From static to ‘dynamic’ and ‘agentic’ resources in mathematics education

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June 2, 2023

VALEDICTORY LECTURE

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Presented on June 2, 2023
at Eindhoven University of Technology

Introduction

The idea of 'resources' has long roots in my work. In my inaugural address (in 2015), I referred to "re-sourcing" mathematics education: I argued that we need a "resourceful approach" in order to enhance mathematics and STEM education. At that time, I started by defining 'resources' as Adler (2000) did:

"It is possible to think about a resource as the verb 're-source', to source again or differently. This term is provocative. The purpose is to draw attention to resources and their use, to question taken-for-granted meanings."

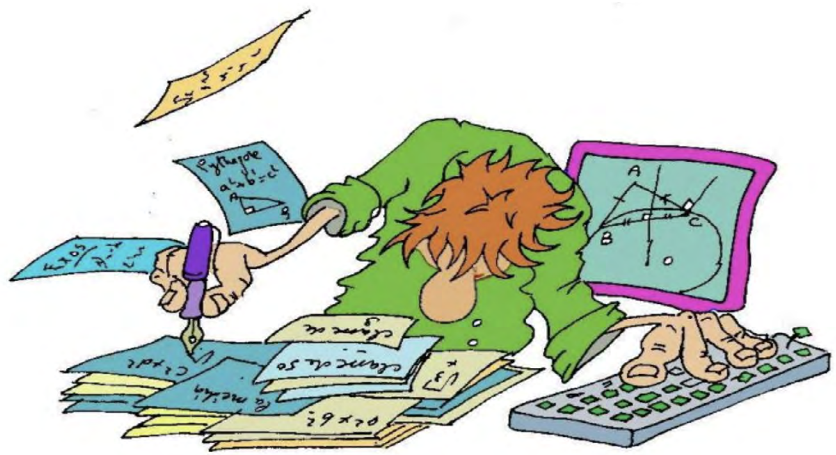
In this farewell speech, I would like to reconstruct my academic path with the concept of 're-sourcing' in mind.

There are two strands in mathematics education research that permeate my academic life: (1) one is related to the international character of my academic work and research in mathematics education; (2) the other is the research on (digital) curriculum resources and their interaction with teachers and students in mathematics education.

Regarding (1), you probably know that I grew up and was educated in Germany. In subsequent years, I lived in France and then in England, where I spent altogether 26 years of my adult life (more years than in any of the other countries, including Germany) and where I was educated and worked as a mathematics educator. Whilst my PhD (in the UK) involved an ethnographic study of mathematics teachers' work and classroom practices in England, France and Germany, one of the results of this study related to the use of mathematics textbooks as the main curriculum resources for mathematics teachers at the time. This set the scene for my international work: several stays in Lyon, France, with Luc Trouche at the Institute of Education of the École Normale Supérieure de Lyon; several visits to Shanghai, China, at Eastern China Normal University; work in the advisory committee of the DZLM (German Centre for Mathematics Teacher Education) in Germany, to name but a few. In subsequent years, when I traveled and worked in France and Norway, this theme of 'curriculum resource', its nature, and mathematics teachers' interaction with such resources became one of the guiding themes in my work. When I arrived in the Netherlands in 2015, I became more interested in student use of and interaction with such resources, as students in the engineering environment used more and more varying resources (than in schools) and in more self-directed ways.

This leads me to the second strand: curriculum resources in mathematics education. There are at least three phases that can be identified in my research about mathematics curriculum 're-sources', their nature and development, and their use by and interaction with teachers and students.

Phase 1: Mathematics textbooks and their use by teachers



In the 1990s, when I started my academic career in mathematics education in the UK, the research literature referred to 'materials' or 'curriculum materials' as support for teachers' teaching (and occasionally for student learning). At that time, the literature was clear about the fact that textbooks were used extensively in classrooms in schools. Keitel *et al*. (1980) claimed that amongst the tools for teaching and learning, the textbook is "one of the most important *orientations* [for the teacher] and the factor which influences the teacher's work in its entirety" (p. 15, free translation). In England, Her Majesty's Inspectorate (HMI) estimated that two-thirds of middle and secondary schools used a commercial mathematics scheme for Year 7 and Year 8 pupils (HMI, 1992) and Millett and Johnson (1996) argued that mathematics has long been regarded by many teachers in Britain as a subject for which the textbook is the main resource. Johnsen (1993) classified teachers as (1) faithful followers of the textbook lesson by lesson, with little or no time for supplementary material, (2) followers of the plan and progression of the textbook but selective in its use or (3) those who break from content and structure and add supplementary material. Although there had not been a systematic study in England, a comment from HMI suggests that little mediation of textbooks might have taken place:

"Many schools used a commercial mathematics scheme, and while they were often valuable in providing structure for mathematics courses there was undue reliance on them in most schools. The consequences included a lack of differentiation; mathematics learning based on 'texts' with pupils carrying out step-by-step instructions rather than on 'contexts'; restricted mathematical thinking; and poorly developed understanding." (HMI, 1991, p. 22)

Evidence from the International Association for the Evaluation of Educational Achievement (IEA) Second International Mathematics Study (Robitaille & Garden, 1989) and the later Third International Mathematics and Science Study (TIMSS, in Schmidt *et al.*, 1996) indicate that this is a worldwide phenomenon.

At that time, as well as today, many researchers (e.g., Apple, 1992) have warned that "texts are not simply 'delivery systems' of 'facts', but that they are the simultaneous results of political, economic, and cultural activities, battles, and compromises" (p. 4). In the same article, Apple (1992) also quoted A. Graham Down of the Council for Basic Education:

"Textbooks, for better or worse, dominate what students learn. They set the curriculum, and often the facts learnt, in most subjects. For many students, textbooks are their first and sometimes only early exposure to books and to reading. The public regards textbooks as authoritative, accurate, and necessary. And teachers rely on them to organise lessons and structure subject matter. But the current system of textbook adoption has filled our schools with Trojan horses – glossily covered blocks of paper whose words emerge to deaden the minds of our nation's youth, and make them enemies of learning." (p. 6).

At that time, at least in the UK, 'teaching by the book' was very much frowned upon (by teacher educators and researchers) and mathematics teachers would rarely talk about textbooks as the main resource in and for their lessons – textbooks were seen as being of mostly poor quality. At the same time, teachers wished for a 'scheme' they could follow (to make their job easier) (Haggarty & Pepin, 2002). Mathematics departments in schools developed their own 'lesson schemes' (including tests) for each grade and differentiated them by attainment group. These schemes were expected to be used by each member of the department. Textbooks

were handed out in class for particular topics and exercises and were collected at the end of each lesson. Hence, students would typically not take a textbook home.

