

# Inaccuracies in the inverse heat conduction problem solution and their effect on the estimation of heat fluxes during quenching

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# Inaccuracies in the inverse heat conduction problem solution and their effect on the estimation of heat fluxes during quenching

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## ABSTRACT

Solving the Inverse Heat Conduction Problem (IHCP) is a common approach to estimate the surface heat fluxes involved in transient, quenching experiments. The inverse problem presents several challenges, such as accounting for the non-uniqueness of its solution, for the effects of noise, or for other practical issues affecting the experimental input data, such as the thermal contact of the internal temperature sensors. In this paper, possible sources of inaccuracy in the IHCP solution and their effect on the estimated surface heat flux in experiments on quenching by water jet impingement are systematically investigated. A “virtual experiment” approach is followed to analyze the effect of a noise cancelling technique, the ambiguity in initial conditions and the quality of the thermocouple contact on the accuracy of the heat flux estimations. The results show that the invalid assumption of perfect thermal contact between thermocouple and test plate leads to overestimation of surface temperature in the initial stages of quenching. Based on these results, two measures are proposed to avoid misinterpretation of quenching heat flux estimations when solving the IHCP.

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## 1. Introduction

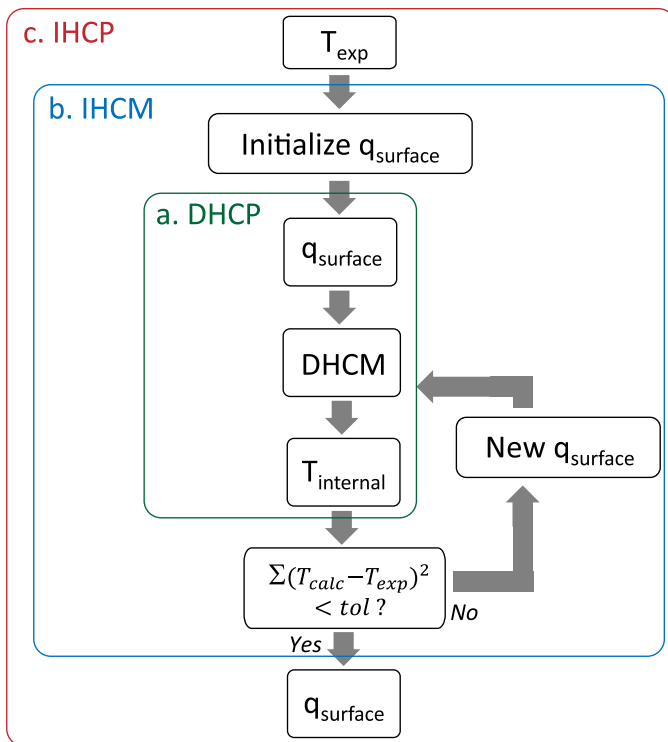
Quenching is the process of fast cooling of hot objects by a liquid. A well-known application is quench-cooling of steel plates by a series of water jets on the Run Out Table (ROT) of steel production plants. In order to control and improve the quality of the steel, knowledge of the details of the cooling process, and hence of the surface heat fluxes involved, is required. Transient quenching experiments have been reported, with the aim to study the heat fluxes during quenching in a controlled environment [1–5]. The results are used to produce the so-called boiling curve, presenting surface heat flux versus surface temperature, at various locations at the surface of the quenched plate. In the specific case of quenching of steel plates by water jet impingement, the boiling curve is used as input for online and offline ROT control systems. The boiling curve resulting from quenching by water jet impingement can be used in the same way as Nukiyama’s pool boiling curve [6] to connect to prevailing boiling regimes or interfacial topologies, be it that the boiling regimes are different for the two curves. It is

noted that the use of boiling curves is not only practical for applications but also partly historical. The use of dimensionless parameters to generalize the findings is of course preferable, but seldom done with primary boiling data. The reason for this is the non-uniqueness of the set of dimensionless criteria involved in two-phase flows with heat transfer.

The surface heat flux and temperature estimations are therefore important to know, both for applications and for the analysis of measurements. At the same time, accurate determination of heat flux and surface temperature is the main challenge of such an experiment. In addition, direct experimental validation is cumbersome. Direct surface temperature measurements have hardly been performed in this research field [7,8]. This is due to the complex machining and weak mechanical stability of surface temperature sensors [7], but also due to their interference with the boiling phenomena occurring at the surface [9]. The mere presence of a sensor at the surface, as well as the adhesives and grooves necessary for installation, affect bubble nucleation and hence the boiling regimes that occur on the sensor. As a result, it is impossible to corroborate if the results are representative of the whole surface or only of the phenomena occurring on the sensor. Consequently, the most used temperature measurement technique is by thermocouples that are welded or fixed inside the steel plate with some thermally conduc-

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**Fig. 1.** Schematic representation of the Direct Heat Conduction Problem (a, DHCP), Inverse Heat Conduction Method (b, IHCM) and Inverse Heat Conduction Problem (c, IHCP).

tive paste [1–3,10–12]. In such experiments, the experimental data consists of the internal temperature histories at various locations inside the plate. These data can be used to estimate the surface heat flux and surface temperature during the experiment.

A typical Direct Heat Conduction Problem (DHCP) consists of the calculation of the internal temperature distribution in a body with known boundary conditions at the plate surface using a Direct Heat Conduction Method (DHCM, Fig. 1a). In the case of a quenching experiment, the situation is reversed: the boundary condition (surface heat flux) is unknown and the internal temperature (experimental data) is known. The unknown boundary condition can be estimated by solving the Inverse Heat Conduction Problem (IHCP, Fig. 1c). The IHCP solution is solved using an Inverse Heat Conduction Method (IHCM, Fig. 1b). The IHCM optimizes the unknown surface heat flux by solving the DHCP iteratively minimizing the difference between the calculated and measured internal temperature data. There are two types of IHCM: in-house algorithms [13] and commercial codes, among which INTEMP is the most often used [14,15].

