Mission possible

Prof.dr. Wil Schilders
June 23, 2023

Mission possible – Mathematics for a better world

TU/e

**EINDHOVEN
UNIVERSITY OF
TECHNOLOGY**

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

VALEDICTORY LECTURE PROF.DR. WIL SCHILDERS

Mission possible – Mathematics for a better world

Presented on June 23, 2023
at Eindhoven University of Technology

Introduction

Dear rector, members of the Executive Board of Eindhoven University of Technology, dear colleagues, dear family, dear friends, some from far away, esteemed audience. It is my great pleasure to find you all interested in listening to my valedictory lecture today.

Almost precisely 20 years ago, I gave my inaugural speech here in the same room. At that time, in Dutch with the title 'De wiskundige leest in bedrijf'. The title had a double meaning. Interpreting the word 'leest' as a verb, it means that a mathematician reads what is happening in industry and acts to see whether there are interesting topics of research. Interpreting 'leest' as a noun, it means a shoemaker's tool, and we have a saying in Dutch that, when translated, reads: 'shoemaker, stick to your last'. In other words: 'stick to what you have been trained for'. Between the lines, it was also a piece of advice to researchers and engineers in other scientific disciplines: leave the mathematics to the mathematicians; they are the experts who know about the latest developments and have a particular way of thinking that helps in solving complex problems. I became very much convinced about this attitude when working in industry from 1980 until 2010. Often, you see ad hoc solutions being used, whereas a deeper insight into the cause of the problems should be the first priority. Only when we understand the real cause can we start to think about solutions. Often, I compare this way of working with that of a doctor. Suppose someone with headaches every day goes to a doctor. One doctor could say, "take this pill every day and the headaches will disappear (for the day)." Another doctor could say, "please sit down and let's analyze your current situation", eventually leading to a sound diagnosis and correct treatment.

Since the introductory lecture in October 2003, a lot has happened. I left industry in 2010 and was fortunate to build a large network of academic and industrial colleagues from all over the world. Gradually, the mission of promoting the use of mathematics for complex challenges was taking shape, owing to my position at this great university in the vicinity of the booming industrial area Brainport, as well as via the position within the newly created Dutch Platform for Mathematics and positions in ECMI, EU-MATHS-IN and ICIAM. Initially, it looked like mission impossible, referring to the great series that we watched in the sixties and early

seventies, with Peter Graves starring as Jim Phelps, director of the Impossible Missions Force.
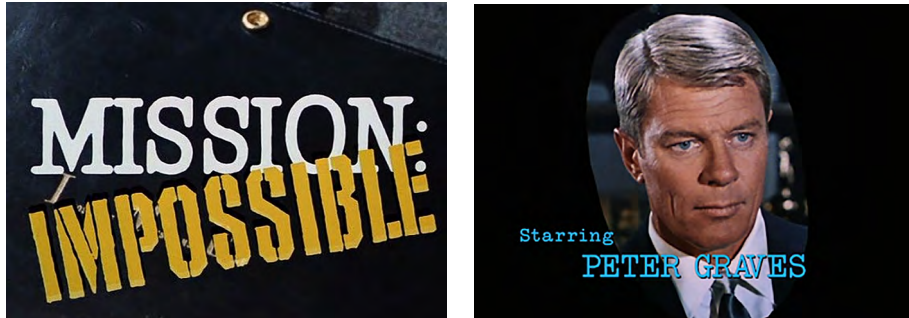
Despite the title of the series, the mission thought to be impossible always turned out to be possible due to smart and intelligent actions. In my opinion, the same is true for the mission of using mathematics to address the complex challenges of our times in industry, society and other sciences. Clearly, this should be teamwork with colleagues from other disciplines who know the context and the backgrounds of the challenges. Together, in this way, we can create a better world with the use of mathematics. This is what I would like to discuss in this valedictory lecture, telling you about my experiences and findings over the years and my view on the future of our mission.

The value of mathematics

When I started to work at Philips Research in 1980, times were much different from this day and age. Software packages, as we know them now, able to perform simulations for a wide variety of different structures and designs, did not exist. Instead, for semiconductor device simulation, engineers made separate software for one-dimensional diodes and one-dimensional transistors. The Mathematical Software Group set out to change this situation and produce software that could cope with arbitrary semiconductor devices in two dimensions. Engineers at Philips Research thought that we were out of our minds; this was absolutely mission impossible according to them. But we did it by developing robust and efficient mathematical methods that could cope with a wide variety of devices. The mission was accomplished within a few years and entirely due to the sound mathematical methods.
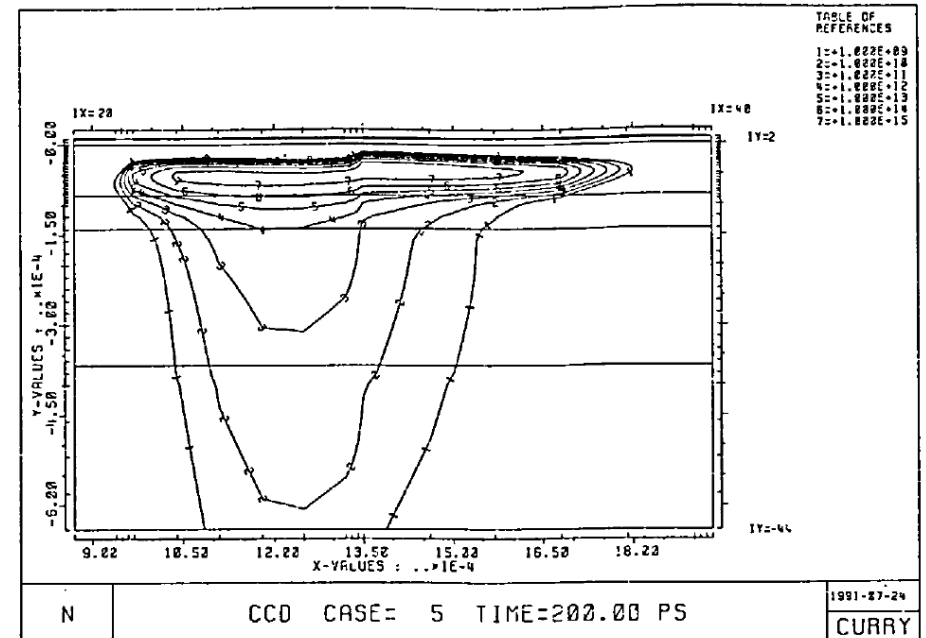
It was the time when software packages were starting to be developed in more places, mostly within industry and for a variety of problems: electromagnetic, mechanical, electronic circuits. This development put a high demand on the available mathematics and new mathematical methods needed to be developed so as to cope with the high demands of these software products. More and more, design was done behind a computer screen rather than by building prototypes and testing these. It led to the emergence of the so-called 'third discipline': simulation, alongside the traditional disciplines of experiment and theory.


