Introduction to the papers of TWG22: curricular resources and task design in mathematics education

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Scope and focus of the working group

At the macro-level, teachers and students work with mathematics curriculum resources, both digital and traditional, inside and outside the classroom. Individually or collectively, teachers select, (re)design, modify, and interact with such resources for lesson preparation, student assessment, and the planning of their courses. These resources (e.g., educative curriculum materials) are the focus of professional development sessions, where mathematics teachers, often with educational researchers, design and transform curriculum resources, including blended materials, and in the process develop design capacity and valuable knowledge for teaching.

At the micro-level, curriculum resources contain mostly tasks derived from textbooks or other sources. The representation of these tasks in resources, their sequencing, and the teachers' actions during their enactment can limit or broaden the cognitive demand the tasks impose and affect students' views of the subject matter. Thus, they can influence the opportunities afforded to students to make mathematical connections, and to develop mathematical concepts, skills, or habits of mind. The literature indicates that tasks play a key role in effective teaching. There has been an upsurge in publications on various aspects of task design (e.g., task features that can help generate certain forms of mathematical activity); methods of task analysis (e.g., analyses of the learning affordances of certain kinds of tasks); and principles for task implementation in conventional and digital learning environments (e.g., factors affecting the fidelity of implementation of tasks in the classroom). Students can also be involved in task design activities to foster their reflections about what they know, understand, and do.

We summarize the papers presented at the conference according to the following predefined themes:

- Empirical research on teachers' and students' interactions with curriculum materials, resources, and tasks, related competencies (e.g., pedagogic design capacity), and influences on this interaction;
- Theoretical foundations and methodologies of task analysis helpful for task design and the design of curriculum resources;

- Studies on the use of carefully designed curriculum materials, resources, and tasks to support the implementation of particular learning goals and to enhance mathematical competence;
- Collaboration between teachers, between teachers and researchers, and possibly also students for designing tasks and resources and for analyzing their implementation;
- Affordances and constraints of digital and conventional tasks and resources.

Issues addressed during the sessions

An important issue in TWG22 was empirical research on teachers' and students' interactions with curriculum materials, resources, and tasks, related competencies (e.g., pedagogic design capacity), and a better understanding of how tasks and resources influence practice. At CERME12, three papers focused on students and their interactions with (digital) curriculum resources:

- Annalisa Cusi and Agnese Ilaria Telloni: Engaging students as designers of digital curriculum resources: Focus on their praxeologies and new awareness.
- Amal Kadan-Tabaja and Michal Yerushalmy: Online feedback designed to support self-reflection while solving fraction example-eliciting tasks.
- Carlos Quiroz, Saba Gerami, and Vilma Mesa: Student utilization schemes of questioning devices in undergraduate mathematics dynamic textbooks.

In these three papers, students assumed the roles of designers and users of digital curriculum resources in addition to those of learners. The papers reflected a combined focus on students with digital rather than traditional curriculum resources, mainly investigating the affordances of the former and how they are used or how they influence the learning process. All three papers related to attempts to foster students' reflection about the study content and learning process, which is regarded as an important aspect in learning.

Quiroz et al. investigated students' use of questioning devices — a new special feature of interactive textbooks, which seeks to engage students in thinking about the content and allows them to type responses to the questions in the textbooks. These answers are immediately shared with the teachers, who can use the information to adjust their teaching. Quiroz et al. identified three classes of situations, in which these questioning devices were used, and one utilization scheme for each situation. The findings reveal that students used the questioning devices for their intended purpose, but also for other purposes, exerting their agency in using curriculum resources for their needs. The term "questioning device" reflects the authors' and teachers' perspective and does not account for the students' perspective on this feature. For students, it is an answering rather than a questioning device. This leads to the question of how students' needs are considered in the design of digital curriculum resources. In an additional poster, Kanwar and Mesa introduced viewing patterns that were mapped to students' use of questioning devices.

Insight into how students think about the affordances of digital curriculum resources in mediating mathematical content was provided in the paper by Cusi and Telloni. Students were asked to design GeoGebra applets related to the content they had learned in a course supporting the transition from secondary school to university. Although students did not intend to become teachers or acquire

pedagogical or didactic knowledge about the subject matter, they showed ability to reflect on the design principles they implemented in the design process and how this influenced their learning. Involving students in the design process of digital curriculum resources is a novel didactic approach afforded by digital resources. On the one hand, it has the potential to provide further insight into students' learning of mathematics. On the other, it seems to be a promising approach to deepening students' understanding of mathematical content. However, open questions remain regarding the timing and the framing of the design activities: Should students design while learning the content or after they have learned it? What happens with the designed resources? What is the goal of designing that is presented to the students? Should students design for other students, for themselves, or for the instructor?