A cross-national study (by my colleague Linda Haggarty and I) of textbooks and their use in England, France and Germany (Pepin & Haggarty, 2001; Haggarty & Pepin, 2002) showed that

"… learners in the different countries are offered different mathematics and given different opportunities to learn that mathematics, both of which are influenced by textbook and teacher." (p. 567; Haggarty & Pepin, 2002)

Around the same time, one of the large-scale international mathematics student achievement studies (TIMSS) suggested that, amongst the curriculum materials, the textbook had been identified as potentially having a large effect:

"Textbooks are commonly charged precisely with the role of translating policy into pedagogy. They represent an interpretation of policy in terms of concrete actions of teaching and learning. Textbooks are the print resources most consistently used by teachers and their students in the course of their joint work." (p. viii; Valverde et al., 2002)

Hence, it made sense to investigate the connections between the worlds of policy, textbooks and teacher instructional practice (e.g., Pepin et al., 2013). It was argued that the mathematics textbook was (and had to be) regarded as the pivotal resource in teachers' resource systems (for curricular practice), even in times of digitization, and as a crucial interface between culture, national policy and classroom practice. This could, to some extent, explain the relative stability of teacher curricular practices, even in times of policy changes (e.g., towards curricular changes such as the integration of ICT into teaching or of inquiry-based teaching).

At that time, the mathematics education research community was mainly concerned with teacher use of curriculum materials and with mathematics tasks in textbooks, including to see in which ways tasks in textbooks offered opportunities for students to appreciate and learn mathematics.

"The tasks in which students engage provide the contexts in which they learn to think about subject matter, and different tasks may place different cognitive demands on students …. Thus, the nature of tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of their subject matter with which they are engaged. Students develop their sense of what it means to 'do mathematics' from their actual experiences with mathematics, and their primary opportunities to experience mathematics as a discipline are seated in the classroom activities in which they engage." (p. 525, Henningsen & Stein, 1997)

Hence, textbooks determine to a large extent what and how students learn in regard to mathematics. It was assumed that the tasks offered to students in curriculum materials, such as textbooks (to be used in and out of school) and teacher mediation of the textbooks and of the mathematics (i.e., tasks) within those books, were the crucial ingredients for the study of and efforts to reform mathematics education (Rezat et al., 2021). Moreover, there were few studies that investigated student use of textbooks (e.g., Rezat, 2009). This meant that mathematics teachers were loaded with a huge responsibility for the enactment of reform efforts and for the support of students in their learning of the (reformed) mathematics. At the same time, little support was provided for teachers (e.g., suitable textbooks and/or in-service teacher education) to shoulder this responsibility.

Phase 2: Levers for change – 'dynamic' digital curriculum resources

In this second phase, the steady increase in the availability of digital curriculum resources (and, more generally, the developments in educational technology) afforded new opportunities and new practices. The digital curriculum resources were different by nature and hence afforded different options for use by and interaction with teachers and students. However, they were also challenging for teachers (and students), who were at times reluctant to let go of their established practices of how mathematics can be learnt. In this scenario, mathematics curriculum resources became mediators of change in terms of new opportunities for the teaching and learning of mathematics.

At this stage, it seems helpful and necessary to distinguish between research on **digital curriculum resources (DCR)** from research on digital technologies in general. In our special issues (Pepin, Choppin, et al. 2017, p. 647), we regarded the main differences as being the particular attention that the DCR pay to:

1. the aims and content of teaching and learning mathematics;
2. the teacher's role in the instructional design process (i.e., how teachers select, revise and appropriate curriculum materials);
3. students' interactions with DCR in terms of how they navigate learning experiences within a digital environment;
4. the impact of DCR in terms of how the scope and sequence of mathematical topics are navigated by teachers and students;
5. the educative potential of DCR in terms of how teachers develop the capacity to design pedagogic activities.

When trying to differentiate DCR from other types of digital instructional tools or educational software programs, it is acknowledged that there is some overlap and that DCR often make use of these other types of tools and software. Indeed, what differentiates them from pre-digital curriculum programs is that they are made accessible on electronic devices and that they often incorporate the dynamic features of digital technologies.

Moreover, in distinguishing between 'static' curriculum resources and DCR, we (Pepin & Gueudet, 2018) defined mathematics curriculum resources as "all the material resources that are developed and used by teachers and students in their interaction with mathematics in/for teaching and learning, inside and outside the classroom" (p. 172/173). Turning to DCR, in a recent chapter (Pepin et al., 2023), we differentiated between:

- digital 'material' resources (e.g., digital/ICT-based curriculum resources like interactive e-textbooks, websites, e-worksheets);
- digital 'non-material' resources (e.g., social and cultural resources such as conversations with tutors, peers and friends);
- cognitive resources (e.g., concepts and techniques in mathematics) used by students and teachers, in addition to
- general, non-curricular digital resources (e.g., Wikipedia).

In the same chapter, we provide examples of such resources:

Table 1. Different types of digital resources

Resource category	Description	Examples
Digital curriculum resources	All the digital resources that are used by the subject in their interaction with mathematics for teaching and learning, inside and outside the classroom. These resources are intended/aligned with the mathematics curriculum (hence curriculum resources).	E-text resources: e.g., e-textbooks, e-worksheets Web-based resources: e.g., digital learning environments Other digital curriculum resources: e.g., Scratch
E-based/digital social resources	Formal or casual human e-based interactions	Web-based conversations with tutors, peers and friends
Cognitive digital resources	E-based mathematical frameworks and concepts subjects work with	Concepts and techniques downloadable from the web
General digital resources	Non-curricular digital resources	E.g., Wikipedia, YouTube, Wolfram Alpha

The mediatory role of curriculum resources (and the mathematical tasks associated with those resources) on teachers and students has been widely acknowledged. This view is anchored in a **socio-cultural perspective**, where the curriculum resource is conceptualized as an artifact that mediates goal-directed activities. Let us view this in more detail, as it can be argued that this has implications for teacher and student interaction with curriculum resources, how artifacts develop into instruments, and for the conceptualization of teacher or student 'design' and what it means to develop 'design capacity'.

Cole (1998) contended that artifacts are simultaneously *ideal* (conceptual) and *material*. This underscores the historical, human-made quality of tool/instrument design and use. Instruments/tools carry the residue of prior needs, questions, problems and solutions, offering both affordances and constraints for the activity. As Pea (1985) stated, "the design of artefacts, both historically by others and opportunistically in the midst of one's activity, can advance that activity by shaping what are possible and what are necessary elements of that activity" (p. 50).

That is, when introducing new tools, this is likely to provoke changes in the (artifact-using) person(s) (Wertsch, 1998). As mentioned earlier, teachers' artifacts for mathematics instruction include curricular resources, such as textbooks that they engage with in the process of planning lessons (e.g., Shield & Dole, 2012). As teachers use, shape and mold these artifacts to plan their lessons, they generate a relationship with the curriculum resources that influences how they view and perceive resources as a means for instructional implementation. Fullan (2007) also made a helpful distinction between three dimensions of teacher change during implementation: in the use of the materials themselves, in related teaching behavior (actions) and in their beliefs. On all three dimensions, one can see mutual adaptation.