All of this was made possible by virtue of mathematical models of the underlying problem, describing its behavior in detail, often by means of a rather complicated set of differential equations. Clearly, such models are made in very close collaboration with scientists that know the details of the underlying problem. Once the mathematical model is known, numerical methods can be developed that will simulate the model accurately. This requires bright mathematical minds, and the semiconductor device problem in particular caused quite a few headaches before a suitable set of algorithms was developed. We did this in close collaboration with companies like IBM and AT&T Bell Labs, as well as with Stanford University.

The developments regarding simulations have accelerated since the 1980s and a lot of software is available nowadays to perform simulations in many domains of science and industry. Design is done behind a screen with the aid of virtual design environments. Mathematics plays a crucial role within this evolving area of computational science and engineering and mathematical methods are indispensable nowadays in the areas of digital twinning, artificial intelligence, energy transition, high-performance computing, biomedicine and many more. Lex Schrijver, esteemed colleague from the Centrum Wiskunde & Informatica (CWI) in Amsterdam, formulated it once in a very nice way:

"Mathematics is like oxygen. You take no notice of it when it's there – if it wasn't, you'd realize you cannot do without it."

Often, people do not realize how much of the mathematics needed to solve a problem or address a challenge is under the surface. For this reason, I often use the term 'mathematical iceberg'.
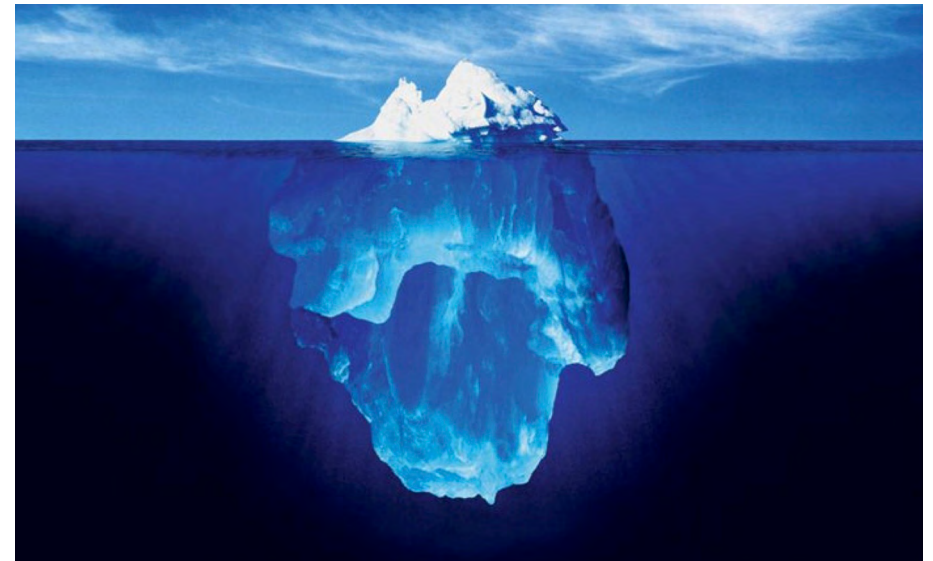
Indeed, the mathematics is often invisible as it is hidden deep in the software package. Success usually comes from applications of the software, i.e., novel designs. Within Philips, engineers were able to design entirely new devices, such as CCD devices for digital cameras, and analyze their behavior thoroughly using software containing fast, robust and efficient mathematical machinery. So, the

statement is justified that **mathematics provides an invisible contribution to visible success**.

A really great example showing the power of sound mathematics is the analysis of disc break squeal by my very valued colleague Volker Mehrmann from TU Berlin. Disc brake squeal is a frequent and annoying phenomenon and the automotive industry has been trying for decades to reduce squeal by changing the design of the brake and the disc. In 2015, detailed mathematical analysis revealed bifurcations because the designs contained highly stiff springs used to avoid rigid connections. The negative effects of this design technique came as a surprise to the industrial partners! All car manufacturers are extremely interested in the results. Clearly, these hidden properties of the underlying system can only be revealed by mathematicians; it is their specialty!
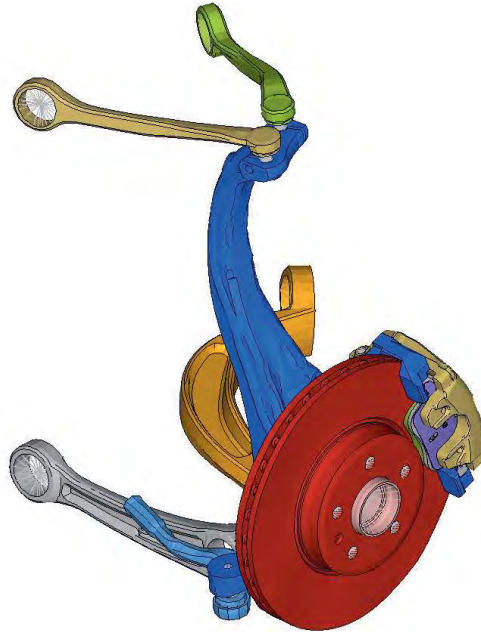
But there is much more to tell about the invisible contributions of mathematics. Everyone knows Moore's Law, which for more than 50 years predicted that the speed and density of transistors double every 18 months. Translated into practical terms, it means that our computers become much faster all the time. Many people then draw the conclusion that the impressive simulations that we see nowadays are entirely due to the greatly improved power of computers. However, this is only half the truth. The mathematics that is hidden in the software packages, enabling the simulations, is outperforming the hardware improvements, often in a considerable way. See the following table, where we summarize the improvements for a number of mathematical methods.

Mathematical method	Period (years)	Hardware improvement	Mathware improvement
Solving large linear systems	35	**10,000,000**	**10,000,000**
Linear programming	16	1600	**3300**
Mixed integer programming	25	6500	**870,000**
Particle simulations	40	100,000,000	**1,000,000,000**

The difference is most pronounced in the case of mixed integer programming: if we relied only on the performance improvements of computers, simulations that would cost an estimated 180 years 25 years ago would now require 10 days. But if we also take the improvements in the mathematical methods into account, the simulation is reduced to one second. Philippe Toint, Emeritus Professor in Numerical Optimization at Namur University, once said: "I would rather use today's algorithms on yesterday's computers than vice versa." Clearly, if we only relied on improvements in hardware, we would now be doing the simulations of the 1990s.

