Error Bounds in Model Reduction of Interconnected Systems

Error Bounds in Model Reduction of Interconnected Systems: A Robust Performance Approach

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Introduction

Complex models of dynamic (multi-)physical systems are often based on an interconnection of subsystems with a high number of states. For such systems, model order reduction (MOR) is required to make controller synthesis, simulation and analysis computationally feasible.

In *modular model reduction,* each of the subsystems models is reduced individually. This preserves the interconnection structure of the model and dividing the problem into multiple smaller problems avoids the computationally challenging reduction of one high-dimensional model. However, although modular reduction leads to accurate subsystem models, it does not guarantee the accuracy of the reduced-order interconnected model. The question arises:

"How can we relate subsystem reduction errors to the resulting error in the reduced interconnected model?"

A method is introduced which provides *a priori error bounds* on the frequency response of the reduced interconnected model given error bounds on reduced subsystem models [1].

Preliminaries

We combine k linear time-invariant subsystems $G_j, j = 1, \dots, k$ in the transfer function $G_b = \mathrm{diag}(G_1, \dots, G_k)$. Inputs u_b and outputs y_b of the subsystems are interconnected via matrix K. All subsystems are reduced to reduced-order subsystems given by $\hat{G}_b = \mathrm{diag}(\hat{G}_1, \dots, \hat{G}_k)$.

The high-order interconnected model G_c and reduced-order interconnected model $\hat{G}_c$ are then both given by a feedback of K with G_b and $\hat{G}_b$, respectively, external input u_c and output y_c, as shown in Fig. 1.

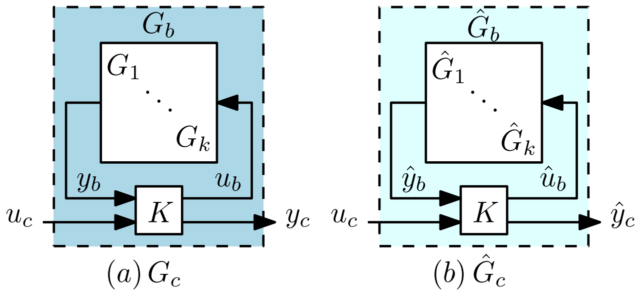

Fig. 1: Block diagram of a) high-order interconnected model G_c and b) reduced-order interconnected model $\hat{G}_c$

Methodology

In this work, we relate $E_j = G_j - \hat{G}_j$ to $E_c = G_c - \hat{G}_c$ by reformulating the problem such that the structured singular value μ, a tool from robust control [2], can be used.

We define a weighted uncertain subsystem for which $E_j \in W_j\Delta_j V_j$ such that $\hat{G}_j \in G_j + W_j\Delta_j V_j$. Then, we model E_c as a feedback of VNW and Δ as shown in Fig. 2. Nominal system N is a function of G_b and K [1]. $V = \mathrm{diag}(V_1, \dots, V_k, V_c)$, $W = \mathrm{diag}(W_1, \dots, W_k, W_c)$ and $\Delta = \mathrm{diag}(\Delta_1, \dots, \Delta_k, \Delta_c)$.

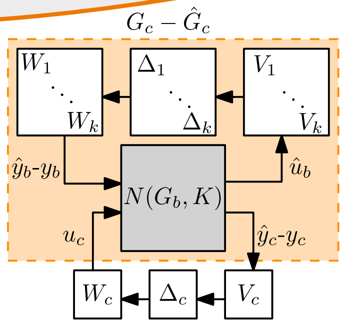

Fig. 2: MOR in a robust performance perspective

Results

From robust performance [2],

$$\sup_{\omega \in \mathbb{R}} \mu_\Delta(VNW) < 1,$$

if and only if the system in Fig. 2 is *internally stable* and

$$\|W_c E_c V_c\|_\infty < 1, \text{for all } E_j \text{ s.t. } \left\|W_j^{-1} E_j V_j^{-1}\right\|_\infty \leq 1.$$

Given that $\mu_\Delta(VNW) < 1$ can be verified computationally, this relation can be used to find a *global* error bound on the interconnected model based on subsystem errors. Similarly, a *frequency-dependent* bound can be computed (see [1]).

Example and conclusion

To illustrate the proposed framework, we apply it to the system illustrated in Fig. 3. In this example, G_1 has order 500 and is reduced using balanced truncation to $\hat{G}_1$ with order 80 (G_2 and G_3 are left unreduced).

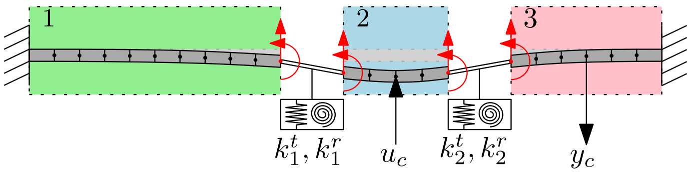

Fig. 3: Example system: Three interconnected beams

Using the proposed approach, an a priori global ε_c and frequency-dependent $\varepsilon_c(\omega)$ error bound is computed (see Fig. 4). For validation, the actual error $|E_c|$ is computed a posteriori. The example shows that the proposed method can be used to compute tight a priori error bounds for modular model reduction of interconnected systems.

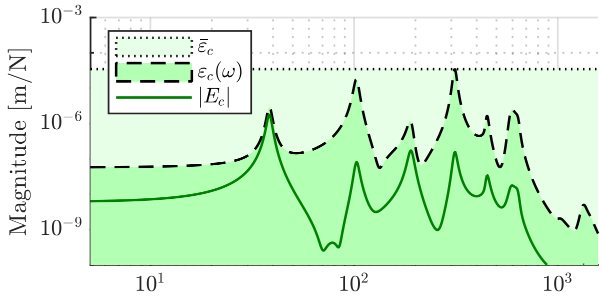

Fig. 4: Actual error $|E_c|$ and a priori global ε_c and frequency-dependent $\varepsilon_c(\omega)$ error bounds on $\hat{y}_c - y_c = E_c u_c$

References

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