Let us turn to 'instruments' now in order to better understand the relationship between *artifact* and *instrument* and the processes that develop when teachers interact with curriculum resources: the interrelated processes of *instrumentation* and *instrumentalization* (e.g., Trouche, 2004). Rabardel and his colleagues (Rabardel & Bourmand, 2003; Verillon & Rabardel, 1995) explain that the *artifact* is the 'bare tool': in our case, the 'material' curriculum resource, which is available to the teacher to, for example, prepare and enhance a lesson. Unless teachers have the goal of improving their lesson and the 'intuition' (and intention) that this particular resource can enhance their lesson, it will not be seen as useful for them. So, only after teachers have become aware of how the artifact/tool can extend their capacities for the enhancement of their lesson and after they know how to use this resource for their lesson (improvement) does the resource/artifact become, from a teacher's perspective, a valuable and useful instrument that mediates their activity of teaching. Typically, the notion of instrument can be summarized as

Instrument = artifact + schemes of usage

Hence, the instrument encompasses both the artifact (e.g., the 'bare tool') and the accompanying 'schemes' that teachers (or students) develop as they use the curriculum resource(s) for accomplishing specific kinds of tasks (e.g., designing a new lesson/series). This implies that instruments are always bound to an individual user or a group of users. It has to be noted that the artifact does not need to be a material object but can also be a digital one, for example. Hence, it is clear that an artifact only develops into an instrument when combined with purposeful use schemes (of the user). In other words, the development of an instrument requires a process of 'intended use' (or interaction or appropriation) which allows the artifact to mediate the activity (a process called *instrumental genesis*). So, in the same way as the affordances and constraints of the artifact shape and influence the (thinking of the) teacher (the *instrumentation* process), the teacher's understanding and preferences shape the ways in which s/he uses the artifact – the artifact is shaped by the teacher (the *instrumentalization* process). This theoretical link is used in the *Documentational Approach to Didactics* (DAD; see Gueudet & Trouche, 2009; Trouche, Gueudet & Pepin, 2018; explained later). This interactive relationship is also well expressed in the term 'mutual adaptation' in the curriculum design literature (e.g., Fullan 2007).

In terms of **analysis of DCR**, we have previously outlined three primary frameworks to inform this analysis. (1) The Digital Typology created by Choppin et al. (2014) in which they suggest three categories when analyzing DCR, i.e., students' learning experiences, curriculum use and adaptation, and assessment systems. They conceptualized the learning space in terms of learning experiences, differentiation/individualization and social/collective features. (2) The second framework, by Choppin and Borys (2017), provides four perspectives (the private sector perspective (e.g., publishers), the designer perspective, the policy perspective and the user perspective) that inform the design, development and dissemination of curriculum resources. The team explores how these perspectives lead to a foregrounding (or backgrounding) of the features described in the Choppin et al. (2014) framework. (3) In the third framework (Pepin et al. 2016), we distinguish between four types of e-textbooks (according to their model of development and their functionality): integrative, e-text, evolving or 'living' e-textbook, and interactive e-textbook.

Relatively recent research has tried to unravel how the participatory relationship between teachers and curriculum resources in designing instruction is shaped by features of the materials. For example, Choppin et al. (2021) analyzed enactments of the US Common Core State Standards Mathematics (CCSSM) curriculum in several states and across different curriculum contexts in terms of rigor of mathematical activity across middle school mathematics lessons. They identified two contrasting sets of design features that they regarded as two opposed endpoints of a conjectured continuum of design perspectives: curriculum resources are characterized as either a 'delivery mechanism' or a 'thinking device'. Their results indicate that non-routine forms of rigor of mathematical activity were found only in classrooms using materials classified as thinking devices. Thus, the authors concluded that the features of the curriculum resources influence the nature of mathematical activity in the classrooms. This has implications for the design of and interaction with DCR by teachers and by students.

In terms of DCR and the **design of instruction**, much has been studied at a theoretical level. Teachers used to be seen as the 'implementers' of the curriculum and the associated curriculum resources (developed by professional curriculum designers and mathematicians), with the teacher's role being that of the mediator of textbook content by 'aligning' with the designers. Now, many studies have shown that teachers use curriculum resources, particularly digital ones, differently to prepare and set up their teaching in class. The relationship between teachers and curriculum resources is nowadays widely characterized as interactive and participatory (e.g., Remillard, 2005; Trouche et al., 2018). The Documentation Approach to Didactics (DAD) provides one of these theorizations.

The DAD has been developed as a theory in mathematics education with the aim of understanding teachers' professional development by studying their interactions with the resources they use and design in and for their teaching. It acknowledges the central role of resources for teachers' work. Teachers search for resources, select relevant resources, modify them, combine them and share them; DAD describes this as the documentation work of the teachers. A key notion is also that of artifacts; any set of resources is conceived as an artifact that teachers can use to accomplish specific teaching tasks characterized by specific objectives. The DAD maintains two main concepts introduced by Verillon and Rabardel (1995): instrumentation and instrumentalization. In performing a teaching task, a teacher interacts with a set of resources. This interaction combines two interrelated processes. First, the process of instrumentation, where the selected resources support and influence the teacher's activity, representing an interface between the knowledge, goals and values of the author and the user. The instrumentation process also encompasses the constraints that resources may place on teachers' documentation work. Second, there is the process of instrumentalization, where the teacher adapts the resources for their needs, requiring craft in their use; the materials are inert objects that come alive only through interpretation and use by a practitioner (Brown 2009). Instrumentalization is also linked with the resource's affordances, the possibilities it opens for teachers' documentation work.

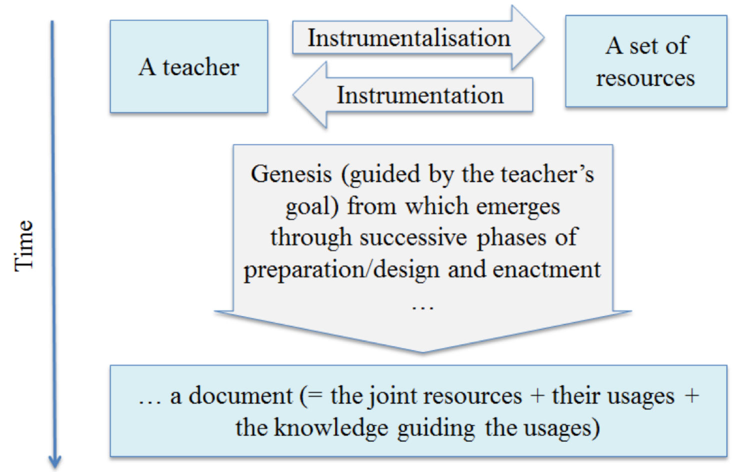


Figure 1. Documentational genesis

Leaning on the DAD, Pepin et al. (2017b) have construed 'mathematics teacher design capacity' as the capacity enabling teachers to engage in the design activities that are based on the following three main components: (1) a clear goal which provides points of reference for (a specific) lesson design (e.g., what the students know/can do and where the teacher is heading in terms of student learning, how this lesson connects to previous and future lessons and grades); (2) a set of design principles (for a specific context and lesson sequence) that are robust (e.g., evidence-informed, rationalized) and flexible (i.e., allow him/her to e.g., adapt to different challenges and contexts); and (3) 'reflection in action' (e.g., lines of action that may develop in the course of instruction). In accordance with Brown (2009), Pepin et al. (2017b) emphasize that mathematics teacher design capacity is always associated with the creation of 'new', deliberate and productive designs that help accomplish teachers' instructional goals. Pedagogical Design Capacity (PDC), a term coined by Brown (2009), can be defined as the capacity enabling teachers to engage in the design activities of teaching through (at least) two actions: perceiving and mobilizing existing resources (personal and curricular) to craft instructional episodes. The notions of Brown's 'design capacity' and of the 'mathematics teacher design capacity' of Pepin et al. are relevant as it has been argued that teacher design capacity can be enhanced with educative curriculum resources (Pepin, 2018; Pepin et al., 2017b). In the example below, a French teacher (called Vera) is supported by educative resources to develop good questioning in her lessons.

