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Noise-induced pulse-timing statistics in an integrated two-section semiconductor laser with saturable absorber

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We have analyzed and explained the generation of irregularly timed, spontaneous-emission triggered optical pulses from a two-section semiconductor laser with saturable absorber, operating near threshold in a regime of excitability. Here we focus on the statistics of the spontaneously emitted pulses. The numerical simulations and analytical theory are based on the Yamada model. The observed irregular pulse train intervals exhibit an initial refractory time, followed by a time interval until the next emitted pulse. The latter is analyzed in terms of a first-passage-time distribution for the intensity to diffuse from its equilibrium value to hit a larger threshold intensity for the first time. Analytic asymptotic short-time and long-time approximations have been derived.

Introduction

For decades, the human brain has been an inspiration to develop artificial neural networks, which consist of a highly interconnected network of neurons arranged in layers. After training the network with a specific dataset, a neural network is used to perform classification of input data. At the heart of such a network is the neuron itself, which effectively performs a non-linear operation. It was previously shown that from a dynamical perspective, a two-section semiconductor laser shows similar dynamics and spiking capabilities as biological neurons when operated close to its threshold [1]–[3]. Consequently, an integrated semiconductor laser may be a suitable candidate for a spiking neural network (SNN).

The dynamics are the result of the intracavity loss element next the gain element in an optical resonator, of which the recovery rate is different compared to the gain section recovery rate. When the laser is operated close the its threshold, it may be triggered by an external optical trigger, which means the laser is excitable. At this operation point, spontaneous emission is relatively high, and thus optical noise is present. The effect of optical noise on the spiking behavior of an excitable two-section laser was studied before in terms of jitter and amplitude deviations [4]. Recently, analytical expressions to describe the time distributions were derived based on the initial relative refractory period and a first passage time (FPT) principle [5]. In this paper, we present simulation results when the level of absorption in an excitable two-section laser is changed using a modified and normalized Yamada model, which includes a term for optical noise injection. We also present how the timing distributions change due to different absorption values and fit the asymptotic approximations based on previously developed theory [5].

Normalized Yamada Rate Equations with Noise Injection

The structure under investigation is a two-section laser, consisting of a gain and saturable absorber section in an optical cavity, as shown schematically in Figure 1.

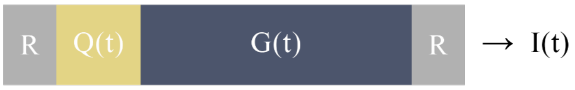

Figure 1: Schematic overview of the two section laser structure model. The optical cavity is formed by the two mirrors R, a loss sections Q, and gain section G. The optical output is marked with I.

Under the assumption the optical intensity is uniform along the longitudinal axis, the loss and gain dynamics as well as the optical intensity can be modeled using the normalized Yamada model, which comprises a set of coupled differential equations for the gain G, loss Q and intensity I [3]:

$$\dot{G} = \gamma_G \left[A - G(t) - G(t)I(t)\right] \tag{1}$$

$$\dot{Q} = \gamma_Q\left[B - Q(t) - aQ(t)I(t)\right] \tag{2}$$

$$\dot{I} = \gamma_I\left[G(t) - Q(t) - 1\right]I(t) + \epsilon f(G) + \theta(t) + F_I(t) \tag{3}$$

with gain A, absorption B, recovery rates γ_G, γ_Q, and $\gamma_I$for the gain, loss and intensity equations respectively, differential absorption relative to the differential gain a, spontaneous emission $\epsilon f(G)$, and optical injection $\theta(t)$. The influence of noise on the excitable responses is modeled using the white-noise fluctuating delta-correlated Langevin noise term $F_I(t)$. This term satisfies the following conditions for the first and second moments [6]:

$$\langle F_I(t)\rangle = 0 \tag{4}$$

$$\langle F_I(t)F_I(u)\rangle = 2D_{II}\delta(t-u) \tag{5}$$

with $D_{II}$the intensity diffusion coefficient. To find the numerical solution to the set of rate equations, on every integration step a random number is drawn from a Gaussian distribution with $\mu = 0$ and $\sigma^2 = 2R_S I\Delta t$. Here, R_S is the average spontaneous emission rate, and Δt the fixed numerical integration timestep.

