$ exhibit the same Gaussian shape. The field intensity decreases when moving away from the center $(x=0, y=0, \tau=0)$. This feature is very different from 1D dissipative Kerr soliton solutions, that sit over a flat intensity background. Increasing C, so does temporal confinement, until a temporally compressed LB with a lentil shape is obtained [see Fig. 1(b)]. To the contrary, if one reduces C, the LB temporally expands, as it

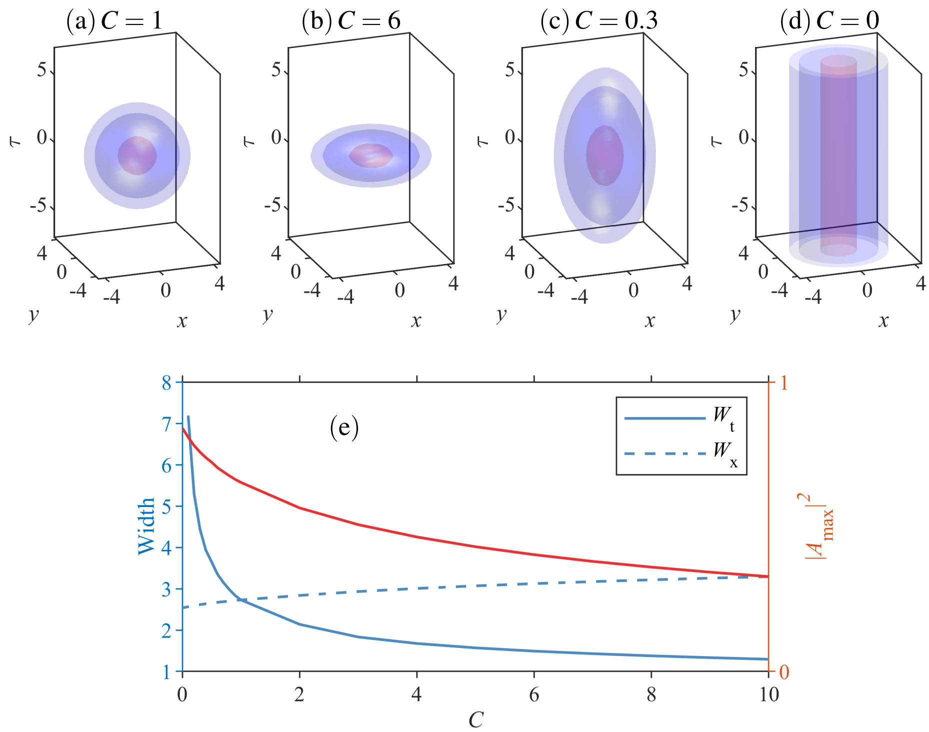

Fig. 1. (a-d) Stable light bullet state evolution with temporal potential strength C. Shells from center to outside represent equal field intensity with values $0.5 \cdot |A_{\max}|^2$, $0.1 \cdot |A_{\max}|^2$, and $0.05 \cdot |A_{\max}|^2$, respectively. (e) Temporal width W_t (full width at half maximum at $(x=0, y=0)$), spatial width W_x at $(y=0, \tau=0)$ and maximum values of field intensity $|A_{\max}|^2$ of light bullets as a function of C. Other parameters are $P=3$, $\delta=3$.

is shown in Fig. 1(c). Finally, if $C=0$, we obtain the tube solution [see Fig. 1(d)], where the field is homogeneous in the temporal domain, while keeping a Gaussian shape in the spatial domain.

Figure 1(e) illustrates the evolution of the temporal width (FWHM) W_t, the spatial width W_x, and the peak field intensity $|A_{\max}|^2$ of the LBs, vs. the chirp C. A larger C decreases both the peak value $|A_{\max}|^2$ and the temporal width W_t, and it slightly increases the spatial width W_x. In the absence of dispersion, nonlinearity, pump, losses and detuning, Eq. (1) possesses Laguerre-Gauss (LG) mode solutions. In a weakly nonlinear regime, the LBs may be considered to be composed by a superposition of LG modes. The fundamental mode is a Gaussian with FWHM $W_{mode\,1} = 1.665$. In our case, the width of the LB $W_x > W_{\mathrm{mode}\,1}$, which means that is composed by the superposition of multiple modes [see Fig. 1(e)].

In conclusion, we predicted the formation of dissipative LBs in externally driven Kerr cavities including a 3D potential. Such LBs are strong attractors for wide baisin of initial states. Moreover, we have shown that the shape of the LBs can be controled by tuning the temporal modulation or potential.

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