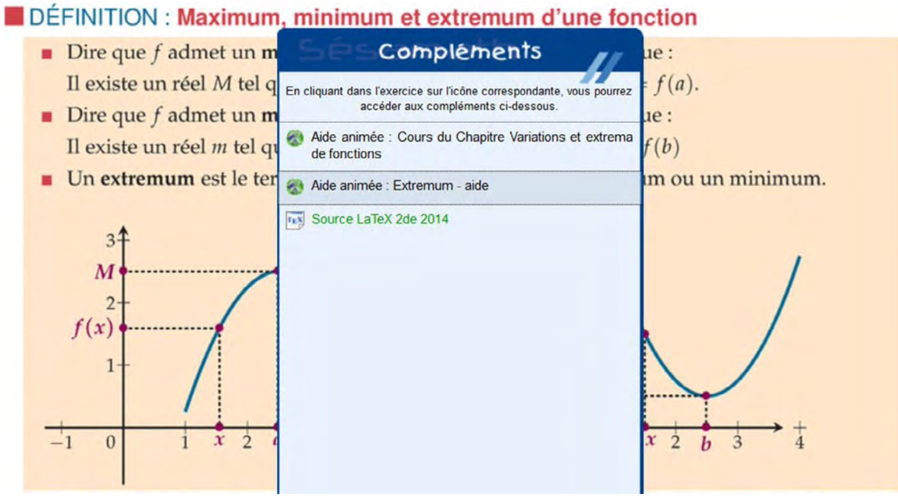


Figure 2. Sésamath e-textbook showing one of its interactive features ('compléments') as an overlay to the text page

Many studies (e.g., Gueudet et al., 2018; Naftaliev, 2016; Pepin et al., 2017a) show teachers' work with e-textbooks, one of the promising DCR promulgated in earlier years. In the study by Pepin et al. (2017b), evidence is provided of how individual teachers (like Vera) worked with the French Sésamath e-textbook: (1) using Sésamath tools and content for lesson sequence design/preparation; (2) using Sésamath digital tools to talk directly to students and provide them with individualized learning exercises/homework; (3) keeping a written record of what students should write in their "lesson books"; (4) writing to the Sésamath association to vary/change an explanation or exercise, to name but a few of the affordances. This was juxtaposed by the work of Cora (a teacher in Norway) with "designer-made" DCR (produced by a European Union-supported team). Both teachers adapted the DCR to their needs; indeed, they "picked and chose" appropriate materials and were inspired by some of the innovative pedagogical features afforded by the digital nature of the materials. For example, Vera used the digital environment of Sésamath to develop differentiated exercises (e.g., in terms of degrees of help for solving the same exercise) for her students. Cora used the DCR (modules of the digital professional development program) to develop questioning for her students. This suggests that these mathematics teachers benefited from work with quality DCR, particularly when they worked with colleagues and teacher educators in professional development sessions.

However, looking closely at the DCR that Cora worked with, it became clear that we were predominantly attending to DCR as tools for teachers to use in their lesson design and teaching, eventually with the potential for enhancing student learning. Less attention was paid to teacher learning with DCR (with some exceptions, e.g., the work of Visnovska, Cobb & Dean, 2012). However, for teacher educators, it is not only a responsibility to investigate teachers' interactions with curriculum resources 'as is' (in theoretical terms, to attend to the schemes of usage, i.e., how they use these materials at present) but also to suggest/design/develop ways for teachers to learn from working with these chosen 'tools'. This brings us to ***educative curriculum resources*** for mathematics teachers.

Hence, we turned to DCR that were educative for teachers, e.g., DCR with educative features – these referred to "the elements in curriculum materials specifically intended to provide support for teacher learning" (p. 294, Davis et al., 2017). In a similar way as Davis and her team had researched and provided (six) "design principles" for educative curriculum materials in science education, we provided 'functions' and 'design specifications' for educative curriculum resources in mathematics education (Pepin, 2018). Following from that, Gueudet, Pepin and Lebeau (2021) conducted a design research study in order to develop and investigate how a curriculum resource could turn into a meta-resource that can support teachers' implementation of reform efforts aimed at increasing students' autonomy through the use of digital resources.

Leaning on Prieur (2016), the term meta-resource has since been re-defined (in Gueudet, Pepin & Rezat, 2023) as a *material resource which has been explicitly designed for supporting mathematics teachers' planning of teaching and learning activities.* Meta-resources can have a digital or non-digital character as long as they support mathematics teachers in becoming creative and didactically effective designers of their teaching. Keeping this in mind, in the literature study by Gueudet et al. (2023), meta-resources were categorized corresponding to six selected aims: (1) choosing resources; (2) designing learning progressions; (3) designing lesson plans (individually); (4) task design; (5) assessment; (6) designing collaboratively in the context of professional development programs. In relation to (1) choosing resources, for example, teachers often have to choose among different applets, freely available on the Internet, that they can integrate into their teaching, and they have to assess their quality.

Phase 3: 'Agentic' resources – who is the agent?

Over the last decades, particularly whilst working at TU/e, we could see the change in the nature (from static to digital and dynamic/interactive) and availability (towards an abundance) of resources for teaching and learning mathematics. Moreover, possibly due to the different affordances of DCR and to the intentions to enact more innovative student-centered teaching and learning approaches (e.g., Challenge-Based Education – CBE), a change of use schemes and users has been noticeable, and users of DCR now include not only teachers but also students themselves, who identify and interact with DCR for their learning, particularly in university (mathematics) education.

In order to make engineering education more relevant and engaging for students, many universities of technology are developing innovative learning environments to create more student-centered forms of engineering education (Pepin & Kock, 2021). In these environments, students are expected to contribute to the solution of societal problems, typically in collaboration with partners from industry. In the literature, no generally agreed upon definition of CBE exists (Gallagher & Savage, 2020), but CBE learning environments are said to offer learning experiences "where the learning takes place through the identification, analysis and design of a solution to a sociotechnical problem" (p. 87; Malmqvist et al., 2015).

At TU/e, Challenge-Based Education (CBE), referring to both teaching and learning approaches, has been adopted and is described in the following way:

"In CBE, the acquisition and production of disciplinary knowledge and the development of professional competencies (e.g., problem resolution, design capacity, ethical awareness, and multidisciplinary collaborative work) are expected to go hand in hand. Groups of students are given a challenge, which is often connected to one of the 'grand challenges' in society and is provided by a stakeholder from an external organization (e.g., a company or a research institute). Based on the challenge, students identify a problem they want to work on, and they develop their knowledge and competences by collaboratively developing a solution to the real-life problem, generally in the form of a working prototype or proof of concept. Often this takes place in a multidisciplinary setting." (p. 1, Kock, Salines-Hernándes & Pepin, 2023)

In what follows, I explain – leaning on the research literature – how I understand the term 'agency', and more particularly 'student agency', in order to lead my way to what I understand by 'agentic' resources and how the use of particular resources has been evidenced in our recent studies.