As in all inverse problems, the IHCP is an ill-posed mathematical problem. This leads to the following numerical issues:

- The solution to the problem is not unique. The input to solve the IHCP is an internal temperature history, but identical internal histories can be found for different boundary conditions. This is clearly explained by Karwa [16], who showed that heat diffusion leads to an internal temperature history where the high frequency component present in the surface heat flux is lost. The loss of high frequency information is worse at locations deeper below the surface. For this reason, the temperature sensors are always installed as closely to the surface as possible.
- Slight noise or measurement error in the temperature history measured inside the plate is amplified in the estimated surface heat flux. A small temperature fluctuation at a location

relatively far from the surface can only be caused by a large heat flux fluctuation at the surface. Installation of the thermocouple as closely to the surface as possible also mitigates this problem, but the inherent presence of noise in the experimental data makes this a crucial issue. The amplification of data noise can be mitigated by two numerical techniques. The first is filtering the temperature data prior to solving the IHCP [17]. The second is the implementation of the Tikhonov regularization technique [14], consisting of a penalty on high frequency components during the optimization of the surface heat flux. In both techniques, it is crucial to find the optimal extent to which the technique needs to be applied: over-filtering (or over-penalizing) leads to loss of physical high frequency components in the surface heat flux, while under-filtering (or under-penalizing) does not remove all non-physical high frequency components [18,19].

Apart from these mathematical challenges, errors can also arise from the following technical issues:

- The initial conditions of the plate are not precisely known. Although accurate determination of the initial temperature profile might sound trivial, in practice this proves to be a challenge. The known initial conditions immediately before quenching correspond to the thermocouple locations, but in order to accurately estimate the surface heat flux during the first instants of quenching the complete temperature distribution before quenching must be defined in the IHCM. In practice, inhomogeneous heating in the oven and cooling by the environment after removal from the oven and before the start of the jet impingement lead to an unknown initial temperature distribution. The general approach is to assume a homogeneous initial temperature distribution based on the thermocouple data before impingement. Errors can be minimized by ensuring that the test plate reaches a stable temperature in the oven and minimizing the time that the test plate spends outside the oven prior to quenching. However, a homogeneous initial temperature distribution is practically impossible to be realized. Although the change in temperature of the plate may be estimated by computation of convective cooling, the values of the heat transfer coefficients are not precisely known.
- When solving the IHCP, the thermocouple is generally considered to have perfect thermal contact with the steel plate, and thus the temperature recorded by the thermocouple is assumed to be equal to the temperature at the thermocouple location if the sensor would not be there. In practice, perfect thermal contact is impossible. Welding of thermocouples might create cracks in the steel or affect the local microstructure of the test plate, which in turn affects the heat transfer in the vicinity of the thermocouple joint. Thermal paste cannot guarantee perfect thermocouple contact either, given the relatively low conductivity of paste materials as compared to steel and the possibility to create cracks or air pockets during the curing of the paste. In addition, given the fact that the thermocouple shield and filling materials have different thermal properties than the steel test plate, the thermocouple affects the heat conduction in the plate and, consequently, the IHCP results [20]. Since the exact values of these resistances are unknown, it is not straightforward to take them into account in the IHCM to reduce the errors in the estimation of the surface heat flux.

In this paper, the effect of the above mentioned issues on the accuracy of the IHCM will be analyzed. Recently published quenching experiments revealed the connection between interfacial topologies, flow patterns and time after the start of jet impingement. The corresponding heat transfer data will be analyzed in Section 2 in order to reveal inconsistencies. On these inconsistencies a hypothesis will be based that is subsequently investigated

in the remainder of the paper. In Sections 3 and 4, a “virtual experiment” approach is used to quantitatively analyze the effect of parameter settings in the Tikhonov regularization technique, of initial conditions, and of thermocouple thermal contact on the accuracy of the IHCM. The main aim is to study whether underestimating the thermal resistances between test plate and thermocouple leads to a systematic underestimation of the surface heat flux and overestimation of the surface temperature in the first stages of quenching. Finally in Sections 5 and 6, two recommendations are proposed to avoid misinterpretation of experimental results and conclusions will be summarized.

## 2. Inverse heat conduction method and possible source of errors

As commented above, solving the IHCP is a challenging task since there are multiple potential sources of errors. An added challenge is that when using experimental data, the exact solution of the IHCP is unknown, making it almost impossible to assess the accuracy of the resulting heat flux and temperature estimations. Fortunately, recent experimental developments have made it possible to couple temperature data to visual observations of the flow patterns on the coolant side, i.e. in the jet. Since the heat transfer coefficients and plate temperatures of these flow regimes can be estimated, the trustworthiness of the outcomes of the IHCM can be judged. This will be done in the present section. Two inconsistencies will be revealed, concerning the estimated temperature of the surface during the initial stages of the experiment and the effect of the initial plate temperature on the boiling curve. In the final part of this section, we will present a hypothesis that would explain these inconsistencies in terms of errors arising from the IHCP.

### 2.1. Overestimation of the surface temperature

When quenching a hot steel plate with subcooled water, the literature agrees that a sharp increase of surface heat flux occurs upon rewetting, or direct water-surface contact. This results in a sharp surface temperature decrease and it is generally reported to occur immediately (or almost immediately) upon jet impingement. This sharp increase of heat flux contains an important high frequency component, which is known to be compromised by the IHCM. Although judging its accuracy is a difficult task, we can use the visual information provided by Leocadio et al. [4,5].

Leocadio uses a borescope to perform direct high speed visualization recordings of the jet stagnation zone in the initial moments of quenching. The high speed recordings are analyzed and compared to the surface heat flux and temperature histories provided by INTEMP, which was used to solve the IHCP. Leocadio et al. report a surface temperature of 703 °C when the borescope recordings show stable nucleate boiling in the complete stagnation zone. However, stable contact of liquid with the plate, as occurs in nucleate boiling, is thermodynamically impossible above the thermodynamic limit of water superheat [21]. The situation of stable nucleate boiling in the stagnation zone reported by Leocadio et al. can only occur if the actual surface temperature is much lower than 703 °C, meaning that their reported IHCM solution overestimates the surface temperature in the first instants of quenching. Leocadio et al. explain this overestimation of the rewetting temperature by an assumption of constant heat flux in a zone where multiple boiling regimes occur simultaneously [5]. However, this explanation is not valid in cases where a single boiling regime is observed in the entire uniform heat flux zone.

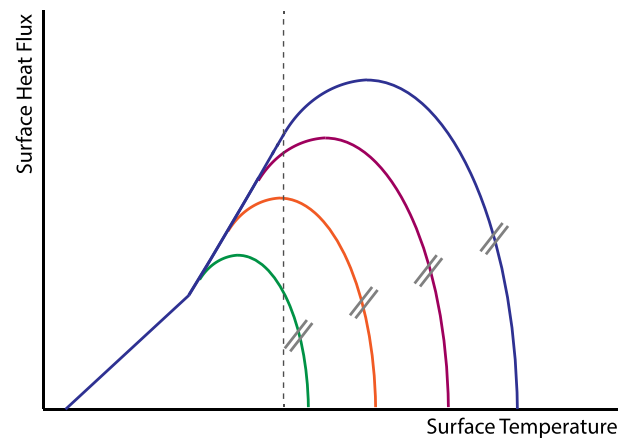


Fig. 2. Schematic representation of the effect of initial temperature in the boiling curve reported in literature.