Kadan-Tabaja and Yerushalmy presented an example of an interactive task with related feedback aimed at fostering students' self-reflection and meta-cognitive skills. The special design of the feedback, which combines automated evaluation of students' solutions with opportunities to reflect on the solutions, seems to be a promising approach for future feedback design.

Two papers theorized student interactions that could manifest through concrete design features of mathematical tasks, problem posing, and working backwards, and suggested finer definitions and characterizations for given student interactions:

- Ling Zhang, Andreas Stylianides, and Gabriel Stylianides: Problematizing the notion of problem posing expertise.
- Daniela Assmus and Torsten Fritzlar: Working backwards revisited: Some theoretical considerations.

Zhang et al. compared problem posing of master's and sixth grade students to demonstrate the challenge of characterizing expertise. Using a data-driven approach, they identified expert problem posers based on participants' problem posing characteristics, such as the number of problems posed, their complexity, and clarity. They found that performance was not aligned with the participants' previous mathematical experience or backgrounds. Their results suggest that defining an expert should be specific to the expertise assessed, considering additional aspects beyond mere mathematical background.

Assmus and Fritzlar demonstrated a range of tasks designed to promote the working backwards heuristic. Although this seems to be a single problem-solving strategy, the tasks analyzed revealed differences stemming from the design of the tasks. Focusing on characteristics such as operations and order, careful analysis shows that task design that incorporates different subsets of these characteristics requires distinct solving processes. These differences can considerably influence problem demands and students' problem-solving processes. In a related poster, Assmus and Forster demonstrated different designs of working backwards problems, showing that distinct solving processes were related to different proportions of elementary school students who had solved them correctly.

Two papers addressed contextual and cultural influences on teachers' and students' interactions with curriculum resources, and urged rethinking the conceptualizations of resources and textbooks:

- Dubravka Glasnović Gracin: Rethinking resource conceptualization in times of pandemic and earthquakes: What is important for (mathematics) education?
- Hendrik Van Steenbrugge: Rethinking the notion of textbooks as mediators between the official curriculum and classroom practice.

Glasnović Gracin analyzed how the relevance of resources varies depending on the context. Van Steenbrugge sought to identify how culture becomes apparent in curriculum resources beyond the influence of the curriculum, analyzing textbooks and teacher guides from different countries. Although different in their research object and methods: case study of two teachers (Glasnović Gracin) vs. document analysis of textbooks and teacher guides (Van Steenbrugge), both studies showed the interrelatedness of curriculum resources with the social and cultural context in which they are developed and used. This raises the issue of how to read and interpret studies about curriculum resources from different social and cultural contexts, and suggests that results from one context cannot easily be transferred to another.

An additional perspective on resources was apparent in papers that studied the affordances and constraints of digital vs. conventional tasks and resources. Three papers at CERME 12 revealed different research perspectives.

- Lisnet Mwadzaangati and Mercy Kazima: An examination of mathematical affordances available in grade 2 teachers' guide and learners' textbook on addition of whole numbers.
- Ayla Carvalho and Rúbia Amaral-Schio: Characterizing the presence of activities using GeoGebra in Brazil's mathematics textbooks.
- Malin Norberg: Students' expressions about working successfully with mathematics textbooks: Multimodality and socio-mathematical norms in early years.

Mwadzaangati and Kazima examined the learning affordances provided for a given learning goal. They analyzed the learning affordances of a teachers' guide for teachers in relation to the addition of whole numbers for grade two students relying on variation theory. The study examined the teacher guides used by teachers in Malawi to choose tasks for implementation in their classrooms. The paper adopted two perspectives, discursive and cognitive, by connecting mathematics discourse in instructional aspects with the cognitive ones of what is said and presented. The tasks in the textbooks were analyzed from a cognitive point of view, based on the additive structure of the tasks and their possible variations (Marton & Pang, 2006). The findings point to the low quality of Malawian mathematics textbooks on the addition of whole numbers suggesting that this might be one of the causes of persistent low performance of learners in mathematics (simple strategy). A general question is: How can the results presented influence textbook design or the classroom?

Carvalho and Amaral-Schio reviewed four printed collections of textbooks (16 textbooks from sixth to ninth grade) used in Brazil to characterize and count the textbook activities using GeoGebra, the main dynamic geometry system tool used in Brazil, and to bring to the fore their potential for exploration to engage students in mathematical discovery. The study shows that GeoGebra was used only for geometry activities in all textbooks. Few activities have been found, and even fewer with the

potential of exploration. Yet the results do not represent the actual use of GeoGebra in classrooms, raising the question of the distance between the learning affordances of a task and effective learning.