In 2014, we asked Deloitte to investigate the value of the mathematical sciences to the Dutch economy, which led to some stunning conclusions. One of these was that the mathematical sciences contribute up to 26% of total employment. Because these are high-income jobs, the economic contribution of the mathematical sciences is even higher, representing around 30% of Dutch national income. Another conclusion is that a strong mathematical sciences foundation is critical to the success of any advanced economy. Better mathematical skills correlate with a more competitive economy and a higher standard of living. Moreover, with the revolutions in computational science, big data, statistics and business analytics, the importance of mathematical sciences to society is likely to increase substantially in the coming decades. These revolutions are driven by ever more powerful computers, the data explosion and improved algorithms.
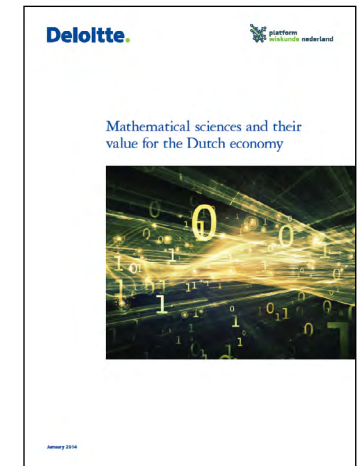
Similar reports were made in the UK, France and Spain, resulting in similar conclusions. In Germany, a great book was produced in 2008, during the national 'Year of Mathematics', entitled 'Mathematics: engine of the economy'. 20 leading companies in Germany contributed to this volume. In the introduction, the following statement is made: *"Mathematics is everywhere. In the past, it was only individuals who clearly recognized this, such as Alexander von Humboldt with his timeless sentence 'Mathematical studies are the soul of all industrial progress' or Werner von Siemens, who gave his brother Wilhelm the urgent advice, 'Your main course of study must now be mathematics'. Today, there is a consensus that without mathematics, people cannot organize their everyday lives."*
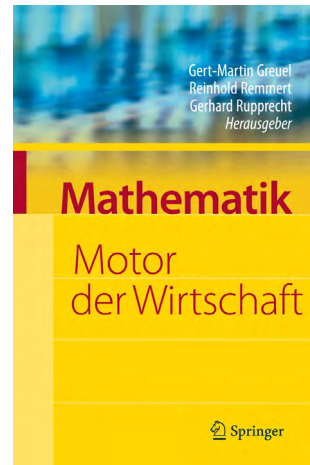
As stated in the Deloitte report, there is a clear correlation between mathematical ability, measured in terms of the PISA score in secondary schools, and the competitiveness of a country. This is clearly demonstrated in the figure below.
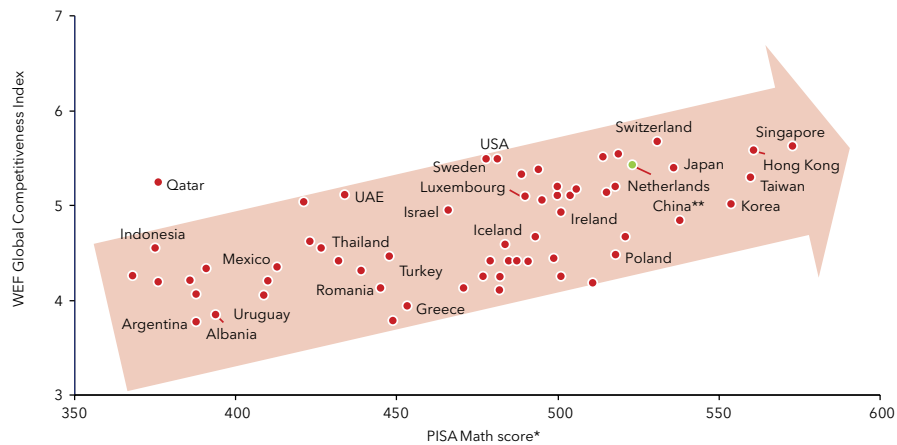
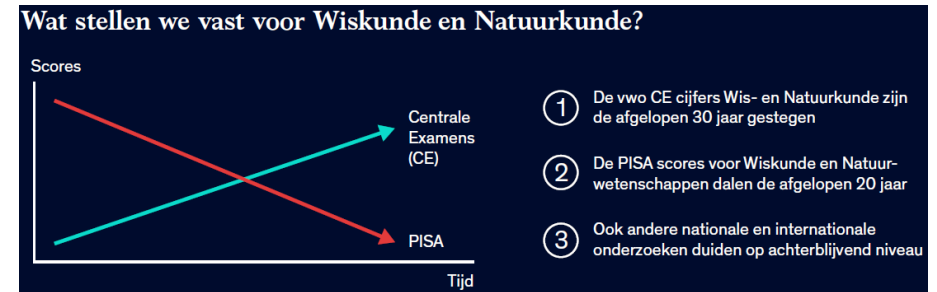Rather worrying in this respect is that in a recent report of McKinsey, it was concluded that the PISA score of the Netherlands has been going down over the past 20 years.

Clearly, action must be taken to counter this development. The committee for Education within the Dutch Platform for Mathematics is expressing its concern to the ministers and the message is quite clear: *"The trend identified by McKinsey seems to be part of the flattening that is taking place in education on many fronts. There is a complex of causes for this. The student population is changing, qualified teachers are hard to find, and teaching time for mathematics has been reduced with the introduction of the second phase. Education has to adapt to changes in society, but the impoverishment that is now occurring has negative consequences for the personal development and socialization of our pupils and for the level at which they enter further education. In STEM studies, they fall short of foreign students in knowledge and work attitude, which weakens the innovative power of our country."*

Let's get back into a more positive mood again! It is clearly important to convince the general public, secondary school pupils, policymakers, scientists and industry of the value of mathematics for our society. There are many ways to do this, such as with the Deloitte report. The booklet entitled 'Success formulas', written by Ionica Smeets and Bennie Mols on behalf of the Dutch Platform for Mathematics, was also very successful. It contains 36 stories about applications of mathematics and a number of interviews with captains of industry and policymakers in order to show the versatility of mathematics. Many do not know that mathematics is present everywhere in our society, and this booklet gives an impression of this omnipresence. Some 20,000 copies were distributed all over the country and, in 2021, we made a German version with many new stories.
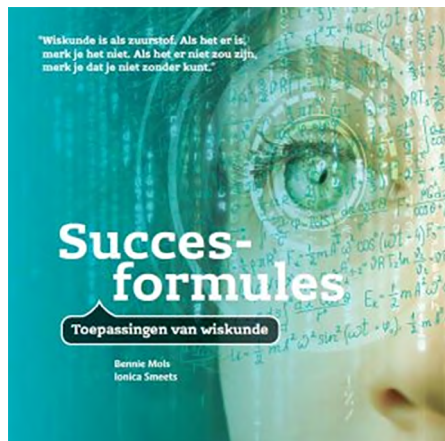
Scientific Computing for Industry

Over the years, I have been able to demonstrate the value of mathematics, especially in industry. When appointed at TU/e, the assignment was 'Scientific Computing for Industry', fitting perfectly with the profile I had in mind. Scientific computing, also referred to as numerical analysis, is a discipline that emerged when computers started to become more powerful, say since the 1950s or 1960s. Initially, it was a field that was occupied with a sound analysis of rounding errors, as computers can only use a limited number of digits to represent numbers, which can lead to a considerable build-up of errors in certain processes.