AGENCY

A report from the OECD (2019) is clear that "when students are agents in their learning, they are more likely to have learned how to learn – an invaluable skill that they can use throughout their lives" (p. 2). In university mathematics education in particular, there are good reasons for supporting students in developing agency in their learning. As understood in the context of the OECD Learning Compass 2030, the concept of agency is based on the conviction that "students have the ability and the will to positively influence their own lives and the world around them" (p.2). According to this OECD report (2019) on "Student agency for 2030", student agency refers to the level of autonomy and power that a student experiences in their learning environment. It represents the ability for students to play a critical role in their own development (what they want to learn), practice (how they are learning what they want to learn) and reflection on what and how they have learnt (or contemplation on what they wanted to learn and how they wanted to learn it) (OECD, 2019). Agency requires the ability to frame a guiding purpose and identify actions to achieve a goal. It is said to be about acting rather than being acted upon, shaping rather than being shaped, and making responsible decisions and choices rather than accepting those determined by others (OECD, 2018). Student agency refers to learning through activities that are meaningful and relevant to students, driven by their interests, and often self-initiated with appropriate guidance from teachers (OECD 2019, p. 5). It gives students a voice and often a choice in how they learn and which resources they use for their learning. This includes the choice of resources that they regard as meaningful for them and how they use them for their mathematics learning and study (e.g., Pepin & Kock, 2019).

Studies of agency examine how resources (mostly artifacts) and people shape our actions and decision-making. Agency is a central construct in educational studies that consider teachers' and students' actions that are perceived to be "channelled by culture, other people and the material world" (including resources) (p. 4, Carlsen et al., 2016). To human and materials agency, Pickering (1995) has added disciplinary agency (of mathematics):

"It is, I shall say, the agency of a discipline – elementary algebra, for example – that leads us through a series of manipulations within an established conceptual system." (p. 115).

He also coined the term 'dance of agency' in performance where agencies "emerge in the temporality of practice and are definitional of and sustain one another" (p. 21; Pickering, 1995). This metaphorical dance can be seen in mathematics lessons/projects with DCR: the teacher may take the lead at the start of the lesson/project but, as the lesson/project proceeds, other agents come into play, e.g., students, DCR or mathematics (Carlsen et al., 2016). Boaler (2003) uses the 'dance of agency' metaphor when illustrating the importance for mathematics learners to have an empowering identity in relation to school mathematics. To know when to draw on mathematical ideas and to be able to solve mathematical problems is a critical part of the dance of agency according to her.

It is important to note that agency is not just individual; it can also be exercised within social practices: "Agency lies in the improvisations that people create in response to particular situations" (p. 279; Holland, Lachicotte, Skinner & Cain, 2003). Another cautionary note relates to the following: promoting student agency should not be confused with providing unlimited autonomy without structure and guidance. A lack of structure and guidance may jeopardize student opportunities to learn successfully, particularly if students are relative beginners in a domain (Kirschner, Sweller & Clark, 2006). This may also be at the cost of student motivation. A productive balance between autonomy and structure/guidance needs to be found, which may be a challenging task for teachers and educational designers (Kock et al., 2013).

In terms of student agency and the learning of mathematics, the education research literature has shown that students who develop agency in their learning are more motivated, experience greater satisfaction in their learning and, consequently, are more likely to achieve academic success (Lin-Siegler, Dweck & Cohen, 2016; Cobb et al., 2009). It has been said that student agency is one of the most transformative ingredients for learning, including of mathematics (Brown, 2009), and that learning mathematics with understanding is best achieved when it is driven by the student/learner (Hiebert et al., 1997). Hence, it can be said that

students developing agency in and for their learning may lead to more motivated, more satisfying and more successful learning.

The OECD (2019) defines agency in a broader sense:

"Student agency relates to the development of an identity and a sense of belonging. When students develop agency, they rely on motivation, hope, self-efficacy and a growth mindset (the understanding that abilities and intelligence can be developed) to navigate towards well-being. This enables them to act with a sense of purpose, which guides them to flourish and thrive in society." (p. 5)

This definition of agency by the OECD (2019) requires students to become self-navigating learners who are capable of making choices, who purposefully make choices about the directions that their mathematical inquiries will take and about the resources (e.g., material, digital, social/human or cognitive) they may need to undertake such inquiries. Hence, one of the main ingredients of agency development relates to the purposeful choice and meaningful use of resources by students.

To provide an example of students' selection and use of resources in such innovative learning environments, Pepin and Kock (2021) have described students' use of (digital) resources in an CBE course at bachelor level. Multidisciplinary student groups were given a real-life challenge formulated by an external stakeholder, typically from a company. The students selected a concrete problem based on the challenge, investigated the problem and developed a prototype solution. In this process, the students exercised considerable agency because they had to not only identify 'their' problem within the challenge but also find and select relevant resources that would help them solve the problem (e.g., scientific papers, software tools). The students were guided by feedback from social resources (e.g., their academic tutors and the stakeholders).

In order to consider providing agency for students by involving them in choosing their own resources in a problem-based project in CBE, it is necessary to see the curriculum and its associated resources not as a product or a fixed set of requirements and tools but as a negotiation process wherein external aims give direction and, moreover, where (teachers and) students influence what is actually experienced in class. As early as 1938, Dewey (1938) opposed the idea that the curriculum is a prescription of what learners have to undergo. He argued that learning cannot happen through the external motivation of a prescribed curriculum and the provided resources but that learning starts with the experiences and interests of the learner and is built up by negotiation (between teacher and student) towards a more systematic growth of knowledge and insights of the learner.

AGENCY AND STUDENT USE OF RESOURCES

In our analysis of the research literature concerning students' use of resources and factors influencing this (see Pepin, Kock & Rezat, 2023), we have shown that students exercise agency in selecting and using resources at primary, secondary and tertiary levels. Whilst at school, students mainly use the prescribed resources (e.g., textbooks); at university, where the institutional framing is often quite different to primary and secondary mathematics education, students get access to and are invited to use a larger number of DCR. Student choice and use of the resources on offer then depend on various factors, amongst them the nature and structure of the course and its associated pedagogical approaches and students' beliefs concerning their learning of mathematics (e.g., 'I succeed by doing lots of exercises') and their actual goals. Whilst in traditional courses the goals tend to be examination-driven, in project/challenge-based courses the goals are defined by the problem (Martin et al., 2007).

In one of our earlier studies (e.g., Kock & Pepin, 2018; Pepin & Kock, 2019), we investigated the resources of and their use by first-year university students in 'traditional' Calculus (CS) and Linear Algebra (LA) courses. In these two courses, students used different/additional resources and they used the available resources differently for LA than for CS. (1) Basically all of the LA curriculum resources offered/provided were used and students worked with them according to the lecturer's guidance. (2) The CS resources seemed to be a large bag of 'tools', a 'pile of bricks', that the students could pick from (according to their needs) and use for their learning. However, how students could orchestrate and align the resources for the learning of CS was not clear. These differences appeared to be related to (a) the size and student audience of the courses (130 students in LA; 2000 in Calculus), which was, in turn, connected to the different organization of the courses (four hours of lectures and three hours of tutorials in LA versus six hours of lectures and one hour of tutorials in Calculus), and (b) the organization and alignment of the resources with the assessment/tests. For example, there was a clear intended (by the lecturer) learning trajectory in LA, with exercises aligned with the examinations. Students mentioned that if they worked according to/with the reader and did "all exercises in the reader" and the obligatory weekly assignments, they could expect to pass the examination. In the CS course, many support tools were proposed (e.g., on the web, in print), with many exercises and tasks that, according to the students, were not always clearly aligned with the examinations. Students said that it was impossible to do all exercises and read all materials provided, and they often had considerable difficulties choosing from the immensity of resources provided.