### 2.2. Effect of initial temperature on the boiling curve

The effect of initial plate temperature on the boiling curve has been studied in several transient quenching experiments and the results are consistent [2,3,10]. The general trend is illustrated in Fig. 2: both the maximum heat flux and the surface temperature at which it occurs increase with increasing initial plate temperature.

Although this effect is widely agreed upon, a physical interpretation has never been provided, as far as the authors are aware. The trend is in fact hard to explain based on common knowledge of boiling heat transfer. If one traces a vertical line at constant surface temperature, for example the dashed line in Fig. 2, the intersection points with the boiling curves correspond to experiments that started at different surface temperatures. At these points, the experiments have equal jet properties and equal instantaneous surface temperature, but still show significantly different surface heat fluxes. The only explanation for different heat fluxes on the coolant side along the dashed line would be different gas-liquid interfacial topologies and different flow dynamics at equal jet and equal surface conditions, which is unlikely. Note that for later stages of the experiment, the trend is as boiling theory dictates: equal surface temperatures correspond to equal surface heat fluxes.

The results reported in literature show another common characteristic in the negative slope side of the boiling curve: the slope in the initial stages of the experiment seems to be independent of the initial temperature [3,10]. However, stagnation zone recordings show significantly different boiling regimes in the first instants of quenching, for different initial plate temperatures. Leocadio et al. [5] report stable nucleate boiling directly after impingement at initial temperatures below 450 °C. For initial temperatures above 450 °C they report film boiling regime to occur during a short time interval after impingement, with increasing duration at higher initial temperatures. Gomez et al. also report significant changes in the boiling regimes during the initial stages of the experiment at different initial plate temperatures [21]. The fact that the boiling curve slope in the initial stages of quenching is independent of the initial temperature in a range in which the boiling regimes vary significantly indicates that this slope might not be physically correct.

### 2.3. Hypothesis

Based on thermodynamic considerations and recent visualization studies of flow regimes during quenching, two inconsistencies were revealed in the previous section: an overestimation of surface



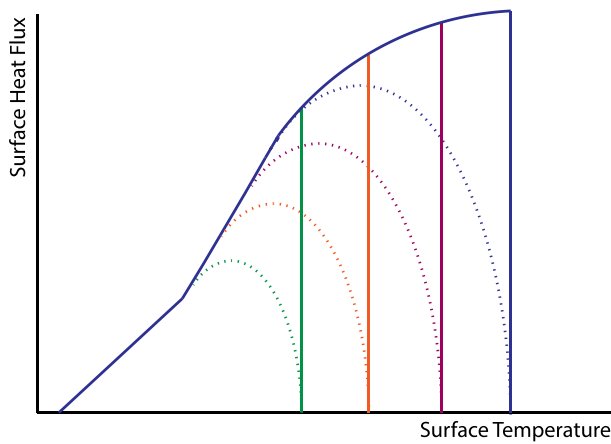


Fig. 3. Schematic representation of the hypothetical effect of the IHCM in the estimated boiling curve.

temperatures in the initial stages of quenching and an irrational effect of initial temperature on the shape of the boiling curves.

Studies reported in literature [10,21] agree that quenching with subcooled water jets results in either immediate rewetting or very short film boiling periods in the order of milliseconds, at least at initial temperatures below 900 °C. As a consequence, a sharp increase of heat flux comparable to a step function is expected. Such a step function rise of the boiling curve has been claimed to be the case for industrial quenching technologies that impede the formation of a stable film boiling regime [22].

Our hypothesis is based on the discrepancies stated in the previous section and is the following. The real boiling curve at different initial temperatures follows the trend illustrated by the continuous lines in Fig. 3. A step-like increase in heat flux occurs upon impingement, granted that rewetting is immediate or almost immediate. A single boiling curve is shared for different initial temperatures. Due to the limitations of the IHCM presented in Section 1, the high frequency component of the step-like heat flux increase at jet impact is lost. As a result, the surface heat flux is underestimated in the initial stages of the experiment, resulting in an overestimation of the surface temperature. This leads to the boiling curve trend illustrated by the dashed lines in Fig. 3. This hypothesis will be investigated in the following sections.

### 3. Virtual experiments procedure

#### 3.1. Definition

In order to assess the performance of the IHCM as data processing tool, the boundary condition at the top of the plate must be known in the course of time. Since this boundary condition is unknown in practical experiments, it is then impossible to assess the accuracy of the solution. In order to circumvent this and evaluate the accuracy of the IHCP solution, a special routine was designed and named “virtual experiment”.

The virtual experiment strategy is illustrated in Fig. 4. A known heat flux history is implemented as boundary condition in a DHCP. The DHCP solution is the internal temperature history at all places in the plate. From the DHCP solution, the temperature history at one or more internal locations is extracted. These internal locations correspond to sensor locations in the test plate and the thermocouple positions in a real experiment. The corresponding temperature histories will be named virtual experiment data. Subsequently, the virtual experiment data are used as input for the IHCP. Note that if analytical solutions would be available for the DHCP, these could be used to produce the virtual experiment data at the se-

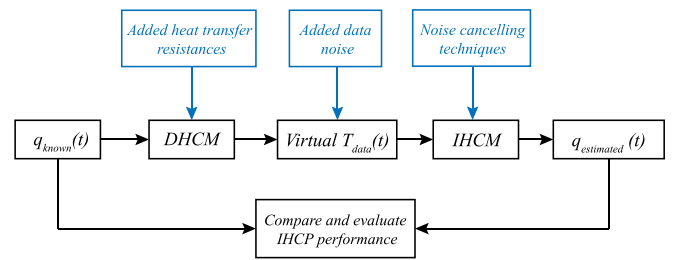


Fig. 4. Schematic representation of the virtual experiment procedure.

