

# Identification of Continuous-Time Systems Utilizing Lebesgue-Sampled Data

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# Identification of Continuous-Time Systems Utilizing Lebesgue-Sampled Data

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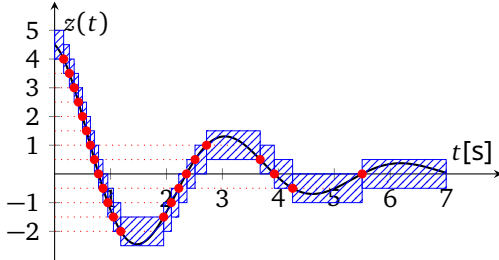
ERNSI 2022



## Lebesgue sampling

**Key idea:** sample only when a signal crosses fixed thresholds

- Reduces data-rate requirements
- Analysis and control design are more complicated
- To avoid spurious sampling, send-on-delta strategy can be used



## Problem formulation

Continuous-time SISO system in state-space form:

$$\dot{x}(t) = Ax(t)dt + Bu(t)dt + dw(t)$$

$$z(t) = Cx(t) + Du(t)$$

- $w(t)$ : Wiener process of incremental covariance  $Q$
- $u(t)$ : known input signal
- $A, B, C, D$  and  $Q$ : unknown constant matrices

Assume  $z(t)$  is sampled as above, with time-stamps  $\{t_l\}_{l=1}^M$  and threshold distance  $h$

How can we estimate  $A, B, C, D$  and  $Q$  using  $\{u(t)\}_{t \in [t_1, t_M]}$  and  $\{z(t_l)\}_{l=1}^M$  and the intersample behavior knowledge?

## EM for Lebesgue-sampled data

**Approach:** small sampling period  $\Delta$  + Delta domain. Relabeling  $k = 1$  to  $k = N := \lfloor \frac{t_M}{\Delta} \rfloor - \lfloor \frac{t_1}{\Delta} \rfloor + 1$  and discretizing:

$$dx_k^+ = \Delta A_{in} x_k + \Delta B_{in} u_k + dw_k^+$$

$$z_k = Cx_k + Du_k + \epsilon_k \leftarrow \text{added to ensure full rank matrices}$$

$$y_k = \mathcal{Q}_h\{z_k\} \leftarrow \text{set-valued function describing Lebesgue sampling}$$

with  $A_{in} = (e^{A\Delta} - I)/\Delta$ ,  $B_{in} = \Delta^{-1} \int_0^\Delta e^{A\tau} B d\tau$ , and noise vector covariance  $\Delta Q_{in} \delta_{k-l}^K$ , where  $Q_{in} = \Delta^{-1} \int_0^\Delta e^{A\tau} Q e^{A^T \tau} d\tau$ .

Note that  $(A_{in}, B_{in}, Q_{in}) \approx (A, B, Q)$  for small  $\Delta$   
 $\implies$  **derive EM for fast-sampled Delta domain equivalent!**  
 Parameter estimates  $A, B, C, D, Q$  described in  $\theta$

## E-step: Q function

$$-Q(\theta, \hat{\theta}_i) = -\mathbb{E} \left\{ \log p(x_{1:N+1}, z_{1:N} | y_{1:N}, \hat{\theta}_i) \right\} \\ \sim L_0(\hat{\theta}_i) + N \log \det(Q_{in})$$

$$+ \frac{1}{\epsilon^2} \left( \begin{bmatrix} C^T \\ D \end{bmatrix}^T \begin{bmatrix} \Gamma_{xx} & \Gamma_{ux} \\ \Gamma_{ux} & \Gamma_{uu} \end{bmatrix} \begin{bmatrix} C^T \\ D \end{bmatrix} - 2 \begin{bmatrix} C^T \\ D \end{bmatrix}^T \begin{bmatrix} \Gamma_{xz} \\ \Gamma_{uz} \end{bmatrix} \right)$$

$$+ \text{tr} \left\{ Q_{in}^{-1} \left( \begin{bmatrix} A_{in}^T \\ B_{in}^T \end{bmatrix}^T \begin{bmatrix} \Gamma_{xx} & \Gamma_{ux} \\ \Gamma_{ux} & \Gamma_{uu} \end{bmatrix} \begin{bmatrix} A_{in}^T \\ B_{in}^T \end{bmatrix} - \begin{bmatrix} A_{in}^T \\ B_{in}^T \end{bmatrix}^T \begin{bmatrix} \Gamma_{x\delta} \\ \Gamma_{u\delta} \end{bmatrix} - \begin{bmatrix} \Gamma_{x\delta} \\ \Gamma_{u\delta} \end{bmatrix} \begin{bmatrix} A_{in}^T \\ B_{in}^T \end{bmatrix} \right) \right\}$$

where  $\Gamma$  matrices are conditional expectations w.r.t.  $y_{1:N}, \hat{\theta}_i$ .

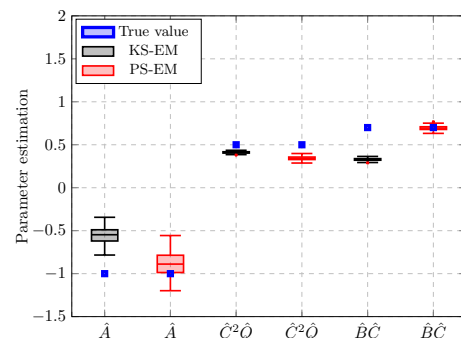
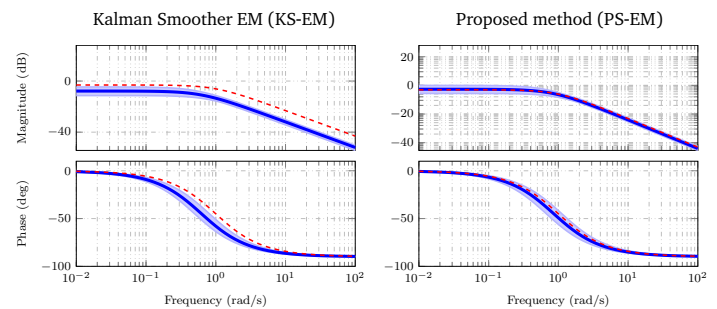
**M-step:** EM iterations  $\hat{\theta}_{i+1} = \arg \max_{\theta} Q(\theta, \hat{\theta}_i)$  are given by

$$\begin{bmatrix} A_{i+1} & B_{i+1} \\ C_{i+1} & D_{i+1} \end{bmatrix} = \begin{bmatrix} \Gamma_{x\delta} & \Gamma_{xz} \\ \Gamma_{u\delta} & \Gamma_{uz} \end{bmatrix}^T \begin{bmatrix} \Gamma_{xx} & \Gamma_{ux} \\ \Gamma_{ux} & \Gamma_{uu} \end{bmatrix}^{-1}$$

$$Q_{i+1} = \frac{\Delta}{N} \left( \Gamma_{\delta\delta} - \begin{bmatrix} \Gamma_{x\delta} \\ \Gamma_{u\delta} \end{bmatrix}^T \begin{bmatrix} \Gamma_{xx} & \Gamma_{ux} \\ \Gamma_{ux} & \Gamma_{uu} \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_{x\delta} \\ \Gamma_{u\delta} \end{bmatrix} \right)$$

How to compute  $\Gamma$ ? Particle filtering/smoothing (or Gaussian Sum filtering/smoothing)

## Simulation studies



## References

[1] R. A. González, A. L. Cedeño, M. Coronel, J. C. Agüero and C. R. Rojas. "Identification of Continuous-time State-Space Systems Utilizing Lebesgue-Sampled Data". In preparation.