

Non-parametric Continuous-time System Identification with Lebesgue-sampled Output Measurements

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Non-parametric Continuous-time System Identification with Lebesgue-sampled Output Measurements

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1 Background

Event-based sampling schemes in control design can lead to improvements in overall performance and resource efficiency [1], with applications in, e.g., network control and incremental encoders [2]. One of the most popular event-based sampling methods is Lebesgue sampling, which consists of sampling the continuous-time signal whenever it crosses fixed thresholds in the amplitude domain.

The problem that is addressed in this work is the estimation of non-parametric continuous-time models from Lebesgue-sampled output data. To this end, we seek an estimator that can 1) provide a *continuous-time* impulse response estimate from possibly noisy and short data records, and 2) exploit the entirety of the information contained in the irregular sampling instants and the bounded intersample behavior.

2 Problem formulation

Consider a linear and time-invariant, continuous-time system G (with impulse response $g(t)$), excited by a known input $u(t)$. The noisy output $z(t)$ is Lebesgue-sampled, as in Figure 1. In practice, $z(t)$ is fast-sampled every Δ [s] and samples are retrieved only when $z(i\Delta)$ crosses a threshold level. Thus, we have access to $y(i\Delta) = [\eta_i, \eta_i + h]$, where η_i is the lower threshold at time $i\Delta$ and h is the threshold amplitude. This setup is presented in Figure 2.

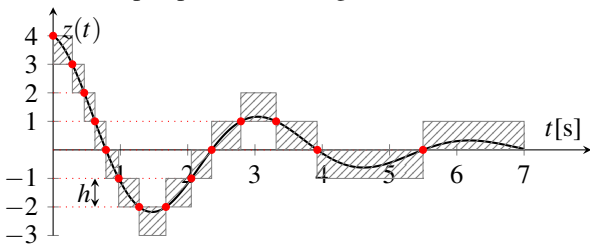


Figure 1: Lebesgue sampling of $z(t)$, with threshold amplitude $h=1$.

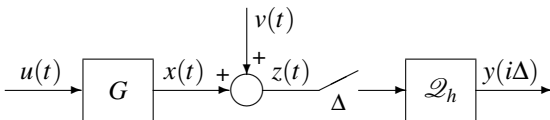


Figure 2: Block diagram of the Lebesgue-sampling scheme. The \mathcal{Q}_h block delivers a set-valued signal y .

The goal is to derive an identification method that estimates $g(t)$ using $\{u(t)\}_{t \in [0, \Delta N]}$ and $\mathbf{y}_{1:N} = \{y(i\Delta)\}_{i=1}^N$.

3 Approach

By adopting a kernel-based approach, the goal is to find a regularized estimator of the form

$$\hat{g} = \arg \min_{g \in \mathcal{G}} (-\log p(\mathbf{y}_{1:N}|g) + \gamma \|g\|_{\mathcal{G}}^2),$$

where $p(\mathbf{y}_{1:N}|g)$ is the likelihood function, and \mathcal{G} is a reproducing kernel Hilbert space (RKHS). The impulse response \hat{g} can be obtained via a *finite-dimensional* optimization problem thanks to the Representer Theorem. The finite-dimensional problem that is derived is solved with MAP-EM (MAP Expectation-Maximization), which provides closed-form iterations that are shown to converge to the global optimum if adequately initialized.

Any kernel-based estimator will depend on the choice of hyperparameters. These are computed with Empirical Bayes in this work, from which MAP-EM iterations are also derived. This time the iterations require computing the second moment of a high-dimensional truncated Gaussian. Such matrix can be estimated with Monte Carlo sampling methods.

4 Results

We consider $G(s) = 1/(0.05s^2 + 0.2s + 1)$. The proposed approach exploits the output intersample knowledge to deliver more accurate models than the standard kernel (Riemann) approach [3], for all values of h .

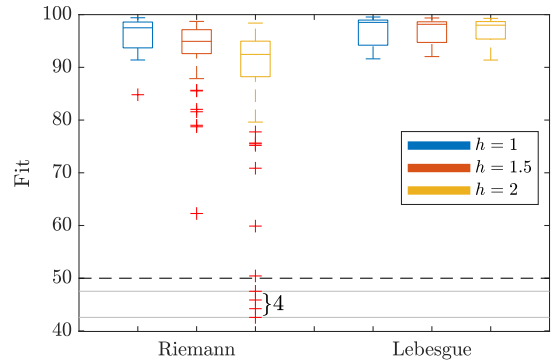


Figure 3: Fit boxplots for the standard Riemann approach (left), and the proposed method (Lebesgue, right) for different values of threshold amplitude h .

Acknowledgments

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