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A priori error bounds for model reduction of interconnected systems

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1 Introduction

Many complex models of dynamic (multi-)physical systems are naturally based on an interconnection of subsystems. The models of interconnected systems often have a high number of states, which makes controller synthesis, simulation and analysis computationally expensive. This motivates the need for model order reduction (MOR) techniques applicable to such systems. *Subsystem reduction*, where each of the subsystems is reduced individually, is a completely modular approach. Hence, the interconnection structure of the high-order model is preserved and the computationally challenging reduction of one very high-dimensional model is avoided by dividing the problem into multiple smaller problems. However, although subsystem reduction methods will lead to accurate reduced-order representations of the high-order subsystems, this does not directly mean that the reduced-order interconnected system is also accurate. Based on [1], we introduce a method which gives an a priori error bound on the frequency response of the reduced interconnected system based on the a priori error bounds of the reduced subsystems.

2 Preliminaries

We study systems of k linear time-invariant (LTI) subsystems that can be aggregated and represented by block-diagonal transfer function $G = \mathrm{diag}(G_1,\ldots,G_k)$. The subsystems are interconnected via matrix K. The external inputs u and outputs y are connected to the subsystems via external input and output matrix L and M, respectively. In subsystem reduction, each of the subsystems G_j is reduced to an approximate reduced-order subsystem $\hat{G}_j$ and the aggregate is given by $\hat{G} = \mathrm{diag}(\hat{G}_1,\ldots,\hat{G}_k)$. The high-order interconnected system G_c and the reduced-order interconnected system $\hat{G}_c$ with $\hat{y} = \hat{G}_c u$ (Figure 1), are then

$$G_c = MG(I - KG)^{-1}L, \ \hat{G}_c = M\hat{G}(I - K\hat{G})^{-1}L. \quad (1)$$

3 Methodology

For balanced truncation, a widely used MOR method, an upper bound on the $\mathscr{H}_\infty$-norm of the difference between the high-order and reduced-order systems is available a priori.

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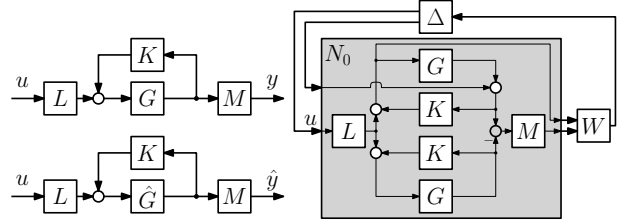
Figure 1: Block diagrams of G_c (top left), $\hat{G}_c$ (bottom left), and $G_c - \hat{G}_c$ represented by N_0, W and Δ (right).

In this work, we extend a priori knowledge of subsystem errors $\|G_j - \hat{G}_j\|_\infty$ to knowledge on $\|G_c - \hat{G}_c\|_\infty$. To do so, we define all subsystem reduction errors as an uncertain system $\Delta := G - \hat{G}$. Then, we can model $G_c - \hat{G}_c$ as a closed loop between N and Δ as shown in Figure 1. Here, the nominal system $N := N_0 W$ is a function of $N_0(G,K,L,M)$ and weighting matrix $W := \mathrm{diag}(w_1 I,\ldots,w_k I,w_c I)$. Using this framework, the structured singular value μ, a powerful tool from robust control, can be used. Namely, if N is internally stable, then

$$\left.\begin{array}{l} \|G_c - \hat{G}_c\|_\infty \le w_c^{-1}, \text{and} \\ \|G_j - \hat{G}_j\|_\infty \le w_j \ \forall \ j = 1,\ldots,k \end{array}\right\} \Leftrightarrow \mu_\Delta(N) < 1. \quad (2)$$

Given that an error bound w_j is available a priori for each subsystem, this relation can be used to find an error bound w_c^{-1} on the interconnected system. A less conservative frequency-dependent version of (2) can be used at the cost of increased computation time. In [1], these bounds are illustrated using an example.

4 Conclusion

With (2), we can directly compute the impact of errors in subsystems to the error of the interconnected system. This result is also the first step towards determining a model accuracy specification for the individual subsystems guaranteeing a required overall system accuracy. Therefore, the method can be used as a significant means for the development of accurate modeling of interconnected systems within a completely modular setting.

References

[1] Janssen, L.A.L. et al., "A Priori Error Bounds for Model Reduction of Interconnected Linear Systems using Robust Performance Analysis", ACC 2022 (accepted for publication).