When computers became more powerful and software packages started to be developed, the discipline of computational science and engineering started to emerge and, within the area of scientific computing, methods needed to be developed that could cope with problems that are rapidly becoming larger. In the early 1980s, the size of the problems was around 1000 variables, but this later grew to tens of thousands, millions and, in recent years, even billions of variables. Especially important is to then develop methods to solve large linear systems of equations, often with a special structure. We were very fortunate that the Netherlands was leading this field, and the work of Henk van der Vorst and Koos Meijerink on the ICCG method was especially revolutionary. We were the first to implement this method, developed around 1977, in industrial software, which enabled us to solve much larger problems than our competitors, as well as much faster.

I very much liked the work on numerical linear algebra and, when at Philips, we had extensive discussions on special systems of equations with our team of mathematical advisers: Bram van der Sluis, Henk van der Vorst and Piet Hemker. We became interested in solving more difficult sets of equations: indefinite linear systems occurring in the area of electronic circuit simulation. It led to an intense collaboration with Andy Wathen of Oxford University in the early years of this century and, in the end, a factorization method for indefinite matrices was developed. The Oxford group named the method 'Schilders' factorization' and I was clearly very honored by this.

The factorization developed is, in fact, one of the many methods that I developed (together with colleagues and students, of course), constituting a common thread over several decades: **mimetic methods**. During my PhD at Trinity College Dublin, we already worked on methods that used information about the underlying problem. The exponential behavior of solutions in singularly perturbed problems was used to develop exponentially fitted finite difference schemes. They performed fantastically in practical situations and had errors that did not depend on the small parameter. Later, at Philips, it turned out that the set of semiconductor device equations was also singularly perturbed, implying that we could use exponentially fitted schemes to discretize the system. The decomposition for indefinite matrices also used information about the underlying problem, namely the fact that there were two different types of variable in the problem. It led to the use of 1x1 and 2x2 blocks in the decomposition process and to a great factorization technique.

I strongly believe in mimetic methods, sometimes also referred to as structure-preserving methods. And I think that we should adapt the curricula in mathematics to include such methods as they are extremely important in practice. At most universities, however, scientific computing is taught by presenting rather general methods based on simple Taylor series expansions, using no knowledge at all about the underlying problem. This leads to the violation of physical principles, such as the conservation of mass and energy, and to less accurate solutions, often requiring much more computation time. During my guest professorship at Bergische Universität Wuppertal (BUW) in 2020-2021, I gave a series of lectures on mimetic methods, showing that these do not only occur within the area of discretization but also within the areas of solving linear and nonlinear problems, meshing, model order reduction and more. Together with my esteemed colleagues from BUW, I am writing a book on mimetic methods.

A very successful example of an industrial project solved with mimetic methods was the ASIVA14 project together with Mentor Graphics (MG). One of the problems was that solving nearly periodic electronic circuits could take several weeks and MG wanted us to develop methods that could considerably reduce this computation time. On investigating this problem, we found that there was a small part of the circuit where much higher frequencies played a role, resulting in extremely small time steps that lead to high computation times. The idea came up to develop an extremely accurate model for this small high-frequency part of the circuit so that the model could be used in the simulation and time steps could be used corresponding to the lower-frequency part of the circuit. It turned out to be

a fantastic idea, leading to speed-ups of more than a factor of 100 and simulation times of hours rather than weeks.

Another successful mimetic method that we developed had its origins within Philips. For strongly nonlinear problems, the well-known Newton's method can often lead to an enormous number of iterations, if it converges at all. The method we developed for semiconductor device simulation is extremely effective and can lead to a very significant reduction of iterations. This is achieved by employing a nonlinear transformation of variables, making use of knowledge about the nonlinear character.

Mimetic principles also played a decisive role within the area of research that I adopted at the start of my career at TU/e and have been working on since then: **model order reduction**. Quite quickly, I started to use the following figure to explain model order reduction to the general public:
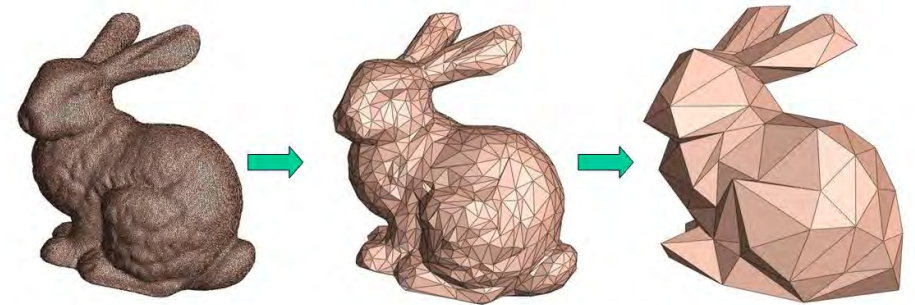
It clearly indicates that it is not necessary to know all details; one can often delete superfluous information and concentrate on the dominant behavior. One will immediately recognize the right picture to be a bunny; one does not need all information contained in the lefthand picture.

Model order reduction is, on the one hand, a flourishing area of research with many researchers contributing to its development. On the other hand, it is one of the most important sets of methods used for the efficient solution of problems in industry and, more generally, in the area of computational science and engineering. With the advent of high-performance computing and the growing power of computers, it is very tempting to simulate larger and larger problems. However, the counter-side is that the energy consumption will be enormous for

all of these simulations. Here, the adagio 'think twice, compute once' is absolutely valid. Using model order reduction, problems can be reduced considerably in size while retaining the accuracy of solutions. Model order reduction extracts the dominant features of solutions, which is perfectly adequate to analyze problems, produce designs and investigate processes. So, I am making a case here to make much more use of all of the advances made in the field of model order reduction over the years before starting any simulations. Together with Peter Benner, an authority in the world of MOR, I set up the European Model Reduction Network in 2014 and, over four years, we gathered a large group of more than 300 European researchers working in the field, had many interesting workshops and, in the end, produced the (open access) Handbook of Model Reduction, consisting of three volumes and considered a standard work in the field now.