In Figure 2 and Figure 3, two simulated timetraces in units of photon lifetimes τ_P for different values of B are shown. Depending on the absorption, the density of intensity pulses changes. From visual inspection it is clear that the time between consecutive pulses (indicated by the inset in Figure 3) seems more consistent at a low level of absorption.

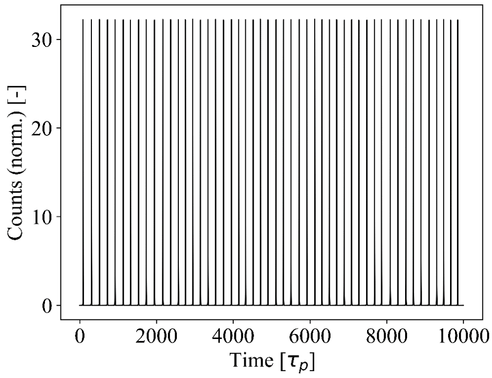

Figure 2: Simulated timetrace using B=3.6540, A=4.5, γ_G=0.05, γ_Q=0.1, γ_I=1, a=5.

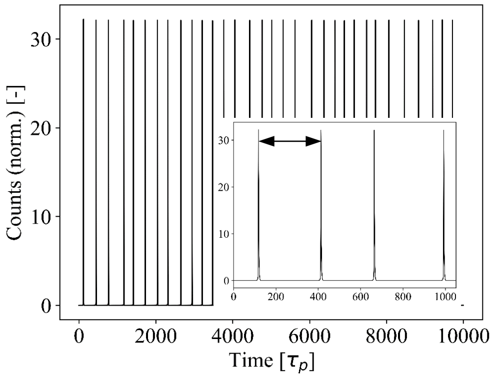

Figure 3: Simulated timetrace using B=3.6740, A=4.5, γ_G=0.05, γ_Q=0.1, γ_I=1, a=5. Inset: zoomed in timetrace indicating the time between consecutive pulses.

Timing Statistics

Next, the timing statistics are further investigated. Figure 4 shows the time distributions for eight different levels of absorption, ranging from *B=3.6500* (low) to *B=3.6780* (high). The distributions are obtained by simulating 170 timetraces while registering the time between consecutive pulses. In all cases, the distributions are asymmetric and positively skewed, which means the distributions are characterized by a sharp onset followed by a long tail. For low absorption (in blue) the distribution is relatively narrow, whereas for high absorption (in yellow) the distribution is spread out in time. The dead-time before the sharp onset occurs is related to the relative refractory period, which is the time needed for the gain and absorption to recover to a steady state, and the first passage time, which is a stochastic quantity. The latter follows a first-passage-time-distribution (FPTD) [5] and from Figure 4 it follows that this quantity is shortest when the absorption is low.

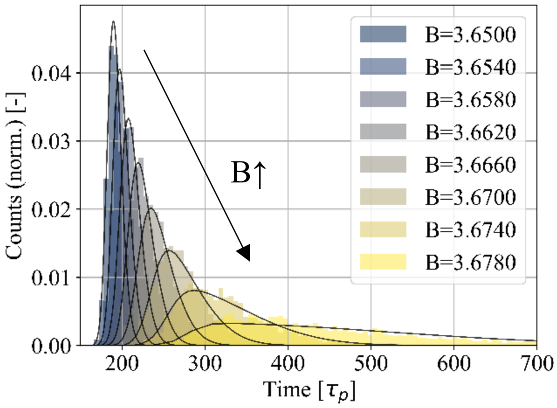

Figure 4: Simulated histograms for 8 values of B, A=4.5, γ_G=0.05, γ_Q=0.1, γ_I=1, a=5, and corresponding fits. The number of samples for every histogram is 8699, 8336, 7869, 7383, 6799, 6056, 5051, and 3497, respectively.