From the interviews based on students' drawings of their resource system (SRRS – Schematic Representation of Resource System; see Pepin et al., 2016), we could identify several *Actual Student Study Path*s, which were the paths students perceived/drew when we asked them which resources they used, the importance of those resources (their role with respect to their perceived study paths) and how they orchestrated them for their learning. The study path of the LA course appeared to be relatively traditional and most students followed it: students could identify core resources (e.g., the lecture, the reader, past examinations, weekly tests) and a particular blending of the different resources was recommended by the lecturer. This would help students to understand the weekly coursework and to pass the final examinations. In addition, students had time to work together (in tutorials) and they also used human resources (e.g., peers, tutors, lecturers) during that time. In this course, the number of tutorials was balanced as compared to the number of lectures (4 + 4).

In contrast, the students on the CS course outlined several study paths based on their individual preferences and experiences and, for each path, different resources came into play and different core resources were described. For example, in the interviews based on their drawings, only two students put the lecture as a center point for their learning. For others, it appeared that the lecture was only for information on what students had to learn: "If I hear them talk about it, it's easier for me to revise/practice when I've already seen it, heard about it" (see student P's drawing in Figure 3).

At the same time, either the lecture or lecture notes were mentioned by all students as a supporting resource for their learning of CS, even if "mostly for orientation and overview" (see Figure 4). Interestingly, a large number of students pointed to human resources, particularly their friends and peers and the tutor, as resources they often used (see Figure 4).

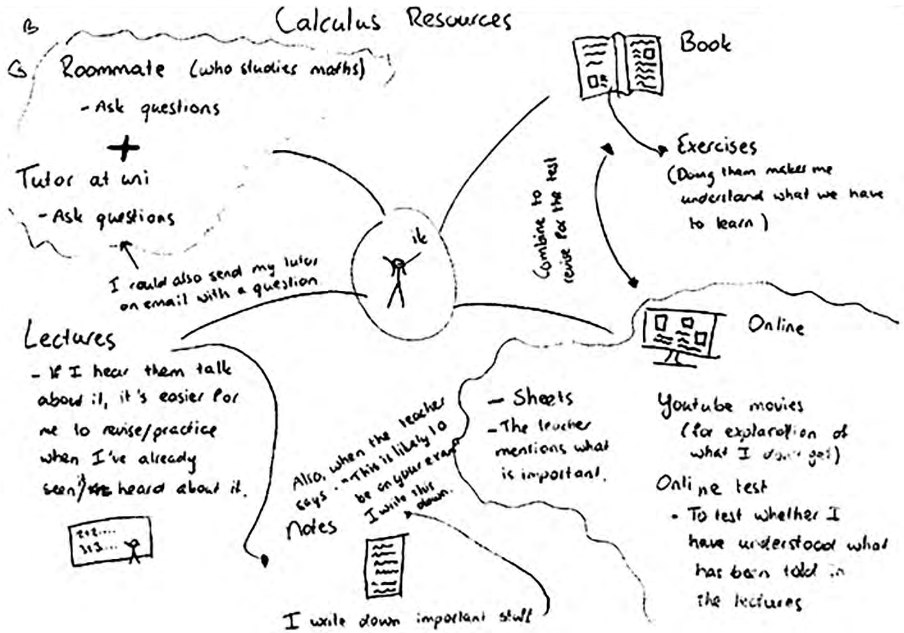


Figure 3. Student P's drawing of her resource system

As perhaps expected, books and tests/quizzes/exercises were mentioned as a huge help, and the digital resources (e.g., YouTube, Khan Academy) seemed to gain importance compared to high school. Altogether, the CS study paths showed a complex picture of students using a mixture and ever-increasing number of external resources, particularly of a human and digital nature. Due to the interviews based on the SRRSs, we could identify a small number of (for students) 'productive' study paths that students self-reported upon (Pepin & Kock, 2019).

Based on our results of this study (Pepin & Kock, 2019), we concluded that it was not sufficient to provide a plethora of curriculum resources – may they be digital, traditional text or human resources – but that serious consideration should be given to how students might work with these resources and orchestrate them into productive *Actual Student Study Paths*. In addition, it was advisable to help (perhaps even to train) students in regard to how to develop such study paths, and these might be different from one subject to another (even from one mathematics course to another). This should be the responsibility of the lecturer/teacher/course designer. Such course design would involve purposeful design, including the development of particular (intended) study paths and the design of particular

Figure 4. Student R's drawing of her resource system

resources supporting such paths. Simply providing access to curriculum resources would not seem to help students to orchestrate the resources on offer, neither to develop their individual study nor learning strategies, but might rather confuse and overwhelm them (due to the immensity of resources on offer) and drive them towards "learning for the test."

In a more recent study (Pepin & Kock, 2021), we compared students' identification and orchestration of resources in different courses, juxtaposing the two more traditional courses (LA and CS, first year) with a CBE course (third-year bachelor end project). The findings show that in the CBE course, students used resources outside the realm of curriculum resources offered to them in traditional courses. These included different 'pieces of knowledge', scientific papers, software, peers and experts in the field. In particular, social resources took on a special role, and the most prominent was the role of the coaches: they gave discipline-specific advice and helped students to refocus on the project aims when the students' ideas went in different directions in an attempt to find solutions to their individual problems.

The structure of the Calculus and Linear Algebra courses were of a different kind to the CBE course. (a) In the CS and LA courses, a number of curriculum resources were suggested by the course leaders and, in the case of the LA course, the learning path and the resources were well-defined and aligned. (b) In the CBE projects, the students were not given resources to pick from; there were no pre-defined resources and no pre-determined learning paths. The multi-disciplinary groups of students had to find their own ways of defining and solving the problems within the open challenges posed by the stakeholders, and it appeared that they very much depended on guidance by social resources (e.g., tutors). From the interviews, the SRRSs and the student reports, it became evident that students also reflected on and used their previously learned (mathematical) knowledge and experiences, within or outside the university, in order to solve the problem at hand (e.g., on data processing in the CBE project). In terms of Actual Student Study Paths, they appeared typically iterative or cyclical: focused on the common project goal, 'diverging' when working in the group and 'converging' with the help of the tutor providing focused advice (see student H's drawing below).