Clearly, in view of the foregoing, our interest was mainly in mimetic model order reduction. We started out by developing several structure-preserving model reduction methods, also following the work of Roland Freund, viz. his method SPRIM in which the 'S' and 'P' stand for 'structure preserving'. At a later stage, we got interested in differential-algebraic equations (DAE), stimulated by the fact that electronic circuits generate systems of equations of that type. The main idea we had was to split the DAE into a differential and an algebraic part, then use one of the many MOR methods to reduce the differential system and use other reduction methods for the algebraic part. In this way, we developed the Index Preserving Model Order Reduction methods of IMOR and IIMOR. Indeed, the so-called index of the problem was automatically conserved due to the procedure followed:
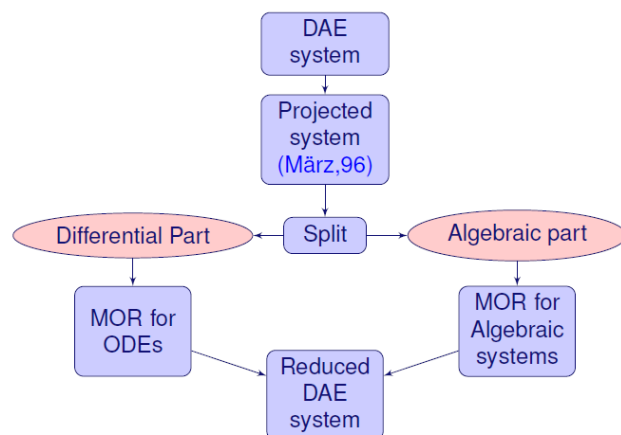
Our work on model order reduction received quite some interest from industry and led to projects with Mentor Graphics, Siemens, austriamicrosystems, MAGWEL, NXP Semiconductors and Signify. Due to the early workshop in the Lorentz Center in 2005 and the European network EU-MORNET, TU Eindhoven was very visible in this area, leading to much interest from industry. Currently, we are involved in projects that aim to develop compact thermo-mechanical models for electronic circuits and compact models for LEDs, with model order reduction playing a major role.

Scientific computing in the future

A few years ago, I was a bit pessimistic about the future of scientific computing. In the area of numerical linear algebra, I did not see any new, ground-breaking developments that would outperform the ICCG method and the multigrid methods developed in the previous century. Some work was done on new preconditioners for special systems of equations, but most of the work was a small improvement on existing methods. In other areas of scientific computing, no new, revolutionary methods emerged either; most of the research was of an incremental nature.
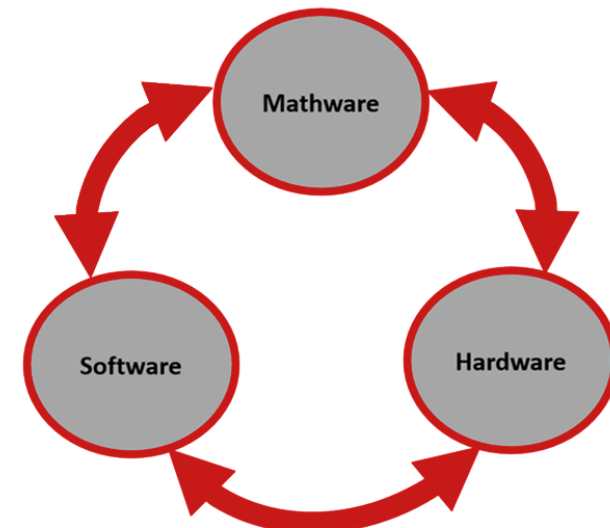
But then, in recent years, the situation changed completely. First of all, **high-performance computing** (HPC) became very popular again. In the 1990s, people had worked on HPC, but Moore's Law was still valid, implying that computers were becoming faster all the time. Hence, there was not a real need for HPC methods. Only recently, this changed dramatically. Moore's Law has come to an end: transistors are not becoming faster anymore and hence the only way to speed up computing systems is to rely on parallel computing.[1] Supercomputers have become much more mainstream now and, indeed, if we wish to speed up our

computations, we will need to resort to parallel machines with many processors as well as GPUs. For the field of scientific computing, this means that we need to concentrate our efforts on methods that are inherently parallel. This is a big challenge for researchers in the scientific computing area. The famous ICCG method, which performs so very well on serial computers, performs extremely badly on supercomputers, as can be seen from the table on the previous page.

As you can see in the column on the right, ICCG only achieves some 2% of peak performance. This is extremely bad and means that researchers in numerical linear algebra have a big challenge ahead of them to develop preconditioners that can bring the performance to a much higher level. So far, I have not seen solutions. Also, in other areas of scientific computing, the growing use of supercomputers implies having to work on novel methods. It is possible that entirely new ideas need to be developed in order to cope with these challenges. It also means that the work of mathematicians in the area of computational science and engineering (CSE) is becoming extremely important. However, people often only speak about software and hardware and the question is then: where is the mathematics? Hidden in the software? Clearly, it is important to first think carefully about the mathematical methods before any software can be written. For this reason, I like to speak about **'mathware'** and isolate this from the software and the hardware:

Rank	Site	Computer	Cores	HPL Rmax (Pflop/s)	TOP500 Rank	HPCG (Pflop/s)	Fraction of Peak
1	DOE/SC/ORNL USA	**Summit**, AC922, IBM POWER9 22C 3.7GHz, Dual-rail Mellanox FDR, NVIDIA Volta V100, IBM	2,397,824	148.60	1	2.926	1.5%
2	DOE/NNSA/LLNL USA	**Sierra**, S922LC, IBM POWER9 20C 3.1 GHz, Mellanox EDR, NVIDIA Volta V100, IBM	1,572,480	94.64	2	1.796	1.4%
3	RIKEN Advanced Institute for Computational Science Japan	**K computer**, SPARC64 VIIIfx 2.0GHz, Tofu interconnect, Fujitsu	705,024	10.51	18	0.603	5.3%
4	DOE/NNSA/LANL/SNL USA	**Trinity**, Cray XC40, Intel Xeon E5-2698 v3 16C 2.3GHz, Aries, Cray	979,072	20.16	6	0.546	1.3%
5	Natl. Inst. Adv. Industrial Sci. and Tech. (AIST) Japan	**ABCI**, PRIMERGY CX2570M4, Intel Xeon Gold 6148 20C 2.4GHz, Infiniband EDR, NVIDIA Tesla V100, Fujitsu	368,640	16.86	10	0.509	1.7%
6	Swiss National Supercomputing Centre (CSCS) Switzerland	**Piz Daint**, Cray XC50, Intel Xeon E5-2690v3 12C 2.6GHz, Cray Aries, NVIDIA Tesla P100 16GB, Cray	387,872	21.23	5	0.497	1.8%
7	National Supercomputing Center in Wuxi China	**Sunway** TaihuLight, Sunway MPP, SW26010 260C 1.45GHz, Sunway, NRCPC	10,649,600	93.02	3	0.481	0.4%
8	Korea Institute of Science and Technology Information Republic of Korea	**Nurion**, CS500, Intel Xeon Phi 7250 68C 563584C 1.4GHz, Intel Omni-Path, Intel Xeon Phi 7250, Cray	570,020	13.93	13	0.391	1.5%
9	Joint Center for Advanced High Performance Computing Japan	**Oakforest-PACS**, PRIMERGY CX600 M1, Intel Xeon Phi Processor 7250 68C 1.4GHz, Intel Omni-Path Architecture, Fujitsu	556,104	13.55	14	0.385	1.5%
10	DOE/SC/LBNL/NERSC USA	**Cori**, XC40, Intel Xeon Phi 7250 68C 1.4GHz, Cray Aries, Cray	622,336	14.02	12	0.355	1.3%

HPCG Benchmark June 2019

[1]   Or quantum computing, but this is so far still a promise.

Mathware researchers should talk to their software and hardware colleagues and discuss the best approach for a certain computational problem. Mathematicians always had quite a tight relationship with software developers and computer scientists, but it will also become more important to speak to the hardware people in the future. For example, one could make use of different representations of numbers: 16, 32 and 64-bit. It is cheap to first perform calculations in 16-bit representations, then one could step it up to 32-bit calculations, finally only having to perform a few calculations with the most expensive 64-bit representation of numbers. This could be an ideal framework to carry out simulations in a much more efficient way. However, not all computers have the right semiconductor devices and chips on board to ease such transformations between 16, 32 and 64-bit representations. Hence, it is urgent to also speak to the hardware developers.

Another important development in recent years is the emergence of the field of **data science** in which abundant amounts of data are generated in many different areas and the challenge is to extract useful information from this. I will not say too much about this, but it is clearly also an area in which mathematicians should play a key role. The Department of Mathematics and Computer Science of TU/e founded the Data Science Center Eindhoven (DSCE) around 2014 and now also has an educational track in data science. Data scientists are in high demand in society and industry, but it is clear that extreme care needs to be taken to extract correct information from the data. Many of the methods used in the area of data science have been well-known for many decades, which is also the reason why Stanford Professor of Statistics David Donoho wrote a somewhat cynical paper some years ago entitled '50 years of Data Science'.

The third development that will change scientific computing in the future is that of **artificial intelligence** (AI) and **machine learning** (ML). Just as with HPC, AI and ML were already quite popular a few decades ago, but the development of methods was only accelerated a few years ago. Deep neural networks were introduced in 2012 and have since taken a giant leap forward. This has changed speech recognition on our phones enormously and also had a big influence in many different areas. In the area of computational science, artificial intelligence, machine learning and artificial neural networks (ANN) also started to be introduced, and clearly this is an extremely interesting development for the area of scientific computing. One of the simplest applications is to generate ANN models of parameters in the problem for which no physical models are available. Often, the physics of the underlying problem is known quite well in the form of ordinary or partial differential equations, but such models may not be available for some

parameters in these equations. I compare it to the situation we encountered at Philips Research: we simulated semiconductor devices by solving the set of drift-diffusion equations, but models for recombination and mobility were provided by the electronic engineers and changed every year. The engineers determined these models by performing many simulations and experiments, thereby generating a lot of data from which they constructed a model by combining curve fitting and physical or engineering insight. We could do the same thing nowadays, using the data to construct adequate neural networks describing the parameters in an accurate way. A problem that arises here is that one can mathematically prove properties of the parameters that should hold, which was not always the case for the manmade models. Clearly, such known properties must also hold for the artificial neural networks, and this is still a big challenge: how to produce ANN that have specified properties?

This is a general and very important question and, in fact, refers back to mimetic methods again: how can we develop artificial neural networks that retain certain properties of the underlying physical system? In other words, we would like to develop **mimetic neural networks**! An additional advantage is that much less data are needed and one can have more confidence of reliable results, including in simulations that are out of the range of training data. One attempt in this direction is provided by physics-informed neural networks, better known as PINNs. These have been developed by the group of George Karniadakis of Brown University in the USA. A sketch of how they work is provided in the following figure.

Partial Differential Equation

Neural Network
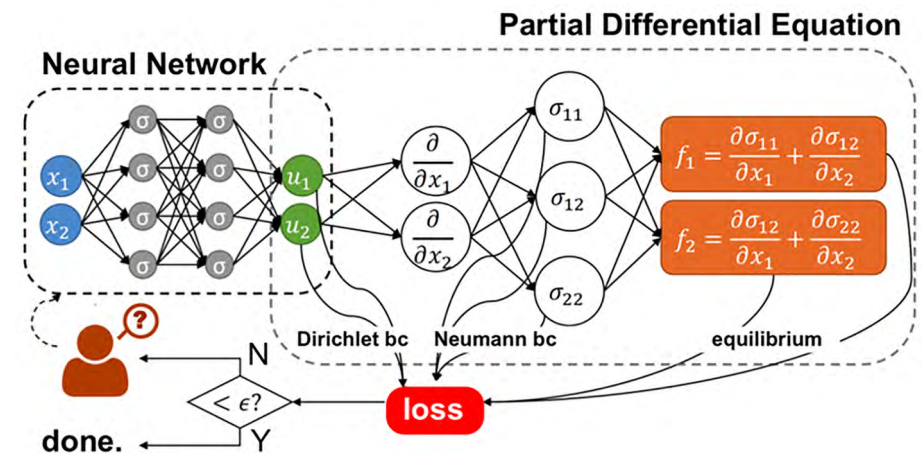
As can be seen here, the partial differential equations are part of a loss function, which means that they will not be solved accurately. Stated differently, the set of differential equations will not be solved, leading to an error that can sometimes be quite substantial, depending on the other components of the loss function.

In our UNRAVEL project, an NWO XL project, we are researching other ways of designing physics-informed neural networks with the explicit demand that the underlying differential equations are satisfied and the vital properties of solutions are retained. A very interesting approach, which originated from work within Philips Research, is that of designing truly dynamic neural networks. For dynamic cases, recurrent networks are often designed. Our dynamic neural networks are truly dynamic, with dynamic actions in the neurons of the network. This approach has enormous potential in our opinion, and we will also explore this further with my team at TU Munich.