The sharp onset and long tail of the simulated histograms in Figure 4 can be expressed by two analytical approximations, which are the short-time Eq. (6) and large-time approximation Eq. (7), and are given by [5]:

$$P_{first}(T; I_{\mathrm{i}}, I_C) = \frac{(I_C - I_{\mathrm{i}})}{\sqrt{4\pi\mathcal{D}T^3}} e^{-\frac{(I_C - I_{\mathrm{i}})^2}{4\mathcal{D}T}} \tag{6}$$

$$P_{first}(T; I_0, I_C) = e^{-\left(1+\frac{T}{T_{I_0,I_C}}\right)} \frac{I_{Bessel,1}\left(2\sqrt{\frac{T}{T_{I_0,I_C}}}\right)}{\sqrt{T_{I_0,I_C}\, T}} \tag{7}$$

with excitability threshold value I_C, initial intensity I_i, diffusion coefficient $\mathcal{D} = 2R_s I_0$, time T, the characteristic time related to the first passage from I_0 to I_C, T_{I_0,I_C}, and the modified Bessel function of the first kind $I_{Bessel,1}$.

For two of the cases shown in Figure 4 the short-time and large-time approximations are fitted to the simulated histograms, which are shown in Figure 5 and Figure 6 using the dashed and dash-dotted lines, respectively. The simulation parameters are the same as mentioned before.

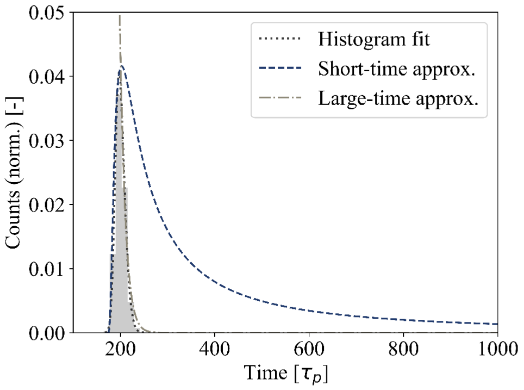

Figure 5: Simulated histogram using B=3.6540 and fitting parameters $I_C=13.0$*,* $I_0=1.8$*,* $R_s=0.18$*, and* $T_{I0,IC}=8.0$*.*

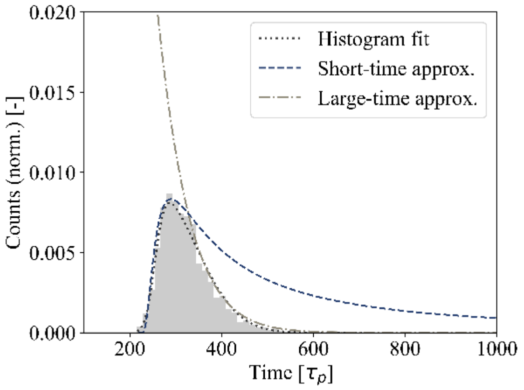

Figure 6: Simulated histogram using B=3.6740 and fitting parameters $I_C=18.6$*,* $I_0=1.8$*,* $R_s=0.18$*, and* $T_{I0,IC}=41.0$*.*

In both cases, the short-time and large-time approximations fit the simulated histogram accurately, indicating the asymptotic models accurately describe the noised induced spiking distributions.

Conclusion

A two-section laser, or integrated optical neuron operated in the excitable regime, close to its lasing threshold, can be triggered by noise to generate an optical pulse. We have shown using the normalized Yamada model with an optical noise term that the pulse density and timing statistics are a function of the level of absorption. Asymptotic analytical models, based on the initial relative refractory period and a first passage time (FPT) principle, accurately describe the short-time and large-time behavior of the simulated timing histograms.

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