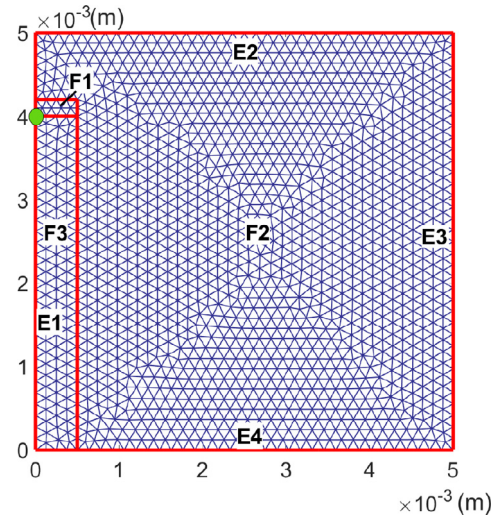


Fig. 5. Numerical domain, mesh and definition of boundaries.

lected sensor locations. The IHCP is ill-posed, because the boundary condition at the top of the plate is unknown. The initial temperature profile is given and constant and the fluxes at the other boundaries are prescribed and constant as well. The way the IHCP is solved will be detailed below.

The solution of the IHCP is an estimated boundary condition (surface heat flux), which in an ideal scenario should match the known boundary condition that was given as input to the DHCP. With the aid of this virtual experiment, the error in the IHCP solution can be objectively quantified.

The virtual experiment does not only allow to evaluate the IHCM itself, but also the effect that external factors may have on its solution. Some of these factors are illustrated in blue in Fig. 4. One can, for example, explore the effect of additional heat transfer resistances in the vicinity of the thermocouple, the effect of data noise or the effect of the noise cancelling techniques.

#### 3.2. Spatial discretization

The domain shown in Fig. 5 is meshed using triangular elements. The maximum edge length was optimized to a value of  $1.4 \cdot 10^{-5}$  m. The heat transfer problem is solved in axisymmetric coordinates with the edge E1 as the symmetry axis.

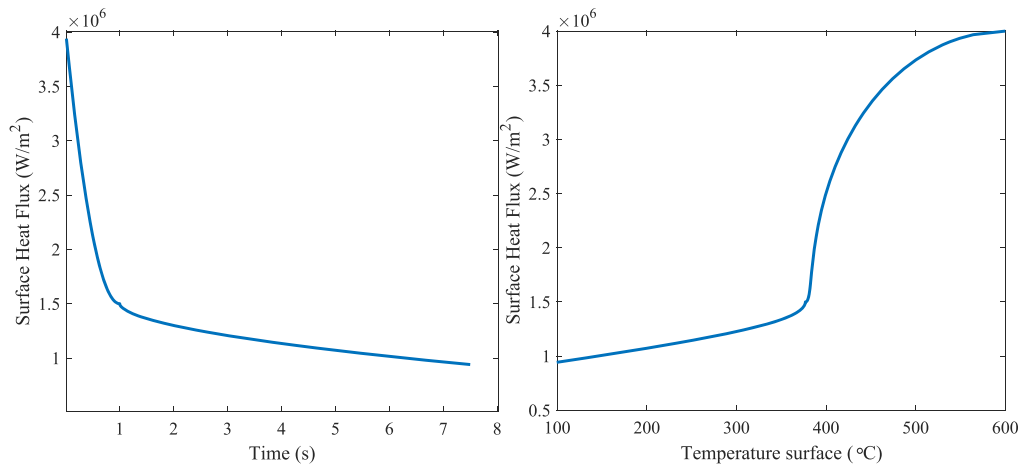
The domain consists of 3 different subdomains (F1, F2 and F3). Subdomain F1 is based on the typical size of the drill tip used to make the thermocouple holes (0.2 mm depth). The dimensions of subdomain F2 are based on the typical distance between 2 thermocouples in quenching experiments (approximately 1 cm, [10]). Subdomain F3 is based on the typical thermocouple dimensions (1 mm diameter). The location of subdomains F1 and F3 is selected such that the thermocouple tip (green marker) is located 1 mm below the top surface and at the axis of symmetry. The materials used in each subdomain for each studied case are summarized

**Table 1**  
Definition of domain materials for each studied case.

Case	DHCP Materials			IHCP Materials		
	F1	F2	F3	F1	F2	F3
Single Material		Steel 304			Steel 304	
Thermal Paste, Ignored	Silver paste	Steel 304	Mullite		Steel 304	
Thermal Paste, Considered	Silver paste	Steel 304	Mullite	Silver paste	Steel 304	Mullite

**Table 2**  
Thermal properties of the defined materials.

Material	Thermal Conductivity (W/mK)	Thermal Diffusivity ( $m^2/s$ )
Stainless Steel 304	22.7	$5 \cdot 10^{-6}$
Thermal Paste	1–10	$1.7 \cdot 10^{-7} - 1.7 \cdot 10^{-6}$
Mullite	5	$2.1 \cdot 10^{-6}$



**Fig. 6.** Heat flux history (left) imposed to solve the DHCP and resulting boiling curve (right).

in Table 1. The material properties are summarized in Table 2. In single material cases, the 3 geometries are assigned stainless steel 304 properties. In cases where the thermocouple and thermal paste are considered, F1 has the properties of silver paste, F2 has the properties of stainless steel AISI/SAE 304 and F3 has the properties of mullite (thermocouple filler material).

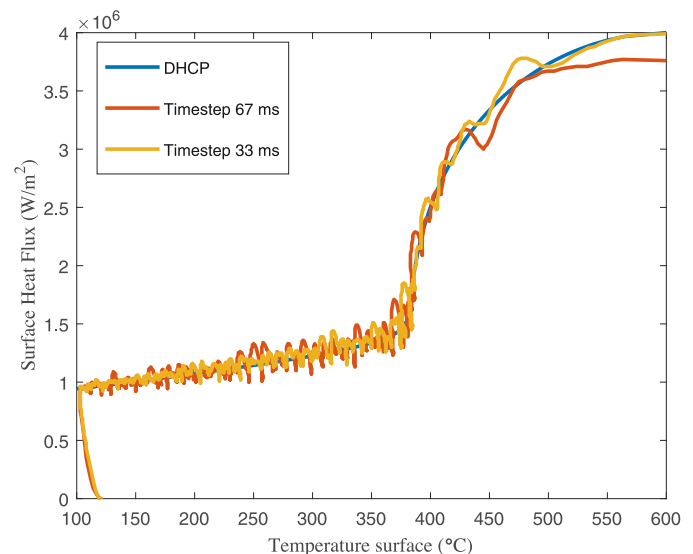
The above spatial discretization is used in the IHCP where the problem is ill-posed due to the unknown heat flux at the top surface and due to the noise in the temperature history given input to the problem. The above spatial grid as well as finer grids have been used in the solution of the DHCP. The choice of grid in the DHCP does not affect the ill-posed-ness of the IHCP since only the temperature history at a single location in the domain will be used for the IHCP. This is further explained in the next section.