Mathematics can and should play a prominent role within the world of artificial intelligence and, to stress this, I always state that

"Real intelligence is needed to make artificial intelligence work."

The future needs computational science and engineering, blending data-driven and physics-based perspectives. The combination of scientific computing and machine learning constitutes a new field of research, termed **scientific machine learning**. Scientific machine learning has been taking the academic world by storm as an interesting blend of traditional scientific modeling with machine learning methodologies like deep learning. While traditional deep learning methodologies have had difficulties with scientific issues like stiffness, interpretability and enforcing physical constraints, this blend of numerical analysis and differential equations has evolved into a field of research with new methods, architectures and algorithms which overcome these problems while adding the data-driven automatic learning features of modern deep learning. Many successes have already been found, with tools like physics-informed neural networks, universal differential equations, deep backward stochastic differential equation solvers for high-dimensional partial differential equations and neural surrogates, showcasing how deep learning can greatly improve scientific modeling practice. Mathematics will be essential in addressing the challenges that we encounter in the rapidly evolving field of scientific machine learning. In this context, it is really great that the national 'AI and Mathematics' (AIM) initiative has been set up. It emphasizes

that mathematicians are very interested in participating in the world of artificial intelligence and machine learning, and I am convinced of a very essential role for mathematics in these fields.

All of the foregoing is necessary to address industrial challenges in the future. The high demand for extremely accurate models that can simulate processes and products in real time or even beyond real time necessitates the solution of coupled systems of equations. Here, we use the terminology of **digital twins**: creating a virtual copy of the real product or process that runs in parallel, is fed with data from sensors and is able to predict failures and recommend precautions to be taken. Huge savings can be obtained if such digital twins are used, but it will take some time before we see true digital twins in industry. Companies like Siemens are heavily investing in digital twinning, acquiring software companies with software for different aspects, such as electromagnetic, thermal and mechanical behavior, just to make sure that every aspect is included.

The future of scientific computing will be bright in all of the aforementioned fields: high-performance computing, data science and artificial intelligence. I fully agree with the statement of Karen Willcox, esteemed colleague from the Oden Institute in Texas: *"It is such an exciting time to be a computational scientist. The field is in the midst of a tremendous convergence of technologies that generate unprecedented system data and enable automation, algorithms that let users process massive amounts of data and run predictive simulations that drive key decisions, and the computing power that makes these algorithms feasible at scale for complex systems and in real-time or in situ settings."*

Mathematics for a better world

Besides the research carried out and described in the foregoing, I also played a role in various organizations, at both the national and international level, to promote mathematics in general as well as mathematics for applications. The impression that people have about mathematics and mathematicians is not always positive; mathematics is often viewed as a rather difficult subject and one often does not have any idea about its usefulness. The mathematics learned in secondary schools is seen as a necessary thing that one needs to do but that one can forget about soon after leaving school. It is important to counter this misperception and show that mathematics is vital in our complex world with many grand challenges.

At the national level, there is the Platform Wiskunde Nederland, supported by NWO and all mathematical institutes. Several reports have been produced since the start in 2010, including the booklet on success stories mentioned before and a Deloitte report detailing the value of mathematics for the Dutch economy. The latter contains stunning figures, indicating the extreme importance of mathematics for society, as mentioned before. Similar reports have been produced in the UK, France and Spain, with corresponding figures. The 'Imaginary' exhibition toured the country in 2016-2017 and 2022-2023, showing the beauty and the power of mathematics. And many other initiatives are being undertaken to demonstrate the unexpected usefulness of mathematics.

At the European level, I have been involved in the European Consortium of Mathematics for Industry (ECMI) and EU-MATHS-IN, the European Service Organization for Mathematics in Industry and Innovation. Both organizations have many researchers on board that work with industry, educate students to have careers in industry and lobby for mathematics at the European level. At this level, it is also rather difficult to convince policymakers in particular of the extremely important role of mathematics. Mathematical methods are often considered tools, like a hammer or a screwdriver, that one can pick from the mathematical toolbox and just use. In my inaugural speech, I indicated that this is not the ideal situation. It is much better to involve mathematicians from the start; they know the methods and their modifications inside out and are in a much better position to judge which methods should be used or what needs to be done. Mathematicians have a very special way of thinking, often in an abstract way, and this is what is needed

to successfully address challenges. Besides this, mathematics is a very versatile science. Methods that have been developed for a certain application can often be abstracted in such a way that application to completely different problems is possible. Often, this is unexpected. Why would a method developed for electronic circuit simulation be useful for a mechanical or civil engineering problem? Mathematicians have the oversight and insight to judge such situations and suggest the use or further development of methods from entirely different fields.

Since 2019, I have been an officer-at-large in the worldwide organization for industrial and applied mathematics, ICIAM, and, in fact, its next president as of October 1, 2023. Yes, indeed, a few months after my (formal) retirement! I would like to continue the work we did at the European level and take it to the larger scale, as it is evident that we can help each other to bring mathematics to the forefront and stress its importance to important challenges in the world like climate change and the energy transition. During the recent SIAM conference on Computational Science and Engineering in RAI Amsterdam, I organized a public evening with the title 'The role of mathematics in solving the world's main challenges'. The presentations can be viewed on YouTube[2], showing impressive examples of how mathematics can contribute to a better understanding of challenges and aid in matching the high demands that society and industry have nowadays.

I am coming to the end of my valedictory lecture. In the last two decades in particular, since my appointment at TU/e, I have considered it my mission to highlight the foregoing ideas and convince researchers and policymakers of the extreme value of mathematics for addressing societal and industrial challenges. In this context, it is enough to quote the famous mathematical physicist Eugene Wigner, Nobel prizewinner for physics in 1963, who said: "The unreasonable efficiency of mathematics in science is a gift we neither understand nor deserve." Let's not worry about the fact that we may not understand or deserve it; instead, let's use this gift! I am convinced that mathematics can contribute to a better world in many ways, and I hope that many will join the mission to convince everyone of this fact. It may seem a mission impossible, but I am sure in the end it will turn out to be a mission possible!