Norberg reported on an ongoing study what was considered successful for 18 7-8-year-old students working with mathematics textbooks. The paper raised the question of socio-mathematical norms related to students' work, and what counts as mathematically sophisticated, accepted, different, efficient, given that norms regulate students' learning. The findings show a tension between using aids and being considered successful in mathematics, which could affect students' possibilities for mathematical learning. They raise questions about what is accepted vs. what is desirable. For instance, using mathematical symbols has been shown to be a way of appearing successful in math, whereas other modes of communication (fingers, gestures, images, diagrams, etc.) are not considered as successful, even if they are efficient. It may be important to consider learning goals in this study as well. And how is it possible to infer mathematical norms from observations? How is it possible to separate the effect of the didactic contract from norms when observing students? Methodologically, what was the influence of the researcher during data collection? In an additional poster, Tutuncu and Hodgen offered a method to analyze the potential of educative materials to offer productive formative assessment by combining analysis of formative assessment practices with the guidance provided to enact these techniques.

Two other issues of TWG22 concern the theoretical foundations and methodologies of task analysis for task and curriculum resource design, and collaboration between teachers, between teachers and researchers, and possibly also students for designing tasks and resources, and for analyzing their implementation. At CERME12, two papers focused on analysis of the selection and characterization of tasks by prospective teachers and teacher collaborative decision-making in task characterization.

- Cengiz Alacaci: Prospective elementary teachers' selection of mathematical tasks.
- Andreas Bergwall, Elisabet Mellroth, Torbjörn, and Johan Nordin: Teachers' characterizations of challenging introductory and enrichment tasks.

Alacaci's exploratory paper viewed mathematical tasks as powerful tools to develop mathematical ideas in the classroom and useful in teacher education. One assumption was that understanding task perceptions of prospective elementary teachers could help predict their eventual modification and appropriation for classroom use, which can affect teachers' practice. It also raised the issue of "good" mathematical tasks — good for what purpose? One of the consequences of the experiment was that prospective teachers were encouraged to think critically about tasks. Bergwall et al. investigated the collaborative characterization by eight teachers of tasks suitable for introduction or enrichment, and presented several dilemmas. Among the results was the observation that introductory tasks should have an easy entry level and not require pre-knowledge of the upcoming concept, while an enrichment task should require relatively deep conceptual pre-knowledge. Teachers' verbalization of task characteristics was one outcome, but not all tasks met all criteria. The teachers were involved in the writing of this paper.

Both papers raise some theoretical and some methodological issues about the choice of the theoretical framework, as well as the selection, characterization, and classification of tasks by teachers with

regards to context, teachers' goals, and implicit characteristics, such as teachers' tacit knowledge (Herbst et al., 2011; Herbst & Chazan, 2011). This highlights the challenge in future attempts to generalize from these studies. In an additional poster, Gustafsson, van Bommel, and Liljekvist suggested pursuing the analysis of teachers' discussions in communities of practice to explore the various facets of mathematical knowledge for teaching that guide these discussions.

A central issue for TWG22 is the use of carefully designed curriculum materials, resources, and tasks to support the implementation of particular learning goals and to enhance mathematical competences. Thus, it is important to investigate how to design tasks and items to provide students with opportunities to make mathematical connections and develop mathematical concepts, skills, and habits of mind. The characteristics of task design can influence the processes that characterize students' interaction with the tasks or items themselves. Among these, reading processes can play an important role because they can direct and determine students' problem-solving processes. At CERME12, two papers focused on how the design of specific tasks and resources influences students' reading processes:

- Valentin Böswald and Stanislaw Schukajlow: Reading comprehension and modelling problems: Does it matter where the question is placed?
- Anneli Dyrvold and Ida Bergvall: The role of dynamic elements in digital teaching platforms: An investigation of students' reading behaviour

Both papers investigate the role played by specific factors in influencing students' reading processes. Böswald and Schukajlow presented a theoretical paper aimed at reflecting on the ways in which the position of the question within the text of a modeling problem can determine students' reading processes. They suggested that placing the question before the text in modelling problems can make the goal clearer for readers, supporting them in distinguishing between relevant and irrelevant information. Dyrvold and Bergvall investigated the influence of the choice of dynamic elements in digital items on students' reading behavior in a digital multimodal environment. Their analysis identified various types of challenges that students may face in working with dynamic elements. In particular, they stressed the potential of dynamic elements to evoke deep engagement in interaction and the risk of misunderstandings or omissions in relation to these elements.

The discussion of these two papers highlighted important aspects in the choice of a task design aimed at supporting students' reading processes. The two main aspects under discussion were: (a) the interrelation between enhancing efficient reading and enhancing students' comprehension and reasoning; (b) the role played by metacognitive processes in guiding students' reading processes when interacting with tasks with certain characteristics; and (c) the effect of students' age and of the focus on different mathematical contents on how the design of tasks and resources influence students' reading processes. Another common issue addressed by both papers was methodological: the use of eye-tracking technology to investigate students' reading processes when interacting with the designed tasks and resources. Some of the questions that arose concerned distinguishing the effects of task design on students' reading behaviors from the effects of other contextual or external factors, and the possible use of artificial intelligence in developing a categorization of reading behaviors.