**3.3. Direct heat conduction problem**

A homogeneous initial temperature equal to 600 °C is prescribed in the complete domain described in Section 3.2. The boundary conditions are a prescribed heat flux at edge E2 and adiabatic edges E3 and E4, see Fig. 5.

The heat flux prescribed at edge E2 is uniform with a history as shown in Fig. 6 (left). This history is designed such that it leads to a boiling curve (Fig. 6, right) close to the one of our hypothesis (Fig. 3).

The DHCP is solved with the Partial Differential Equation Toolbox of Matlab for a total physical time of 7.5 s with a time step of 25 ms. The mesh described in Section 3.2 is used but only the time history predicted at the thermocouple location, the green point in Fig. 5, is used in the inverse heat problem. It has been verified that



**Fig. 7.** Boiling curve solution of the direct problem and effect of the time step on solutions of the IHCP.

further reduction of the time step and that further refinement of the spatial grid do not change the boiling curve in Fig. 7.

**3.4. Inverse heat conduction problem**

The Inverse Heat Conduction Problem (IHCP) is solved using IN-TEMP on the mesh shown in Fig. 5. The materials used in the IHCP

in each geometry for each studied case are summarized in Table 1. Edge E1 is defined as the symmetry axis and edges E3 and E4 are defined as adiabatic. The temperature profile at the thermocouple location resulting from the DHCP is used as input in INTEMP. In order to simulate the effect of data noise, random white noise is added to the DHCP temperature profile in a range of  $\pm 0.75\%$  of the temperature data, with an amplitude based on the error range of grounded K-type thermocouples [10]. A linear congruential pseudo-random number generator of the recursive type was used, which generates nearly uniformly distributed values on the interval chosen. In this way the stochastic variation in actual temperature measurements as registered by thermocouples is simulated in the temperature history at the thermocouple location that is the starting point for the inverse heat problem computations. The uniform surface heat flux at edge E2 is essentially the solution of the IHCP. The time step of 67 ms was chosen based on the timestep of 25 ms of the direct problem. A timestep of 33 ms yields fairly optimistic agreement with the direct solution results, see Fig. 7. The timestep of 67 ms that has been chosen for the IHCP does not yield unrealistically optimistic agreement with the direct solution result. This satisfies the requirement of Kaipio and Somersalo to prevent the so-called inverse crime [23] (pages 5 and 24). It has been verified that the timestep of 67 ms facilitates quick convergence of the optimization algorithm of INTEMP.

The equation of heat diffusion that is solved in the IHCP takes the following general form:

$$\rho c_p \frac{\partial \tau}{\partial t} = \frac{\partial}{\partial x} k_x \frac{\partial \tau}{\partial x} + \frac{\partial}{\partial y} k_y \frac{\partial \tau}{\partial y} \quad (1)$$

where the heat capacity per unit volume,  $\rho c_p$ , and the heat conductivities,  $k_x$  and  $k_y$ , may vary spatially. Time  $\tau$  is discretized with  $N_t$  fixed timesteps each of duration  $\Delta t$  and either a Crank-Nicolson or a fully implicit scheme is chosen to discretize the heat equation governing the temperature field  $\tau$ . In the implicit formulation, the conduction model can be written as

$$(C_i + K_i \Delta t) \cdot T(t_{i+1}) = C_i \cdot T(t_i) + \Delta t q_{i,known} + \Delta t P \cdot q_{i,unknown} \quad (2)$$

where the suffix  $i$  indicates values at time  $t_i$ ,  $C$  is a diagonal capacitance matrix,  $T$  is the vector of all  $n$  nodal temperatures,  $K$  is a symmetric conductance matrix,  $q_{i,known}$  is a vector of size  $n$  of the known heat fluxes (zero at many places),  $q_{i,unknown}$  is a vector of size  $(n_q \times 1)$  of all unknown heat fluxes ( $n_q = 1$  in the examples below) and  $P$  is a participation matrix of size  $(n \times n_q)$ . The input data is in our case one single temperature history  $\{d_j\}_j$ . Intemp solves for the fluxes  $q_{i,unknown}$  by minimizing the sum

$$E - \text{norm} + F - \text{norm}$$

with E-norm the least square error of the predicted and input temperature history at the thermocouple location:

$$E - \text{norm} = \sum_{j=1}^{N_t} (UT(t_j) - d_j)^2, \quad (3)$$

where  $U$  is a matrix, in our case of size  $(1 \times n)$ , identifying the thermocouple location where the input data are retrieved, and with F-norm the regularization term:

$$F - \text{norm} = B \sum_{j=1}^{N_t} (f_j)^2. \quad (4)$$

Coefficient  $B$  is named the regularization coefficient.

The above regularization is named Tikhonov's regularization method and is based on penalizing high frequency components in the surface heat flux. The degree of penalization is controlled by the value of a parameter  $B$ . The higher the value of  $B$ , the stronger

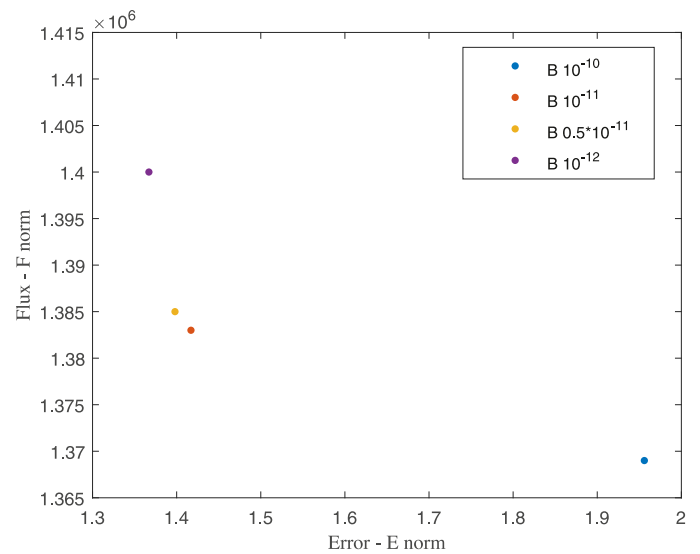


Fig. 8. L-curve of the case study with thermal paste.

is the penalization for high frequency changes in the heat flux estimation. If  $B$  is too high, the physical high frequency components are underestimated, or even not resolved at all. If  $B$  is too low, the noise in the temperature data is translated to non-physical high frequency components.