---

[2]   https://www.youtube.com/playlist?list=PLnhN7EbOlxIpa-nSzok3GTiYS_EEadDOw

Closing words and thanks

Looking back on my life, a feeling of great gratitude prevails. I come from a warm Catholic nest, where respect for my surroundings was taught and where I was encouraged to have a mission in life. At the Catholic University in Nijmegen, nowadays Radboud University, I had excellent and inspiring teachers such as Arnoud van Rooij and Ronald Kortram, and studying mathematics was a sincere pleasure. With John Miller, I had the privilege of a very inspiring teacher in the young field of numerical mathematics, later a colleague and friend. At Philips, I was able to continue doing mathematics every day thanks to the inspiring working environment in the Mathematical Software Group created by Simon Polak. He also taught me the pleasures in the life of a researcher, going to conferences (and great hotels) and meeting many interesting people. Bob Mattheij was a great inspiration for me at TU Eindhoven, he set up relations with Philips, laying the fundaments for my position at TU/e. It was a sincere pleasure to work very closely with Hans van Duijn during the first decisive years of the Platform Wiskunde Nederland, I learned a lot from him, especially taking up tasks immediately.

After my appointment at TU/e, I thoroughly enjoyed spending my professional career in the field of scientific computing for industry and I have many more to thank for that. I became known to colleagues as the traveling mathematician[3] and an expert in food in many places in the world. The longer and many short trips were a tremendous enrichment of my life and work, sometimes also for family members. Organizing symposia and conferences, collaborating in consortia and all kinds of board work associated with that have led to much satisfaction. Being allowed to transfer knowledge to students and seeing them grow into accomplished researchers and mathematicians who apply their qualities for the benefit of industry has also been a fascinating task.

I have many to thank for this fantastic professional career. In first place, TU/e for trusting me to develop the field, for providing an excellent research and social environment and for giving the academic freedom to pursue interesting topics together with many nice people. I'm deeply indebted to all of the academic and support staff of the Department of Mathematics and Computer Science and, in

particular, to my colleagues in the Centre for Analysis, Scientific Computing and Applications. This is abbreviated to CASA and it has really felt like a home to me. I would like to thank the people who worked with me for their commitment and the fine cooperation all these years. I would also like to thank my colleagues in the Netherlands, Europe and beyond for the very pleasant collaborations and aid in the joint mission we have.

Last but absolutely not least, I want to thank my family and friends for being there for me when needed. Thanks are due to my parents for encouraging me in my studies and laying the foundation for me to grow from. But most of all, my heartfelt thanks and love go to my wife, Truus, and my children and grandchildren…too many to mention here!

Ik heb gezegd.

---

3   A book with that title will appear sometime after my retirement, devoted to Marcello Anile, a traveling mathematician from Sicily who unfortunately passed away far too young

References

E.P. Doolan, J.J.H. Miller, and W.H.A. Schilders, Uniform numerical methods for problems with initial and boundary layers, Boole Press, Dublin (1980, translated into Russian and Chinese)

S.J. Polak, C. den Heijer, W.H.A. Schilders, and P.A. Markowich, "*Semiconductor device modelling from the numerical point of view*" (**review paper**, International Journal for Numerical Methods in Engineering, vol. 24, pp. 763-838 (1987)

W.H.A. Schilders, J.J.H. Miller and S. Wang, "*Application of finite element methods to the simulation of semiconductor devices*" (**review paper**), Reports on Progress in Physics, vol. 62, pp. 277-353 (1999)

W.H.A. Schilders and E.J.W. ter Maten (eds.), Numerical methods in electromagnetics, special volume in Handbook of Numerical Analysis (P.G. Ciarlet, ed.), vol. XIII, North Holland (2005)

H.S. Dollar, N.I.M. Gould, W.H.A. Schilders and A.J. Wathen, "*Using constraint preconditioners with regularized saddle-point problems*", Comp. Opt. and Appl.,vol. 36, no. 2-3, pp. 249-270 (2007)

W.H.A. Schilders, H.A. van der Vorst and J. Rommes, "Model order reduction: theory and practice", series Mathematics in Industry, vol. 13, Springer Verlag, Heidelberg (2008)

W.H.A. Schilders, *"Solution of indefinite linear systems using an LQ decomposition for the linear constraints",* Linear Algebra and its Applications, vol.431 (30-4), pp. 381-395 (2009)

J. Rommes and W.H.A. Schilders, "*Efficient methods for large resistor networks*", IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, vol. 29, pp. 28-39 (2010)

G. Alí, N. Banagaaya, W.H.A. Schilders and C. Tischendorf, "*Index-aware model order reduction for index-2 differential-algebraic equations with constant coeffcients*", SIAM J. Sci. Comput., vo. 35, no. 3, pp. A1487–A1510 (2013)

Peter Benner, Stefano Grivet-Talocia, Alfio Quarteroni, Gianluigi Rozza, Wilhelmus H. A. Schilders and Luis Miguel Silveira (eds), Handbook of Model Order Reduction", Volume 1,2,3, Walter De Gruyter GmbH, Berlin (2020-2021)

W.H.A. Schilders and M. Guenther (eds), "Novel mathematics inspired by industrial challenges", Springer Verlag, Heidelberg (2021)

Curriculum Vitae

Prof.dr. W.H.A. (Wil) Schilders was appointed part-time professor of
Scientific Computing for Industry at the Department of Mathematics
and Computer Science at Eindhoven University of Technology (TU/e) on
December 1, 1999.

Wil Schilders (1956) received his MSc degree (1978) from the Catholic
University of Nijmegen and PhD degree (1980) from Trinity College Dublin,
both in Mathematics, then worked at Philips Research (1980-2006) and NXP
Semiconductors (2006-2010). At the end of 1999 he was appointed at TU/e and
gradually extended this position from 2010 onward. In 2010, he also became
the director of the Dutch Platform for Mathematics (PWN). He has been active
in international organizations, being the president of ECMI (2010-2011) and
EU-MATHS-IN (2015-2020) as well as chair of the European Model Reduction
Network (2014-2018). From October 1, 2023, he will be president of the worldwide
organisation ICIAM. He was the 4th Mittelsten-Scheid guest professor at Bergische
Universität Wuppertal (2020-2021) and is currently a Hans Fischer senior fellow at
the Institute for Advanced Study of TU Munich. In 2022, he was elected fellow of
the European Academy of Sciences, became a fellow of SIAM and received the
NWO-ENW Stairway to Impact award. Recently, he organised SIAM CSE 2023,
the largest mathematics conference ever in The Netherlands.

Visiting address
Building 1, Auditorium
Groene Loper, Eindhoven
The Netherlands

Navigation
De Zaale, Eindhoven

Postal address
PO Box 513
5600 MB Eindhoven
The Netherlands
Tel. +31 (0)40 247 9111
www.tue.nl/map

TU/e

**EINDHOVEN
UNIVERSITY OF
TECHNOLOGY**