In two additional posters, other design principles were suggested to achieve specific aims: Vytautas argued that problems that do not explicitly state the concepts needed to solve the problem can be defined as epistemologically potent. These tasks can still be approached by students, because the unknown concepts are not mentioned in the wording of the problem, leading to a meaningful learning process. Stenberg, Haavold, and Sriraman suggested employing pathologies to create uncertainty in order to catalyze creativity for mathematics students.

Another important issue for TWG22 was the design of various types of materials, tasks, and resources to be used in given learning environments. The choice of materials, the sequencing of tasks, and the actions performed by the teacher can affect students' learning processes. The role of the design of resources, materials, and learning environments was a common theme of the following papers:

- Henrik Stigberg: Digital Fabrication for Mathematics Education: A Critical Review of the Field
- Johanna Zöchbauer, Markus Hohenwarter, and Zsolt Lavicza: Improving the GeoGebra classroom tool to better accommodate online educational resource development based on the SAMR model
- Sofía Paz-Rodríguez, Armando Cuevas-Vallejo, and José Orozco-Santiago: A hypothetical learning trajectory for linear combination of vectors in R2

Stigberg proposed a critical review of digital fabrication for creating manipulatives in mathematics education research, stressing the role played by these technologies in enabling teachers to create context-sensitive manipulatives for teaching activities. Zöchbauer et al. investigated how the GeoGebra classroom tool can be improved to better accommodate a set of online educational resources to be used in combination with tangible tools. They used the case study approach to examine the implementation of these resources, highlighting the need for improvements, for example, making the sharing of resources with students easier and faster or enabling the teacher to have an overview of the number of students who answered the questions posed to them. The investigation presented by Paz-Rodríguez et al. enabled them to stress some limitations in the approach they adopted, which focused on the design of technology-mediated tasks, following a hypothetical learning trajectory aimed at supporting university students' conceptualization of linear combination. The analysis of the results of a teaching experiment in an online linear algebra class enabled the authors to identify elements that could support a redesign of this hypothetical learning trajectory.

Among the issues the discussion in these three papers raised, we mention the role played by frameworks and theoretical models to support both the design process (as in the papers by Zöchbauer et al. and Paz-Rodríguez et al.) and the research on this design (as documented in the paper by Stigberg). In particular, reference to the models guiding the design process is a common aspect in both the research presented by Zöchbauer et al., where the SAMR model (Puentedura, 2006) supported the design of an environment aimed at fostering the integration of digital media into the classroom, and the research presented by Paz-Rodríguez et al., where the C&P principles (Cuevas & Pluvinage, 2003) provided a set of criteria aimed at supporting the task design process.

The discussion on these papers also raised some reflections about another fundamental issue related to the design of learning environments, in particular when the focus of the design is also on the role of tangible tools or physical objects, considered as products of digital design (like in the paper by Stigberg) or as tools to be combined with digital ones (like in the paper by Zöchbauer). The paper by Stigberg raised the issue of teachers' learning when they make, share, and use manipulatives, suggesting the need to adopt communities of practice as a framework for understanding teachers' learning of digital fabrication for mathematics education. The paper by Zöchbauer et al. introduced initial reflections on the role played by the teacher's orchestration in combining the use of digital and tangible tools. They noted that teachers should make the connection between digital and tangible tools clearer if they want students to work effectively with a combination of these tools.

Possible directions for future research

TWG22 topics at CERME12 shared many research themes with CERME11 (e. g. different theoretical perspectives leading task design and analysis) and and also added novel themse as mentioned above. When discussing these studies TWG22 discussions often took the opportunity to discuss possible ways to enhance and deepen the topics studied in TWG22. The following directions for future research emerged from the discussions:

(a) Developments of means to communicate over curriculum materials and the wide perspective that takes into consideration different agents, open new opportunities to focus on the role of students in all stages stemming from task and curriculum design and implementation;

(b) The different theories employed in various studies underline the neeed for finer definitions and moving toward more precise and fine-grained analysis of processes and practices, which are theorized only in general forms;

(c) There is place for reports focusing on issues of scalability: many reports focused on a micro-process of design or implementation; large-scale studies or design considerations for scaling-up could contribute to a wider research perspective;

(d) While many conference research reports focus on short processes, long-term and longitudinal studies are needed for broader perspective. We acknowledge that long-term and longitudinal studies are probably challenging in the format of a conference research report; therefore intermediate stage reports are also welcome;

(e) More Research reports are needed to deepen the investigation of the teachers' role in their design and co-design interaction with resources and environments, in the context of professional development programs or research projects.

References*

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* all the papers mentioned in this document are within the references lists of the papers presented at TWG22, except for the one in the list.