The optimal  $B$ -value has been chosen with the aid of the so-called L-curve method, see [15]. A typical example of L-curves of this study is shown in Fig. 8 below. The way to select a  $B$ -value is to minimize both the E-norm and the F-norm, which for this figure yields the  $B$ -value of  $1.0 \cdot 10^{-11}$ .

The method used to minimize the functional is dynamic programming with the aid of recurrence formula. The details are given in the appendix of the article "Numerical solution to a two-dimensional inverse heat conduction problem" by Busby and Trujillo [14].

## 4. Exploring the IHCM limitations

### 4.1. Effect of data noise and noise cancelling technique

In this section, the effect of data noise and the effect of the noise cancelling technique itself are studied. First, a virtual temperature data set is generated by solving the DHCP corresponding to a pure steel test piece with perfect thermocouple contact and application of the predefined heat flux history shown in Fig. 6. The resulting temperature data at a depth of 1 mm is then used to solve the IHCP in noiseless conditions. In order to study the effect of data noise, we add white noise in a range of  $\pm 0.75\%$  of the temperature. Since INTEMP requires the definition of a value of  $B$  even in noiseless conditions, the effect of the value of  $B$  is presented when using both noiseless and noisy data.

Fig. 9 shows the IHCP solutions for using noiseless data with different values of  $B$ . Even in noiseless conditions, the heat flux estimations are affected by the choice of  $B$ . In this case, the optimum  $B$  value according to the L-curve method ( $1 \cdot 10^{-12}$ ) shows the most satisfactory estimation.

Fig. 10 shows the results when noise is added to the temperature. In this case, the effect of Tikhonov's regularization method is clearer. The optimum value of  $B$  obtained by the L curve method is equal to  $1 \cdot 10^{-11}$ . The optimum  $B$  value leads to the most accurate heat flux estimations, and shows only small amplitude oscillations as a consequence of the noise. When using a larger  $B$  ( $1 \cdot 10^{-10}$ ), the heat flux is underestimated in the initial stages of the virtual

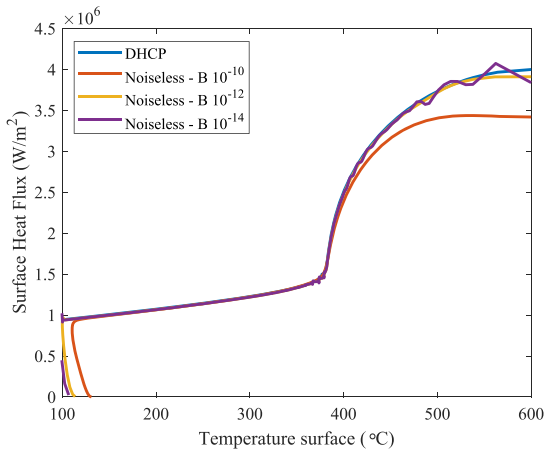


Fig. 9. Effect of B on the IHCP solution when using noiseless temperature data.

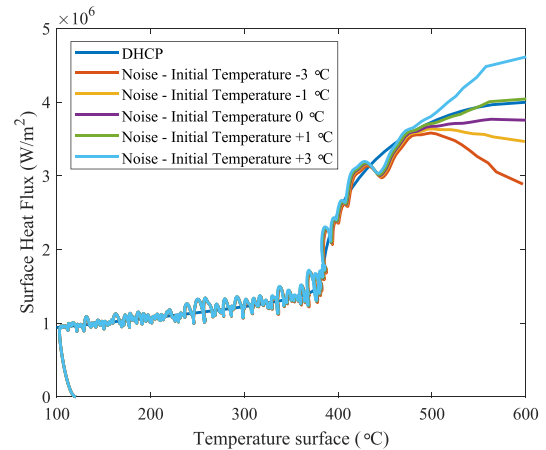


Fig. 11. Effect of inaccurate initial condition on the IHCP solution.

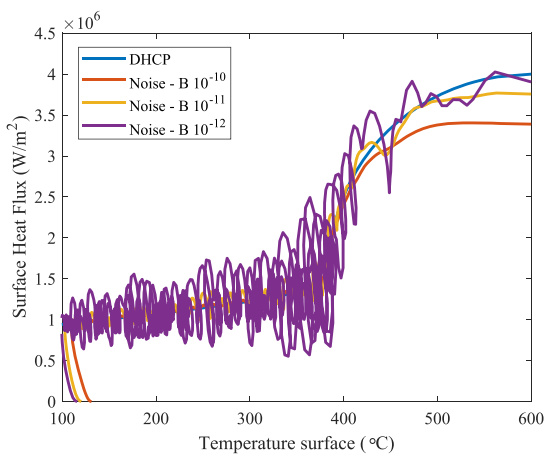


Fig. 10. Effect of B on the IHCP solution when using temperature data with noise.

experiment. On the other hand, a lower B ( $1 \cdot 10^{-12}$ ), leads to fitting of the data noise and results in non-physical heat flux oscillations. For later reference, it is important to note that Tikhonov's regularization cannot distinguish physical from non-physical heat flux oscillations. If the IHCP yields oscillations, it cannot be distinguished whether they result from a too low value of B or whether they are physical. Moreover, if the physical frequencies are in the same order as the noise frequencies, they will be smoothed even when using the optimum B value. The authors obtained similar results when using data filtering as a noise cancelling technique.

#### 4.2. Effect of the IHCP initial conditions

As discussed in Section 1, the IHCP solution depends on the chosen initial conditions. Determination of the real initial conditions is impossible, since thermocouple measurements before impingement do not reveal temperature gradients across the test plate. Since the initial conditions used in the IHCP are usually based on the data obtained at the thermocouple location, the effect of a small temperature difference across the plate is amplified. In this section we study the effect of wrongly assumed initial conditions in the IHCP.

The noiseless data set generated for the previous section to solve the IHCP with the ideal B ( $1 \cdot 10^{-12}$ ) is utilized. The initial homogeneous temperature distribution in the complete domain used in the IHCP is changed with  $-3$ ,  $-1$ ,  $+1$  and  $+3$  °C with respect to the DHCP initial temperature. The effect of this offset in the initial

temperature is presented in Fig. 11. As expected, a negative offset leads to underestimation of the surface heat flux in the initial stages of quenching. A positive offset leads to overestimation of the surface heat flux. The strong deviation in the heat flux in the initial stages of the experiment is a result of the correction that the IHCP must make in order to compensate for the difference between the temperature defined in the bulk of the simulated domain and the input temperature data.

Surprisingly, a larger offset does not affect a bigger part of the boiling curve: the IHCP solution seems to stabilize at the same point on the boiling curve independently of the change in the initial temperature. Similar to the effect of B in the previous section, the most affected period is the initial stage of quenching.