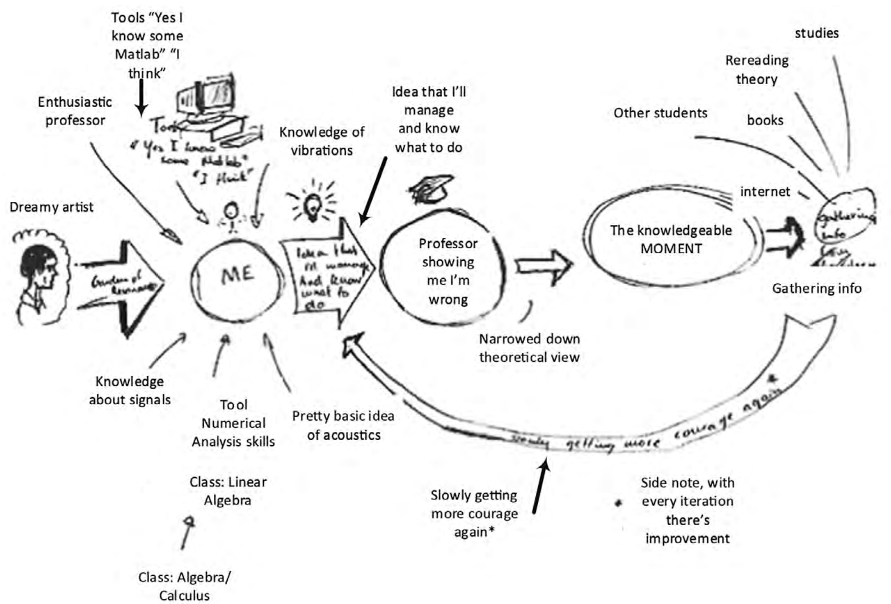


Figure 5. Student H's drawing of his resource system

In our university, CBE has become an important innovative approach to student-centered engineering education. As some people have doubts about the appropriateness of CBE for disciplines such as mathematics, in a recent study we have investigated student learning experiences in three mathematics-oriented CBE courses and how the different resources that the students reported to have used were related to their work in the CBE environments. Students in all three courses reported disciplinary learning in mathematics, but with differences amongst students according to their role in the team. Examples of student self-reported learning included the following:

- Learning to deal with the complexity of the real world and its translation into mathematical terms.
- Learning to choose between modeling approaches.
- Learning new concepts and techniques.
- Improving the skills to implement a model in the form of a computer program.

Some students perceived their work as "only applying" their (previously learnt) mathematical knowledge (Kock, Salinas-Hernandez & Pepin, 2023).

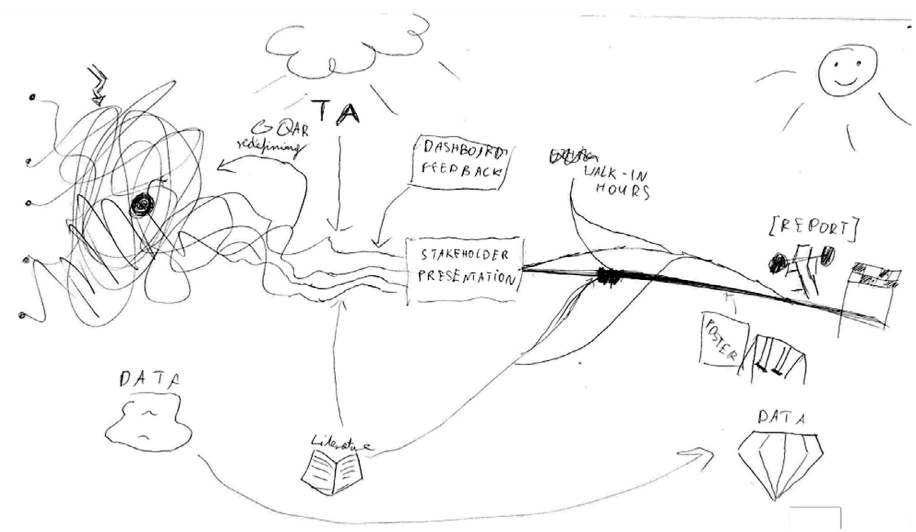


Figure 6. Student J's drawing of his path in a CBE course

Most students appreciated the combination of disciplinary and professional learning (e.g., solving real-life problems; better communication and collaboration) as it "brought things together" in the sense that the things they had learned earlier "were actually useful" and "are getting us somewhere." In that respect, the student learning experiences also contributed to students' self-confidence in their discipline. The tutors and stakeholders (social resources) appeared essential to the success of the student groups, in line with what was found by Pepin and Kock (2021). However, peers could also be social resources, as selected students introduced new mathematical techniques to their groups (which they had learnt previously in other courses). In two of the three courses, particular digital curriculum resources (e.g., Dashboard) became important as they supported a structured way of working and providing feedback – they helped students to formulate promising questions and problems and to keep making progress with their tasks. Interactions with stakeholders/problem owners helped to steer the students in successful directions and increased their sense of working on a meaningful real-life project (Kock, et al., 2023).

In our review, we identified four factors that are likely to influence students' choice and use of resources: (1) the availability and nature of resources (which would include e.g., the structure and visual appearance of resources or the nature of the tasks implemented in digital resources); (2) the nature and structure of the course (including e.g., the nature of problems as part of the course) and its associated pedagogical approaches (e.g., teacher mediation of the use of resources); (3) institutional framing (e.g., emphasis, or not, on innovative student-centered teaching and learning approaches); and (4) student beliefs (e.g., about the nature of mathematics and its learning) and goals. These factors could serve as hypothetical variables to be systematically implemented in learning environments to foster students' agency in selecting and using resources.

In pedagogical terms, considering our literature survey and the four factors named above, we (Pepin, Kock & Rezat, 2023) have proposed a number of characteristics of learning environments that appear beneficial for students developing and exercising agency – we called such learning environments '**agentic**': (1) student-centeredness; (2) active engagement of students; (3) authenticity of tasks/problems; (4) forms of working that foster dialogue/communication amongst students; and (5) the nature of the DCR itself that fosters student agency. These, we have contended, are learning environments where students are provided with opportunities to exercise and develop agency. Under these circumstances, we now understand the term 'agency' as 'distributed agency' – distributed over the different 'agents' involved (in the activity): e.g., students, teachers, (digital) resources (referred to/involved in the learning environment) or the mathematics.

Moreover, in relation to the instrumentation-instrumentalization process (explained earlier), we can consider '**resource agency**', which we think is related to the student – environment affordances. As outlined earlier, the resource's affordances (and constraints) are likely to have an (unintended) influence on practice (of the student as well as the teacher). Hence, together with student (and teacher) agency, we consider 'resource agency' (and its influence on practice), which might be underestimated. Referring to the Vygotskian mediational triangle (subject/tool/object), Cole (1998, p. 119) claims that "the incorporation of tools into the activity creates a new structural relation in which the cultural (mediated) and the natural (unmediated) routes operate synergistically." We interpret this as a case of distributed agency (and mediation in activity); we have argued that more explicit attention should be paid to the 'distribution of agency' over the different agents, particularly the 'resource agency'. How exactly the different agents exercise their agency and how distributions of agency can be optimized to learn mathematics in different educational situations have not been fully explored yet and are topics for further research.

We realize that, at this moment, new resources (e.g., ChatGPT) are also providing opportunities for new visions of student use of resources when learning mathematics. For example, during our recent fieldwork, some students talked about using ChatGPT ("a chatbot based on Artificial Intelligence technology which has been trained to provide a detailed textual response to a user prompt or question" – OpenAI, 2022). The students said that they had started using ChatGPT in a playful way and then continued more seriously for their project. After they had experienced its usefulness in one situation, it was "nice to like see that it could be useful for other stuff." The students did not mindlessly accept ChatGPT's results; they were aware of the errors made by the software and developed a critical attitude. We conjecture that the students' involvement in a Challenge-Based Learning environment has contributed to their critical attitude towards ChatGPT, as they needed results that really worked for their project.

