Non-parametric Continuous-time System Identification with Lebesgue-sampled Output Measurements

Document status and date:
Published: 21/03/2023

Document Version:
Accepted manuscript including changes made at the peer-review stage

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Download date: 04. Oct. 2023

Non-parametric Continuous-time System Identification with Lebesgue-sampled Output Measurements

Rodrigo A. González
TU/e
r.a.gonzalez@tue.nl

Koen Tiels
TU/e
k.tiels@tue.nl

Tom Oomen
TU/e and TUD
t.a.e.oomen@tue.nl

1 Background

Event-based sampling schemes in control design can lead to improvements in overall performance and resource efficiency [1], with applications in, e.g., network control and incremental encoders [2]. One of the most popular event-based sampling methods is Lebesgue sampling, which consists of sampling the continuous-time signal whenever it crosses fixed thresholds in the amplitude domain.

The problem that is addressed in this work is the estimation of non-parametric continuous-time models from Lebesgue-sampled output data. To this end, we seek an estimator that can 1) provide a *continuous-time* impulse response estimate from possibly noisy and short data records, and 2) exploit the entirety of the information contained in the irregular sampling instants and the bounded intersample behavior.

2 Problem formulation

Consider a linear and time-invariant, continuous-time system G (with impulse response $g(t)$), excited by a known input $u(t)$. The noisy output $z(t)$ is Lebesgue-sampled, as in Figure 1. In practice, $z(t)$ is fastly-sampled every Δ [s] and samples are retrieved only when $z(i\Delta)$ crosses a threshold level. Thus, we have access to $y(i\Delta)=[\eta_i, \eta_i+h]$, where η_i is the lower threshold at time $i\Delta$ and h is the threshold amplitude. This setup is presented in Figure 2.
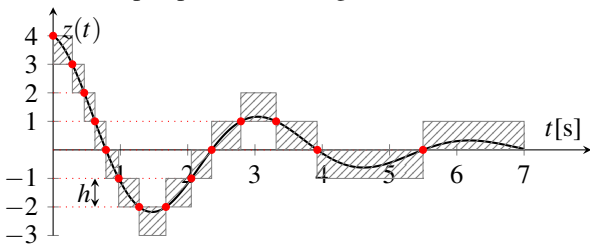

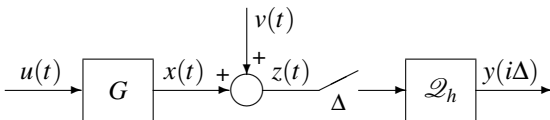**Figure 1:** Lebesgue sampling of $z(t)$, with threshold amplitude $h=1$.

Figure 2: Block diagram of the Lebesgue-sampling scheme. The $\mathcal{Q}_h$ block delivers a set-valued signal y.

The goal is to derive an identification method that estimates $g(t)$ using $\{u(t)\}_{t\in[0,\Delta N]}$ and $\mathbf{y}_{1:N} = \{y(i\Delta)\}_{i=1}^{N}$.

3 Approach

By adopting a kernel-based approach, the goal is to find a regularized estimator of the form

$$\hat{g} = \arg\min_{g\in\mathscr{G}} \left(-\log \mathrm{p}(\mathbf{y}_{1:N}|g) + \gamma\|g\|_{\mathscr{G}}^{2}\right),$$

where $\mathrm{p}(\mathbf{y}_{1:N}|g)$ is the likelihood function, and $\mathscr{G}$ is a reproducing kernel Hilbert space (RKHS). The impulse response $\hat{g}$ can be obtained via a *finite-dimensional* optimization problem thanks to the Representer Theorem. The finite-dimensional problem that is derived is solved with MAP-EM (MAP Expectation-Maximization), which provides closed-form iterations that are shown to converge to the global optimum if adequately initialized.

Any kernel-based estimator will depend on the choice of hyperparameters. These are computed with Empirical Bayes in this work, from which MAP-EM iterations are also derived. This time the iterations require computing the second moment of a high-dimensional truncated Gaussian. Such matrix can be estimated with Monte Carlo sampling methods.

4 Results

We consider $G(s) = 1/(0.05s^2 + 0.2s + 1)$. The proposed approach exploits the output intersample knowledge to deliver more accurate models than the standard kernel (Riemann) approach [3], for all values of h.
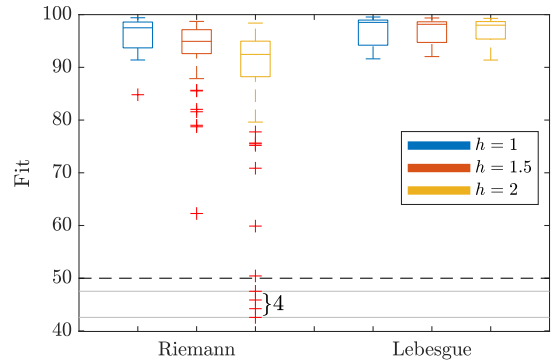

Figure 3: Fit boxplots for the standard Riemann approach (left), and the proposed method (Lebesgue, right) for different values of threshold amplitude h.

References

[1]   K. J. Åström, and B. Bernhardsson. Systems with Lebesgue sampling. In *Directions in Mathematical Systems Theory and Optimization*, pp. 1–13. Springer, 2003.

[2]   N. Strijbosch and T. Oomen. "Iterative learning control for intermittently sampled data: Monotonic convergence, design, and applications". *Automatica*, Article 110171, 2022.

[3]   G. Pillonetto and G. De Nicolao. "A new kernel-based approach for linear system identification". *Automatica*, 46(1), pp. 81–93, 2010.