The most likely error in practice is an underestimation of the initial temperature. Prior to jet impingement, the plate must be removed from the oven and placed in the setup. During this time, the plate surfaces cool down more than its interior. Since the thermocouple is located close to the surface, it is expected that the thermocouple reading is lower than the temperature in the interior of the plate.

#### 4.3. Effect of thermocouple contact

The assumption of perfect thermocouple contact is commonly made when solving the IHCP. In this section, the effect of this assumption on the IHCP solution is studied. Using the same prescribed heat flux history as before, an extra thermal resistance around the thermocouple tip location is implemented in the DHCP. The extra resistance is added as a different material in the subdomain F1, corresponding to thermal silver paste. Its thermal conductivity is set to 2 W/mK, which is within the range of typical thermal conductivities of this material (based on information provided by suppliers). The silver paste is added to a region equivalent to 0.2 mm around the thermocouple tip, which is based on the dimensions of a 1.1 mm drill with triangular tip used for thermocouple installation by Leocadio et al. [5]. This thermal resistance added to the DHCP corresponds to the actual extra resistance of the thermal paste. The resulting temperature profile at the thermocouple tip location is used to solve the IHCP after addition of noise and with an ideal value for B of  $1 \cdot 10^{-11}$ .

Fig. 12 shows the difference between the IHCP solution with and without considering the thermal paste. The comparison shows the consequence of assuming perfect thermocouple contact when an additional resistance is in fact present during the experiment. As can be seen, the boiling curve is strongly affected in the initial stages of the experiment, where the step-like heat flux in-



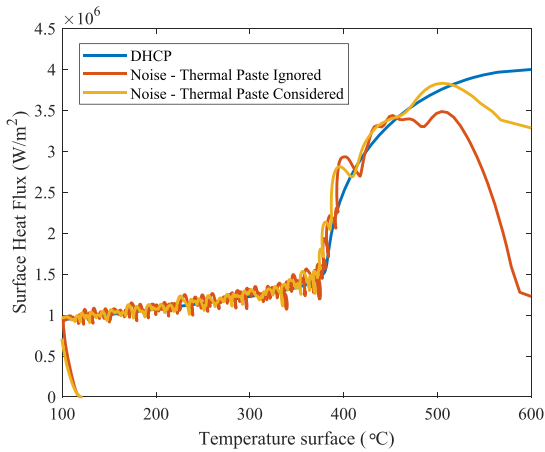


Fig. 12. Effect of wrongly assumed perfect thermocouple contact on the IHCP solution. Boiling curves.

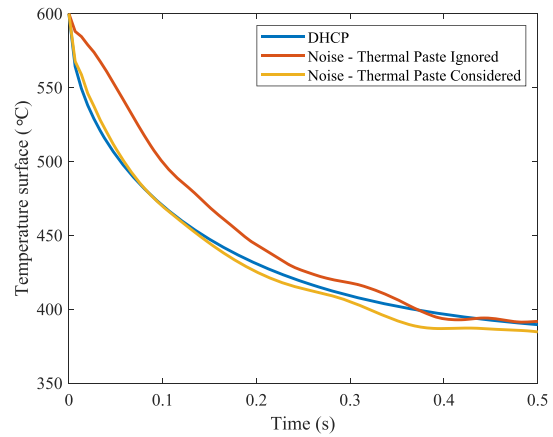


Fig. 14. Effect of wrongly assumed perfect thermocouple contact in the IHCP solution. Surface temperature history in the initial stages of experiment.

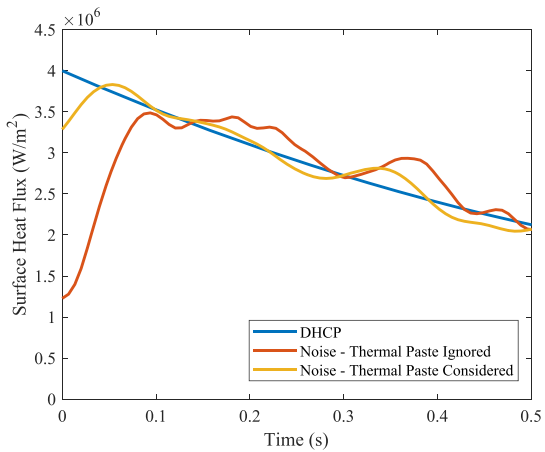


Fig. 13. Effect of wrongly assumed perfect thermocouple contact on the IHCP solution. Heat flux history in the initial stages of experiment.

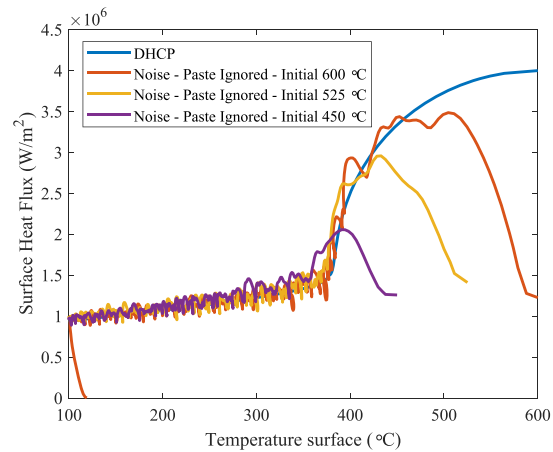


Fig. 15. Effect of initial temperature in the boiling curve with wrongly assumed perfect thermocouple contact.

crease occurs. At later stages of the experiment, the estimation of lower frequency heat flux components is accurate despite the ignored added resistance. The reason for this is physical and is that at later times the Fourier number corresponding to the thickness of the plate is large which implies that temperature profile variations are mainly in radial direction. Ignoring the silver paste resistance leads to a boiling curve that strongly resembles the typical trend published in literature when quenching by subcooled water jet impingement [3,10]. The quench cooling studies published in literature interpret the negative slope section as representation of a physical phenomenon, usually a transition boiling regime similar as the one reported for pool boiling. However, direct visualizations of the stagnation zone do not support this interpretation and show that the maximum heat flux occurs well after stable nucleate boiling is reached in the complete stagnation zone [5].

Fig. 12 also shows the effect of including the silver paste in IN-TEMP. When considering the additional heat transfer resistance in the IHCP, the estimations are significantly closer to the actually imposed boiling curve.