Of course, we were immediately interested in which ways ChatGPT could help our engineering students, as it may contribute to their cognitive development. It appeared that when using the tool, the students gradually developed different utilization schemes. As it is so new, not much is known yet about the development of student utilization schemes regarding ChatGPT in specific situations. We hypothesized that an understanding of the ways in which students use this tool may provide insights into the educational potential of AI technologies such as ChatGPT. This is a line of research that I believe would be worth following, as the nature of this resource is so different to many of the other resources we have investigated.

Theoretically, the line of thinking ('distribution of agency') extends previous work on 'resources' and 'agency' in the sense that agency is now considered to be 'distributed' across the learning environment, lying with the different 'ingredients' of the learning environment: the teacher, the student(s) and the resource(s). It also provides a way to investigate how students appropriate and adapt the resources they use for and in their learning in a larger theoretical context. The focus on the processes of selection and use of digital resources allows the identification of which affordances, constraints and knowledge guide the decisions that students make, individually and collectively, about which resources to use and when to use them.

Another important dimension for student use of resources is the 'institutional dimension' that regulates the availability of resources and their social uses and that relates to other aspects, such as the beliefs and goals of the students and of their teachers and the university at large. This exhibits the complexity of mathematics students' work involving personal orientations, feelings, identity, beliefs and responsibilities. We contend that this is an important and necessary dimension to understanding students' work in mathematics education. This organic view of students' uses of resources also highlights the social nature of resources and the mathematical work itself. Further research into this relationship could deepen our insights into more and less successful student strategies of exercising agency.

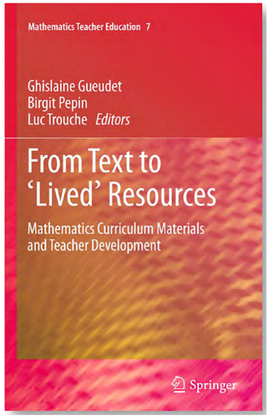
Conclusions

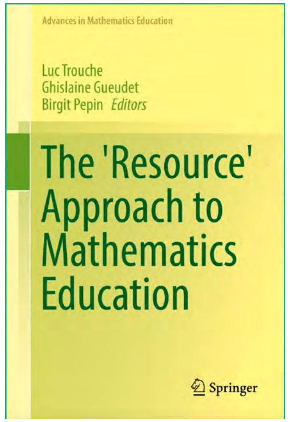
Summarizing and looking back over my work, I consider two aspects to have influenced me considerably. First, the international/cross-national character of my work resulted in a 'comparative mind'. It has been argued (e.g., Stigler & Perry, 1988; Pepin, 2000) that by comparing, one can identity similarities and differences, which helps researchers and educators to step aside and take a distance from the learning environment that they 'know', from the norms and expectations that are widely shared in one culture and that have been passed on from one generation to the next – this makes them nearly invisible to members within one culture. When we see something different and compare it with what we know, these accepted (and sometimes unquestioned) cultural models are revealed. In turn, this leads researchers and educators to a more explicit understanding of their own implicit theories about how children learn mathematics. For me, this comparative mind has helped to understand why in some educational environments, students are offered different opportunities to learn mathematics than in others and are supported by different resources (e.g., textbooks).

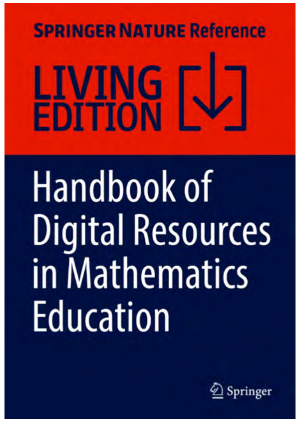
Second, the belief that in order to study such delicate and complex notions as mathematics learning and teaching, one needs a solid 'intermediate', something that is robust and at the same time flexible, leaning towards how both students and teachers work with it as well as towards the mathematics itself. I found this 'intermediate', or 'boundary object' as some would call it, in (digital) curriculum resources, hence the 'lens of resources'.

Our lines of thinking are evidenced in journal articles and special issues (that I have referred to in the course of this text) and in the three books we published:

- "From text to 'lived' resources" (2012)
- "The resource approach to mathematics education" (2018)
- "Handbook of digital resources in mathematics education" (2023)







As I have laid out, the nature of curriculum resources has changed over the years of my work and with it the possibilities (and constraints) of their use by teachers and students and the theories interpreting student-resource and teacher-resource interactions. Empirical studies have provided evidence that new societal and engineering challenges call for new learning and teaching approaches and for new tools to 're-source' the new teaching and learning environments.

More generally, I argue that the focus on students' use of resources helped us to develop a deeper understanding of the support that students need when the curriculum changes from a teacher-centered to a student-centered one. When the CBE education approach becomes more widely enacted at the university, a next step in our research should be to investigate student learning trajectories and to measure learning outcomes and experiences in CBE education experiments with larger numbers of students.

The study of the different courses, particularly the three recent ones, shows that different ways to enact CBE are possible in which mathematics (particularly mathematical modeling) plays an important role and in which students can develop competences relevant to their professional lives. The courses give rise to learning experiences related to similar themes but with different emphases. In CBE, student groups have considerable autonomy to follow their own approach and questions remain, for example, in which ways first-year courses (e.g., in LA or CS) could be taught with a CBE approach. In order to foster student development and success, the designers of these courses face important decisions, for example on:

- the competences (e.g., mathematical and professional) they want to emphasize,
- how to formulate the challenges (together with the problem owners);
- how to create opportunities for stakeholder interaction;
- how to provide efficient tutor feedback (particularly in large courses);
- whether to provide curriculum resources, data and initial models that can guide student thinking.

Moreover, policymakers need to support the design of CBE courses by providing time and professional development resources for course leaders concerning how to design the study/learning paths that include Challenge-Based Learning approaches and the design of particular resources supporting the intended paths. Simply providing access to curriculum resources is not likely to result in coherent learning trajectories, developing neither students' individual studying nor their learning strategies, nor does it change the mindsets of course leaders to provide a student-centered curriculum. As a team, we have provided results from our research and shown examples of how such decisions play out in practice.

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Curriculum Vitae

Prof.dr. Birgit Pepin was appointed as full professor of Mathematics/STEM Education at the Eindhoven School of Education (ESoE) of Eindhoven University of Technology (TU/e) on January 1, 2015.

Birgit Pepin (1956) received her MSc in Physics/Meteorology in Kiel, Germany. She subsequently moved from Germany to France and then to England, where she was educated as a teacher of mathematics at Oxford University (1990). After some years of teaching mathematics in secondary schools, she received a scholarship (ESRC) to study mathematics teacher mobility across Europe for her PhD (1998). During her subsequent career in higher education, she worked as an educator of mathematics teachers in several UK institutions (e.g. Manchester University) before taking up her first full professorship in Trondheim, Norway, in 2009. In January 2015, she joined TU/e as full professor of Mathematics/STEM education at ESoE. Birgit has published widely in prestigious mathematics education journals and handbooks and has served on (and chaired) many national and international projects and boards in mathematics education. Internationally, she has been a visiting professor at the ENS in Lyon, France (three times), and ECNU in Shanghai, China (twice), and she has been and still is visiting professor of Mathematics Education at NTNU in Trondheim, Norway.

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Bart van Overbeeke Photography, Eindhoven

Design
Grefo Prepress, Eindhoven

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