Fig. 13 shows the heat flux profiles in the first 0.5 s of the experiment, which lasts a total of 7.5 s. It can be seen that the heat flux peak occurs after only 0.2 s in the ignored thermal paste case. Although the affected section of the boiling curve in Fig. 12 seems to describe a significant part of the experiment, it actually lasts a relatively short time compared to the total experiment length. As

shown in the above, the IHCM results in the initial stages of the experiment are very susceptible to errors and should be carefully analyzed before drawing any conclusions.

As a result of the underestimation of the heat flux in the initial stages of the experiment, the surface temperature is strongly overestimated (Fig. 14). In the virtual experiment, the maximum surface temperature overestimation is 50 °C after 0.05 s. In a real experiment, the overestimation of the surface temperature can even be stronger due to a combination of an ignored or underestimated thermal resistance of the thermal paste and an inaccurate initial temperature.

In order to confirm this, the virtual experiment presented in Fig. 12 was repeated at different initial temperatures with equal predefined boiling curves. The results (Fig. 15) confirm the hypothesis and show that ignoring the silver paste thermal resistance might lead to the effect of initial temperature reported in literature. Moreover, and in agreement with the cited experimental findings, the negative slope of the boiling curve is similar for different initial temperatures.

Virtual experiments were performed ignoring thermal contacts with different thermal conductivities (Fig. 16). As could be expected, the closer the thermal conductivity is to that of steel, the better the estimation of the boiling curve. This shows that optimizing the thermal contact is crucial to obtain accurate heat transfer estimations.

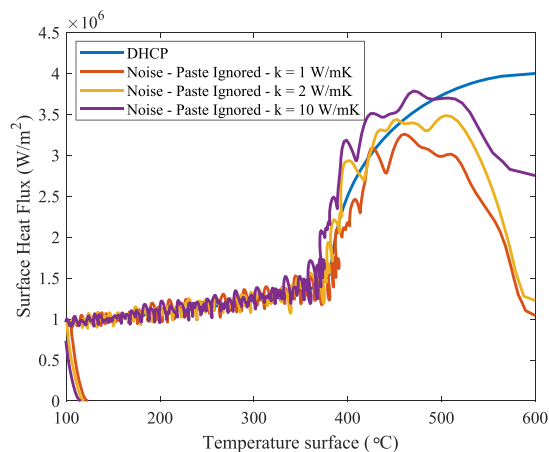


Fig. 16. Effect of thermal paste conductivity on the boiling curve with wrongly assumed perfect thermocouple contact.

## 5. Recommendations for temperature measurement and analysis of the IHCP solution

This manuscript presents a study on the limitations and issues affecting the performance of the IHCM when using INTEMP. An important remark is that the same results were obtained when using an IHCM coded in-house using Matlab [19], showing that the conclusions drawn from this study not only apply to results obtained by INTEMP, but also to other IHCM algorithms. It is therefore important to address the consequences these IHCM limitations have on real experimental results.

In this study, the additional thermal resistance in the thermocouple contact is associated to the presence of thermal silver paste. However, additional thermal resistances can also arise from other installation methods, such as thermocouple welding. Changes in steel microstructure or generation of cracks during welding of the thermocouple tip to the steel plate also result in additional thermal resistances.

It was found that including the extra thermal resistances in the IHCM leads to a significant improvement in the accuracy of the IHCP solution. However, this requires accurate quantification of these resistances. In practice, the resistance is unknown: exact thickness of thermal layer, quality of contact between different materials, composition of new steel phases resulting from welding, size and density of air cracks, etc. cannot be accurately determined. As a result, the effect of the non-perfect thermocouple contact is unknown and accurate heat flux estimations in the initial stages of quenching cannot be obtained.

The main consequences of this inaccuracy are overestimation of the surface temperature in the initial stages of the experiment and the non-physical peak in the boiling curve. Therefore, we recommend two measures when dealing with this type of experimental data. The first one concerns the interpretation of flow pattern visualizations and heat flux estimations in the initial stages of the experiment. Investigators tend to make a relation between flow patterns and the surface temperature estimations resulting from the IHCP solution [4,5]. However, our results show that the IHCP solution is undoubtedly physically suspect and untrustworthy in the initial stages of the experiment. It is recommended not to make such interpretations, since the IHCP solution will result in surface temperatures much higher than expected for the observed boiling regimes, as discussed in Section 2.1. The second measure is not to treat the surface heat flux estimations in the initial stages of quenching as physical, i.e. transition boiling. The heat flux estimations in the initial stages of quenching, prior to the heat flux peak,

should be treated as non-physical and should be plotted for reference using a discontinuous line [2,24].

## 6. Conclusions

It is well known that ambiguities are inherent to inverse heat problems and that uncertainties can be large. In this paper the application of inverse heat conduction methods to quench cooling experiments was systematically analyzed. The careful analysis of experimental quenching studies presented in this manuscript shows inaccuracies in the IHCP solution that were not reported until now. A hypothesis was put forward and its validity was examined by means of virtual experiments. There are unavoidable, technical and mathematical issues inherent to quenching experiments, such as an imperfect thermal contact between the test plate and thermocouple, an inaccurate initial temperature distribution and the noise cancelling techniques. These factors affect the IHCP outcome, that yields boiling curves with high inaccuracy. Specifically, this inaccuracy is reflected in the part of the curve that corresponds to the instants directly after jet impingement and leads to a significant overestimation of the surface temperature during this period. Our results indicate that a quench cooling boiling curve for a certain set of governing process conditions (jet velocity, jet temperature, thickness of the plate, etc.) is independent of the initial plate temperature and would have a shape that is close to that of the curve in Fig. 3 with an initial sudden rise in the form of a step function, corresponding to the rewetting phenomenon. Two recommendations were distilled from these findings. First, it is not recommended to relate the surface temperature estimations and boiling regime observations in the first instants of experiments. And second, it is recommended to treat the heat flux increase during the initial stages of quenching as a non-physical artifact.

## Declaration of Competing Interest

None.

## CRediT authorship contribution statement

**Camila F. Gomez:** Conceptualization, Methodology, Investigation, Formal analysis, Writing – original draft. **Rens Nieuwenhuizen:** Methodology, Investigation, Formal analysis, Writing – original draft, Visualization. **Cees W.M. van der Geld:** Conceptualization, Methodology, Validation, Writing – review & editing. **Hans G.M. Kuerten:** Methodology, Validation, Writing – review & editing. **Mustafa Bsibsi:** Methodology, Validation, Writing – review & editing. **Bart P.M. van Esch:** Supervision, Conceptualization, Funding acquisition, Methodology, Validation, Writing – review & editing.

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